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# Dynamics of Bianchi type-II anisotropic dark energy cosmological model in the presence of scalar-meson fields

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**Abstract:** Today dark energy cosmological models have become essential to explain the accelerated expansion of the universe. Motivated by this we investigate, in this paper, a dark energy model in the presence of mass less scalar field in the frame work of locally rotationally symmetric (LRS) Bianchi type-II space-time. In order to find a deterministic model of the universe we use the hybrid expansion law to solve Einstein's field equations. We, also, use a relation between metric potential for this purpose. The dynamical cosmological parameters of the model are determined and their physical discussion is presented through graphical representation. It is observed that our model being a scalar field model varies in the quintessence region and exhibits a cosmic transition of the universe from early deceleration to late time acceleration. This fact is in agreement with the observations of modern cosmology.



**Keywords:** Bianchi type-II model; Dark energy model; Scalar-meson fields; Cosmological model; Hybrid expansion law.

## 1. Introduction

Astronomical observations of type1a supernovae (SN1a) [1-2], observations of large scale structure [3] and measurements of the cosmic microwave background (CMB) anisotropy [4-5] confirm that our universe is currently undergoing an accelerated expansion. It is believed that this is caused by a mysterious energy dubbed as dark energy (DE) with strong negative pressure. In order to explain this several dark DE candidates have been put forward from time to time. Significant among them are the quintessence scalar field models [6-7], the phantom model [8-10], k-essence [10-12], tachyon field [13-14], dark energy models including Chaplygin gas

[15-16] and so on. In spite of several attempts to identify the candidates for dark energy still cosmic acceleration is a challenge for modern cosmology.

Investigation of dark energy models in the presence of scalar fields is important because of the fact scalar fields play a significant role in inflationary cosmology. In particular spatially homogeneous anisotropic Bianchi type cosmological models in the presence of scalar fields play a vital role to study the possible effects of anisotropy in the early universe [17]. Some such works are given in [18-22].

The study of interacting fields (one of the fields being scalar meson field) is basically an attempt to look into the yet unsolved problem of the unification of the gravitational and quantum theories. In fact, there are two types of scalar meson fields, namely, zero rest mass scalar fields and attractive massive scalar fields. The zero rest mass scalar fields represent long range interactions while massive scalar fields describe short range interactions. The investigation of Einstein field equations in the presence of scalar meson fields has attracted the attention of many workers [23-31]. In particular Singh [32] has studied the static plane symmetric vacuum solutions of Einstein's field equations coupled with zero mass scalar fields while Chatterjee and Roy [33] have discussed the plane symmetric perfect fluid in the presence of mass less scalar fields. Subsequently, the investigation of interacting scalar fields has gained importance because of the fact that the observations reveal that spatially homogeneous and anisotropic cosmological models provide a richer structure of our early universe, both geometrically and physically, than the FRW models of the present day universe. Mohanty and Pradhan [34] have discussed some non-static solutions of Einstein field equations in the presence of viscous fluid and attractive massive scalar field. Singh and Shri Ram [35] have obtained plane symmetric cosmological models with viscous fluid and massive scalar field as the source. Singh [36] presented some inhomogeneous cosmological models in Bianchi type-V space time filled with viscous fluid in the presence of a massive scalar field. Mohanty et al. [37] have studied massive scalar field in Bianchi type-I space-time. Singh [38] discussed Bianchi type-V cosmologies in Lyra [39] manifold in the presence massive scalar field while Rahaman et al. [40] obtained Kantowski-Sachs cosmological model in the presence of mass less scalar field. Singh et al. [41] investigated Bianchi type-III cosmological models in Lyra geometry in the presence of massive scalar field. Spatially homogeneous and anisotropic Bianchi-V DE models in the presence of mass less scalar meson fields have not been studied recently by Reddy [42]. It is well known that exact solutions of general relativity for homogenous and anisotropic space times belong to either Bianchi type or Kantowski-Sachs. It may be mentioned here that Kantowski-Sachs models in scalar-tensor theories have been investigated by many authors [43-46]. Adhav et al. [47] have obtained exact solution for Kantowski-Sachs model in general relativity. Panigrahi and Sahu [48] studied micro and macro cosmological models in the presence of mass less scalar fields interacting with perfect fluid while Tiwari and Dwivedi [49] have studied cosmological models coupled with perfect fluid and mass less scalar fields in general relativity. Shri Ram et al. [50], Rao et al. [51], Rao and Prasanthi [52] and Santhi et al. [53] have discussed anisotropic dark energy models in modified theories of gravitation. However, anisotropic Bianchi type-II dark energy models in the presence of scalar meson fields have not been, so far, investigated.

