

**$\eta$  and  $\eta'$  Meson Mixing in  $U(6)$  Symmetry**

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On the standpoint of the urbaryon model with approximate  $U(6)$  symmetry, several relations among the differential cross sections relevant to  $\eta$  and  $\eta'$  mesons are obtained. It is shown that the overlapping integral between the spatial wave functions of the singlet and the 35-plet mesons plays an important role. Obtained relations well agree with experiments for the case of the quadratic mass formula, but not for the linear mass formula. Decays of  $\eta$  or  $\eta'$  meson and heavy meson decays to  $\eta$  or  $\eta'$  meson are also discussed.

**§ 1. Introduction**

The composite model in which the baryon and the meson are composed of urbaryons as  $(uuu)$  and  $(\bar{u}\bar{u})$  respectively has brought us systematic understanding of the hadron spectrum and their interactions. At high energies, several sum rules among differential cross sections of various hadronic reactions have been derived<sup>1)</sup> from counting the urbaryon rearrangement amplitudes with the Okubo-Lizuka rule<sup>2)</sup> which forbids the disconnected urbaryon diagram. The sum rules well agree with experiments.

Recently it has been argued<sup>3),4)</sup> that the sum rules among the differential cross sections concerning  $\eta$  or  $\eta'$  meson derived from the simple counting rule do not agree with experiments, i.e., the singlet octet ratio  $S = \langle |\eta_1| \rangle / \langle |\eta_8| \rangle$  is introduced as a free parameter and its value is found to be much smaller than  $\sqrt{2}$  obtained from the simple counting rule when high energy hadronic reactions and strong decays are concerned.

From the standpoint of the urbaryon model, a correction to the simple counting rule is naturally interpreted as an effect of the overlapping integral between the hadrons and it is related to the mass formula of pseudoscalar mesons.

In the  $U(6)$  symmetry theory, one of the pseudoscalar mesons with  $I=Y=0$  belongs to **1** of  $U(6)$  multiplets and the other eight pseudoscalar mesons and nine vector mesons belong to **35**. It can be considered that, in the first approximation, the spatial wave functions of the **35** states are equal to each other and different from that of the **1** state and the  $U(3)$  invariant mass terms split into **1** and **35**. The  $U(3)$  symmetry breaking terms  $T_8^3$  well reproduce the observed mass splitting among mesons as well as baryons. On these assumptions, Dalitz and Sutherland gave<sup>5)</sup> a mass matrix of  $I=Y=0$  pseudoscalar mesons as

$$M = \begin{pmatrix} m_8 + 2\delta & -\sqrt{2}\xi\delta \\ -\sqrt{2}\xi\delta & m_1 + \delta \end{pmatrix}, \tag{1.1}$$

where  $\xi$  is the overlapping integral between the spatial wave functions of the **1** and **35** states and  $m_8 = m_\pi^2$  (or  $m_\pi$ ),  $\frac{3}{2}\delta = m_K^2 - m_\pi^2$  (or  $m_K - m_\pi$ ). The masses of  $\eta$  and  $\eta'$  mesons are derived from diagonalization of  $M$ :

$$\begin{aligned} m_{\eta^2} \text{ (or } m_{\eta}) &= \frac{m_8 + m_1 + 3\delta}{2} + \frac{m_8 + \delta - m_1}{2} (\cos 2\theta - \sin 2\theta), \\ m_{\eta'^2} \text{ (or } m_{\eta'}) &= \frac{m_8 + m_1 + 3\delta}{2} - \frac{m_8 + \delta - m_1}{2} (\cos 2\theta - \sin 2\theta), \end{aligned} \tag{1.2}$$

where

$$\tan 2\theta = \frac{2\sqrt{2}\xi\delta}{m_8 - m_1 + \delta}. \tag{1.3}$$

Using the observed mass values of four pseudoscalar mesons, we obtain that  $\xi = 0.54$  and  $\theta = -11^\circ$  for the quadratic mass formula (Q.M.F.) and that  $\xi = 0.45$  and  $\theta = -24^\circ$  for the linear mass formula (L.M.F.).

In consideration of the overlapping integral, high energy  $\eta$  or  $\eta'$  meson production process in small  $t$  regions and the decays of  $\eta$  or  $\eta'$  meson are discussed in §§ 2 and 3 respectively. Conclusion and remarks are given in § 4.

