DYNAMICS OF CHARGED PARTICLES AND THEIR RADIATION FIELD

This book provides a self-contained and systematic introduction to classical electron theory and its quantization nonrelativistic quantum electrodynamics. The first half of the book covers the classical theory. It discusses the well-defined Abraham model of extended charges in interaction with the electromagnetic field, and gives a study of the effective dynamics of charges under the condition that, on the scale given by the size of the charge distribution, they are far apart and the applied potentials vary slowly. The second half covers the quantum theory, leading to a coherent presentation of nonrelativistic quantum electrodynamics. Topics discussed include nonperturbative properties of the basic Hamiltonian, the structure of resonances, the relaxation to the ground state through emission of photons, the nonperturbative derivation of the g-factor of the electron, and the stability of matter.

Suitable as a supplementary text for graduate courses, this book will also be a valuable reference for researchers in mathematical physics, classical electrodynamics, quantum optics, and applied mathematics.

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To the memory of my parents Ortrud Knopp and Karl Spohn

Contents

Preface

Physical theories, while devised to model a particular range of phenomena, are evidently linked in a hierarchical fashion. It is this structure which keeps fascinating me. In statistical mechanics, my scientific home-town, the link between atomic and macroscopic properties is one central issue. There we are taught that the emergence of a more restricted theory from a more general one has a richer structure than merely letting some parameter tend to infinity. I understood at some point, by accident, that similar issues appear in the dynamics of classical charges coupled to the Maxwell field. Since I could not find a satisfactory discussion in the literature, I decided to write up my own account. The theory so covered is the classical electron theory, a subject which is commonly regarded as settled with some modest revival through astrophysical applications. On the other hand, the quantized version of this theory is more lively than ever through the amazing advances in atomic physics and quantum optics. It thus seemed to me a welcome opportunity to expand my enterprise and to cover also nonrelativistic quantum electrodynamics, stressing its classical counterpart more than is done usually.

The research which has led to this book goes back about seven years and in part much longer. I am grateful for the constant help from my collaborators Volker Betz, Brian Davies, Rolf Dümcke, Detlef Dürr, Christian Hainzl, Masao Hirokawa, Fumio Hiroshima, Frank Hövermann, Matthias Hübner, Valery Imaikin, Sasha Komech, Markus Kunze, Joel Lebowitz, József Lőrinczi, Robert Minlos, Gianluca Panati, and Stefan Teufel. In this list I also include Michael Kiessling for many illuminating observations. In addition I thank him for a careful reading of a draft of the book.

As the project expanded I received comments, criticisms, remarks, and questions which in their total sum shaped my understanding of the subject and the way things were written down eventually. All I can do here is to deeply thank Robert Alicki, Asao Arai, Volker Bach, Gernot Bauer, Jens Bolte, Thomas Chen, Stephan

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This book is dedicated to my parents in deep gratitude for a wonderful childhood. My father furnished stability and my mother cared for the three boys, encouraging our curiosity to learn about the world around us. This gift constitutes a marvellously complex lasting source of joy.

Herbert Spohn
München
May 2004

Symbols

Symbols for physical quantities

A	quantized vector potential
$\boldsymbol{A}$	vector potential
$\mathcal{A}$	action
$A_t(x)$	fluctuating vector potential
$A_\parallel$, $A_\perp$, $E_\parallel$, $E_\perp$	longitudinal, tranverse fields
$\boldsymbol{B}$	magnetic field
$\boldsymbol{E}$	electric field
$\mathcal{E}$	total energy
$E(\boldsymbol{p})$	energy–momentum relation
$\boldsymbol{E}^0$, $\boldsymbol{B}^0$, $\boldsymbol{q}^0$, $\boldsymbol{v}^0$	initial conditions
E_s	soliton energy
E_bin	binding energy
$\boldsymbol{E}_\mathrm{ex}$, $\boldsymbol{B}_\mathrm{ex}$	external fields
$\boldsymbol{E}_\mathrm{ini}$, $\boldsymbol{B}_\mathrm{ini}$	initial fields
$\boldsymbol{E}_\mathrm{out}$, $\boldsymbol{B}_\mathrm{out}$	outgoing fields
$\boldsymbol{E}_\mathrm{ret}$, $\boldsymbol{B}_\mathrm{ret}$	retarded fields
$\boldsymbol{E}_\mathrm{sc}$, $\boldsymbol{B}_\mathrm{sc}$	scattered fields
E_self	self-energy
$\boldsymbol{E}_{\boldsymbol{v}}$, $\boldsymbol{B}_{\boldsymbol{v}}$	soliton fields
$\mathbf{F}$	electromagnetic field tensor
$\boldsymbol{F}$	force
H	Hamiltonian
H_f	field Hamiltonian
H_p, $H(p)$	Hamiltonian at fixed total momentum p
H_sp	spin Hamiltonian
I_b, I_f	moment of inertia