It is well known that the observable universe is almost homogeneous and isotropic. Hence, the Friedmann– Robertson–Walker (FRW) models have attracted considerable attention in cosmology. But, cosmologists and theoretical physicists strongly believe that cosmic microwave background anisotropies at small angular scales are the key to the formation of discrete structures of the universe. Also the theoretical argument and recent experimental data support the existence of an anisotropic phase that approaches an isotropic phase (in the future). Hence, these leads us to consider the Bianchi type models which are homogeneous and anisotropic.

The motivation to study Bianchi type-II dark energy cosmological model in the presence of gravitational field coupled with zero mass scalar fields has come from the above discussion. The present work is organized as follows. Section 2 takes care of the metric and gravitational field equations. Sec. 3 discusses the solution of the field equations and resulting model of the universe. The cosmological parameters of the model are determined in Sec. 4 and their physical significance is highlighted in Sec. 5. The last section deals with summary and conclusions.

## 2. Basic Field Equations

A spatially homogeneous anisotropic LRS Bianchi type-II metric can be written as

$$ds^2 = -dt^2 + X^2(dx - z\,dy)^2 + Y^2(dy^2 + dz^2) \tag{1}$$

where X and Y are functions cosmic time t only.

Einstein field equations in the presence of anisotropic DE fluid and scalar meson fields are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -T_{ij} - (V_{,i} V_{,j} - \frac{1}{2} g_{ij} V_{,k} V^{,k}) \tag{2}$$

where V is the zero mass scalar field and the scalar field satisfies the Klein –Gordon equation

$$g^{ij} V_{;ij} = 0 \tag{3}$$

where a semi colon denotes covariant derivative and a comma indicates ordinary derivative. The modern cosmological observations confirm the cosmic acceleration which is supposed to be because of dark energy. Quintessence theories advocate the role of dynamical scalar fields for the above cosmic expansion. A scalar field, more generally negative equation of state were implemented as substitute of cosmological constant [7]. This fact has motivated us to consider the quintessential object.

The energy momentum tensor for DE fluid $T_{ij}$ is defined as

$$T_j^{\,i} = diag[-\rho_\Lambda, p_x, p_y, p_z] = diag[-1, \omega_x, \omega_y, \omega_z]\rho_\Lambda \tag{4}$$

where $\rho_\Lambda$ is the DE density and $p_x, p_y$ and $p_z$ are the pressures of DE fluid along *x, y* and *z* axes respectively.

Here
$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} \tag{5}$$

is the equation of state (EoS) parameter of the fluid and $\omega_x, \omega_y, \omega_z$ are the EoS parameters in the directions of *x, y* and *z* axes respectively. The energy momentum tensor of DE fluid can be parameterized as

$$T_j^{\,i} = diag[-1, \omega_\Lambda, (\omega_\Lambda + \delta), (\omega_\Lambda + \delta)]\rho_\Lambda \tag{6}$$

For the sake of simplicity we choose $\omega_x = \omega_\Lambda$ and skewness parameter $\delta$ is the deviation from $\omega_\Lambda$ along *y* and *z* axes. Here $\omega_\Lambda$ and $\delta$ need not be constants but can be functions of cosmic time.