### § 2. Productions of $\eta$ and $\eta'$ mesons at high energy

Discussion is limited to high energy inelastic reactions  $0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{1}{2}+}$  (or  $0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{3}{2}+}$ ) involving  $\eta$  and  $\eta'$  meson productions. The forward scattering amplitudes include the overlappings between the wave functions of initial and final mesons and baryons respectively. As far as the above processes are concerned, the overlappings between baryons for the processes can be taken to be equal to each other, since both the baryons belong to the same  $U(6)$  multiplet **56**. The meson overlappings are not equal to each other because the final meson is a superposed state of different multiplets **35** and **1** for  $\eta$  and  $\eta'$  meson productions. In

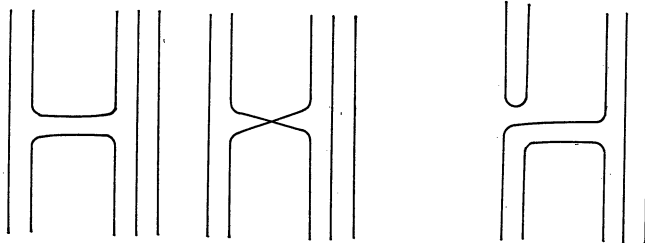


Fig. 1.

Fig. 2.

Fig. 1. Urbaryon rearrangement diagrams counted by unity for  $P_8 B_8 \rightarrow P_8 B_8$  and counted by  $\xi$  for  $P_8 B_8 \rightarrow P_1 B_8$ .

Fig. 2. Diagrams forbidden from the Okubo-Iizuka rule.

counting the urbaryon rearrangement diagrams (Fig. 1), we must take into account the ratio of the **1-35** to **35-35** overlapping  $\xi$ . Disconnected diagrams (Fig. 2) are forbidden from the Okubo-Iizuka rule.

Here we assume that the overlapping parameter in the scattering is the same  $\xi$  appearing in the mass formula of the pseudoscalar mesons. The overlapping integral may, in general, vary with the momentum transfer squared  $t$ , but we assume that the  $t$ -dependence of  $\xi$  is negligible and use  $\xi=0.54$ (Q.M.F.) or  $\xi=0.45$ (L.M.F.) obtained from the mass formula independently of  $t$ . As is seen below, this is the case in small  $|t|$  regions.

1) *Charge exchange processes*

We denote the (integrated or differential) cross section for  $\pi^-p \rightarrow \pi^0n$ ,  $K^-p \rightarrow \bar{K}^0n$ ,  $K^+n \rightarrow K^0p$ ,  $\pi^-p \rightarrow \eta n$  and  $\pi^-p \rightarrow \eta' n$  by  $\sigma(\pi)$ ,  $\sigma(\bar{K})$ ,  $\sigma(K)$ ,  $\sigma(\eta)$  and  $\sigma(\eta')$  respectively. In consideration of the overlapping integral, the counting rule of urbaryon rearrangement diagram gives several relations among these cross sections in terms of  $\xi$  and  $\theta$  as

$$\frac{\sigma(\eta')}{\sigma(\eta)} = \left| \frac{a_2}{a_1} \right|^2 \tag{2.1}$$

and

$$\frac{\sigma(\eta)}{\sigma(\eta_8)} = |a_1|^2, \tag{2.2}$$

where

$$\begin{aligned} a_1 &= \cos \theta - \sqrt{2}\xi \sin \theta, \\ a_2 &= \sin \theta + \sqrt{2}\xi \cos \theta \end{aligned} \tag{2.3}$$

and

$$\sigma(\eta_8) = \frac{1}{3}[\sigma(K) + \sigma(\bar{K}) - \sigma(\pi)]. \tag{2.4}$$

The  $\sigma(\eta_8)$  is the cross section for the pure octet  $I=Y=0$  pseudoscalar

Table I. Ratios predicted from the counting rule with overlapping  $\xi$ .

	Q.M.F. ( $\theta=-11^\circ$ ) $\xi=0.54$ $\xi=1$		L.M.F. ( $\theta=-24^\circ$ ) $\xi=0.45$ $\xi=1$		Experiments ( $P_L$ [GeV/c])
$\frac{\sigma(\eta')}{\sigma(\eta)}$	0.24	0.91	0.02	0.34	0.2 (3~4) 0.42±0.07 (3.8, 6, 8, 12) 0.22±0.04 (5) $\Delta^{++}$ 0.24±0.02 (7) $\Delta^{++}$
$\frac{\sigma(\eta)}{\sigma(\eta_8)}$	1.27	1.57	1.38	2.23	0.90 $\begin{matrix} +0.47 \\ -0.28 \end{matrix}$ (10) 1.19 $\begin{matrix} +0.40 \\ -0.30 \end{matrix}$ (12.3)

$\Delta^{++}$  The experimental ratios for  $\pi^+p \rightarrow \eta(\eta')\Delta^{++}$  is shown instead of  $\pi^-p \rightarrow \eta(\eta')n$ .

