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Dynamics of a Gear System with Faults in Meshing Stiffness

GRZEGORZ LITAK[1,*] and MICHAEL I. FRISWELL[2]
[1]*Department of Applied Mechanics, Technical University of Lublin, Nabystrzycka 36, 20-618 Lublin, Poland;* [2]*Department of Aerospace Engineering, University of Bristol, Queens Building, Bristol BS8 1TR, U.K.;* **Author for correspondence (e-mail: g.litak@pollub.pl; fax: +48-81-5250808)*

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Abstract. Gear box dynamics is characterised by a periodically changing stiffness. In real gear systems, a backlash also exists that can lead to a loss in contact between the teeth. Due to this loss of contact the gear has piecewise linear stiffness characteristics, and the gears can vibrate regularly and chaotically. In this paper we examine the effect of tooth shape imperfections and defects. Using standard methods for nonlinear systems we examine the dynamics of gear systems with various faults in meshing stiffness.

Key words: gear system, meshing errors, nonlinear vibrations

1. Introduction

Gears are very common systems, and practically impossible to replace in various applications where mechanical power must be transferred. Time varying mesh stiffness due to multiple teeth contact and a backlash between the teeth give rise to complex behaviour [1–6]. In consequence, under a dynamic load, a typical gear system is a nonlinear oscillator, exhibiting a range of complex behaviour including chaos [4, 7–13]. During operation the geometric parameters of the gears may change, and this causes the corresponding nonlinear response to change [4, 5, 14, 15]. Choy et al. [16] and Kuang and Lin [17] examined the effect of tooth wear. The vibration response of gear systems to stochastic forces has been analysed [4, 14, 15, 18].

In practice it is important to minimise the effect of noise and keep the machine as close as possible to a stable response. In this paper we classify meshing faults and examine the effect of broken teeth and meshing stiffness fluctuations on the vibration response. The possibility of amplitude jumps in systems with meshing defects is demonstrated.

2. Modelling of Gear Dynamics

Consider the single gear-pair system shown in Figure 1. In non-dimensional form, the equation of motion can be written [4, 9, 12] as

$$\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}x + \frac{2\zeta}{\omega}\frac{\mathrm{d}}{d\tau}x + \frac{k(\tau)g(x,\eta)}{\omega^2} = B(\tau) = \frac{r_1 M_1(\tau)}{I_1} + \frac{r_2 M_2(\tau)}{I_2}, \tag{1}$$

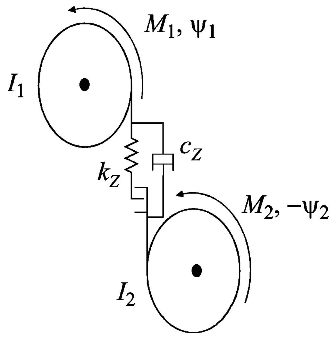

Figure 1. One stage gear system. Note, the relative displacement is $x = r_1\psi_1 - r_2\psi_2$.

where

$$\begin{aligned} \tau &= \omega t, \\ 2\zeta &= c_z\left[\frac{r_1^2}{I_1} + \frac{r_2^2}{I_2}\right], \end{aligned} \tag{2}$$

ω, ζ, $k(\tau)$, $g(x, \eta)$, η and $B(\tau)$ [4] and the other symbols are defined in Table 1 and shown in Figure 1. Often the excitation torque is assumed to be sinusoidal, and in this case $B(\tau)$ will take to form,

$$B(\tau) = \frac{B_0 + B_1\cos(\tau + \Theta)}{\omega^2}. \tag{3}$$

In the analysis that follows, the stiffness functions $k(\tau)$ and $g(x, \eta)$ need special attention. $g(x, \eta)$ has a piecewise character due to the backlash η, and is shown in Figure 2. $k(\tau)$ is the meshing stiffness arising from the interaction of a single-pair or multiple teeth in contact. For an ideal gear

Table 1. Symbols and parameters used in the analysis.

I_1, I_2	Moments of inertia
ψ_1, ψ_2	Rotational angles
$x = r_1\psi_1 - r_2\psi_2$	Relative displacement
r_1, r_2	Radii of gear wheels
v	Relative velocity
x_0, v_0	Initial conditions
M_1, M_2	External torques
ω	Excitation frequency
τ	Dimensionless time
ζ	Damping
η	Backlash
$k(\tau)$	Meshing stiffness
$g(x, \eta)$	Nonlinear stiffness function
B, B_0, B_1	External excitation
δ_i	Distance between increasing teeth contacts
σ_δ, σ_k	Standard deviations

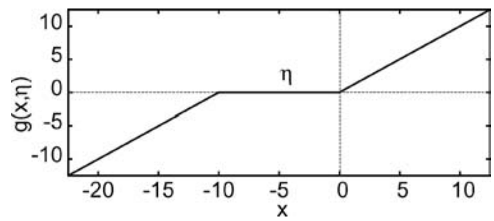

Figure 2. The nonlinear stiffness function $g(x, \eta)$ with an assumed backlash parameter $\eta = 10$.