Explicit Einstein field equations (2) for the metric (1) with the help of Eq. (6) can be written as

$$2\frac{\dot{X}}{X}\frac{\dot{Y}}{Y} + \left(\frac{\dot{Y}}{Y}\right)^2 - \frac{1}{4}\frac{X^2}{Y^4} - \frac{\dot{V}^2}{2} = \rho_\Lambda \tag{7}$$

$$2\frac{\ddot{Y}}{Y}+\left(\frac{\dot{Y}}{Y}\right)^2-\frac{3}{4}\frac{X^2}{Y^4}+\frac{\dot{V}^2}{2}=-\omega_\Lambda\rho_\Lambda \tag{8}$$

$$\frac{\ddot{X}}{X}+\frac{\ddot{Y}}{Y}+\frac{\dot{X}\dot{Y}}{XY}+\frac{1}{4}\frac{X^2}{Y^4}+\frac{\dot{V}^2}{2}=-(\omega_\Lambda+\delta)\rho_\Lambda \tag{9}$$

The Klein-Gordon equation for the metric (5) takes the form

$$\ddot{V}+\dot{V}\left(\frac{\dot{X}}{X}+2\frac{\dot{Y}}{Y}\right)=0 \tag{10}$$

and the conservation equation for the DE fluid is

$$\dot{\rho}_\Lambda+\left(\frac{\dot{X}}{X}+2\frac{\dot{Y}}{Y}\right)(1+\omega_\Lambda)\rho_\Lambda=0 \tag{11}$$

Here an overhead dot indicates differentiation with respect to comic time t.

## 3. Solutions of field equations and the DE model

Here we solve the field equations (7)-(11) which are a system of four independent equations [Eq. (11) being the consequence of Eqs. (7)-(9)] in six unknowns $X$, $Y$, $V$, $\rho_\Lambda$, $\omega_\Lambda$ and $\delta$. Hence we need two more conditions to get a determinate solution. We use the following physically viable conditions:

(i) The scalar expansion of the space time is proportional to shear which gives a relation between metric potentials [54] given by

$$X=Y^k \tag{12}$$

where $k\neq 1$ is a positive constant which takes care of the anisotropy of the space time and

(ii) Several authors have used a constant deceleration parameter to construct the cosmological models which gives a power law for the metric potentials [55–58]. The positive value of the deceleration parameter shows the early deceleration of the universe, whereas the negative value of the deceleration parameter exhibits the late time acceleration of the universe. Recent observations from Type Ia supernova and CMB anisotropies suggest that the universe is not only expanding, but also accelerating at present and having decelerated phase at early times. Therefore, the deceleration parameter must show this transition by its signature changing. That is why the deceleration parameter is variable in time, not a constant. This motivates us to choose the following average scale factor which provides a time dependent deceleration parameter the hybrid expansion law, for the average scale factor, given by Akarsu et al. [59]

$$a(t) = a_0 t^{\alpha_1} e^{\alpha_2 t} \tag{13}$$

where $\alpha_1$ and $\alpha_2$ are non-negative constants and $a_0$ is the present day value of $a(t)$. Recently many authors in the literature have constructed cosmological models using this hybrid expansion law [60-63].

For the metric (1) the average scale factor is given by

$$a = (XY^2)^{\frac{1}{3}} \tag{14}$$

Now using Eqs. (12) and (14) we obtain the metric potentials as

$$X = \left(a_0 t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{3k}{k+2}}$$
$$Y = \left(a_0 t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{3}{k+2}} \tag{15}$$

Through a proper choice of coordinates and constants ($a_0$=1one can write the metic (1) using Eqs. (15) as

$$ds^2 = -dt^2 + \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{6k}{k+2}} (dx - z\, dy)^2 + \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{6}{k+2}} (dy^2 + dz^2) \tag{16}$$

Using Eqs. (10) and (16) we obtain the zero mass scalar field as

$$V = V_0 \int \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{-3} dt + c_0 \tag{17}$$

where the constant of integration $c_0$ can be set equal to zero. Now, Eq. (16) along with Eq. (17) represents LRS Bianchi type-II DE model in the presence of zeo mass scalarfields.

