Dynamics of Networks if Everyone Strives for Structural Holes

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Published in:
American Journal of Sociology

DOI:
10.1086/590674

Dynamics of Networks if Everyone Strives for Structural Holes[1]

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When entrepreneurs enter structural holes in networks, they can exploit the related benefits. Evidence for these benefits has steadily accumulated. The authors ask whether those who strive for such structural advantages can maintain them if others follow their example. Burt speculates that they cannot, but a formal demonstration of this speculation is lacking. Using a game theoretic model of network formation, the authors characterize the networks that emerge when everyone strives for structural holes. They find that the predominant stable networks distribute benefits evenly, confirming that no one is able to maintain a structural advantage in the long run.

INTRODUCTION

The view that social networks are a form of capital because they can facilitate economic activity is now generally accepted. The structure of an individual's social environment has been shown to matter in a number of ways. Job search through weak ties is more successful (Granovetter [1974] 1995). Nonexcludable trading parties have larger profit margins (Cook and Emerson 1978; Willer 1999). Dense structures and closure in

[1] The order of the authors is alphabetical. We thank the four *AJS* reviewers for extraordinarily useful reviews. We thank Robb Willer, with whom we initially developed the idea for this article. We also thank Ronald Burt, Michael Macy, Werner Raub, Marcel van Assen, Erik Volz, Jeroen Weesie, and members of CREED, Amsterdam, for helpful comments and suggestions. This article is part of the Polarization and Conflict Project (CIT-2-CT-2004-506084) funded by the European Commission–DG Research Sixth Framework Programme. Additional funding was provided by Utrecht University through the High Potentials 2004 subsidy for the research program Dynamics of Cooperation, Networks, and Institutions. Direct correspondence to Vincent Buskens, Department of Sociology, Faculty of Social Sciences, Utrecht University, Heidelberglaan 2, 3584 CS Utrecht, Netherlands. E-mail: v.buskens@uu.nl

0002-9602/2008/11402-0003$10.00

networks facilitate trust (Coleman 1988; Raub and Weesie 1990; Buskens 2002). And ties between otherwise unconnected groups, spanning so-called structural holes, benefit the broker (Burt 1992). This last example will be the focus of this article.

From the realization that relationships have value follows an important implication: it gives rise to incentives to relate to others for personal gain rather than on the basis of liking or social obligation. In modern society, then, people have not only the means for strategic partner choice, in the form of increased mobility and information technology. They also have the ends. There is a rationale for being selective when establishing personal and business relationships. If some networks are more beneficial than others, actors can be expected to modify the less beneficial ones to their advantage (Flap 2003, pp. 12–13). The network becomes a "device to be manipulated consciously for an actor's own ends" (Watts 1999, p. 495). People invest in new friendships with others who can give them valuable information. Traders actively seek out alternative trading partners so as to enhance their bargaining positions. The friendship and trading networks we observe, then, must be the ones to which no one has found it worth making any further alteration.

This places strategic social networking on the agenda as an important explanatory mechanism of changes in the social fabric and cohesion of societies. The transformation from normatively to purposively constructed patterns of *formal* relations that Coleman (1993) spoke of generalizes to *informal* relations. Networking can help explain why traditionally separate social circles are increasingly crosscutting (Blau and Schwartz 1984), why the world is shrinking in terms of social distance (Watts 1999), or why dense clusters of relationships, old bases of societal trust, erode (Putnam 1995).

Assessing the macrostructural consequences of strategic social networking at the microlevel is therefore critical to our understanding of contemporary society, but at the same time is a daunting task. By the mere nonatomic nature of networks, the type of action networking involves is interdependent—one can only broker as long as two brokees remain unconnected—and collective—it takes two to connect. Each choice to connect or disconnect then becomes contingent upon another's previous choice and is a precursor of a following change (see also Willer 2007). An initial network change may thus trigger nontrivial network evolution. Sociologists have only recently begun to develop the methodological tools that such an assessment requires (e.g., Doreian and Stokman 1997; Snijders 2001, 2005). In this article, we contribute to the methodology needed to explore this link between the ever-growing autonomy society is giving its members to choose for themselves with whom to interact and the ever-disintegrating group-based interaction patterns of the past. In doing so, we attempt to further develop theoretical insight into which networks can be expected to emerge when actors purposively choose their relationships. In particular, we introduce a model in which actors strive for structural holes.

When Burt (1992) launched his idea of structural holes, he went beyond structuralism. He did not assume that actors would simply reap the fruits from structural advantages that they happened to have over others. He suggested the possibility that entrepreneurs, just as they can strategically put financial and human resources to work, exploit social resources and turn them into profit: "You enter the structural hole between two players to broker the relationship between them" (Burt 1992, p. 34). Burt even went so far as to argue that social capital *implies* prior strategic networking: "I will treat motivation and opportunity as one and the same . . . a network rich in entrepreneurial opportunity surrounds a player motivated to be entrepreneurial. At the other extreme, a player innocent of entrepreneurial motive lives in a network devoid of entrepreneurial opportunity" (Burt 1992, p. 36). Burt's "structural entrepreneur personality index" (2005, p. 34) is an attempt to quantify this inclination to exploit social resources.

Nevertheless, the agency component in Burt's argument was never as fully developed as the structure component. Burt proposed a precise measure of structural disadvantage, the "constraint" formula (Burt 1992, p. 54), but the network dynamics he sketched in his book are instructions on how to unilaterally reduce one's score on the constraint measure. He thereby remained at the microlevel, showing how an ego network would change with structural holes as the driving force, assuming cooperation and passivity on behalf of all alters. Thus neutralizing the interdependence between actors, he precluded possible cascades of subsequent network adaptations by other actors. Burt (2005, chap. 5) dealt with some of these issues informally. He speculated that as more and more people strategically add and remove ties, less and less structural advantage is obtained in equilibrium (Burt 2005, pp. 230–33). Although his considerations are very insightful, they lack a strict theoretical deduction. For example, he speculated on the emergence of stable networks if network benefits are extended beyond brokerage to include closure, and if these benefits and also the costs of ties are heterogeneous among actors. We think that such speculations are unwarranted. We show that with much simpler assumptions, analysis is already challenging.

We will limit ourselves to formalizing network dynamics with a single type of benefit and with homogeneous actors. Clearly, these are strong, simplifying assumptions. Strategic networking will be more salient or feasible in some settings than in others, and in any particular setting, not everybody will be equally interested in structural holes. Still, our model provides an insightful benchmark that can straightforwardly be modified to accommodate more complex assumptions. In the discussion section, we indicate which assumptions are crucial for our findings and which can be relaxed without changing the substantive results.

In Burt's typical example of a network after entrepreneurial activity, the majority of benefits are held by a single individual. However, in most of Burt's examples, no one else is granted an opportunity to add and delete ties. The environment around the focal actor is held fixed. We consider series of network changes by interdependent actors who together equilibrate toward or around a certain stable end network. It is not obvious what this end network is. Nor is it obvious that if everyone wants to follow the initial entrepreneur's example, his structural advantage automatically disappears. Some economic studies of network dynamics in information and communication settings have demonstrated that it is possible for everyone to have perfectly identical preferences and abilities to strategically alter the network and still equilibrate toward a highly asymmetric network with both winners and losers. Jackson and Wolinsky (1996), Bala and Goyal (2000), and Goyal and Vega-Redondo (2007) all identified as a stable network the "star," a network in which one central actor brokers everyone else. In this article, we show that the star is not stable if everyone tries to minimize his network constraint. We come to this conclusion by pursuing the following aims.

Our first aim is to extend and improve the methodology for answering questions about which networks can be expected to emerge, given that we know how actors benefit from network positions in a certain context. This involves the development of a model of network formation and a comparison of existing and new stability concepts indicating to which networks strategic actors will not make any further changes. In addition, we develop a tool for determining which of the stable networks are more or less likely to emerge. Our second aim is to apply this methodology to answer the question, What networks would emerge if strategic actors, in their choice of social contacts, were exclusively concerned with access and control benefits from brokerage? Would these be dense or sparse, and full or void of social closure? And in these networks, how would benefits from brokerage be distributed? Would a minority broker the rest and claim a majority of benefits, or would everyone share the profits?

Our specification of how actors benefit from brokerage is based on extensive empirical research into the economic advantages of occupying structural holes. Thus, we model network evolution, not employing a stylized utility function, as the aforementioned economic studies have done, but instead assuming a relationship between network structure and profit that has solid empirical backing. Burt (1992) provides us with such an empirically tenable measure of access and control benefits, as we will argue in the next section. In this way, we build bridges between the sociological and economic literatures in this area.

We first review the structural hole argument. Then we introduce a model of structural entrepreneurship, explain the stability concepts we use, and identify several classes of stable networks. Using simulation, we show that networks of one particular class—balanced complete bipartite networks—evolve with a much higher likelihood than any of the others. These structures can be characterized as follows: everybody has a strong network position in terms of structural holes, but no one has a structural advantage over others. Our model thus predicts a *network race* in which it is unlikely that anyone maintains an advantaged position. Thus, in competitive environments in which many others employ *social networking strategies* as well, such as in certain business environments, the intended structural advantages of networking may never accrue despite the effort. In addition to distributing benefits equally, the stable networks we identify are rather dense, and distances between actors are small. Yet despite their density and closeness, the emerging networks are void of closure; many relationships are present, but there are no friendship cliques. Social circles are perfectly crosscutting, and no triadic bases of trust exist.

STRUCTURAL HOLES

Structural holes are "disconnections or nonequivalencies between players" and hence "entrepreneurial opportunities for information access, timing, referrals, and control" (Burt 1992, pp. 1–2). There is a structural hole between two players if there is a potential for beneficial information flow between them. The word *disconnections* in the above definition refers to the absence of a tie or path through which the information can flow.

A network rich in structural holes thus contains many exploitable brokering opportunities: "The structural hole is an opportunity to broker the flow of information between people and to control the form of projects that bring together people from opposite sides of the hole" (Burt 1997, p. 340). The structural entrepreneur recognizes these opportunities and places himself in the hole by initiating ties with both players. Just as the investment banker and the human resource manager generate returns from financial and human capital, so the structural entrepreneur seeks profit in information structure. "When you take the opportunity to be the *tertius*, you are an entrepreneur in the literal sense of the word—a person who generates profit from being between others" (Burt 1992, p. 34). Occupying the hole and making himself essential to the information flow between the two players, the entrepreneur can charge a brokering fee.