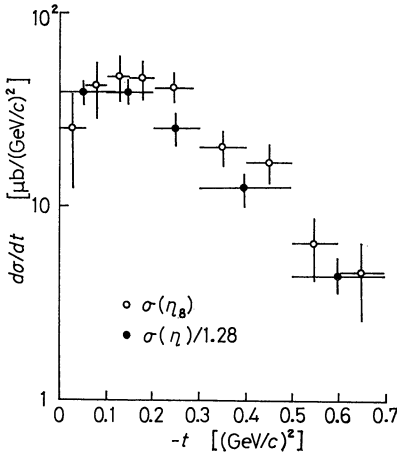


Fig. 3. Test of  $\sigma(\eta_s) = \sigma(\eta)/1.28$  predicted in the case of the Q.M.F. ( $\xi = 0.54$ ). Open (closed) circles show the L.H.S. (R.H.S.) evaluated from the data<sup>7)</sup> of the  $d\sigma/dt$  for  $\pi^-p \rightarrow \pi^0n$  at  $P_L = 13.3$  GeV/c,  $\pi^-p \rightarrow \eta n$  at 13.3 GeV/c and  $K^-p \rightarrow \bar{K}^0n$  at 12.3 GeV/c in the approximation  $\sigma(K) = \sigma(\bar{K})$ , which is proper at  $P_L \approx 12$  GeV/c from the comparison with the  $d\sigma/dt$  for  $K^+n \rightarrow K^0p$  at  $P_L = 13$  GeV/c.

meson production. It is noted that these derivations are free from  $F/D$  ratios and the line reversed relation. The same relations can be derived for the processes where the final baryon belongs to the decuplet state with spin parity  $\frac{3}{2}^+$ . Equation (2.1) is calculated for each case of Q.M.F. and L.M.F. Comparison with experiments<sup>6)</sup> is made in Table I. In the case of Q.M.F. the calculated value of the right-hand side of (2.1) is 0.24 and consistent with experimental values of  $\sigma(\eta')/\sigma(\eta) = 0.2 \sim 0.5$ . On the other hand, in the case of L.M.F. it is 0.02, which is too small. As for  $\sigma(\eta)/\sigma(\eta_s)$ , both of the calculated values 1.28 and 1.36 for Q.M.F. and L.M.F. are consistent with the experimental values.<sup>7)</sup>

To examine the  $t$ -dependence of  $\xi$ , we compare the differential cross section  $\sigma(\eta_s)$  to  $\sigma(\eta)/1.28$  at about  $P_L = 13$  GeV/c in Fig. 3. No significant  $t$ -dependence of  $\xi$  is observed in the region  $|t| < 0.6$  (GeV/c)<sup>2</sup>.

## 2) Hypercharge exchange processes

We denote the (integrated or differential) cross sections for  $K^-p \rightarrow \pi^0A$ ,  $\pi^-p \rightarrow K^0A$ ,  $K^-p \rightarrow \eta A$  and  $K^-p \rightarrow \eta' A$  by  $\sigma(\pi A)$ ,  $\sigma(KA)$ ,  $\sigma(\eta A)$  and  $\sigma(\eta' A)$  respectively. We get the following sum rule:

$$6[a_2a_4\sigma(\eta A) - a_1a_3\sigma(\eta' A)] = (a_1a_4 - a_2a_3)[2a_1a_2\sigma(\pi A) - a_3a_4\sigma(KA)], \quad (2.5)$$

where  $a_1$  and  $a_2$  are defined in (2.3) and

$$\begin{aligned} a_3 &= -2 \cos \theta - \sqrt{2}\xi \sin \theta, \\ a_4 &= -2 \sin \theta + \sqrt{2}\xi \cos \theta. \end{aligned} \quad (2.6)$$

This derivation is also free from the values of  $F/D$  ratios and the line reversed relation.

For the case of Q.M.F., the sum rule (2.5) becomes

$$\sigma(\eta A) + 2.82 \sigma(\eta' A) = 0.765 \sigma(\pi A) + 1.24 \sigma(KA). \quad (2.7)$$

The experimental integrated cross sections<sup>8)</sup> of  $K^-p \rightarrow \pi^0A$ ,  $K^-p \rightarrow \eta A$ ,  $K^-p \rightarrow \eta' A$  at  $P_L = 3.95$  GeV/c and  $\pi^-p \rightarrow K^0A$  at  $P_L = 3.93$  GeV/c give  $149 \pm 22$  ( $\mu b$ ) and  $136 \pm 11$  ( $\mu b$ ) for the left- and right-hand sides of (2.7) respectively. The differential cross sections  $d\sigma/dt$  of both sides of (2.7) are compared in Fig. 4. It is seen that the sum rule is well satisfied independently of  $t$ .

