DYNAMICS AND STABILITY OF RACING BOATS WITH AIR WINGS

Nikolai Kornev, University of Rostock
Lutz Kleinsorge, University of Rostock
Gunther Migeotte, Icarus South Africa

SUMMARY

The aim of this paper is to derive a mathematical model for predicting the longitudinal stability of racing boats with aerodynamic support. The theory is based on a combination of stability theories developed for planing boats and wing in ground effect craft. Influence of different geometric and mass boat parameters on the stability is investigated.

1 INTRODUCTION

Since the inception of planing hulls, speeds of racing boats have been increased from speed of about 50 knots half a century ago to speed of 150 knots that are common place today with modern planing boats, see Figure 1. With continuously increasing speed, stability of these craft has become a more important consideration. The International Towing Tank Conference (ITTC) [1] has identified the following different forms of instability that affect planing craft:

- Take-off
- Loss of GM due to wave system
- Course keeping and lateral stability
- Bow diving and plough-in
- Porpoising
- Chine tripping
- Spray rail engulfing resulting in plough-in
- Effect of critical speed in shallow water

Most of these different forms of instability are quite well understood and/or mathematical models exist for predicting the onset of such instabilities and [1] gives a good list of reference works on each of these instabilities. Notably however, the problem of take-off, which is normally associated with very high speed catamarans, has according to the ITTC not been well addressed to date. Fig. 2 shows a series of video stills [2] of an Offshore Class-1 catamaran pitching-up and then taking off.

There is a very fine balance between the aerodynamic, hydrodynamic and propulsive forces at the high-speeds the boats travel at (up to 200 knots for some hydroplanes). The stability can be easily upset by waves, wind gusts, turning (asymmetrical flow). Instability is usually onset due to a pitch up motion that results in the hull taking-off and pitching about the propeller. Once airborne the vessel quickly flies out of control often with catastrophic consequences as indicated in Figure 2. Englar et al. [3] have studied this form of instability for racing hydroplanes.

The primary design consideration of such catamarans is usually high-speed. Analysis of the resistance characteristics of the vessels shows that the lowest resistance, and in turn the highest speed, is obtained by maximizing the aerodynamic lift while keeping the hydrodynamic forces to a minimum. When considering the balance of forces and moments (see Section 2) it is clear that the aerodynamic forces are the source of the instability of such boats as the center of aerodynamic lift is located ahead of the longitudinal center of gravity (LCG) of the boat. Thus increasing the aerodynamic lift component is inherently coupled with decreasing stability of the boat.

The design of such vessels is therefore a compromise between aero- and hydrodynamic considerations and retaining a fine balance between the various parameters that influence the stability. At present the stability of these vessels is usually evaluated using simple balance of moments and some simple design rules [4]. Such simple methods have however been shown to inadequate to ensure stability as pitch-up and take-off stability remains an important problem and is the cause of many accidents.

Take off and pitch tendencies are strongly associated with the aerodynamics of such hulls and are therefore only a consideration when the aerodynamic lift produced becomes a significant portion of the total lift. Typically this occurs at speeds in excess of 60 knots for most of the craft in operation today. The stability of such craft is similar to the take-off stability of Wing-In-Ground(WIG) craft and in essence the same methods can be applied to determine the stability of catamarans. Morch [5] discussed some details of the aero- and hydrodynamics of very high speed catamarans (80 knots) but discussion of stability is given. The results of his experiments and Computational Fluid Dynamics(CFD) computations however indicate that, for the 7.5m catamaran traveling at 80 knots, the aerodynamic lift forces were over 50 per cent during normal operation of the craft and that the center of aerodynamic lift is very sensitive to the running trim angle of the vessel.

The most common way to increase speed on such high speed catamarans is to run at a higher trim angle but this brings the vessel closer to its stability limits and often such crafts run in a marginally unstable condition with the pilot providing continuous correction to the running attitude. Constant vigilance is therefore

Fig. 1: Racing boat Qatar

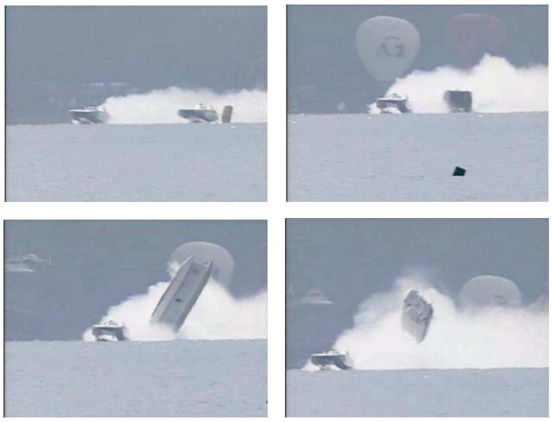

Fig. 2: Crash due to loss of the longitudinal stability [2]

required by the pilot to prevent the boat from taking off. Such boats often include some emergency measures such as water ballast tanks in the bows that can be filled with water in a very short time if the boat cannot be controlled and wants to take off.

The critical nature of the stability of these crafts is clearly evident. Proper design tools in order analyze stability of such crafts would be valuable to be able to develop designs that can possibly extend the operable limits of these crafts further. A longitudinal theory is proposed below which meets this requirement. The theory is based on two theoretical developments. The first development is the stability theory of planing boats proposed in a series of theoretical and experimental works performed during more than two decades in 60s and 70s at the Central Aerohydrodynamic Institute (TSAGI). The most valuable achievement is the simple and very robust mathematical model for the calculation of hydrodynamic forces acting on a planing surface at both steady and unsteady flow conditions. This model has been thoroughly tested in various measurements [6]. Implementation of this model within the linear stability theory results in the characteristic equation of the fourth order having a couple of conjugate roots. As Kovrizhnykh [6] and Lotov[7] shown the oscillatory instability is the most serious problem for the planing boats whereas the aperiodic instability has never been observed. Kovrizhnykh obtained the areas of the planing boat instability. At a given speed the stability gets worse as the angle of attack increases. The planing boat becomes unstable when the angle of attack attains a definite critical value. Surprisingly there is a narrow area of stability at large angles of attack which quickly disappears when the angle gets even larger. The presence of this stability region is confirmed in measurements with freely towed planing boat models [8].