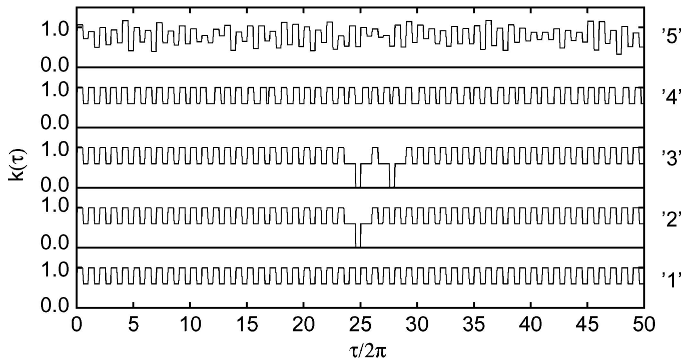

Figure 3. Various realizations of meshing stiffness, $k(\tau)$, used in the simulations. '1' corresponds to the ideal system without errors while '2' and '3' show the meshing stiffness with one and two broken teeth. '4' has a randomised distance δ_i between increasing teeth contacts with a standard deviation of $\sigma_\delta = 0.2\bar{\delta}_i$ ($\bar{\delta}_i = 0.8\pi$ in the non-dimensional time domain), and '5' is a randomised meshing stiffness. Here the amplitude changes with a standard deviation $\sigma_k = 0.1$ related to the maximum deterministic value $k_{\max} = 1$.

system we have followed references [4, 12] and assumed that this meshing stiffness changes periodically. Possible variations from the ideal case, and other possible meshing errors, are plotted in Figure 3. Note here that the nonlinearities arise from the piecewise linear character of the nonlinear function $g(x, \eta)$. Nonlinear dependence of the nonlinear external and parametric excitation functions on $\tau = \omega t$ (Equation (1)) appears only as a phase modulation to the periodic functions $\cos(\tau)$ and $k(\tau)$ (plot '1' in Figure 3), and, as usual, the additional dimension corresponding to the τ variable is wrapped. However, other models include different nonlinear effects such as impacts with a restitution coefficient [15, 19–23]. Schmidt [24] and Warmiński et al. [25, 26] included nonlinear self-excitation effects caused by dry friction between the gear teeth when the lubrication layer fails. Warminski et al. [4, 25, 26] examined nonlinear corrections due to a Duffing type stiffness.

Figure 4 shows the results of simulations of the model given by Equation (1), with time dependent meshing stiffness but without errors. We have used following system parameters: $\omega = 1.5$, $\zeta = 0.08$, $B_0 = 1.0$, $B_1 = 4.0$, $\eta = 10$. Note that the backlash $\eta = 10$ was chosen to produce the chaotic solutions found in earlier papers [4]. In any nonlinear system multiple solutions may coexist, and the solution obtained depends on the initial conditions. With the above parameters, there are indeed multiple solutions for the gear model, and this effect was examined in detail in a previous paper [27]. In Figures 4a and b we show regular and chaotic solutions, depending on the initial conditions for $[x_0, v_0] = [-9, 1]$ and $[x_0, v_0] = [-9, -1]$, respectively. Figure 4c shows the two coexisting attractors, obtained for various initial conditions, on the same graph.