Burt introduced a formula for quantifying the benefits from spanning structural holes, the *constraint* measure. Your entrepreneurial opportunities are considered to be constrained if there exists a feasible alternative road along which the information you are intending to broker can travel: "Contact j constrains your entrepreneurial opportunities to the extent that: (a) you've made a large investment of time and energy to reach j, and (b) j is surrounded by few structural holes with which you could negotiate to get a favorable return on the investment" (Burt 1992, p. 54).[2] The constraint measure c_i captures the extent to which this is the case for each contact j of actor i:

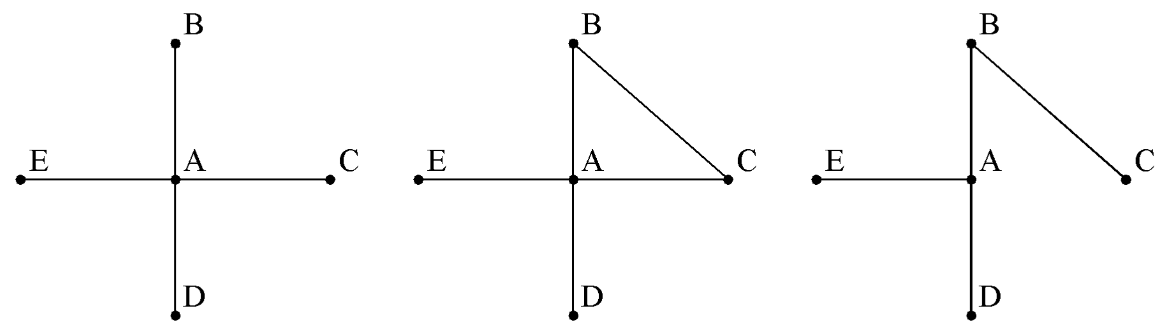
$$c_i \equiv \sum_{j \neq i} \Big(p_{ij} + \sum_{k \neq i, k \neq j} p_{ik} p_{kj} \Big)^2 , \tag{1}$$

where p_{ij} is the proportion of time that i has invested in contact j. Burt assumes that an actor distributes his time equally over his contacts: if i is connected to j, then $p_{ij} = 1/d_i$, where d_i is actor i's degree—that is, the number of ties of i. If i and j are not connected, $p_{ij} = 0$. The constraint measure c_i in equation (1) lies between 0 and 9/8 (see theorem 1 in app. A). The constraint measure is not well behaved for isolates; it takes on a value of 0 for isolates, which implies that actors without ties have the lowest constraint. It is more plausible, though, that it is better to be connected in some way than not to be connected at all, and we think Burt did not intend isolates to be least constrained. Therefore, we additionally assume that $c_i = 2$ for isolates.[3]

The higher the score on the constraint measure c_i, the more structural opportunities are constrained and, as a result, the lower the network benefits. Compare, as an example, actor A's positions in the three networks displayed in figure 1. In the network on the left, actor A is essential for all information flow. His constraint score is $c_A = 4 \times (\frac{1}{4} + 0)^2 = \frac{1}{4}$. In the middle network, B and C can also communicate directly rather than through A. This constrains the relations between A and B as well as

[2] In recent work (e.g., Burt 2005), Burt multiplies his index by 100 and rounds to compare integer values of the index. We will use the original formulation in this article. Of course, both formalizations are equivalent. As an alternative, we have found that our results stay the same if we use a *relative* version of the constraint formula as a measure of benefits, where an actor's network benefits equal the reciprocal of his constraint score divided by the sum of all actors' constraint scores, representing the idea that an entrepreneur wants a "better" position than the others in the network.

[3] The precise value is irrelevant for the results of the model as long as the value is larger than 9/8. The further consequence of this assumption is, as we will see below, that actors always want to connect with isolates and isolates want this as well. This property is in correspondence with the more general property that we will derive, that actors are always willing to connect if the connection does not create any closed triads. Note that this does not imply that they never want to connect if it does create closed triads.



$c_A = 4 \cdot (1/4 + 0)^2 = 1/4$
$c_B = 1 \cdot (1/1 + 0)^2 = 1$
$c_C = 1 \cdot (1/1 + 0)^2 = 1$
$c_D = 1 \cdot (1/1 + 0)^2 = 1$
$c_E = 1 \cdot (1/1 + 0)^2 = 1$

$c_A = 2 \cdot (1/4 + 0)^2 + 2 \cdot (1/4 + 1/4 \cdot 1/2)^2 = 13/32$
$c_B = (1/2 + 1/2 \cdot 1/4)^2 + (1/2 + 1/2 \cdot 1/2)^2 = 61/64$
$c_C = (1/2 + 1/2 \cdot 1/4)^2 + (1/2 + 1/2 \cdot 1/2)^2 = 61/64$
$c_D = 1 \cdot (1/1 + 0)^2 = 1$
$c_E = 1 \cdot (1/1 + 0)^2 = 1$

$c_A = 3 \cdot (1/3 + 0)^2 = 1/3$
$c_B = 2 \cdot (1/2 + 0)^2 = 1/2$
$c_C = 1 \cdot (1/1 + 0)^2 = 1$
$c_D = 1 \cdot (1/1 + 0)^2 = 1$
$c_E = 1 \cdot (1/1 + 0)^2 = 1$

FIG. 1.—The constraint measure: three example networks

between A and C. A's constraint score is now $c_A = 2 \times (\frac{1}{4} + 0)^2 + 2 \times (\frac{1}{4} + \frac{1}{4} \times \frac{1}{2})^2 = 13/32$. A network without one of the two ties with B and C would be better for A. In the network on the right, A's constraint is lower, namely, $c_A = 3 \times (\frac{1}{3} + 0)^2 = \frac{1}{3} < 13/32$. Therefore, actor A is willing to give up his relation with C in the middle network of figure 1 and move to the network on the right. In this article, we search for networks that are stable in the sense that if all actors consider their possible changes in ties as actor A above does, no one wants to change a relationship.

The constraint formula has been found to be negatively related to a wide range of objective indicators of success (see Burt 2000, 2002, and 2005 for extensive reviews). Producer profit margins are larger for firms in buyer-supplier networks (Talmud 1994; Yasuda 1996; Burt et al. 2002). Jobs are more desirable (Bian 1994; Leenders and Gabbay 1999; Lin 1999; Lin, Cook, and Burt 2001). Salaries are higher (Burt 1997, 1998; Podolny and Baron 1997; Burt, Hogarth, and Michaud 2000; Mehra, Kilduff, and Brass 2001; Mizruchi and Sterns 2001). And negative correlations have been found with positive performance evaluations, peer reputations, promotions, and good ideas (Gabbay 1997; Burt 2001, 2004). Given that this evidence indicates a (negative) association between the constraint formula and network benefits, we use constraint as an (inverse) indicator for the utility that actors can extract from a network.

A MODEL OF STRUCTURAL ENTREPRENEURSHIP

What the evidence above does *not* tell us is whether investments in brokerage relations pay off. Burt has suggested that, in a passive environment, a structural entrepreneur can thrive by removing ties that are costly and adding ties that are beneficial, eventually obtaining the returns on these investments. In an active environment in which others similarly strategize on partner selection, current network manipulations that lead to an improved network position in the short run may trigger subsequent changes by others that could ultimately make an entrepreneur worse off. This temporal interdependence in decision making is not trivial, and its examination requires an explicit model. Recently, some models have been proposed to examine such dynamics. A number of reviews of models of network dynamics already exist (e.g., Weesie and Flap 1990; Doreian and Stokman 1997; Stokman and Doreian 2001; Breiger, Carley, and Pattison 2003; these all provide extensive overviews in sociology). But this line of research has also been receiving more and more attention recently within economics and physics (see Dutta and Jackson 2003; Jackson 2004; Newman, Barabási, and Watts 2006; Goyal 2007).

We use a model that assumes that actors optimize a utility function through choices in their ego networks (for a similar approach, see also Robins, Pattison, and Woolcock [2005]). We chose this modeling approach because Burt's structural entrepreneurs are precisely such optimizing actors with brokerage benefits as network-derived utility. One such model has been independently proposed by Snijders (1996, 2001) in sociology and by Watts (2001) in economics. We refer to this as the SW model. A related model was introduced in sociology by Gould (2002) and in economics by Myerson (1991, p. 448). To ensure the continuity of this text, we describe the Gould-Myerson (GM) model in appendix B, in which we also show how it can be unified with the SW model.

Let us first introduce the necessary notation. The number of actors is indicated by $n \geq 2$, $N = \{1, 2, \ldots, n\}$ is the set of actors, and ij indicates a tie between actors i and j. In our model, actors cannot have ties with themselves (ties are nonreflexive); if i has a relation with j, then j also has a relation with i (ties are undirected); and ties are either present or absent (ties are unvalued). Let g^N denote the complete network of all nonreflexive, undirected, and unvalued connections ij; $g + ij$ denotes network g with the tie ij added to it, and $g - ij$ denotes network g with tie ij removed; and $u_i(g)$ is the utility of the network g to actor i. Specifying the model for the special case of Burt's constraint-based utility, actor i's utility is a decreasing function of the constraint measure c_i as indicated above—for example, $u_i(g) = -c_i(g)$.[4]

The SW model asks what an actor would change, given the current status of the network, if he were offered that possibility. An actor is allowed to delete a tie or add a tie with permission from the new contact—as long as it does not make this other actor worse off. Stability is reached if no actor can profitably delete a tie or add an acceptable tie. Jackson and Wolinsky (1996) call this pairwise stability:

Definition 1.—*A network* $g \subseteq g^N$ *is* pairwise stable *if both* (a) *for all* $ij \in g$, $u_i(g) \geq u_i(g - ij) \wedge u_j(g) \geq u_j(g - ij)$ *and* (b) *for all* $ij \notin g$, $u_i(g + ij) > u_i(g) \Rightarrow u_j(g + ij) < u_j(g)$.