The second development used in the present paper concerns WIG craft. With the development of WIG craft and Ekranoplans in the USSR, much work was done on the stability of high-speed craft making use of aerodynamic support [9]. Both the lateral and longitudinal stability of WIG craft had been thoroughly tested and well understood. In this paper we restrict ourselves on the longitudinal stability theory for WIG craft as developed by Irodov in 1970 in USSR [10] and independently by Staufenbiel in 1971 in Germany [11]. They derived the criterion of the static stability in two different forms but with the same physical meaning. Both criteria can be reduced to the same form after simple algebraic transformations. According to Irodov the WIG craft is statically stable when the aerodynamic center in height $x_h = m_z^h/C_y^h$ lies in front of the aerodynamic center in pitch $x_\vartheta = m_z^\vartheta/C_y^\vartheta$ where h is the height of flight, ϑ the pitch angle, C_y and m_z are respectively the lift coefficient and the pitching moment coefficient, $C_y^{\vartheta,h}$ and $m_z^{\vartheta,h}$ are their derivatives. According to experience, if the criterion of the static stability referred to the mean aerodynamic chord is between 0.05 and 0.12 the statically stable WIG craft is stable dynamically as well. An excessive static stability can result in the dynamic instability. A weak positive static stability is not admissible because of too weak damping of perturbations. Another important requirement widely used in the design of Russian WIG craft is the reciprocal position of aerodynamic centers and center of mass of the vehicle. The LCG shold be located between both aerodynamic centers x_h and x_ϑ closer to the aerodynamic center in height x_h [9]. In this case the dynamical properties of the WIG craft are favourable and the response of the craft to perturbations is mild. The longitudinal dynamic stability is investigated using three equations describing the translatory motions in x and y directions and pitching motions, see Figure 3. The procedure which is quite usual in the linear stability analysis leads to the characteristic equation of the fifth order which has one real root and a couple of two conjugate roots. A typical mutual position

of roots for the stable WIG craft is presented in [12]. Most important is the couple with the minimal real part which is responsible for the appearance of the dynamic oscillatory instability.
These two stability theories are used in this paper for developing the complex stability theory of planing craft with aerodynamic support.

2 THEORY OF LONGITUDINAL STABILITY

2.1 STEADY EQUILIBRIUM CONDITION

A necessary requirement for stability is that the planing boat is in a equilibrium condition. This means that the sum of vertical forces has to be zero:

$$mg - (Y_0 + C_y \frac{\rho}{2} U_a^2 S) = 0 \qquad (1)$$

where Y_0 is the steady hydrodynamic lift evaluated in Section 2.4. The moment around the Z-Axis can be neglected, because an equilibrium of moments can be easily derived for every operation point by an interceptor, or an elevator unit. Equation (1) is used to determine the floating position (draught) of a racing boat at a given speed and trim angle.

2.2 MOTION EQUATIONS

When the steady equilibrium condition is fulfilled, the stability is determined through analysis of roots of characteristic equation derived from a linearized equations system describing longitudinal perturbed motion.
Equations of three-dimensional dynamics of racing boats can be obtained directly from the second law of Newton and can be stated in fixed, speed, connected or semi-connected coordinate system [13] (Figure 3). For formulation of dynamics equations the choice of coordinate system is defined by requirements of simplicity of form and convenience in presentation of forces. Most appropriate in this sense is the semi-connected system of coordinates.
A complete system of equations of three-dimensional motion is (designations see in Tab. 1):

$$\begin{aligned}
\dot{U}_d &= f_1 T - f_2 {U_a}^2 \left(C_x - C_z \beta_a\right) \\
&\quad - f_1 R_{x,hydr}(t) \\
\dot{U}_{ycg} &= f_2 {U_a}^2 C_y + f_1 T \vartheta_T \\
&\quad - 9.81 + f_1 R_{y,hydr}(t) \\
\dot{\beta}_d &= f_2 U_a \left(C_z + C_y \gamma + C_x \beta_a\right) + \omega_y \\
&\quad - T \beta_a \\
\dot{\omega}_x &= f_3 {U_a}^2 \left(m_x + f_4 m_y\right) \\
\dot{\omega}_y &= f_5 {U_a}^2 \left(m_y - f_6 m_x\right) \\
\dot{\omega}_z &= \left(f_7 {U_a}^2 m_z - f_8 T\right) + \frac{m_{z,hydr}(t)}{J_z}
\end{aligned}$$

For formulation of the equations additional parameters are used:

$$\dot{h}_{cg} = U_{y_{cg}},\ \dot{\gamma} = \omega_x - \omega_y \left(\vartheta - \vartheta_0\right),\ \dot{\psi} = \omega_y, \dot{\vartheta} = \omega_z$$

$$\beta_a = \beta_d - \frac{w_z(t)}{U_a},\ U_y = U_{y_{cg}} - w_y(t), U_a = U_d + w_x(t)$$

$$\psi_d = \psi - \beta_d,\ h = \frac{h_{cg}}{b} - \left(1 - \overline{x}_{cg}\right)\vartheta - \overline{y}_{cg}$$

$$\vartheta_T = \vartheta + \Theta_T - \vartheta_0,$$

$$f_1 = \frac{1}{m}, f_2 = \frac{\rho S}{2m}, f_3 = \frac{\rho S b}{2 J_{xc}}, f_5 = \frac{\rho S b}{2 J_{yc}}, f_7 = \frac{\rho S b}{2 J_z}$$

$$f_8 = \frac{y_T}{J_z}$$

$$f_4 = \left[1 - \frac{J_{xc}}{J_{yc}}\right] \tan\left(\vartheta + \varphi_c - \vartheta_0\right)$$

$$f_6 = \left[1 - \frac{J_{yc}}{J_{xc}}\right] \tan\left(\vartheta + \varphi_c - \vartheta_0\right)$$

$$\varphi_c = \frac{1}{2} \arctan\left[\frac{2 J_{xy}}{J_y - J_x}\right]$$

$$J_{xc} = J_x \cos^2\varphi_c + J_y \sin^2\varphi_c - 2 J_{xy} \cos\varphi_c \sin\varphi_c$$

$$J_{yc} = J_y \cos^2\varphi_c + J_x \sin^2\varphi_c - 2 J_{xy} \cos\varphi_c \sin\varphi_c$$

Since this paper is dealing only with the longitudinal stability the full motion system can be reduced to the following three equations:

$$\dot{U}_d = f_1 T - f_2 {U_a}^2 C_x - f_1 R_{x,hydr}(t) \qquad (2)$$

$$\begin{aligned}
\dot{U}_{ycg} &= f_2 {U_a}^2 C_y + f_1 T \vartheta - 9.81 \\
&\quad + f_1 R_{y,hydr}(t)
\end{aligned} \qquad (3)$$

$$\dot{\omega}_z = \left(f_7 {U_a}^2 m_z - \frac{y_T}{J_z} T\right) + \frac{m_{z,hydr}(t)}{J_z} \qquad (4)$$

Here *hydr* stands for hydrodynamics.