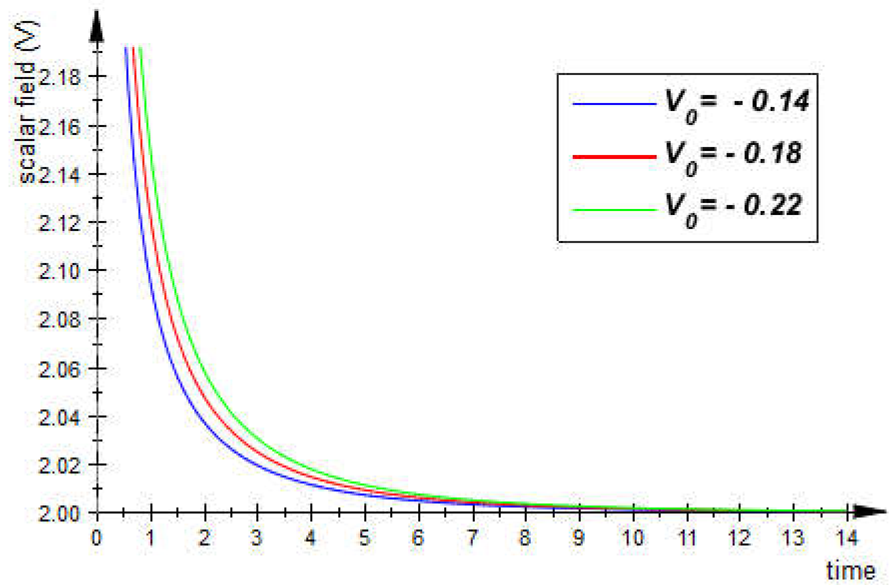


Fig. 1: Plot of scalar field versus cosmic time t for $c_0 = 2,\ \alpha_1 = 0.6$ and $\alpha_2 = 0.06$.

## 4. Cosmological parameters

The cosmological parameters which are useful for the physical discussion of the model(16) are defined and determined in the following:

The spatial volume v is

$$v = XY^2 = \left(t^{\alpha_1} e^{\alpha_2 t}\right)^3 \tag{18}$$

The average Hubble parameter is

$$H = \frac{1}{3}\left(\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y}\right) = \frac{\alpha_1}{t} + \alpha_2 \tag{19}$$

The scalar expansion is

$$\theta = 3H = \left(\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y}\right) = 3\left(\frac{\alpha_1}{t} + \alpha_2\right) \tag{20}$$

The shear scalar is

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y}\right)^2 = \left(\frac{k-1}{k+2}\right)^2 \left(\frac{\alpha_1 + \alpha_2 t}{t}\right)^2 \tag{21}$$

The anisotropy parameter is

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = 8\left(\frac{k-1}{k+2}\right)^2 \tag{22}$$

The deceleration parameter is

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{\alpha_1}{(\alpha_1 + \alpha_2 t)^2} - 1 \tag{23}$$

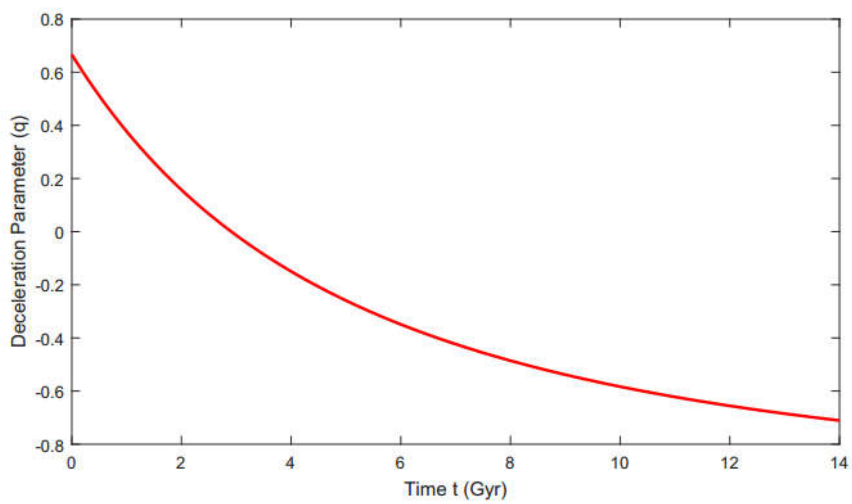


**Fig. 2**: Plot of Deceleration parameter versus cosmic time t for $\alpha_1 = 0.6,\ \alpha_2 = 0.06$.

From Eqs. (16), (17) and (7) the DE density in the model is

$$\rho_\Lambda = \left[\frac{36(2k+1)(\alpha_1 + \alpha_2 t)^2 - (k+2)^2 t^2 \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{6(k-2)}{k+2}} - 2V_0^{\ 2}(k+2)^2 t^2 \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{-6}}{4(k+2)^2 t^2}\right] \tag{24}$$