Condition (*a*) states that no actor wants to sever a tie, and condition (*b*) states that no pair of actors wishes to add a tie. On the one hand, pairwise stability has the advantage that pairs of actors can add ties if they both want to have a tie. On the other hand, the disadvantage of

[4] We make the strong assumption here that reducing network constraint is the only utility argument for all actors and that the utility derived from a lack of constraint is the same for all actors. We thereby neglect other utility arguments related to, e.g., closure (Burt 2005, chap. 3) and indirect brokerage (Burt 2007). Neither do we investigate the effects of heterogeneity among actors (see Burt 2005, chap. 1.4). We will consider these issues in more detail in the discussion section. Note that utility should be interpreted here as the extent to which an actor is expected to be able to extract benefits from the network. A network position is not considered beneficial in itself.

pairwise stability is that it does not consider the simultaneous removal of multiple ties. Networks can be pairwise stable despite profitable deviations that involve multiple ties, even though such deviations arguably make a network less stable.

Gilles et al. (2006) introduce the stronger concept of *strong pairwise stability* for the SW model. Strongly pairwise stable networks are networks in which (1) no actor can become better off by deleting *any subset of his ties* and (2) no pair of actors wants to add a tie between them.

However, there is an asymmetry here: an actor can delete any number of ties but can add only one tie—and only if that tie does not make the other actor worse off. Even more problematic is the assumption that actors do not simultaneously add and delete ties. For example, strong pairwise stability implies that an actor does not contemplate improving his network position by replacing one contact with another. This, however, seems a rather straightforward change in a network.

To resolve the problems of asymmetry and nonsimultaneity in the deletion and addition of ties, we introduce *unilateral stability*. We first define *unilateral obtainability*:

Definition 2.—*A network* $g' \subseteq g^N$ *is* unilaterally obtainable *from* g *by* i *through* $S \subseteq N\backslash\{i\}$ *if* (*a*) *all ties that are in* g' *but were not in* g *involve actor* i *and an actor in* S, *and* (*b*) *all ties that are not in* g' *but were in* g *involve actor* i.

In words, one network is unilaterally obtainable from another by a proposing actor i and through a subgroup S if each tie that is added or deleted involves actor i and if each tie that is added also involves a member of S. Now we can define *unilateral stability*.

Definition 3.—*A network* $g \subseteq g^N$ *is* unilaterally stable *if for all* i, $S \subseteq N\backslash\{i\}$, *and* $g' \subseteq g^N$ *unilaterally obtainable from* g *by* i *through* S, $u_i(g') > u_i(g) \Rightarrow u_j(g') < u_j(g)$ *for some* $j \in S$.

In words, a network is called unilaterally stable if no actor i can change the ties that he is involved in himself such that two conditions are fulfilled: (1) i is strictly better off, and (2) none of the actors in S to whom actor i proposes a new tie is worse off than in the original network. The definitions above imply that as we move from pairwise stability to strong pairwise stability and on to unilateral stability, we must render additional changes in a network undesirable before considering it stable. It follows that all unilaterally stable networks are strongly pairwise stable, and all strongly pairwise stable networks are pairwise stable.

NETWORKS OF STRUCTURAL ENTREPRENEURS

We now identify networks to which no entrepreneur can profitably make any further change. Our first result states that two actors will always connect if they have no shared contacts.

THEOREM 2.—*Adding a tie without creating closed triads is always beneficial for both actors involved in the new tie.*

For purposes of legibility, we moved all proofs to appendix A.

Theorem 2 establishes the unconditional benefits from brokerage. If an actor adds a tie without creating a closed triad, then this actor will be on the shortest path between the new contact and all the contacts the actor already had. And vice versa: the new partner comes to mediate the information the focal actor receives and passes this along to his old contacts. Scores on the constraint measures of both actors drop. The added value of an additional tie decreases as more ties are added, because an actor has to distribute his time among more neighbors and can thus broker less information per pair of neighbors, but this marginal utility never becomes zero. As we will see below, the reverse of theorem 2 is not true. Sometimes actors want to add ties that close triads, and networks with closed triads can even be pairwise stable.

COROLLARY 1.—*The shortest path between any pair of actors in a pairwise stable network has length less than or equal to 2.*

The shortest path between two actors can be of length 2 or less only if both actors are directly connected or can reach each other through a broker. If neither condition obtains for some pair of actors, then these actors can add a tie without creating a closed triad, which is profitable, by theorem 2. Note that since pairwise stability is a weaker stability concept than unilateral stability, corollary 1 also holds for unilaterally stable networks.

COROLLARY 2.—*A network of disconnected parts cannot be pairwise stable.*

A network of disconnected parts contains many brokerage opportunities. Every entrepreneur wants to add a tie to someone in another part, because that tie will never create a closed triad. Such networks can therefore not be pairwise stable (nor, consequently, strongly pairwise stable or unilaterally stable, either). The following set of definitions describes a family of networks that includes important stable networks:

DEFINITION 4.—

(a) *An m-*partite network *is a network in which the actors can be divided into m groups such that there are no ties within these groups.*

(b) *The* complete m-partite network $K_{n_1,n_2,...,n_m}$ *is the m-partite network in which all the possible ties between the actors in the m groups, which have sizes* $n_1, n_2, \ldots, n_m$ *(see Wasserman and Faust* 1994, *p. 120), are present. (The complete tripartite network $K_{2,2,2}$, or octahedron, is shown on the right-hand side of fig. 2.)*

(c) *A* balanced m-partite network *is an m-partite network such that the difference between the number of actors in the largest group and the number of actors in the smallest group is at most 1—that is, groups are as equal as possible given the number of actors in the network.*

(d) *If $m = 2$, m-partite networks are called* bipartite networks. *(The balanced complete bipartite network $K_{3,3}$ is shown in fig. 2.)*

(e) *If $m > 2$, m-partite networks are called* multipartite networks.

(f) *The complete bipartite network $K_{1,k}$ is also called the k-*star.

Now we can formulate the following corollary of theorem 2:

COROLLARY 3.—*Pairwise stable networks that are bipartite networks are necessarily complete bipartite networks (otherwise, some actors are at a distance greater than 2 and one can add ties without creating closed triads).*

Corollary 3 says that in order for a bipartite network to be stable, it will have to be complete. Otherwise, there would be a brokerage opportunity. As the reader can verify, eliminating any tie in the first network in figure 1 would make both actors involved in that tie worse off: actor A would have constraint 1/3 instead of 1/4, and the other actor would become an isolate. The next result implies that the first network in figure 1 is *not* strongly pairwise stable, despite its completeness.

THEOREM 3.—*A complete bipartite network of size n is strongly pairwise stable, unless it is a k-star with $k > 3$. The k-star with $k > 3$ is also not pairwise stable.*

Theorem 3 identifies an important class of strongly pairwise stable networks, and thus also of pairwise stable networks. As soon as both groups in a bipartite network consist of at least two actors and the network is complete, it is strongly pairwise stable. The reasoning behind this result is as follows. No pair of actors wants to add a tie, because it would cause many closed triads to form. Moreover, no removal of any number of ties is beneficial, as we know from theorem 2. No bipartite network contains any closed triads, so no subnetwork of a bipartite network can contain any closed triads either. As a result, no actor wishes to delete any number of ties, because there are no closed triads.

If, by contrast, one of the groups consists of only one actor, only networks with at most three peripheral actors can be stable. The first network in figure 1 is a 4-star, and by theorem 3 it is not pairwise stable. The reason is that any two peripheral actors wish to connect: B and C lower their constraint from 1 to 61/64 if they connect, even though this tie creates a closed triad. In some cases, when the broker has many ties and can therefore spend little time passing information, it is better to establish a direct connection. This is what happens in this example network. Actors

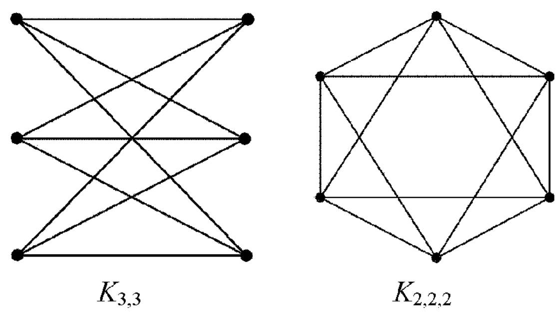


FIG. 2.—Examples of a balanced complete bipartite network and a multipartite network

B and C connect because A spends too much time with D and E. The middle network in figure 1 evolves. Given the argument for the instability of larger stars, it is surprising that in other unbalanced complete bipartite networks, actors in the larger groups never have an incentive to connect. If one broker can be too busy, then why is it that two cannot? This is one of a number of idiosyncrasies of Burt's constraint measure that we discuss in the last section.

Complete bipartite networks have another nice property—namely, that they are efficient in the Pareto sense.

DEFINITION 5.—*A network is* Pareto efficient *if there is no other network in which no actor is worse off and at least one actor is better off.*

If our actors could cooperate and enforce agreements—which we have assumed they cannot—then they would not be able to leave a Pareto-efficient network. There would always be some actor vetoing a transition. The following theorem tells us that the class of stable networks identified in theorem 2 is Pareto efficient:

THEOREM 4.—*Complete bipartite networks are Pareto efficient.*

We could not prove the reverse of theorem 4—namely, that complete bipartite networks are the only efficient networks. We verified that there do not exist other Pareto-efficient networks that involve eight or fewer actors.

We now specify a necessary and sufficient condition under which complete bipartite networks are not only strongly pairwise stable but also unilaterally stable.

THEOREM 5.—*A complete bipartite network is unilaterally stable if and only if it is balanced.*

Theorem 5 can be intuitively grasped by considering a particular type of network change. In any *unbalanced* complete bipartite network, a member of the larger group can give up all relations to the other group and initiate new relations to all actors in his own group. In this way, this actor leaves the large group to become a member of the smaller group. This actor strictly increases his number of ties, while keeping the number of closed triads at zero. This network change is therefore profitable to that actor, as we know from theorem 2. Moreover, the new contacts—the remaining members of the larger group—are happy to connect, because they also each gain a contact without the creation of closed triads, and so their utility increases as well. The members of the smaller group are worse off, but they do not have a say in this network change because no ties need to be added to the members of the smaller group. In *balanced* complete bipartite networks, by contrast, such a network change does not result in an increase in the number of ties and is therefore not a utility-increasing change.

Theorems 4 and 5 together identify a class of networks that are efficient and stable under the strictest stability condition. In a balanced complete bipartite network, no actor can profit from deleting and adding permitted ties to other actors in any combination without making at least one of the new contacts worse off. These stable networks contain no closed triads. This suggests the intuitive conclusion that the striving for structural holes by all actors should eliminate closed triads from the network. Actors terminate all contacts they are already indirectly connected to and initiate contacts they do not indirectly have. One would intuitively expect that if everyone followed this example, no closed triads would remain. This intuition turns out to be wrong. We now identify a second class of pairwise stable networks that are not unilaterally stable. Surprisingly, this class of stable networks contains many closed triads.