2.3 AERODYNAMICS

The coefficients of aerodynamic forces can be represented as (see [9]):

$$\begin{aligned}
C_x &= C_x(\vartheta, h) + C_x^{\dot{\vartheta}}(\vartheta, h)\, \omega_z \frac{b}{U_a} \\
&\quad + C_x^{\dot{h}}(\vartheta, h) \frac{U_y}{U_a} \\
C_y &= C_y(\vartheta, h) + C_y^{\dot{\vartheta}}(\vartheta, h)\, \omega_z \frac{b}{U_a} \\
&\quad + C_y^{\dot{h}}(\vartheta, h) \frac{U_y}{U_a}
\end{aligned} \qquad (5)$$

$$\begin{aligned}
m_z &= m_z(\vartheta, h) + m_z^{\dot{\vartheta}}(\vartheta, h)\, \omega_z \frac{b}{U_a} \\
&\quad + m_z^{\dot{h}}(\vartheta, h) \frac{U_y}{U_a}
\end{aligned} \qquad (6)$$

Tab. 1: Nomenclature

b	$[m]$	Chord of the wing
C_x		Aerodynamic drag coefficient
C_y		Aerodynamic lift coefficient
C_z		Aerodynamic side force coefficient
C_T^U		Derivative of thrust coefficient on speed
C_W		Coefficient of hydrodynamic resistance
g	$[m/s^2]$	Acceleration of gravity
H	$[m]$	Submergence of the boat under center of gravity
H_0	$[m]$	Submergence at the transom of the racing boat
h	$[m]$	Height of flight
h_{cg}	$[m]$	Height of center of mass
h_t	$[m]$	Height of the boat at the transom
$J_x;\ J_y;\ J_z$	$[kgm^2]$	Mass moment of inertia
$k(\beta)$		Coefficient of added mass
L	$[m]$	Span of the wing
l_0	$[m]$	Wetted length of the hull
LCG	$[m]$	Longitudinal position of center of gravity, measured from the transom of the boat
m_0	$[kg]$	mass
$m_{hull};\ m_{wing}$	$[kg]$	Masses for estimation of J_z
m_{hydr}	$[kg]$	Added mass of planing boat cross section
$m_x;\ m_y;\ m_z$		Coefficients of aerodynamic moments around x,y,z axes
$m_{z,hydr}(t)$	$[Nm]$	Trim hydrodynamic moment
M_W	$[Nm]$	Trim moment of hydrodynamic resistance
$R_{x,hydr}(t)$	$[N]$	Hydrodynamic drag force
$R_{y,hydr}(t)$	$[N]$	Hydrodynamic lift force
S	$[m^2]$	Area of the wing
S_0	$[m^2]$	Wetted surface of the hull
$S_{wing};\ S_{hull}$	$[m^2]$	Areas for estimation of J_z
T	$[N]$	Thrust of the boat
U_a	$[m/s]$	Boat speed with wind perturbations
U_d	$[m/s]$	Boat speed
U_{ycg}	$[m/s]$	Velocity of center of gravity in vertical direction
$x_{cg};\ y_{cg}$	$[m]$	Position of center of gravity
Y_0	$[N]$	Steady hydrodynamic lift
y_T	[m]	Thrust arm of the engine
W	$[N]$	Hydrodynamic resistance
$w_x; w_y; w_z$	$[ms^{-1}]$	Wind perturbations
α		$\vartheta_0 + \vartheta$
β		Deadrise angle
β_a	$[rad]$	Drift angle with wind perturbations
β_d	$[rad]$	Drift angle
ψ	$[rad]$	Angle of course
γ	$[rad]$	Angle of roll
λ		Aspect ratio of the air wing
$\mu;\ i_z$		Dimensionless mass and mass moment of inertia
η_0	$[m]$	Distance between keel and center of gravity
ρ	$[kg/m^3]$	Density of air
ρ_W	$[kg/m^3]$	Density of water
θ_T	[rad]	Setup angle of the engine
ϑ	[rad]	Pitch or trim angle
ϑ_0	[rad]	Mean trim angle
ξ_0	$[m]$	Distance between stern and center of gravity
$\omega_x;\ \omega_y;\ \omega_z$	$[1/s]$	Angular velocities

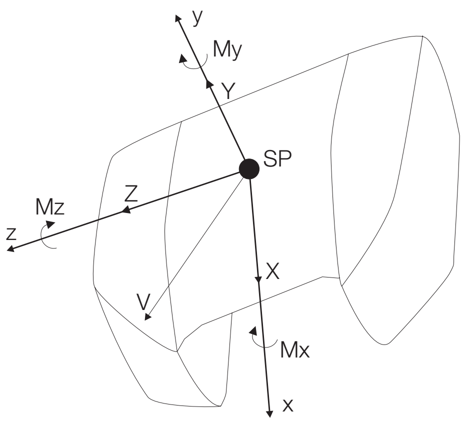

Fig. 3: Coordinate system

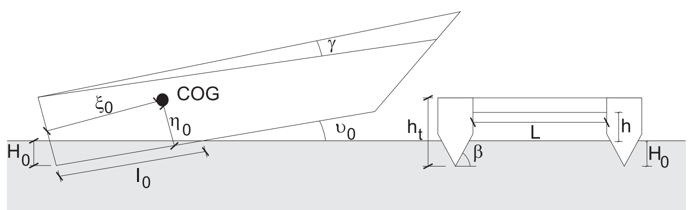

Fig. 4: Main dimensions of planing boat

The determination of aerodynamic characteristics of air wings in semi-connected coordinate system is performed using the program Autowing.