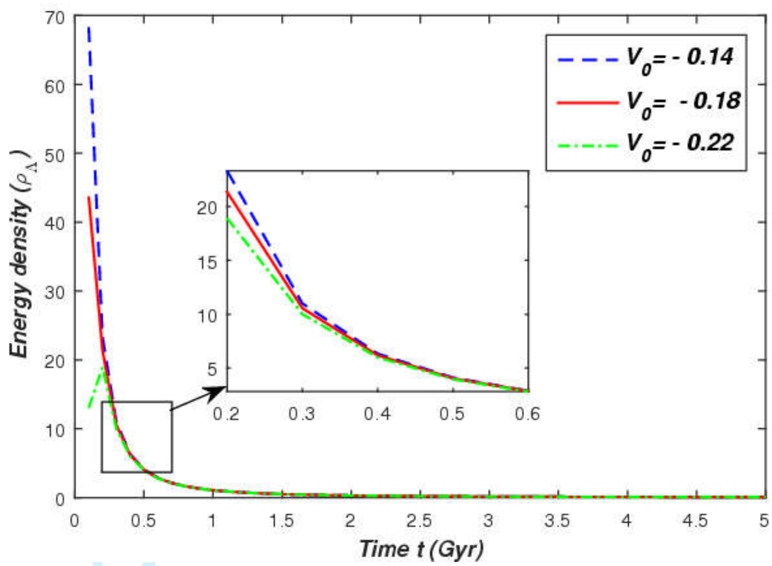


**Fig. 3**: Plot of energy density of dark energy versus cosmic time t for $\alpha_1 = 0.6,\ \alpha_2 = 0.06$ and *k=0.98*.

The EoS parameter can be found from Eqs. (8), (16), (17) and ( 24) as

$$\omega_\Lambda = -\left[\frac{108(\alpha_1+\alpha_2 t)^2 - 24\alpha_1(k+2) - 3(k+2)^2 t^2\left(t^{\alpha_1}e^{\alpha_2 t}\right)^{\frac{6(k-2)}{k+2}} + 2V_0^{\,2}(k+2)^2 t^2\left(t^{\alpha_1}e^{\alpha_2 t}\right)^{-6}}{36(2k+1)(\alpha_1+\alpha_2 t)^2 - (k+2)^2 t^2\left(t^{\alpha_1}e^{\alpha_2 t}\right)^{\frac{6(k-2)}{k+2}} - 2V_0^{\,2}(k+2)^2 t^2\left(t^{\alpha_1}e^{\alpha_2 t}\right)^{-6}}\right] \quad (25)$$

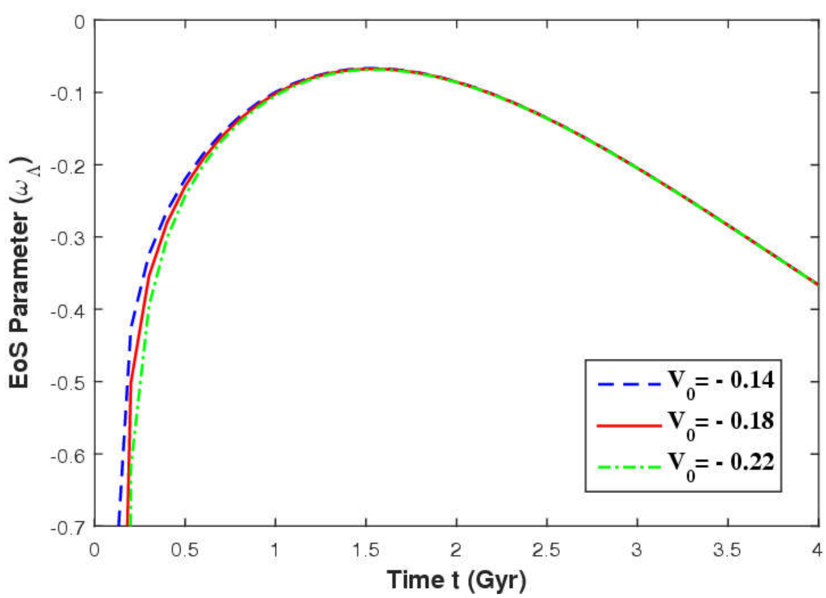


**Fig. 4**: Plot of EoS parameter versus cosmic time t for $\alpha_1 = 0.6,\ \alpha_2 = 0.06$ and *k=0.98*.

Using Eqs. (16), (17), (8), (9), (24) and (25) we obtain the skewness parameter as