Theorem 6.—*All complete multipartite networks are pairwise stable if the groups are of equal size and contain more than one actor.*

One example of a network that meets the requirement of theorem 6 is the six-actor complete tripartite network $K_{2,2,2}$ (see fig. 2). This network is neither unilaterally stable nor strongly pairwise stable. If an actor deletes two ties, then the actor's constraint drops from 9/16 to 1/2. The network is nevertheless pairwise stable because if an actor is allowed to delete only a single tie, he prefers to keep it. His constraint then increases to 43/72. The limited gain actors get from removing one tie is mainly due to the fact that removing one tie will not remove all closed triads with any of their neighbors. As a result, the gain from removing some closed triads is relatively small. However, by removing multiple ties, actors can gain some neighbors with whom they do not have any closed triads. Because the deletion of two ties makes an actor better off, this is also an indication that pairwise stability might be a bit too weak as a stability concept. Moreover, the $K_{2,2,2}$ is extremely inefficient. All actors would fare better in the $K_{3,3}$, which gives each actor constraint 1/3. More generally, this class of complete multipartite networks consists of Pareto-inefficient pairwise stable networks.

SIMULATION

We have identified two main classes of pairwise stable networks to which structural entrepreneurship might give rise. The first class consists of complete bipartite networks in which there are no closed triads, and, if Burt's constraint formula correctly quantifies brokerage benefits, they are Pareto efficient. No alternative network makes one actor better off without making another worse off. We have also shown that a subset of these networks satisfy stronger stability requirements. Second, in the multipartite networks of theorem 6, all pairs are brokered, and in addition, the majority of pairs are directly connected. The latter networks contain many closed triads and are Pareto inefficient. Every actor would fare better in the balanced complete bipartite network of the same size. Yet these multipartite networks are pairwise stable. No entrepreneur can profitably delete a single tie, and no pair of actors can profitably add a single tie.

On the basis of our analysis, we might expect either type of network—or yet another—to arise in a world in which entrepreneurs pursue access and control benefits by changing ties one by one. Simulating such a world enables us to investigate which of the two types of networks is more likely to emerge. Such simulations also help us identify potential pairwise stable structures that the two classes of networks mentioned above do not cover.

The simulation we built executes the following steps:

1. Start from some network.
2. Randomly order all actors in the network. Pick the first actor in this ordering and continue with step 3.
3. The chosen actor considers, in a random order, for each absent or existing tie whether he can or cannot decrease his constraint by changing the status of the tie. If he cannot decrease his constraint, he will stick to the present situation. Otherwise, he decreases his constraint either by adding the tie because he wants to have it *and* the other actor involved also wants to have it, or by removing the tie because he prefers not to have it. If the actor does not change this tie, step 3 is repeated until either the actor finds a tie that he can profitably change or he has considered all his ties without seeing any opportunity to decrease his constraint. After a tie change, the simulation returns to step 2. If the actor has considered all his ties without making a change, the process proceeds to step 4.
4. Pick the next actor in the ordering created in step 2 and return to step 3. If all actors have had their chance to make a change and no

one has made a change, the process stops.

Networks that are formed after this simulation process stops are necessarily pairwise stable; otherwise, they could not have passed the stopping rule in step 4.

The analysis of the simulation consists of two parts. First, we investigate to what extent the analytic results above exhaust the pairwise stable networks that exist. As we will see, some other stable networks exist. Second, we study the likelihood that given network size, a particular pairwise stable network emerges.

We ran simulations starting with each of the 13,597 nonisomorphic networks of sizes 2–8. This allowed us to identify all pairwise stable networks for these network sizes, since simulations starting from pairwise stable networks end instantly. We drew a sample of all networks stratified on density for network sizes 9–25. We decreased the number of networks per network density for larger network sizes, in order to have comparable numbers of networks per network size. In this way, we attempt to minimize bias toward networks of a particular density while keeping the set feasibly small (for a complete overview of the sampling procedure, see Buskens and Snijders [2008]). For each emerging pairwise stable network, we checked whether it was strongly pairwise stable or unilaterally stable as well.

Table 1 shows the numbers of stable networks by network size and stability concept. Figure 3 displays the pentagon, the wheel, and other pairwise stable networks that do not belong to one of the two general classes derived from the theorems analytically. For $n = 2$, we have the connected pair (or 1-star) as the only pairwise stable network. For $n = 3$, the 2-star is the only pairwise stable network. For $n = 4$, the $K_{2,2}$ and the 3-star are the pairwise stable networks. For $n = 5$, there are two pairwise stable networks, the pentagon (see fig. 3) and the $K_{2,3}$, which are both unilaterally stable. For $n = 6$, there are four pairwise stable networks: the $K_{2,4}$ and the $K_{3,3}$, as well as the bag (see fig. 3) and the $K_{2,2,2}$. The $K_{2,2,2}$ is not strongly pairwise stable, and only the $K_{3,3}$ is unilaterally stable. For $n = 7$, there are three pairwise stable networks: the $K_{2,5}$, the $K_{3,4}$, and the $PS_{2^1,3^6}$ (see fig. 3; the name indicates the degree-distribution—i.e., there is one actor with two ties and six with three ties). The $K_{3,4}$ is also unilaterally stable. For $n = 8$, there are 10 pairwise stable networks: the $K_{2,6}$, the $K_{3,5}$, the $K_{4,4}$, the wheel (see fig. 3), the $K_{2,2,2,2}$, and the five remaining networks in figure 3 that have not yet been mentioned. The three densest networks, including the $K_{2,2,2,2}$, are not strongly pairwise stable. The wheel is the second unilaterally stable network for $n = 8$, in addition to the $K_{4,4}$. It is a regular structure in which everyone has three ties and occupies a regularly equivalent (Wasserman and Faust 1994, pp. 473–74) position with all the others.

TABLE 1
NUMBER OF NETWORKS (FOUND) FOR VARIOUS STABILITY CRITERIA

n	Nonisomorphic	Connected	Pairwise Stable	Strongly Pairwise Stable	Unilaterally Stable
2	2	1	1	1	1
3	4	2	1	1	1
4	11	6	2	2	1
5	34	21	2	2	2
6	156	112	4	3	1
7	1,044	853	3	3	1
8	12,346	11,117	10	7	2
9	274,668	261,080	9	7	1
10 ...	12.01×10^{6}	11.72×10^{6}	14	9	2
11 ...	10.19×10^{8}	10.07×10^{8}	15	10	1
12 ...	16.51×10^{10}	16.41×10^{10}	27	12	1
13 ...	50.50×10^{12}	50.34×10^{12}	14	7	1
14 ...	29.05×10^{15}	29.00×10^{15}	20	10	1
15 ...	31.43×10^{18}	31.40×10^{18}	26	13	2
16 ...	64.00×10^{21}	63.97×10^{21}	28	16	2
17 ...	24.59×10^{25}	24.59×10^{25}	25	14	1
18 ...	17.88×10^{29}	17.87×10^{29}	33	21	1
19 ...	24.64×10^{33}	24.64×10^{33}	35	18	1
20 ...	64.55×10^{37}	64.55×10^{37}	40	25	2
21 ...	32.22×10^{42}	32.22×10^{42}	43	24	1
22 ...	30.71×10^{47}	30.71×10^{47}	48	26	1
23 ...	55.99×10^{52}	55.99×10^{52}	58	31	1
24 ...	19.57×10^{58}	19.57×10^{58}	58	28	1
25 ...	13.13×10^{64}	13.13×10^{64}	68	31	2

NOTE.—For $n \leq 10$, we checked all possible structures; for $n > 10$, we report the networks as found in our simulations.

In addition, we checked which of the more than 12 million nonisomorphic networks of sizes 9 and 10 fulfilled a specific stability condition. In this way, we found nine pairwise stable networks for $n = 9$ and 14 pairwise stable networks for $n = 10$. The unilaterally stable structures for $n = 10$ are the $K_{5,5}$ and another network in which every actor has four ties. Finally, for sizes 11–25, we checked whether the networks that resulted from the simulations were strongly pairwise stable or unilaterally stable as well. The results are also summarized in table 1. For network sizes larger than 10, it is not guaranteed that every stable network has been found, because not all starting networks were considered.[5] In fact, we know from the analytical results that some networks that we did not find in the simulations are nevertheless pairwise stable. Still, these results

[5] Checking all structures for $n = 10$ took about five days with our software and computers, which implies that for $n = 11$ it would take about 500 days.

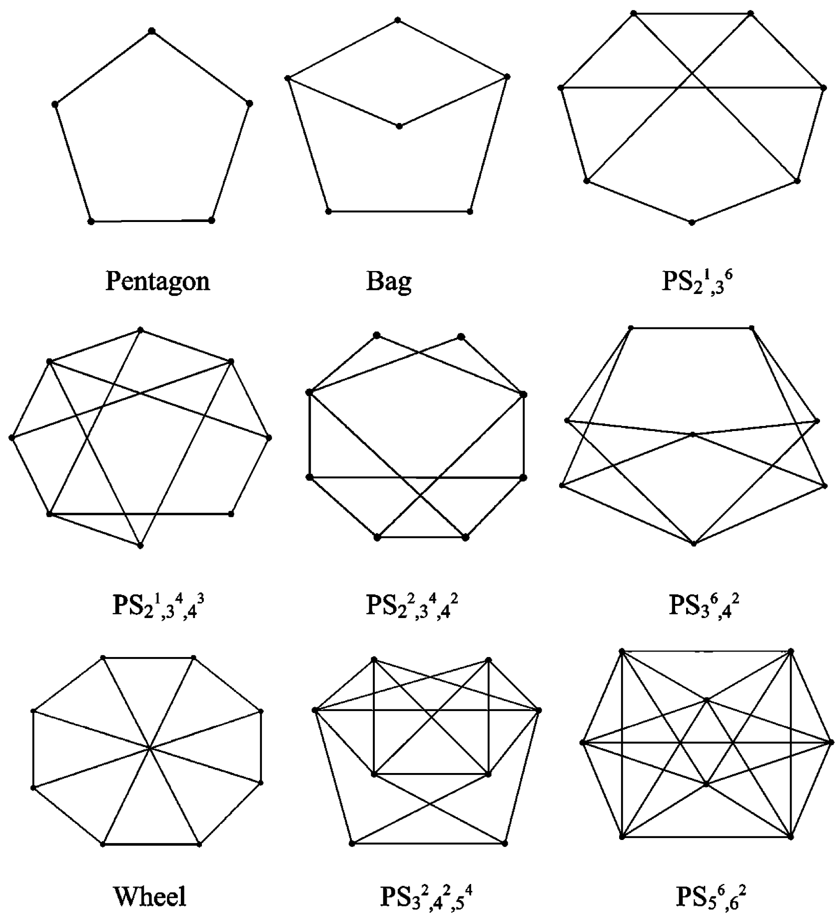


FIG 3.—Pairwise stable networks not belonging to the class of complete bipartite networks nor the class of multipartite networks.

show that the number of pairwise stable networks per network size is very small and increases only slowly with network size. There is no network size for which we identified more than two unilaterally stable networks, which suggests that the relative number of unilaterally stable networks increases even less with network size than the number of pairwise stable networks.