2.4 HYDRODYNAMICS

For calculation of hydrodynamic forces on a planing part of the boat a simple strip model proposed by Kovrizhnykh [6] and described in details by Lotov [7] is applied. The derivation of Kovrizhnykh starts from the Newtons second law for the local force f acting on a cross section of the planing surface:

$$f = (m_{hydr} U_n) \frac{d\alpha}{dt}$$

where U_n is the vertical velocity of the cross section. Integrating the last formulae over the whole wetted length one obtains the total lifting force acting on the hull. Taking into account that the hydrodynamic added mass for a prismatic shaped hull is

$$m_{hydr} = k(\beta)\, \rho_W h_1^2$$

the local force can be written in the form:

$$\begin{aligned} f &= \rho_W k(\beta) \left(h_1^2 U_n\right) \frac{d\alpha}{dt} \\ &= \rho_W k(\beta) \left(2 h_1 \dot{h_1} U_0 + h_1^2 \dot{U_n}\right) \end{aligned} \qquad (7)$$

Here h_1 is a local submergence of the cross section as a function of the longitudinal coordinate ξ and the unsteady angle of trim $\alpha = \vartheta_0 + \vartheta$, where ϑ_0 is the mean trim angle and ϑ is increment with respect to ϑ_0,

$$h_1 = (l - \xi_0 - \xi)\, \alpha,$$

Tab. 2: Lift and trim moment on the planing hull.

parameter	Lift derivatives F	Moment derivatives M
0	$\rho_W k(\beta) U_0^2 l_0^2 \vartheta_0^3$	$\rho_W k(\beta) \left(\frac{l_0}{3} - \xi_0\right) U_a^2 l_0^2 \vartheta_0^3$
h	$-2\rho_W k(\beta) U_a^2 l_0 \vartheta_0^2$	$-2\rho_W k(\beta) \left(\frac{l_0}{2} - \xi_0\right) U_a^2 l_0 \vartheta_0^2$
$\dot{h}$	$-2\rho_W k(\beta) U_a l_0^2 \vartheta_0^2$	$-2\rho_W k(\beta) \left(\frac{l_0}{3} - \xi_0\right) U_a l_0^2 \vartheta_0^2$
$\ddot{h}$	$-\frac{1}{3}(2 - cos(\beta)) \rho_W k(\beta) l_0^3 \vartheta_0^2$	$-\frac{1}{3}(2 - cos(\beta)) \rho_W k(\beta) \left(\frac{l_0}{4} - \xi_0\right) l_0^3 \vartheta_0^2$
ϑ	$2\rho_W k(\beta) \left(\frac{l_0}{2} + \xi_0\right) U_a^2 l_0 \vartheta_0^2$	$-2\rho_W k(\beta) \xi_0^2 U_a^2 l_0 \vartheta_0^2$
$\dot{\vartheta}$	$2\rho_W k(\beta) \xi_0 U_a l_0^2 \vartheta_0^2$	$-2\rho_W k(\beta) \left(\frac{l_0^2}{12} - \frac{l_0 \xi_0}{3} + \xi_0^2\right) U_a l_0^2 \vartheta_0^2$
$\ddot{\vartheta}$	$-(2 - cos(\beta)) \rho_W k(\beta) \frac{l_0}{3} \left(\frac{l_0}{4} - \xi_0\right) l_0^2 \vartheta_0^2$	$-(2 - cos(\beta)) \rho_W k(\beta) \frac{l_0}{3} \left(\frac{l_0^2}{10} - \frac{l_0 \xi_0}{2} + \xi_0^2\right) l_0^2 \vartheta_0^2$

Tab. 3: Resistance and its trim moment.

parameter	Resistance derivatives	Moment derivatives
0	$c_W S_0 \frac{\rho_W U_a^2}{2}$	$-c_W \frac{\rho_W U_a^2}{2} S_0 (\eta_0 - H_0)$
h	$-c_W \frac{\rho_W U_0^2 S_0}{H_0}$	$c_W \rho_W U_a^2 \frac{S_0}{H_0} \left(\eta_0 - \frac{3}{2} H_0\right)$
ϑ	$-c_W \frac{\rho_W U_0^2}{2} S_0 \frac{H_0 - 2\xi_0 \vartheta_0}{H_0 \vartheta_0}$	$c_W \frac{\rho_W U_a^2}{2} \frac{S_0}{\vartheta_0} (2H_0 - \eta_0)$

ξ_0 is the length between the stern and the longitudinal center of gravity. Expressing U_n through $\dot{h}$

$$\begin{aligned} U_n &= \dot{h_1} = U_0 \alpha - \dot{y} - \xi \dot{\vartheta} \\ \dot{U}_n &= 2U_0 \dot{\vartheta} - \ddot{y} - \xi \ddot{\vartheta} \end{aligned} \tag{8}$$

and substituting (8) into (7) gives

$$\begin{aligned} f(\xi) = &\rho_W k(\beta) [2(l - \xi_0 - \xi) \alpha \left(U_0 \alpha - \dot{y} - \xi \ddot{\vartheta}\right)^2 \\ &+ (2 - cos(\beta)) \alpha^2 (l - \xi_0 - \xi)^2 \left(2U_0 \dot{\vartheta} - \ddot{y} - \xi \ddot{\vartheta}\right)] \end{aligned} \tag{9}$$

The factor $(2 - cos(\beta))$ is a correction factor proposed by Logvinovich [7].
To get the resulting moment and the resulting force, the $f(\xi)$ function has to be integrated over the ship wetted length

$$\begin{aligned} Y_{hydr} &= \int_{-\xi_0}^{l-\xi_0} f(\xi) d\xi \\ m_{z,hydr} &= \int_{-\xi_0}^{l-\xi_0} f(\xi) \xi d\xi \end{aligned} \tag{10}$$

The wetted length l can also be written as

$$l = l_0 - \frac{y}{\vartheta_0} - \frac{(l_0 - \xi_0)}{\vartheta_0} \vartheta. \tag{11}$$