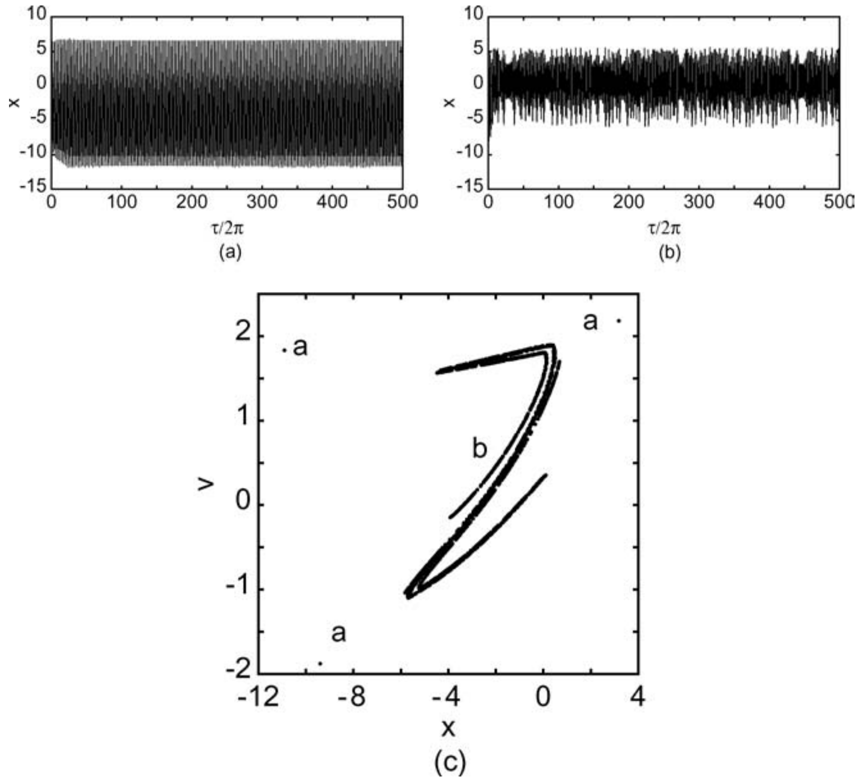

Figure 4. Time series and the Poincaré map for an ideal system for various initial conditions $[x_0, v_0]$; regular motion for $[x_0, v_0] = [-9, 1]$ (a) and chaotic motion for $[x_0, v_0] = [-9, -1]$ (b), respectively. Poincaré map for various initial conditions (c). The characters a and b in figure (c) denote the regular and chaotic attractors given by time series (a) and (b).

3. Errors in Meshing Stiffness

In this section we examine the effect of meshing stiffness errors. First consider a gear with one or two next neighbour teeth missing on one of the gear wheels, where each gear wheel has 50 teeth. The meshing stiffnesses, $k(\tau)$, are given in Figure 3 as '2' and '3', respectively, and should be compared to the ideal case, '1'. The response is simulated using the model given in Equation (1) with the meshing stiffness $k(\tau)$ given by curves '2' and '3' (Figure 3). Although the effect of one broken tooth appears to be fairly benign in terms of the gears dynamics, if two adjacent teeth are broken the result is a complex response of the system showing the characteristic amplitude jump phenomenon as the solution changes from the regular to the chaotic attractor. This is visible in Figure 5, which shows a time history for this case, and also shows the reverse jump from the chaotic to the regular attractor. Comparing to the ideal cases without any stiffness errors presented in Figures 4a–c (note the corresponding amplitudes of vibrations) it is clear that the system stays for a longer time in the chaotic attractor with intermittent regular motion. This result confirms previous results on stochastic jumps [4, 14] in systems with a stochastic force. However, the system modelled here is fully deterministic (Equation (1)) and the broken teeth act as additional parametric excitation (Figure 3).

We have also investigated vibrations of gears with a random distance δ_i between their increasing teeth contact (Figure 3, '4'). The results for two different noise levels and two different initial conditions $[x_0, v_0]$, which correspond to different attractors in the deterministic case (Figure 4), are shown in Figures 6a–d. Interestingly, for weak noise the system chooses the chaotic attractor. This conclusion

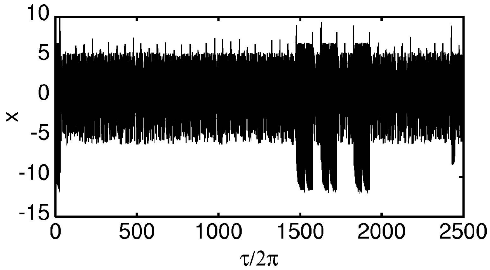

Figure 5. Time series for a gear system with two broken neighbouring teeth. Initial conditions: $[x_0, v_0] = [-9, 1]$.