J	total angular momentum
J_{f}	field angular momentum
$L, L_{\mathrm{at}}, L_{\mathrm{f}}, L_{\mathrm{int}}$	Liouvillean
$L, \mathcal{L}$	Lagrangian
L_{D}	Davies generator
M_{e}	electric moment
M_{m}	magnetic moment
N	number of particles
$\boldsymbol{N}$	torque
$\mathcal{P}$	total momentum
$\boldsymbol{P}_{\mathrm{s}}$	soliton momentum
$\boldsymbol{P}_{\mathrm{f}}$	field momentum
$\mathcal{P}_{\mathrm{f}}, P_{\mathrm{f}}$	field momentum
$\mathcal{S}$	soliton manifold
T	temperature
V_{coul}	Coulomb potential
V_{dar}	Darwin potential
Z	partition function, nucleon charge
a^*, a	creation, annihilation operators
c	velocity of light
$e\varphi$	charge distribution
e	electric charge
e_{λ}	polarization vectors
$\mathbf{f}$	Minkowski force
f_{α}	distribution function
g	g-factor
$g_{\mu\nu}$	metric tensor
$\hbar$	Planck's constant
$\boldsymbol{j}$	current density
$\mathbf{j}$	four-current
$\boldsymbol{k}$	momentum
k_{B}	Boltzmann's constant
m	mass
m_{b}	bare mass
m_{f}	field mass
m_{g}	gyrational mass
m_{eff}	effective mass
$\widehat{n}$	unit vector
$\mathbf{p}$	four-momentum

p, $\boldsymbol{p}$, P	momentum
q, $\boldsymbol{q}$	position
$\mathbf{q}(\tau)$	world line
$\boldsymbol{r}$	position
r_{B}	Bohr radius
s	spin angular momentum
$\mathbf{t}$	Minkowski torque
t	time
$\boldsymbol{u}$	four-velocity
$\mathbf{u}$, $\boldsymbol{u}$	velocity
v, $\boldsymbol{v}$	velocity
$\mathbf{x}$	four-space vector
x, $\boldsymbol{x}$	space
Δ	Laplacian
Λ	ultraviolet cutoff
Ω	four-gyration
$\Omega^{\pm}$	wave operator
α	fine structure constant
β	inverse temperature
γ	relativistic velocity factor
$\delta^{\perp}$	transverse delta function
λ_{c}	Compton wavelength
μ	magnetic moment
ρ	charge distribution
ρ	density matrix
σ	Pauli spin matrices
τ	eigentime
ϕ	electrostatic potential
ϕ, π	scalar field, scalar momentum field
ϕ_{ex}, A_{ex}	external potentials
$\widehat{\varphi}$	form factor
ψ	wave function
ψ_{g}	ground state wave function
ω	angular velocity
ω_{c}	cyclotron frequency
ω_{s}	spin precession frequency
ω	free-field dispersion relation
ω_{β}	KMS state
$\widehat{\omega}$	unit vector

Mathematical symbols

$A(q, p)$	operator-valued function
$B(\mathcal{H})$	bounded operators on $\mathcal{H}$
C, $C(\mathbb{R}, \mathbb{R}^d)$	continuous functions on $\mathbb{R}$ with values in $\mathbb{R}^d$
C^∞	infinitely often differentiable functions
C^k	k times differentiable functions
$\mathbb{C}$	complex numbers
$D(\cdot, \cdot)$	Dirichlet form
$D(A)$	domain of operator A
$\mathbb{E}$	expectation
$\mathcal{F}$	Fock space
$\mathcal{H}_\mathrm{f}$	field Hilbert space
$\mathcal{H}_\mathrm{p}$	particle Hilbert space
L^2, $L^2(\mathbb{R}^3, \mathbf{d}^3x)$	Hilbert space of square-integrable functions on $\mathbb{R}^3$
$\mathcal{M}_N$	algebra of $N \times N$ matrices
$\mathbb{N}$	positive integer numbers
$\mathbb{P}$	probability measure
$\mathbb{R}$	real numbers
Ran A	range of operator A
$\mathcal{T}_1(\mathcal{H})$	trace class operators on $\mathcal{H}$
$\mathcal{T}_2(\mathcal{H})$	Hilbert–Schmidt operators on $\mathcal{H}$
$\mathcal{W}_\varepsilon$	Weyl quantization
$\mathbb{Z}$	integer numbers
$d(\cdot, \cdot)$	metric
$\widehat{f}$	Fourier transform of f
ℓ, r	left, right representation
tr	trace
Ω	Fock vacuum
$\sigma(H)$	spectrum of operator H
$\Vert \cdot \Vert$	Hilbert space norm
$\Vert \cdot \Vert_1$	L^1-norm
$\Vert \cdot \Vert_\infty$	L^∞-norm
$\Vert \cdot \Vert_R$	local energy norm
$\langle \cdot, \cdot \rangle_\mathcal{H}$	Hilbert space scalar product
$\langle \cdot \vert \cdot \rangle$	scalar product for Hilbert–Schmidt operators
$: \;:$	normal order, Wick order
$\{\cdot, \cdot\}$	Poisson bracket
$[\cdot, \cdot]$	commutator
$\int \mathbf{d}q_s$	stochastic integration
$\sharp$	Moyal product
∇	nabla operator