Therein the index 0 stands for the steady state value. The wetted length l_0 is calculated as

$$l_0 = H_0 \vartheta$$

where the submergence of the stern H_0 is calculated iteratively from the equilibrium condition at given speed and trim angle (see 2.1). Substituting (9) and (11) into (10) allows one to represent the hydrodynamic forces and moments in form of a truncated Taylor series with respect to y, $\dot{y}$, $\ddot{y}$, ϑ, $\dot{\vartheta}$ and $\ddot{\vartheta}$

$$\begin{aligned} Y_{hydr}\left(y, \dot{y}, \ddot{y}, \vartheta, \dot{\vartheta}, \ddot{\vartheta}\right) &= Y_0 + F^y y + F^{\dot{y}} \dot{y} + F^{\ddot{y}} \ddot{y} \\ &\quad + F^{\vartheta} \vartheta + F^{\dot{\vartheta}} \dot{\vartheta} + F^{\ddot{\vartheta}} \ddot{\vartheta} \\ m_{z,hydr}\left(y, \dot{y}, \ddot{y}, \vartheta, \dot{\vartheta}, \ddot{\vartheta}\right) &= M_0 + M^y y + M^{\dot{y}} \dot{y} + M^{\ddot{y}} \ddot{y} \\ &\quad + M^{\vartheta} \vartheta + M^{\dot{\vartheta}} \dot{\vartheta} + M^{\ddot{\vartheta}} \ddot{\vartheta} \end{aligned} \tag{12}$$

Coefficients of the series are presented in Table 2.
The hydrodynamic resistance can also be represented in the form of the Taylor series:

$$W = W_0 + W^y y + W^{\vartheta} \vartheta \tag{13}$$

The hydrodynamic moment M_W caused by W is calculated as:

$$M_W = -W(\eta_0 - H) \tag{14}$$

where H is the submergence at the position of the center of gravity: $H = H_0 - y + \xi_0 \vartheta$ and η_0 is the height of the center of gravity above keel.
The moment can also be represented in a form of the Taylor series:

$$M_W = M_{W0} + M_W^y y + M_W^{\vartheta} \vartheta \tag{15}$$

The coefficients are given in Table 3. The wetted surface of the hull S_0 can be calculated from

$$S_0 = \frac{\pi}{2} \frac{H_0^2}{\vartheta_0 \sin \beta}$$

2.5 STABILITY ANALYSIS

Substituting representations (5),(6),(12), (13) and (15) into the system (2),(3) and (4) and using the

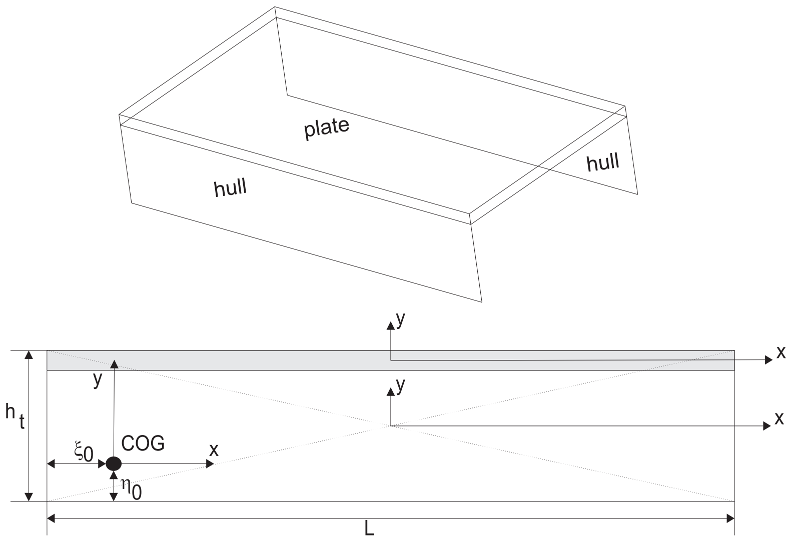

Fig. 5: Model for determination of mass moment of inertia I_z

Tab. 4: Coefficients of linearized system.

parameter	a	b	c
11	$2C_W \frac{\rho_W}{\rho} - C_T^{\overline{U}} \frac{\rho_W}{\rho} + 2c_x^0$	0	0
12	$C_x^h - 2C_W \frac{\rho_W}{\rho} \frac{S_0}{S} \frac{1}{\widetilde{H_0}}$	$C_x^{\dot{h}} \frac{1}{\mu}$	0
13	$C_x^\vartheta - C_W \frac{\rho_W}{\rho} \frac{S_0}{S} \frac{\widetilde{H_0} - 2\widetilde{\xi_0}\vartheta_0}{\widetilde{H_0}\vartheta_0}$	0	0
21	$\left(2C_y^0 + C_T^{\overline{U}} \frac{\rho_W}{\rho} \vartheta_T\right) \mu$	0	0
22	0	$C_y^{\dot{h}} - 2\kappa$	$1 + \frac{1}{3}(2 - \cos\beta)\,\widetilde{l_0}\kappa \frac{1}{\mu}$
23	$\left(C_y^\vartheta + 2\kappa \left(\frac{\widetilde{l_0}}{2} + \widetilde{\xi_0}\right) \frac{1}{\widetilde{l_0}}\right) \mu$	$C_y^{\dot{\vartheta}} + 2\kappa\widetilde{\xi_0}$	$-(2 - \cos\beta)\,\kappa \frac{\widetilde{l_0}}{3} \left(\frac{\widetilde{l_0}}{4} - \widetilde{\xi_0}\right) \frac{1}{\mu}$
31	$\left[-\widetilde{y_T} \frac{\rho_W}{\rho} C_T^{\overline{U}} + 2\left(C_T^0 \frac{\rho_W}{\rho} - C_W \frac{\rho_W}{\rho} \frac{S_0}{S} \left(\widetilde{\eta_0} - \widetilde{H_0}\right)\right)\right] \frac{\mu}{i_z}$	0	0
32	$\left(m_z^h - 2\kappa \left(\frac{\widetilde{l_0}}{2} - \widetilde{\xi_0}\right) \frac{1}{\widetilde{l_0}} + 2C_W \frac{\rho_W}{\rho} \frac{S_0}{S} \frac{\widetilde{\eta_0} - \frac{3}{2}\widetilde{H_0}}{\widetilde{H_0}}\right) \frac{\mu}{i_z}$	$\left(m_z^{\dot{h}} - 2\kappa \left(\frac{\widetilde{l_0}}{3} - \widetilde{\xi_0}\right)\right) \frac{1}{i_z}$	$\left(m_z^{\ddot{h}} - \frac{1}{3}\kappa (2 - \cos\beta) \left(\frac{\widetilde{l_0}}{4} - \widetilde{\xi_0}\right) \widetilde{l_0}\right) \frac{1}{\mu i_z}$
33	$\left(m_z^\vartheta - \kappa \frac{\widetilde{\xi_0^2}}{\widetilde{l_0}} + C_W \frac{\rho_W}{\rho} \frac{S_o}{S} \frac{2\widetilde{H_0} - \widetilde{\eta_0}}{\vartheta_0}\right) \frac{\mu}{i_z}$	$\left(m_z^{\dot{\vartheta}} - 2\kappa \left(\frac{\widetilde{l_0^2}}{12} - \frac{\widetilde{l_0}\widetilde{\xi_0}}{3} + \widetilde{\xi_0^2}\right)\right) \frac{1}{i_z}$	$1 + \kappa (2 - \cos\beta) \frac{\widetilde{l_0}}{3} \left(\frac{\widetilde{l_0^2}}{10} - \frac{\widetilde{l_0}\widetilde{\xi_0}}{2} + \widetilde{\xi_0^2}\right) \frac{1}{\mu i_z}$