In the case of L.M.F., (2.5) becomes

$$\begin{aligned} \sigma(\eta A) + 7.0 \sigma(\eta' A) \\ = 0.54 \sigma(\pi A) + 2.6 \sigma(KA). \end{aligned} \quad (2.8)$$

This sum rule leads  $330 \pm 47 = 196 \pm 13 (\mu\text{b})$  and it is not satisfied.

Here we note on the sharp dip at  $t \simeq -0.4 (\text{GeV}/c)^2$  of the  $d\sigma/dt$  for  $K^-p \rightarrow \eta A$  and  $K^-p \rightarrow \eta \Sigma^0$  in energy regions  $P_L = 3 \sim 4 \text{ GeV}/c$ .

Martin and Michael<sup>9)</sup> calculated the ratio  $\sigma(\eta A)/\sigma(\pi A)$  using the strong exchange degenerate Regge pole model. The predicted ratio  $\sigma(\eta A)/\sigma(\pi A)$  at  $t \simeq -0.4 (\text{GeV}/c)^2$  is larger than experiments for the case of Q.M.F. and they cannot reproduce the sharp dip.

What condition is required to reproduce the sharp dip? The  $d\sigma/dt$  for  $K^-p \rightarrow \eta A$  and  $K^-p \rightarrow \eta \Sigma^0$  are described as follows from counting the urbaryon rearrangement diagrams:

$$\sigma(\eta A), \sigma(\eta \Sigma^0) \propto |a_1 X + a_2 H|^2. \quad (2.9)$$

The strong exchange degenerate Regge amplitudes have successfully explained the data of two-body reactions concerning the helicity flip parts in energy regions where the line reversed relation recovers, so that we assume the helicity flip  $H$ - and  $X$ -amplitudes to be

$$\begin{aligned} H &= h e^{-i\pi\alpha(t)}, \\ X &= x, \end{aligned} \quad (2.10)$$

where  $h$  and  $x$  are real and their signs are the same and  $\alpha(t)$  is  $K^*-K^{**}$  degenerate trajectory. Equations (2.10) show that the strong exchange degeneracy and the line reversed relation ( $|H|=|X|$ ) appear if  $h=x$ .

Requirement  $\sigma(\eta A) = 0$  at  $\alpha(t) = 0$  derives the strong breaking of line reversed relation in the case of Q.M.F. as follows:

$$x \simeq \sqrt{3}h \quad \text{at} \quad \alpha(t) \simeq 0. \quad (2.11)$$

Equations (2.10) and (2.11) are realized for the helicity flip amplitudes. The dip of  $d\sigma/dt$  for  $\pi^-p \rightarrow \pi^0 n$  at  $t \simeq -0.5 (\text{GeV}/c)^2$  predicted from Eqs. (2.10) are observed and  $\sigma(K) \simeq 3 \sigma(\bar{K})$ <sup>9)</sup> at  $P_L = 3 \text{ GeV}/c$  in the regions  $0.2 \leq |t| \leq 0.5$

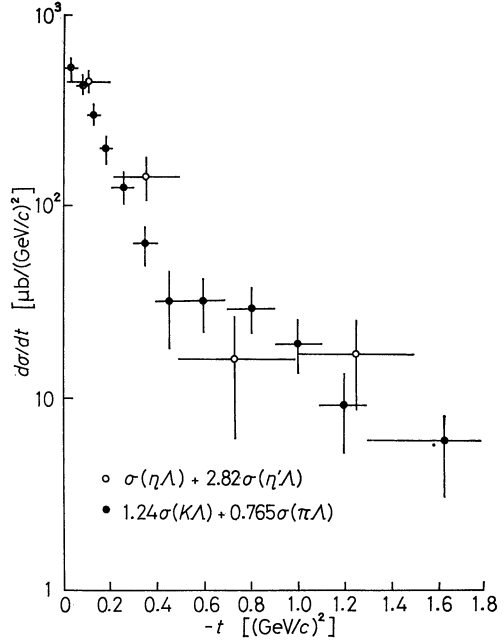


Fig. 4. Test of Eq. (2.7). Open (closed) circles show the L.H.S. (R.H.S.) evaluated from the data<sup>9)</sup> of the  $d\sigma/dt$  for  $K^-p \rightarrow \eta A$ ,  $K^-p \rightarrow \eta' A$  and  $K^-p \rightarrow \pi^0 A$  at  $P_L = 3.95 \text{ GeV}/c$  and  $\pi^-p \rightarrow K^0 A$  at  $P_L = 3.93 \text{ GeV}/c$ .