The additional unilaterally stable networks that we found fall into two classes that the simulation enabled us to discover. The first of these classes comprises networks with a number of actors that is a multiple of five—say, $5m$—and that are generalizations of the pentagon. The actors are divided into five equally sized groups, and the groups are organized in a pentagon. There are no ties within these groups, but all actors are connected with all actors in the two neighboring groups around the pentagon. The second network in figure 4 is the example of a generalized pentagon with 10 actors. In these networks, every actor has $2m$ ties, which makes them inefficient. By comparison, in the balanced complete bipartite network of the same size (the first network in fig. 4), which is also void of closed triads, an actor has approximately $2.5m$ ties and therefore a lower score on the constraint measure. The second additional class of unilaterally stable networks are networks with a number of actors that is a multiple of eight, say, $8m$ actors. The actors are divided into eight groups of size m, and these groups are ordered along a circle. All actors are then connected to all actors in the two neighboring groups as well as to all actors in the group right across the circle. There are no ties within the groups. The third network in figure 4 is the example of such a network with 16 actors. These networks are generalizations of the wheel in figure 3. Each actor has $3m$ ties in these networks, which is clearly inefficient in comparison with the balanced complete bipartite network, in which each actor has $4m$ ties. We found the cases $m = 1$ and $m = 2$ as results of simulations on which we report below, but not the 24-actor network with $m = 3$.[6]

Now we turn to the analyses about the *likelihood* that a certain stable network emerges in the simulation. We introduced noise to investigate the extent to which our results depend on whether actors sometimes make mistakes in their decisions.[7] We ran more noise levels and more replica-

[6] The formal proof that these two classes of networks are unilaterally stable is even more laborious than the proof of theorem 4, because more different situations have to be distinguished. The proof is available from the authors, but was not added to app. A because it does not provide substantially new insights. We thank Jurjen Kamphorst for assistance in completing this proof.

[7] Step 3 in the simulation is a bit more complicated in the case of noise. For each absent or existing tie an actor considers, there are now two possibilities: (*a*) The actor makes a mistake with probability $1 - (1 - noise)^{1/(n-1)}$. This implies that he increases his constraint either by adding the tie while he does not want to have it, by removing the tie while he would like to keep it, or by staying in the situation in which he already is while he would be better off otherwise. Mistaken additions do not require consent. If a tie is proposed by mistake, it is always accepted. After any mistake, the simulation returns to step 2 of the simulation process. (*b*) With probability $(1 - noise)^{1/(n-1)}$, the actor does not make a mistake. In this case, if he cannot decrease his constraint by changing the status of the tie, he will stick to the present situation. Otherwise, he decreases his constraint either by adding the tie because he wants to have it *and* the other actor involved also wants to have it, or by removing the tie because he prefers not to have it. If the actor does not change this tie, step 4 is repeated until either the actor finds a tie that he can profitably change, or he makes a mistake, or he has considered all his ties without seeing any possibility to decrease his constraint. As soon as the actor changes a tie or makes a mistake, the process returns to step 2. If the actor has considered all his ties without making a change or a mistake, the process

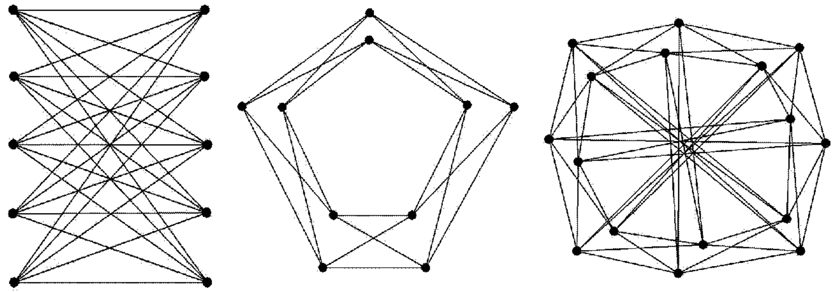


FIG. 4.—Representatives of three classes of unilaterally stable networks: balanced complete bipartite network, generalized pentagon, and generalized wheel.

tions of the same starting network and noise level for small networks to obtain more reliable estimates of the likelihoods to converge to a specific structure. To maintain feasibility, we reduced the number of repetitions as well as the noise levels for larger networks. One can see from the results that variations with noise are smaller for larger networks. The complete overview is provided in table 2. Convergence to pairwise stability always occurs, and it does so reasonably fast, although time to convergence increases quickly with noise. For $n = 25$, the maximum number of iterations to reach a pairwise stable network is 347 without noise, but the iterations exceed 3,000 for $n = 16$ and a noise level of 0.2.[8]

Examining the entire range of n from size 2 through 25 in table 3, we can make several important observations. The number of pairwise stable networks increases as n increases, although not entirely monotonically. For any size, the number of pairwise stable networks is small compared to the total number of networks of that size. The balanced complete bipartite network is by far the most likely to emerge from the simulations. The less equal the group sizes of a complete bipartite network are, the less likely it emerges in the simulation. Table 3 shows the proportions of simulations from which the most equal and the second most equal complete bipartite networks emerge as pairwise stable networks. These two networks cover more than 90% of the resulting networks for $n > 9$ without

proceeds to step 4. We chose the probability for a mistake such that an actor does *not* make a mistake with probability $[(1 - noise)^{1/(n-1)}]^{(n-1)} = 1 - noise$ after considering all his ties. Thus, he makes a mistake with a probability *noise*, which we will call the noise level. The noise level is varied from 0 (no mistakes) to 0.3, in steps of 0.1.

[8] We did not extend the analyses to larger networks for two reasons. First, the patterns of the results are clear and change only gradually while network size is increasing. Therefore, increasing size to networks with 30 or even 40 actors would not lead to substantially new insights. Second, simulation time increases exponentially with network size, which makes running a considerable number of starting networks with 50 or more actors infeasible.

TABLE 2
Simulation Design

n	Repetitions[a]	Noise Levels
2–3	4	0, 0.1, 0.2, and 0.3
4–5	250	0, 0.1, 0.2, and 0.3
6	25	0, 0.1, 0.2, and 0.3
7–8	4	0, 0.1, 0.2, and 0.3
9–16	2	0, 0.1, and 0.2
17–25	2	0 and 0.1

[a] Per network and per noise level.

noise and for all n except $n = 5$ and $n = 7$ if there is enough noise. If n is odd, the balanced complete bipartite network alone even accounts for over 80% of the resulting pairwise stable networks, except for $n = 7$. There are no other pairwise stable networks that obtain in a large proportion of simulations (especially with noise) except for the pentagon (19%) and the $\mathrm{PS}_{2}{}^{1}{}_{,3}{}^{6}$ for $n = 7$ (32%). For $n \geq 8$, no other network occurs in more than 4% of the simulations, although the complete bipartite networks that are two steps from balanced gain some territory for larger even-sized networks. The other unilaterally stable networks that are not bipartite do not emerge in substantially larger percentages than other pairwise stable networks (except for the pentagon). The wheel, for example, occurs in only 1% of the simulations for $n = 8$. Complete multipartite networks are obtained in only a negligible number of cases. It turns out that adding noise to the dynamical process increases the likelihood that a network converges to a complete bipartite network, mostly to a balanced complete bipartite network.

Additional analyses show that it is very unlikely that our main results depend on the set of starting networks. Since we used all possible structures for $n < 9$, we reweighed our results by counting every network with the number of isomorphic structures that exist for the network.[9] In this way, we obtain statistics that resemble statistics for starting from a random network. It turns out that table 3 would hardly change despite such a rather drastic reweighing of cases. In addition, the correlation that does exist between the density of the starting networks and the density of the resulting networks is small for simulations without noise and completely disappears when noise is added.

Although only a bit more than half of the pairwise stable networks we discovered in the simulations are also strongly pairwise stable, it turns out that virtually all simulations (98.5% without noise and up to 99.8%

[9] Numbers of isomorphisms were determined using Nauty 2.2 (see McKay 1990).