dimensionless time τ

$$t = \tau \frac{2m}{\rho S U_0}$$

we obtain the following linearized motion equations (see also [9]):

$$\begin{aligned} &\Delta\dot{\overline{U}} + a_{11}\Delta\overline{U} + b_{12}\Delta\dot{\widetilde{h}} \\ &\quad + a_{12}\Delta\widetilde{h} + a_{13}\Delta\vartheta = 0 \\ &a_{21}\Delta\overline{U} - c_{22}\Delta\ddot{\widetilde{h}} + b_{22}\Delta\dot{\widetilde{h}} + a_{22}\Delta\widetilde{h} + c_{23}\Delta\ddot{\widetilde{\vartheta}} \\ &\quad + b_{23}\Delta\dot{\widetilde{\vartheta}} + a_{23}\Delta\vartheta = 0 \\ &a_{31}\Delta\overline{U} + c_{32}\Delta\ddot{\widetilde{h}} + b_{32}\Delta\dot{\widetilde{h}} + a_{32}\Delta\widetilde{h} - c_{33}\Delta\ddot{\widetilde{\vartheta}} \\ &\quad + b_{33}\Delta\dot{\widetilde{\vartheta}} + a_{33}\Delta\vartheta = 0 \end{aligned} \tag{16}$$

The dimensionless parameters are introduced according to the following relations:

$$\Delta\overline{U} = \frac{\Delta U}{U_0};\, \dot{\vartheta} = \frac{\rho S U_0}{2m}\dot{\widetilde{\vartheta}};\, \ddot{\vartheta} = \left(\frac{\rho S U_0}{2m}\right)^2 \ddot{\widetilde{\vartheta}}$$

$$\widetilde{h} = hb;\, \dot{h} = \frac{\rho S U_0 b}{2m}\dot{\widetilde{h}};\, \ddot{h} = \left(\frac{\rho S U_0}{2m}\right)^2 b\ddot{\widetilde{h}}$$

The coefficients a_{ij}, b_{ij} and c_{ij} are given in Table 4 where the following dimensionless parameters are used

$$\mu = \frac{2m_0}{\rho S b}\ ;\ i_z = \frac{J_z}{mb^2}\ ;\ C_T^{\overline{U}} = \frac{2T^U}{\rho U S}$$

$$\kappa = 2\frac{\rho_W}{\rho} k(\beta)\, \widetilde{l_0^2} \frac{b^2}{S} \vartheta_0^2$$

$$\widetilde{l_0} = \frac{l_0}{b}\ ;\ \widetilde{H_0} = \frac{H_0}{b}\ ;\ \widetilde{\xi_0} = \frac{\xi_0}{b}\ ;\ \widetilde{\eta_0} = \frac{\eta_0}{b}$$

According to the procedure of the linear stability analysis a differentiation operator is introduced

$$p = \frac{d}{dt}, p^2 = \frac{d^2}{dt^2}$$

into the system (16). Replacing derivatives of kinematic parameters by p and p^2 and grouping terms proportional to these parameters, one obtains the system of algebraic equations with respect to U, h and ϑ with the determinant:

$$\begin{vmatrix} p + a_{11} & b_{12}p + a_{12} & a_{13} \\ a_{21} & -c_{22}p^2 + b_{22}p + a_{22} & c_{23}p^2 + b_{23}p + a_{23} \\ a_{31} & c_{32}p^2 + b_{32}p + a_{32} & -c_{33}p^2 + b_{33}p + a_{33} \end{vmatrix}$$

Calculation of the determinant results in the characteristic equation of the system (16):

$$D_5p^5 + D_4p^4 + D_3p^3 + D_2p^2 + D_1p + D_0 = 0$$

This equation is quintic and has five roots. All of the real parts of these roots have to be negative for a stable planing.
Necessary and sufficient conditions of stability are [9]:

$$D_i > 0, (i = 1, 2, 3, 4, 5) ; D_1D_2 - D_3 > 0;$$

$$\begin{aligned} R_5 =& (D_1D_2 - D_3)(D_3D_4 - D_2D_5) \\ &- (D_1D_4 - D_5)^2 > 0 \end{aligned}$$

The boundary of dynamic (oscillatory) stability is determined by equation $R_5 = 0$, and the boundary of static (aperiodic) stability $D_5 = 0$ with other conditions of stability being fulfilled.