$$\delta=\left[\frac{12(k-1)(k+2)[(\alpha_1-3(\alpha_1+\alpha_2 t)^2]-4(k+2)^2 t^2\left(t^{\alpha_1}e^{\alpha_2 t}\right)^{\frac{6(k-2)}{k+2}}}{36(2k+1)(\alpha_1+\alpha_2 t)^2-(k+2)^2 t^2\left(t^{\alpha_1}e^{\alpha_2 t}\right)^{\frac{6(k-2)}{k+2}}-2V_0^{\,2}(k+2)^2 t^2\left(t^{\alpha_1}e^{\alpha_2 t}\right)^{-6}}\right] \tag{26}$$

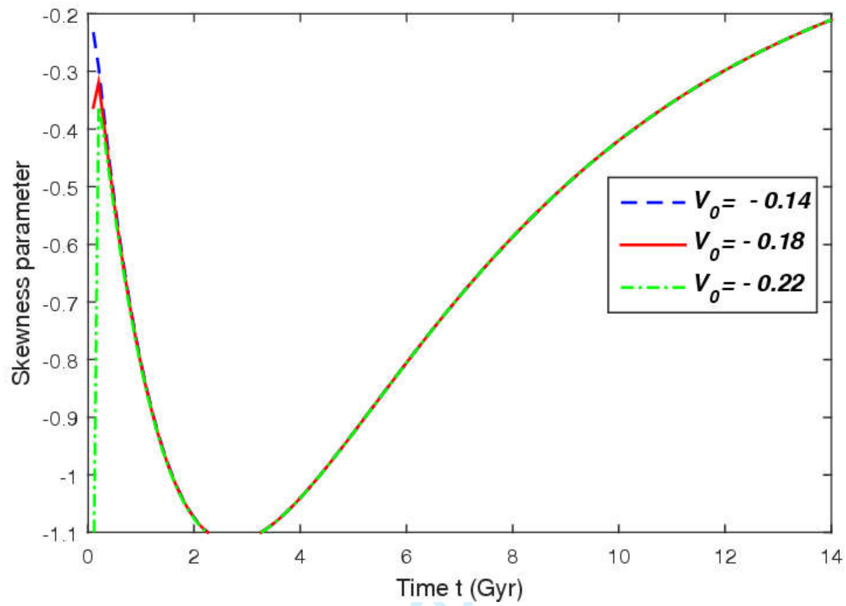


**Fig. 5**: Plot of skewness parameter versus cosmic time t for $\alpha_1=0.6,\ \alpha_2=0.06$ and *k=0.98*.

The jerk parameter is defined as the third derivative of the scale factor with respect to the cosmic time and is given by [64]

$$j(t)=\frac{1}{H^3}\frac{\dddot{a}}{a}=q+2q^2-\frac{\dot{q}}{H}=1-\frac{3\alpha_1}{(\alpha_1+\alpha_2 t)^2}+\frac{2\alpha_1}{(\alpha_1+\alpha_2 t)^3} \tag{27}$$