TABLE 3

SIMULATION RESULTS

n	No. Starting Networks	Proportion Balanced Complete Bipartite ($K_{\lfloor n/2 \rfloor \lceil n/2 \rceil}$)*				Proportion $K_{\lfloor (n-2)/2 \rfloor \lceil (n+2)/2 \rceil}$*				Occurence of Other Networks at Highest Noise Level (Proportion)
		0	0.1	0.2	0.3	0	0.1	0.2	0.3	
2 ……	2	1	1	1	1	. . .	. . .	. . .	. . .	no other
3 ……	4	1	1	1	1	. . .	. . .	. . .	. . .	no other
4 ……	11	.79	.83	.85	.87	.21	.17	.15	.13	no other
5 ……	34	.80	.82	.82	.80	. . .	. . .	. . .	. . .	pentagon (.20)
6 ……	156	.70	.76	.83	.84	.13	.10	.07	.06	bag (.09)
7 ……	1,044	.52	.62	.68	.68	.02	.01	.00	.00	$PS_{2,3}^{1,6}$ (.32)
8 ……	12,346	.61	.70	.80	.86	.12	.12	.10	.07	$PS_{2,3,4}^{1,4,3}$, $PS_{3,4}^{6,2}$ (.03)
9 ……	9,292	.86	.92	.96	. . .	.01	.01	.01	.01	$PS_{3,4}^{2,7}$ (.02)
10 ….	10,070	.68	.72	.73	. . .	.24	.25	.26	. . .	none > .01
11 ….	10,898	.91	.95	.97	. . .	.03	.03	.02	. . .	none > .01
12 ….	10,930	.61	.64	.70	. . .	.33	.33	.30	. . .	none > .01
13 ….	5,078	.88	.92	.96	. . .	.07	.06	.04	. . .	none > .01
14 ….	5,700	.57	.61	.70	. . .	.35	.35	.30	. . .	none > .01
15 ….	6,358	.86	.90	.94	. . .	.07	.07	.05	. . .	none > .01
16 ….	7,062	.58	.61	.63	. . .	.35	.35	.36	. . .	none > .01
17 ….	2,346	.86	.90	. . .	. . .	.09	.07	. . .	. . .	none > .01
18 ….	2,666	.55	.58	. . .	. . .	.39	.38	. . .	. . .	$K_{7,11}$ (.01)
19 ….	3,006	.85	.88	. . .	. . .	.10	.09	. . .	. . .	none > .01
20 ….	3,366	.53	.55	. . .	. . .	.42	.42	. . .	. . .	$K_{8,12}$ (.01)
21 ….	3,746	.84	.86	. . .	. . .	.13	.12	. . .	. . .	none > .01
22 ….	4,146	.52	.54	. . .	. . .	.41	.42	. . .	. . .	$K_{9,13}$ (.02)
23 ….	4,566	.82	.83	. . .	. . .	.14	.15	. . .	. . .	none > .01
24 ….	5,006	.50	.50	. . .	. . .	.43	.44	. . .	. . .	$K_{10,14}$ (.04)
25 ….	5,466	.80	.81	. . .	. . .	.16	.16	. . .	. . .	none > .01

* Broken down by noise level.

with noise = 0.3) end in a strongly pairwise stable network. The dominant stable network is the balanced complete bipartite network—an efficient and egalitarian network. This result is robust throughout all analyzed network sizes and all noise levels.

DISCUSSION

We have attempted to illuminate the relationship between, on the one hand, the increased autonomy actors have in the selection of interaction partners in modern society and, on the other hand, contemporary changes in societal cohesion. We have done so by characterizing the networks that a simple model of strategic networking produces. We chose actors in this model to be of an ideal-type. They are "structural entrepreneurs" in the way Burt (1992) intended: they optimize relationships in terms of brokerage opportunities, initiate relationships with others who are otherwise unconnected, and resolve relationships if they are not cost effective in terms of access and control benefits. More specifically, we have assumed that everyone tries to minimize his network constraint, Burt's measure for brokerage. We then answered the question, What networks will evolve?

The answer we found is that most of the time, balanced complete bipartite networks evolve. These networks consist of two groups of similar size with all intergroup ties and no intragroup ties present. Such networks meet the unilateral stability criterion, most simulations generated such networks, and this result was robust across noise levels and network sizes. The balanced complete bipartite network strongly contrasts with the outcome of strategic networking activity, where one actor brokers multiple dense, otherwise separated groups. Moreover, it contrasts with some economic models of network dynamics in information and communication settings that identify stars as the stable networks. *It confirms Burt's (2005) speculation that when the monopoly on structural entrepreneurship is lifted, structural advantages most likely disappear.*

The difference between our predicted networks and more star-like networks is considerable not only in terms of structure, but also in the distribution of benefits among the entrepreneurs. Burt's single-broker structure and the star are both winner-take-all networks. Balanced complete bipartite networks, by contrast, are egalitarian. They benefit each entrepreneur equally. The reason is that everyone is as little a broker as anyone else. Even though each entrepreneur attempts to occupy a brokering position, two-step information flow between any two people travels through at least $\frac{1}{2}(n - 1)$ third parties in balanced complete bipartite networks. Thus, betweenness centrality (Freeman 1979; Wasserman and

Faust 1994, pp. 189–91) is not particularly high for any single actor. Although unbalanced complete bipartite networks emerge with a smaller likelihood than the balanced ones, it is important to realize that, in these strongly pairwise stable networks, the actors in the smaller group do have a comparative advantage. The smaller the small group is, the larger this advantage.

As for global network properties, we have proven that all stable networks necessarily have a maximal path length of 2. It should be noted that had we introduced some stricter form of budget or time constraint in the model, networks would have been less dense and distances between actors would have been larger. Notwithstanding the density of the stable networks, our simulations overwhelmingly produced networks void of social closure. Strategic social networking can thus explain decreased closure in certain economic spheres of society where networking is prevalent (Putnam 1995). And last, clustering is minimal in all of the stable networks we found. The mechanism of structural entrepreneurship thus eliminates any signature of social groups. Socioeconomic circles of the type considered here become maximally crosscutting (Blau and Schwartz 1984).

We identified three other classes of stable networks, namely, symmetric multipartite networks, generalizations of the pentagon, and generalizations of the wheel. These classes are all egalitarian as well. Every actor is equally well off. In addition, these networks are inefficient in terms of their network constraint. Especially the even-sized multipartite networks that are divided in $n/2$ groups of size 2 contain numerous closed triads despite their pairwise stability. The generalizations of the pentagon and the wheel, by contrast, are inefficient because of sparseness. Balanced complete bipartite networks of the same size give each actor more ties without adding closed triads.

Another property of balanced complete bipartite networks is that they are not stars. Economists have also recently modeled network dynamics as a process in which actors maximize information-based utility. Jackson and Wolinsky (1996), Bala and Goyal (2000), and Goyal and Vega-Redondo (2007), using three distinct utility functions, all find the star to be the dominant equilibrium network. Goyal and Vega-Redondo even contend that their utility function is a measurement for the richness of structural holes in someone's network. Their model does not use the constraint measure as proposed by Burt (1992). A theoretical reason for the difference between the two models is that control benefits are not subject to decay over longer paths in Goyal and Vega-Redondo's model. This implies that brokerage of indirectly received information is as valuable as brokerage of directly received information. By contrast, Burt's constraint measure implies that brokerage of indirectly received information is worthless, and only brokerage of directly received information creates value. Although both assumptions are quite extreme, we choose to insist strictly on using the constraint measure for three reasons. First, actors who take indirect brokerage benefits into account must have information on the structure of the entire network. Their model is thus scope-limited to settings in which such information is readily available. The actors in our model need to know only which of their contacts are in contact with one another and which are not, and how many relations their direct contacts have in the network. Second, the constraint measure has empirically been shown to explain success (see the evidence discussed at the start of this article). And third, Burt (2007) demonstrates in a recent paper that the returns on indirect brokerage are in some contexts not visible at all, and if they are found, they are considerably smaller than returns on direct brokerage. This provides empirical evidence for an aspect of the constraint measure that is quite crucial for our results—namely, that additional routes to indirect contacts are always cost effective. The other studies do not have a body of empirical evidence to back up their utility functions.

Still, the fact that different formalizations of theoretically the same concept yield such different results raises the question, How robust are our results for changes in assumptions on the utility function? Three properties of the network constraint are crucial for our finding. First, network ties are in principle cheap, so if they are well chosen, an actor wants as many ties as possible. Second, closed triads are bad, so that an actor almost never wants to create a tie if it closes a triad. Third, alternative information channels to distant others are never superfluous.

These three properties are maintained if we change the utility function to be decreasing not in the absolute but in the relative constraint score—the absolute constraint of an actor divided by the sum of the absolute constraints of the other actors. This alternative measure would be applicable to settings in which benefits are arguably zero-sum, as in Burt's (1992) example of promotions, where only one out of a pool of candidates can be promoted. We analyzed and simulated a model with this alternative utility function. Results were very similar. Also under this utility function, the balanced complete bipartite networks are the dominant stable networks. They are pairwise stable and emerge in the majority of simulations.

In Burger and Buskens (2008) and Kleinberg et al. (2008), the utility function is reduced to its basic principles. In Burger and Buskens (2008), there are (marginally decreasing) benefits of ties and linear costs for ties and closed triads. This much simpler utility function, which also has the three properties mentioned above, led to complete bipartite networks' being the most prominent class of networks to emerge and also to a higher likelihood for balanced complete bipartite networks. Kleinberg et al. (2008) consider linear benefits and costs of ties, as well as linear benefits of direct brokerage that decrease in the number of brokers. This utility function satisfies the first and third property but is open-triad seeking rather than closed-triad avoiding. Interestingly, Kleinberg et al. (2008) find the multipartite networks of our theorem 6 to be equilibrium networks.

In addition, Robins et al. (2005) find in a *stochastic* network evolution context that networks converge to the networks we also predict if they introduce a low probability that closed triads are formed and relatively small costs for having ties in general. Their figure 12 (Robins et al. 2005, p. 931) shows two representations of a complete bipartite network (the top one is balanced), which indicates that even with changed other parameters, as long as triads are unlikely enough and direct relations are likely enough, complete bipartite networks also emerge in this set-up. Our conclusions would also not change if we added indirect constraint, as is done by Burt (2007; an actor's indirect constraint is the average of the constraints of his neighbors)—we would still find the same networks to be stable because indirect constraint is optimized, given that each individual constraint is optimized.

The main results of our article will change if we change one of the three crucial properties mentioned above. First, results change if we take into account that there is limited new information in multiple contacts if these are linked to many of the same third parties (see Reagans and Zuckerman, in press). Complete bipartite networks are full of such redundancies over two steps and, therefore, are unlikely to be stable if such redundancies are taken into account. Second, if relations are so expensive that actors do not want to connect to at least half of the group, complete bipartite networks will no longer be stable. If this were the only change to the utility function, networks would remain to stabilize in bipartite structures, but these structures would not be complete. Third, stability results will—obviously—change if we assume not that closed triads are costly but rather that actors value closed triads in a positive way. This will lead to either complete networks or, if direct ties are relatively expensive, networks that segment into different complete subnetworks (see Burger and Buskens 2008).