3 RESULTS OF THE STABILITY ANALYSIS

The analysis presented above was implemented into the Fortran program called STABBI and intended for the longitudinal stability analysis of racing boats with aerodynamic support. Because of luck of information on the mass moment of inertia, it was calculated under assumption that the planing boat consist of three parts, two hulls and the wing between them. They are modeled as flat rectangular plates with uniform mass distribution on areas $S_{wing} = Lb$ and $S_{hull} = L \cdot h_t$. Figure 2.4 shows this geometric model and three different coordinate systems. For a plate the mass moment of inertia around the lateral axis is defined as

$$J_z = \int (x^2 + y^2)dm$$

The mass moment of inertia J_z can be transferred to the coordinate system of the craft with the origin in the center of gravity by Steiners theorem. This results in:

$$\begin{aligned} J_z =& \frac{2}{12} m_{hull} \left(L^2 + h_t^2\right) + \frac{1}{12} m_{plate} L^2 \\ &+ 2m_{hull} \left(\left(\frac{L}{2} - \xi_0 \right)^2 + \left(\frac{h_t}{2} - \eta_0 \right)^2 \right) \\ &+ m_{plate} \left(\left(\frac{L}{2} - \xi_0 \right)^2 + (h_t - \eta_0)^2 \right) \end{aligned}$$

The mass of the hull part and the wing is then calculated by:

$$m_{hull} = \frac{S_{hull}}{S_{hull} + S_{wing}} m$$

$$m_{wing} = \frac{S_{wing}}{S_{hull} + S_{wing}} m$$

. The influence of the following kinematic and geometric parameters of the racing boats on stability was studied:

- γ [deg] - setup angle of the air wing with respect to the planing surface;
- β [deg] - deadrise angle of the planing surface;
- b [m] - chord of the air wing;
- L [m] - span of the air wing between end plates;
- ξ_0 and η_0 [m] - coordinates of the center of gravity measured from the transom and the planing surface;
- h_t [m] - height of the racing boat at the transom
- m [kg] - mass;
- J_z [kgm^2] - mass moment of inertia;
- U [m/sec] - speed of motion;

Based on these parameters the following dimensionless parameters can be proposed for further investigations of the stability:

$$\gamma \; ; \; \beta \; ; \; \widetilde{\xi_0} = \frac{\xi_0}{b} \; ; \; \widetilde{\eta_0} = \frac{\eta_0}{b} \; ; \; \mu = \frac{2m_0}{\rho Sb}$$

$$i_z = \frac{J_z}{m_0 b^2} \; ; \; \lambda = \frac{L}{b} \; ; \; \widetilde{h_t} = \frac{h_t}{b} \; ; \; \widetilde{U} = \frac{U}{\sqrt{\frac{m_0 g}{\rho L b}}}$$

The dimensionless parameters were varied in the range typical for modern racing boats (see Table 5).

3.1 INFLUENCE OF SPEED AND TRIM ANGLE

The diagrams of stability were obtained by variation of the speed and the trim angle. The curves of the diagrams show the border between stable and unstable planing. Beneath each curve the planing is stable at a given speed U and trim angles ϑ whereas above the line it is unstable. The stability decreases with increasing speed, because aerodynamic and hydrodynamic lifts are getting larger and the submerged part of the hull contributing to the stability becomes smaller. The same effect takes place when the trim angle is growing.

3.2 INFLUENCE OF AIR WING

Here a boat with the parameters from Table 5 was investigated. Concerning the contribution of the air wing to the stability it was found that this contribution is usually negative. Figure 6 illustrates this fact. It happens because the submerged part of the boat becomes smaller. A part of the boat weight is carried by the air wing which is unstable. In fact, the stability of wing in ground effect craft is secured mostly by the large tail unit. The WIG wing alone is unstable. The area of the stability of the boat with air wing is clearly smaller than that of the boat consisting only planing part. Only at small speed U when the influence of the aerodynamics is negligible the stability is the same for both boats.

3.3 INFLUENCE OF DIMENSIONLESS MASS

Figure 7 shows the diagram in which the value of μ was varied. For a small μ the area of stable planing is also small. When μ rising, the stability is getting better. Increase of μ is conducted by increase of the submerged part of the boat which contributes to the stability. Therefore, increase of the mass helps to avoid porpoising instability.

3.4 INFLUENCE OF DEADRISE ANGLE

Increase of the deadrise angle β influences stability in the same way as the dimensionless mass increase. Figure 8 shows a diagram for different deadrise angles β. When β is getting larger, the area of stable planing also increases.

3.5 INFLUENCE OF LONGITUDINAL POSITION OF THE CENTER OF GRAVITY

Figure 9 shows the diagram illustrating the influence of the longitudinal position of the center of gravity. The largest area of stability is observed at the smallest value of $\widetilde{\xi}$. When the longitudinal center of gravity is moved aft and $\widetilde{\xi}$ is decreased, the stability becomes better.

3.6 INFLUENCE OF VERTICAL POSITION OF THE CENTER OF GRAVITY

The diagram in Figure 10 shows that a change of $\widetilde{\eta}$ does not influence the stability very much. For different $\widetilde{\eta}$ the border curves between stable and unstable planing are nearly the same.

3.7 INFLUENCE OF THE HEIGHT OF THE BOAT AT TRANSOM

The height of the boat at the transom determines the largest flight height for the racing boat without loosing contact with the water surface. The diagram in Figure 11, in which the parameter $\widetilde{h}_t$ is varied, shows no significant change of stability for different heights of the transom.

3.8 INFLUENCE OF THE MASS MOMENT OF INERTIA

The results of the stability estimations show (see Figure 12), that the stability area for different i_z is almost the same. It has to be noted, that there were no reliable information on this parameter available. It might be that the real mass moment lies outside of the range investigated in this paper. Therefore this parameter has to be investigated more thoroughly in future works.

3.9 INFLUENCE OF THE ASPECT RATIO

The aspect ratio λ of the air wing has a great influence on stability. With increase of λ (increase of the span at constant chord) the stability area decreases sufficiently (see Figure 13). The increase of the aspect ratio leads to the increase of aerodynamic forces which reduce the submerged hull and enhance the instability.

4 CONCLUSION

A mathematical model and corresponding computer program have been developed to estimate the longitudinal stability of racing boats with aerodynamic support. It was shown that the aerodynamic forces acting on racing boats contribute to the dynamic instability. The stability can be sufficiently improved by increase of the deadrise angle, dimensionless mass and by positioning the center of gravity as close as possible to the stern. Influence of the vertical position of the center of gravity, height of the boat at the transom and mass moment of inertia is negligible. Increase of the aspect ratio of air wings enhances the instability.