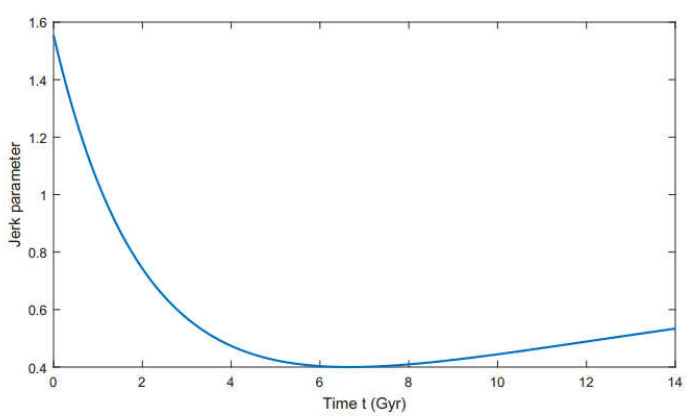


**Fig. 6**: Plot of jerk parameter versus cosmic time t $\alpha_1=0.6,\ \alpha_2=0.06$.

## 5. Physical discussion

The above results facilitate to discuss the behavior of our model (16). It may be noted that the spatial volume vanishes at t=0 and it increases with the increase in time which shows that the universe evolves with zero volume at t=0 and expands with cosmic time. It can be observed that the average Hubble parameter approaches zero for infinitely large time. As $t \to \infty$, the scale factors X(t) and Y(t) tend to infinity. The scalar expansion and shear scalar tend to zero for infinitely large values and they diverse at t=0. The mean anisotropy parameter is uniform throughout the evolution of the universe for $k \neq -2$ but for *k=-2* tends to infinity.

Fig. 1 shows the behavior of the scalar field. It may be observed that as $V_0$ increases the scalar field in the model decreases. Also, at late times we find the effect of variation of $V_0$ is negligible. Fig. 3 describes the behavior of dark energy density. It can be seen that the dark energy density is positive and decreases with cosmic time for different values of $V_0$. Initially, as the scalar field increases the dark energy density decreases and at late times the effect of scalar field on dark energy density is almost negligible. Fig. 5 explains the variation of skewness parameter versus cosmic time *t* which shows the anisotropic behavior of dark energy throughout the evolution of the universe. It is interesting to note that at sufficiently large cosmic time the skewness parameter vanishes. Also there is no significant effect of scalar field on the skewness parameter. It can be observed from the Fig. 2 that the deceleration goes from positive to negative region with cosmic time t and approaches the present value $q_0$=-0.703. Also from Fig. 6 it can be seen that the jerk parameter is positive throughout the evolution of the universe. This shows that the universe exhibits a smooth transition of the universe from early deceleration to the present accelerated phase. The above results are in agreement with the present scenario and observations of modern cosmology.

Fig. 4 gives the behavior of EoS parameter with time. It may be observed that the dark energy model always varies in the quintessence region and it can be seen that the effect of scalar field is not quite significant at late times. That is we are obtaining a quintessence scalar field model.

In the following we, now, discuss other problems which are relevant to our present work and compare our dark energy model with the other models. The work of Sarkar [65] on Bianchi-type-I interacting dark energy model of the universe with variable deceleration parameter also supports the behavior of the EoS parameter of our model. Rao and Prasanthi [66] have discussed

Bianchi-type-I and –III dark energy models in Saez-Ballester scalar-tensor theory of gravitation, and they have obtained a model which starts the evolution from the phantom region and ultimately reaches the quintessence region. Das and Sultana [67] have studied the Bianchi-type-$VI_0$ dark energy model in general relativity with sign-changeable interaction and obtained a similar result from an analysis of the EoS parameter. Also, Mishra et al. [68] found a quite similar result from the analysis of the EoS parameter in their study of Bianchi-type-V string cosmological model with anisotropic distribution of dark energy. Rao et al. [69] have investigated non-static plane symmetric universe with dark energy within the framework of Saez-Ballester theory of gravitation. In this model, the EoS parameter varies from matter dominated to phantom region by crossing phantom divide line and then goes towards quintessence region in the latter epoch. Reddy [70] found a similar result from the analysis of the EoS parameter in his study of Bianchi type-*V* dark energy model with scalar meson fields in general relativity. It can also be observed that the behavior of the model obtained, here, is in good agreement with the recent observational data [71-73].

## 6. Summary and conclusions

In this work, we have investigated an anisotropic dark energy model coupled with zero mass scalar fields in LRS Bianchi type-*II* space-time. An exact solution of the Einstein field equations is presented using a relation between metric potentials and hybrid expansion for the average scale factor of the universe. Using this solution a DE model is presented and the dynamical cosmological parameters corresponding to the model are determined and their physical significance is discussed through graphical representation. It is observed that our model evolves from initial epoch, expands and exhibits a transition from decelerated phase to accelerated state. The behavior of skewness parameter shows that the dark energy always remains anisotropic throughout the evolution of the universe and ultimately vanishes for vary large time. The DE density is positive, decreases with cosmic time. Our model, being scalar field model, always varies in the quintessence region. Thus we have obtained a cosmological model which is in agreement with the modern cosmological observations.

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