An important next question is how we can empirically test the implications of our theoretical results. The settings to which Burt's structural entrepreneurship (and thus also our results) apply are competitive settings where firsthand information is important. We can imagine two examples of settings that fulfill these conditions to some extent. The first example is colleagues within firms in which the competition for promotion is very high. For instance, consider a firm like the one in Burt and Ronchi (2007), in which a large group of employees is trained in structural hole theory. We would expect performance to increase after training because of more efficient information flows, but we would expect the effects on promotion changes to be less striking (as is also found by Burt and Ronchi), given that if everyone strives for structural holes, structural advantages can be expected to disappear. A second example can be firms in competitive and innovative sectors in which well-chosen alliances with other firms are an important precondition for securing competitiveness within the sector. A testing of the theory does not need to concentrate only on whether the ultimate stable networks emerge. Looking at the microlevel, our model also provides predictions for which relations are more likely to be established or broken than others. Using longitudinal network data, we could investigate the extent to which the model predicts changes in the network even when a stable network is not yet formed. This would imply testing whether the network constraint has an effect on tie formation, using statistical models such as those developed by Snijders (2001, 2005). In a laboratory experiment, Burger and Buskens (2008) show that if the incentive structure resembles the one related to structural holes, networks are very likely to emerge as balanced complete bipartite networks. Although this is a test in a rather artificial setting, it creates some confidence that if we know the incentives at the individual level well, models such as the one presented in this article are able to predict the emerging structure.

To conclude, we want to emphasize that this article provides a benchmark for research on the emergence of networks. Using a combination of stability and simulation analysis, we have shown how one can derive stable networks and study the likelihood of the emergence of these networks, and thus how one can derive hypotheses on the structures that can be expected given specified network benefits. The theoretical methodology allows for many possible extensions. A seemingly obvious one would be adding explicit costs for maintaining ties, as is common in the literature. This would be particularly interesting if we assumed heterogeneity between actors in costs of bridging ties. Some actors might be natural entrepreneurs, while others may not have the inclination or courage to step up to strangers and build bridging ties, or they just may not observe these brokerage opportunities. Another way to include heterogeneity among actors in the model might be to assume that structural holes are not the only things that matter. In many settings, other competing incentives will be present, such as balance in friendship networks. As Burt (2005, chap. 5) notes in the last chapter of his recent book, stability might emerge in networks even with many brokerage opportunities still open, since a considerable number of actors are not interested in brokerage or are not able to observe these structural holes. Finally, considering the two types of social capital that Burt (2005, chap. 3) distinguishes, it may be fruitful to use a utility function that is a hybrid of a brokerage-based utility function and a closure-based utility function. One could then make the relative importance of closure a parameter and study the consequences for network stability. Sato (1997) has already taken a first step in this direction. Results would change, because complete bipartite networks do not include any closed triads. Likely, actors who care little about structural holes but a lot about friendship and trust end up in networks full of unexploited brokerage opportunities, but such speculations are unwarranted in the absence of a formal foundation.

APPENDIX A

Proofs

For the convenience of the reader, the theorems are reproduced here along with their proofs.

THEOREM 1.—*For the Burt constraint measure, it holds that* $c_i \leq 9/8$ *if* $d_i > 0$ *for all actors i in the network.*

Proof. We can rewrite the constraint measure as

$$c_i = \frac{1}{d_i^2}\sum_j \left(1 + \sum_k \frac{1}{d_k}\right)^2,$$

where j is the index for neighbors of i, and k is the index for neighbors of i that are also connected to j. This constitutes the product of $1/d_i^2$ with a sum of squares of d_i numbers that are greater than or equal to 1. Consider the sum of these numbers:

$$\left(d_i + \sum_j \sum_k \frac{1}{d_k}\right).$$

We rearrange the terms in this double summation, realizing that for each neighbor j of i, the term $1/d_j$ is included exactly once for each common neighbor k. In addition, the number of neighbors that j shares with i is smaller than or equal to $d_j - 1$ *and* smaller than or equal to $d_i - 1$. Therefore,

$$\left(d_i + \sum_j \sum_k \frac{1}{d_k}\right) = \left(d_i + \sum_j \frac{\sum_k 1}{d_j}\right) \leq \left[d_i + \sum_j \frac{\min(d_i, d_j) - 1}{d_j}\right]$$
$$\leq [d_i + (d_i - 1)] = 2d_i - 1.$$

We maximize a sum of squares of nonnegative numbers while holding the sum constant by assigning a value as close to 0 as possible to all elements but one and assigning the remainder to this last element. Since the sum of the d_i numbers is smaller than or equal to $2d_i - 1$, and the numbers are always larger than or equal to 1, the maximum sum of squares is $d_i^2 + (d_i - 1) \times 1^2 = d_i^2 + d_i - 1$. Hence, we can write

$$c_i = \frac{1}{d_i^2} \sum_j \left(1 + \sum_k \frac{1}{d_k}\right)^2 \leq \frac{d_i^2 + d_i - 1}{d_i^2} \leq \frac{5}{4}.$$

For $d_i = 2$, we know that c_i reaches a maximum when the two neighbors are connected: $c_i = 9/8 < 5/4$. Since the previous formula tells us that for $d_i > 6$, c_i is strictly lower than 9/8, inspecting all seven-actor networks and not finding a value for c_i of at least 9/8 implies that 9/8 is indeed the maximum value for c_i. The argument for this last implication is that every network position for a focal actor with six or fewer neighbors that can occur will occur in a seven-actor network. In larger networks, this can only be complemented with neighbors who have more neighbors themselves outside the original seven actors, but this will only decrease the constraint of the focal actor. Q.E.D.

Theorem 2.—*Adding a tie without creating closed triads is always beneficial for both actors involved in the new tie.*

Proof. We rewrite the constraint of actor i as

$$c_i = \frac{1}{d_i^2} \sum_j \left(1 + \sum_q \frac{1}{d_q}\right)^2,$$

where d_i is the number of actors i is linked to, j is the index for neighbors of i, and q is the index for neighbors of i that are also connected to j. This can be done because $p_{ij} = 1/d_i$ for all neighbors j of i. Suppose that two actors i and r can add a tie without creating a closed triad. Neither before nor after tie addition are there any actors q who are connected to both i and r. Let c_i denote the network constraint of i before and c_i* that after the initiation of the new tie, and let j continue to stand for the index of neighbors before tie addition. Then,

$$c_i^* = \frac{1}{(d_i + 1)^2} \left[1 + \sum_j \left(1 + \sum_q \frac{1}{d_q}\right)^2\right].$$

Using straightforward calculations, this implies that

$$\begin{aligned} c_i^* - c_i &= \frac{1}{(d_i + 1)^2} \left[1 + \sum_j \left(1 + \sum_q \frac{1}{d_q}\right)^2\right] - \frac{1}{d_i^2} \sum_j \left(1 + \sum_q \frac{1}{d_q}\right)^2 \\ &= \frac{1}{(d_i + 1)^2} - \frac{(d_i + 1)^2 - d_i^2}{d_i^2 (d_i + 1)^2} \sum_j \left(1 + \sum_q \frac{1}{d_q}\right)^2 \leq \frac{1}{(d_i + 1)^2} - \frac{(d_i + 1)^2 - d_i^2}{d_i (d_i + 1)^2} \\ &= \frac{-d_i - 1}{d_i (d_i + 1)^2} = -\frac{1}{d_i (d_i + 1)} < 0. \end{aligned}$$

Thus, the addition of the new tie necessarily decreases actor i's network constraint and hence increases his utility. Similarly, the constraint decreases for actor r. Q.E.D.

THEOREM 3.—*A complete bipartite network of size n is strongly pairwise stable, unless it is a k-star with* $k > 3$.

Proof. We proceed by showing that no change to such a network is profitable and feasible. Removing one or more ties is not an option in a $K_{k,l}$ because that would create a shortest path longer than 2, and hence it cannot be an improvement, by corollary 1. Therefore, we need to consider only conditions under which group members create a tie within their group. Without loss of generality, assume $k \leq l$. The constraint in the complete bipartite network equals $1/k$ for actors in the group of size l and $1/l$ for actors in the group of size k. Creating a tie in the larger group of l actors changes the constraint of the two actors involved in that tie to

$$\frac{k}{(k+1)^2}\left[1+\frac{1}{(k+1)}\right]^2+\frac{1}{(k+1)^2}\left(1+\frac{k}{l}\right)^2$$
$$=\frac{1}{(k+1)^2}\left[\frac{k(k+2)^2}{(k+1)^2}+\frac{(l+k)^2}{l^2}\right],$$

because these actors now have one common neighbor with all the actors in the group of size k and k common neighbors with each other. In order for the network to be strongly pairwise stable, this expression must be larger than $1/k$, or

$$\left[\frac{k(k+2)^2}{(k+1)^2}+\frac{(l+k)^2}{l^2}\right]>\frac{(k+1)^2}{k} \Leftrightarrow k^2l^2(k+2)^2+k(k+1)^2(k+l)^2>(k+1)^4l^2$$
$$\Leftrightarrow k(k+1)^2(k+l)^2>(2k^2+4k+1)l^2. \quad \text{(A1)}$$

Thus, if $k = 1$, $4 \times (l+1)^2 > 7l^2 \Leftrightarrow l < 4$ should hold. Therefore, stars are stable only if there are fewer than four peripheral actors. If $k > 1$, then the inequality above is always implied by $k(k+1)^2 > 2k^2 + 4k + 1 \Leftrightarrow k^3 - 3k - 1 > 0$, and this condition is always fulfilled for $k > 1$.

The same expression should hold for actors in the small group, but then with k and l reversed:

$$\left[\frac{l(l+2)^2}{(l+1)^2}+\frac{(l+k)^2}{k^2}\right]>\frac{(l+1)^2}{l}. \quad \text{(A2)}$$

Inequality (A2) is satisfied for any $l > 1$, $l \geq k \geq 1$, by reason of symmetry, because (A1) holds for all $k > 1$. Note that the case $l = k = 1$ is irrelevant because no tie can be added. Q.E.D.

THEOREM 4.—*Complete bipartite networks are Pareto efficient.*

Proof. Consider an actor i from the smaller group of $k \leq l$ actors.