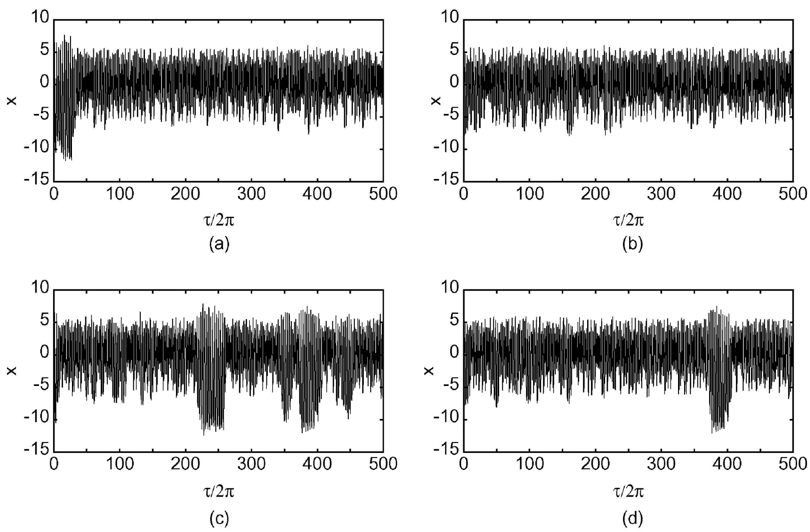

Figure 6. Time series for a gear system with a randomised distance δ_i between increasing teeth contacts with two different standard deviations: $\sigma_\delta = 0.2\bar{\delta}$ in (a) and (b), and $\sigma_\delta = 0.3\bar{\delta}$ in (c) and (d); and two different initial conditions $[x_0, v_0] = [-9, 1]$ in (a) and (c), and $[x_0, v_0] = [-9, -1]$ in (b) and (d).

differs from that obtained in the paper by Warmiński et al. [4] but the assumptions about the noise are different. Warmiński et al. [4] used an external stochastic force generated by stochastic Langevin simulations rather than the stochastic stiffness modelling in the present paper. For stronger noise the motion shows an intermittent character with short jumps to the regular attractor, as in the previous case with broken teeth (Figure 5).

Figure 7 corresponds to the meshing stiffness with a randomised amplitude but a regular distance (Figure 3, '5'), for $\sigma_k = 0.1$. Here we observe that neither attractor is favoured for these conditions.

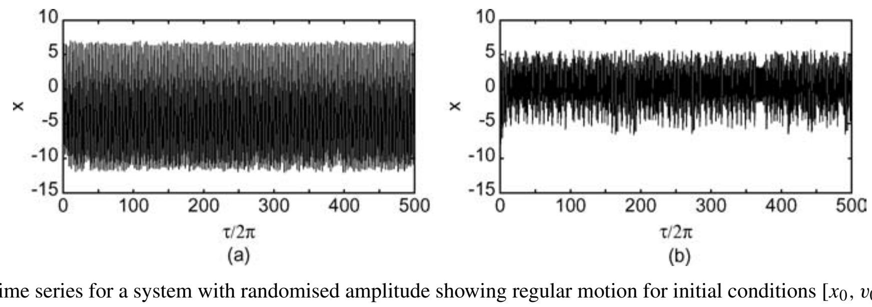

Figure 7. Time series for a system with randomised amplitude showing regular motion for initial conditions $[x_0, v_0] = [-9, 1]$, (a), and chaotic motion for $[x_0, v_0] = [-9, -1]$, (b).

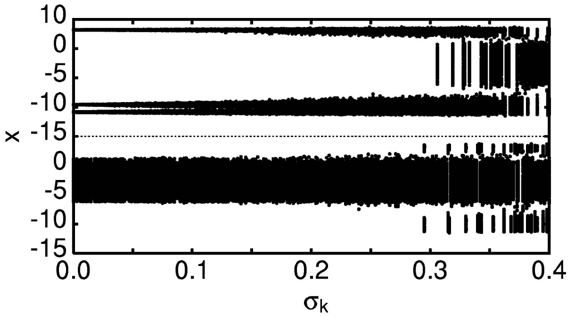

Figure 8. Bifurcation diagram for the case of randomised meshing stiffness amplitude: x is plotted stroboscopically against the square deviation of fluctuating meshing stiffness σ_k for two different initial conditions $[x_0, v_0] = [-9, 1]$ (the upper panel) and $[x_0, v_0] = [-9, -1]$ (the lower panel). Note the characteristic jumps between the attractors appear in both cases at $\sigma_k \approx 0.3$.

Eventually, for a sufficiently large noise level ($\sigma_k \approx 0.3$) the response returns to the intermittent behaviour with jumps between the regular and chaotic attractors. This effect is shown clearly in Figure 8, which shows the bifurcation diagram x against noise level σ_k.

4. Conclusions

We have examined the dynamics of gears in the presence of meshing faults. Such faults may arise due to wear during operation, or incorrect tolerances during production. The analysis of various types of errors and tooth faults highlights the presence of a dynamic jump phenomenon. Such jumps between different types of motions, namely chaotic and regular, can be crucial for the system reliability. In this respect our results are consistent with earlier results [4, 14, 18]. Moreover, the system is more sensitive to errors in the distance between teeth than fluctuations in the stiffness magnitude, although the qualitative effect is similar. One broken tooth has little influence on the dynamics of the gears, although two broken teeth can have a significant effect.

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