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Tab. 5: Standard parameter(dimensional and nondimensional)

parameter	value
m	$4800kg$
J_z	$74436kgm^2$
γ	$0deg$
b	$12m$
L	$2m$
ξ	$4.2m$
η	$1m$
h_t	0.65

parameter	value
μ	33
i_z	0.108
β	20
$\widetilde{\xi}$	0.35
$\widetilde{\eta}$	0.083
λ	0.166
h_t	0.0542

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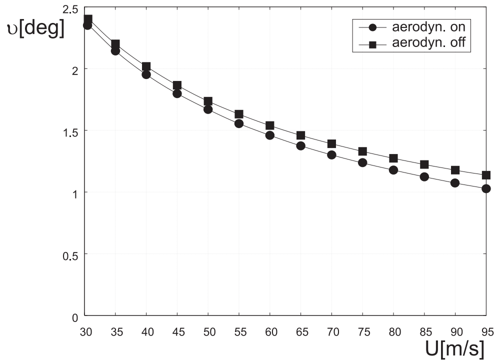

Fig. 6: Stability with and without aerodynamics

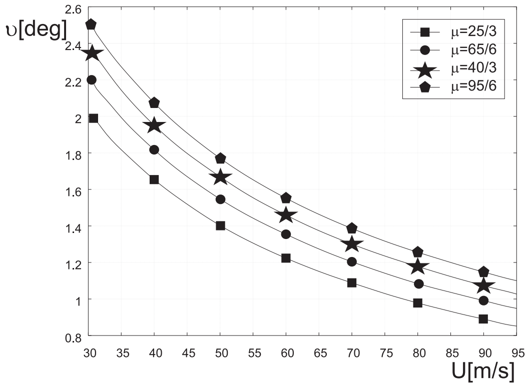

Fig. 7: Influence of μ on stability

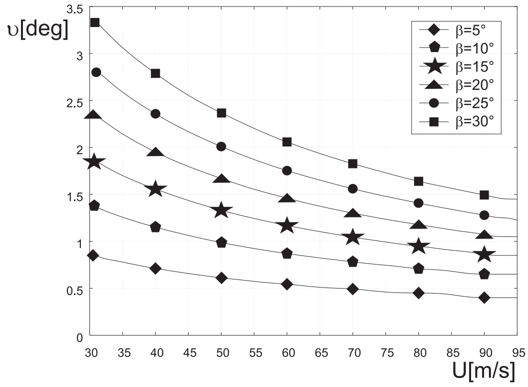

Fig. 8: Influence of β on stability

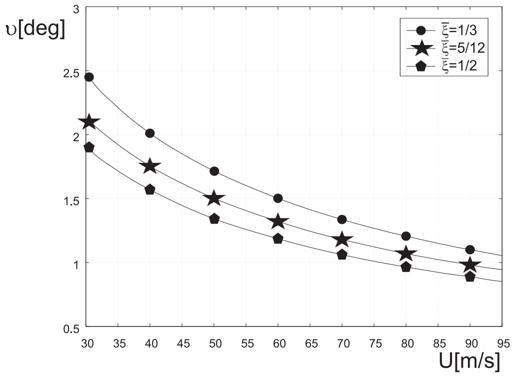

Fig. 9: Influence of $\widetilde{\xi}$ on stability

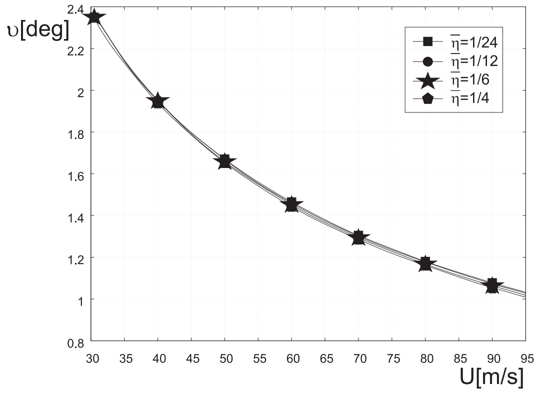

Fig. 10: Influence of $\widetilde{\eta}$ on stability

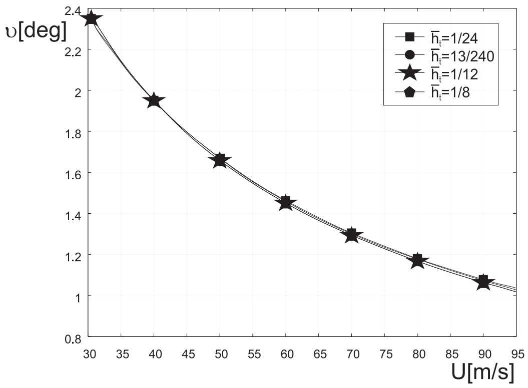

Fig. 11: Influence of $\widetilde{h}_t$ on stability

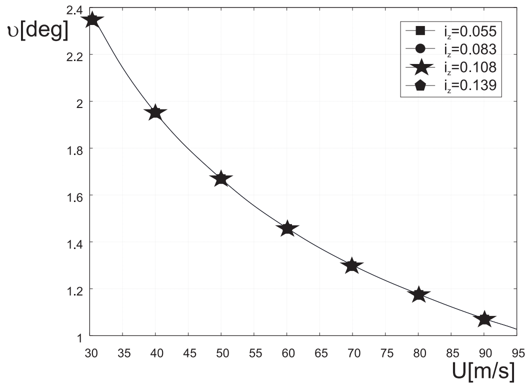

Fig. 12: Influence of i_z on stability

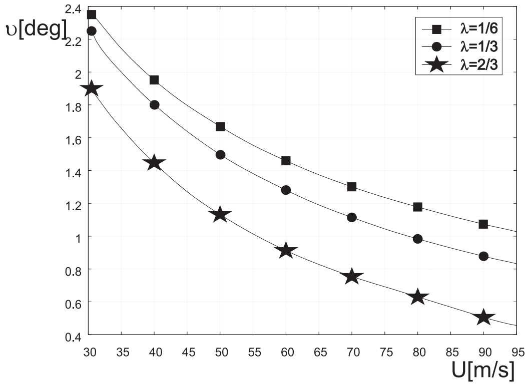

Fig. 13: Influence of λ on stability