The network constraint of i can be lower in another network than in the focal complete bipartite network only if he has more than l ties in that other network. This is so because the minimal constraint one can have with l ties, namely, in the absence of closed triads, is

$$c_i = \frac{1}{d_i^2}\sum_j \left(1 + \sum_q \frac{1}{d_q}\right)^2 = \frac{1}{l^2}l = \frac{1}{l}.$$

Let $a \leq k - 1$ be this additional number of ties of actor i; let j be the index for neighbors of i in the new network, and q the index for actors that i and j share as neighbors; and let π_j indicate the proportion ties of j with other neighbors of i out of all ties of j, and $\bar{\pi}_j$ the average of all $l + a$ proportions π_j. Then, for i to have a lower network constraint in the new network, the following inequality must hold:

$$\begin{aligned} \frac{1}{l} &> \frac{1}{(l+a)^2}\sum_j \left(1 + \sum_q \frac{1}{d_q}\right)^2 \geq \frac{1}{(l+a)^2}\sum_j \left(1 + \sum_q \frac{1}{d_q}\right) \\ &= \frac{1}{(l+a)^2}\sum_j \left(1 + \sum_q \frac{1}{d_j}\right) \\ &= \frac{1}{(l+a)^2}\sum_j (1 + \pi_j) = \frac{1}{(l+a)}(1 + \bar{\pi}_j) \Rightarrow \pi_j < \frac{a}{l} \text{ for some } j. \end{aligned}$$

The most difficult step in the derivation above is the first equality, which is implied by the fact that for each j the number of times $1/d_j$ should be added on account of closed triads is equal to the number of common neighbors that j has with i.

Note that for each j, in order to be at least as well off in the new network as in the complete bipartite network considered, his degree d_j must be at least k. Only $k - a - 1$ of j's connections can be to actors to whom actor i is not connected, thereby excluding i himself. For each j, π_j may therefore be no less than a/k:

$$\pi_j \geq \frac{d_j - k + a}{d_j} \geq \frac{a}{k} \geq \frac{a}{l} \text{ for all } j.$$

We have reached a contradiction. Thus, to lower i's network constraint, at least one actor j must be given fewer than k neighbors, and this actor is consequently strictly worse off in the new network than in the complete bipartite network considered.

A potential Pareto improvement must therefore leave the network constraints of all actors with l ties unchanged, giving them precisely l ties and no closed triads. However, this can be done only in the complete bipartite network considered or, in the case of $k = l$, in another complete bipartite network with two groups of size k. This renders the assumed Pareto improvement impossible. Q.E.D.

THEOREM 5.—*A complete bipartite network is unilaterally stable if and only if it is balanced.*

Proof. *If*. Consider an actor i from the group of k actors. We know from the proof of theorem 3 that we cannot make this actor better off without letting one of his neighbors have a degree lower than k. But leaving the ties that do not involve actor i unchanged, all his neighbors have at least degree k. Actor i can therefore not lower his constraint by only changing his own ties. In the even case, in which $k = l$, by symmetry, this impossibility of unilateral improvement extends to actors of the group of size l. The single remaining possibility for unilateral improvement is therefore a permitted decrement of the constraint of actor i from the group of l actors in the odd case, in which $k = l - 1$. Let $0 \leq b_k \leq k$ be the number of ties actor i has with actors from the group of size k in the new network, and let $0 \leq b_l \leq k$ be the number of ties he has with actors from the group of size l. Again, leave the ties that do not involve actor i unchanged. Because these ties constitute the balanced complete bipartite network with $k = l - 1$ actors in each group, the following inequality must hold:

$$\frac{1}{k} > \frac{1}{(b_k + b_l)^2}\left[b_k\left(1 + \frac{b_l}{k+1}\right)^2 + b_l\left(1 + \frac{b_k}{k+1}\right)^2\right] = \frac{b_k(b_l + k + 1)^2 + b_l(b_k + k + 1)^2}{(b_k + b_l)^2(k+1)^2}$$

$$\Leftrightarrow b_l^2(k+1)^2 + 2b_k b_l(k+1)^2 + b_k^2(k+1)^2$$

$$> b_k k(k+1)^2 + 2b_k b_l k(k+1) + b_k b_l^2 k + b_l k(k+1)^2 + 2b_k b_l k(k+1) + b_l b_k^2 k$$

$$\Leftrightarrow b_l^2(k+1)^2 - 2b_k b_l(k-1)(k+1) + b_k^2(k+1)^2$$

$$> b_k k(k+1)^2 + b_k b_l^2 k + b_l k(k+1)^2 + b_l b_k^2 k$$

$$\Leftrightarrow [b_k(b_k - k) + b_l(b_l - k)](k+1)^2$$

$$> b_k b_l[(b_k + b_l)k + 2(k-1)(k+1)].$$

The left-hand side of this last inequality is never strictly positive, and the right-hand side is never strictly negative. Hence, it cannot be satisfied.

Only if. If $l - k > 1$, an actor from the larger group of l actors can delete all her ties with actors from the smaller group of k actors and add $l - 1$ ties to the other actors from the larger group of l actors. By doing so, she decreases her constraint from $1/k$ to $1/(l-1)$. By permitting this change, the $l - 1$ actors see their constraint fall from $1/k$ to $1/(k+1)$. Q.E.D.

THEOREM 6.—*All complete multipartite networks are pairwise stable if the groups are of equal size and contain more than one actor.*

Proof. Let $n_2 = n/m > 1$ be the size of each group. Then, for a complete m-partite network with equal groups of size n_2 with constraint c_i, the following inequality should hold such that no one wants to sever a tie to obtain a network with constraint c_i^* (note that we need to confirm only one inequality because all actors have automorphically equivalent positions):

$$c_i - c_i^* = \frac{1}{(n-n_2)^2}(n-n_2)\left(1+\frac{n-2n_2}{n-n_2}\right)^2$$
$$-\frac{1}{(n-n_2-1)^2}\left[(n-2n_2)\left(1+\frac{n-2n_2-1}{n-n_2}\right)^2 + (n_2-1)\left(1+\frac{n-2n_2}{n-n_2}\right)^2\right] < 0.$$

Multiplying the inequality above by $(n-n_2-1)^2(n-n_2)^3$, and after some tedious rearranging of terms, one obtains

$$(n-2n_2)(4n-6n_2-1)(n-n_2)-(n-n_2-1)(2n-3n_2)^2 < 0,$$

which is equivalent to

$$4(n-2n_2)(n-n_2)\left(n-\frac{3}{2}n_2-\frac{1}{4}\right) - 4(n-n_2-1)\left(n-\frac{3}{2}n_2\right)^2 < 0.$$

This is always true, because $(n-2n_2)(n-n_2) < (n-\frac{3}{2}n_2)^2$ and $(n-\frac{3}{2}n_2-\frac{1}{4}) < (n-n_2-1)$ if $n_2 \geq 2$.

For no actor to benefit from adding any of his equivalent potential ties in this complete n/n_2-bipartite network with equally sized groups, the following inequality must hold:

$$\frac{1}{(n-n_2)^2}(n-n_2)\left(1+\frac{n-2n_2}{n-n_2}\right)^2$$
$$-\frac{1}{(n-n_2+1)^2}\left[(n-n_2)\left(1+\frac{1}{n-n_2+1}+\frac{n-2n_2}{n-n_2}\right)^2 + \left(1+\frac{n-n_2}{n-n_2}\right)^2\right] < 0.$$

Multiplying by $(n-n_2-1)^4(n-n_2)^3$, we derive that the above inequality is equivalent with $x^4(7-6n_2) + x^3(12-18n_2) + x^2(4-16n_2) - 4xn_2 + 2x^3n_2^2 + 5x^2n_2^2 + 4xn_2^2 + n_2^2 < 0$, where $x = n - n_2$. Since $n_2 < x = n - n_2$, the inequality above is implied by (replacing n_2^2 with xn_2) $x^4(7-4n_2) + x^3(12-13n_2) + x^2(4-12n_2) - 3xn_2 < 0$, which is true because $n_2 \geq 2$ and $x > 0$. Q.E.D.

APPENDIX B

The Gould-Myerson Model

In the Gould-Myerson (GM) model, each actor simultaneously proposes the complete set of other actors he wants to be connected to. We use $s_i \in \{0, 1\}^n$ to indicate a strategy of actor i in which s_{ij} indicates whether or not i proposes a link with j. Because actors cannot connect to themselves, $s_{ii} = 0$. The utility function $u_i(s)$ assigns a numerical value to each set of strategies $s = \{s_i \mid i \in N\}$. Gould considers *Nash equilibrium* to be the stability concept.

DEFINITION.—*A set of strategies* $s^* = \{s_i^* \mid i \in N\}$ *is a* Nash equilibrium *if* $u_i(s^*) \geq u_i(s_i, s_{-i}^*)$ *for all i and* s_i, *where* s^*_{-i} *is the set of all strategies in* s^* *excluding the one of i*.

We say that the set of strategies s *induces* the network g if $ij \in g \Leftrightarrow s_{ij} = s_{ji} = 1$; that is, only ties that are proposed by both actors are part of the network. Given that ties are costless, the utility function $u_i(s)$ is the same for combinations of strategies that induce the same network. Formally, the utility function has the property that $u_i(s') = u_i(s'')$ if s' and s'' induce the same network g. With some abuse of notation, we can also write $u_i(g)$ as the utility of a certain network g, given that it does not matter what strategies induce this network. Now, we can also define a stability concept related to networks rather than strategies:

DEFINITION.—*A network* g^* *is a* Nash network *if some* s^* *inducing* g^* *is a Nash equilibrium*.

The GM model shortcuts any network dynamics. Instead of representing network evolution as a continuous process in which one actor can react to changes by other actors elsewhere in the network, as in the SW model, it assumes that all actors make their decisions simultaneously and that these decisions are binding. A Nash network is a rather weak stability concept for undirected networks. There are often numerous equilibria, and the lack of a specification of the network evolution process then leaves us unable to identify the network that is most likely to evolve. From an evolutionary viewpoint, many of these equilibria can hardly be considered stable networks. If one actor does not propose a tie to another, the second actor has no incentive to propose a tie to the first, because a tie is formed only if it is proposed by both actors at the same time. No one can increase his utility through any proposal if no one else is proposing ties, which makes "nobody proposing any tie" a Nash equilibrium. The network induced by this set of strategies is the empty network, and this is a Nash network. Given our utility function, every *pair* of actors wants to initiate the first tie in the empty network, because isolates have the lowest possible utility. Thus, many networks are Nash because of trivial coordination problems.

The good news, however, is that the intersection of pairwise stable networks in the SW model and Nash networks in the GM model can easily be characterized. It is exactly the set of strongly pairwise stable networks (see Cálvo-Armengol 2004). Therefore, unilateral stability, being a refinement of strong pairwise stability, can also be recast as a refinement of Nash equilibrium in the GM model (see Van de Rijt and Buskens 2008). By using unilateral stability, we can analyze stability in both models while considering only the SW model in the main text.

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