(GeV/c)<sup>2</sup> which leads us to Eq. (2.11). (These charge exchange processes are dominated by the helicity flip amplitudes.) The sharp dip can be reproduced naturally, if the contribution of the helicity non-flip amplitudes to  $d\sigma/dt$  for  $K^-p \rightarrow \eta A(\Sigma^0)$  at  $t \simeq -0.4(\text{GeV}/c)^2$  is small. The dip is expected to lose its sharpness as energy increases since, then, the breaking of line reversed relation becomes smaller.

### § 3. Resonance decays including $\eta$ or $\eta'$ meson

In this section, the effect of the overlapping integral on the resonance decays of mesons is discussed. We assume that the singlet components of these decay amplitudes are suppressed by  $\xi$  also.\*) By counting the diagrams in Fig. 5, the ratios of reduced decay widths  $\tilde{\Gamma}$  free from the kinematical factors are given in terms of  $\theta$  and  $\xi$  as follows:

$$\begin{aligned} \tilde{\Gamma}(\phi \rightarrow \eta\gamma) / \tilde{\Gamma}(\omega \rightarrow \pi\gamma) &= \frac{4}{27} |\sqrt{2} \cos \theta + \xi \sin \theta|^2, \\ \tilde{\Gamma}(\omega \rightarrow \eta\gamma) / \tilde{\Gamma}(\omega \rightarrow \pi\gamma) &= \frac{1}{27} |\cos \theta - \sqrt{2}\xi \sin \theta|^2, \\ \tilde{\Gamma}(\eta \rightarrow 2\gamma) / \tilde{\Gamma}(\pi^0 \rightarrow 2\gamma) &= \frac{1}{3} |\cos \theta - 2\sqrt{2}\xi \sin \theta|^2, \\ \tilde{\Gamma}(\eta' \rightarrow 2\gamma) / \tilde{\Gamma}(\pi^0 \rightarrow 2\gamma) &= \frac{1}{3} |\sin \theta + 2\sqrt{2}\xi \cos \theta|^2, \\ \tilde{\Gamma}(A_2 \rightarrow \eta\pi) / \tilde{\Gamma}(A_2 \rightarrow K\bar{K}) &= \frac{2}{3} |\cos \theta - \sqrt{2}\xi \sin \theta|^2, \\ \tilde{\Gamma}(A_2 \rightarrow \eta'\pi) / \tilde{\Gamma}(A_2 \rightarrow K\bar{K}) &= \frac{2}{3} |\sin \theta + \sqrt{2}\xi \cos \theta|^2. \end{aligned} \quad (3.1)$$

Here the disconnected diagrams shown in Fig. 6 are forbidden from the Okubo-Lizuka rule.

The calculated results are summarized in Table II, for the values of  $\theta$  and  $\xi$  in the cases of Q.M.F. and L.M.F. In Table II the experimental values of  $\tilde{\Gamma}$  have been obtained on the assumption that the kinematical factors are  $k_f^3$  for

\*) The overlapping parameter appearing in the amplitudes of radiative decays  $V \rightarrow P\gamma$  and  $P \rightarrow 2\gamma$  is suggested to have roughly the same value as the one in the mass formula from the following discussion. Decay amplitudes of  $V \rightarrow P\gamma$  are given as  $f_{ab}(k) \sim \int \psi_a(x) \exp(-\frac{1}{2}ikx) \psi_b(x) d^3x$  with non-relativistic approximation. Using the harmonic oscillator model, it leads  $f_{ab}(k) \sim \exp(-\alpha^2 k^2/16) \cdot \int \psi_a \psi_b d^3x$  for ground states  $a$  and  $b$ , where  $\alpha^{-1} = (\alpha_a^{-1} + \alpha_b^{-1})/2$ ,  $\alpha_a(\alpha_b)$  is a parameter of hadron extension. Mass of hadron is written as  $m_{ab} = \int \psi_a M_{ab} \psi_b d^3x \propto \int \psi_a \psi_b d^3x$ . Thus the overlapping of  $V \rightarrow P\gamma$  amplitudes is the same as the one in mass, except for a factor  $\exp(-\alpha^2 k^2/16)$ , which are within 0.8~1 for the concerned decays. For decays  $P \rightarrow 2\gamma$ , the same result is obtained from regarding these processes as  $P \rightarrow "V"\gamma$ .

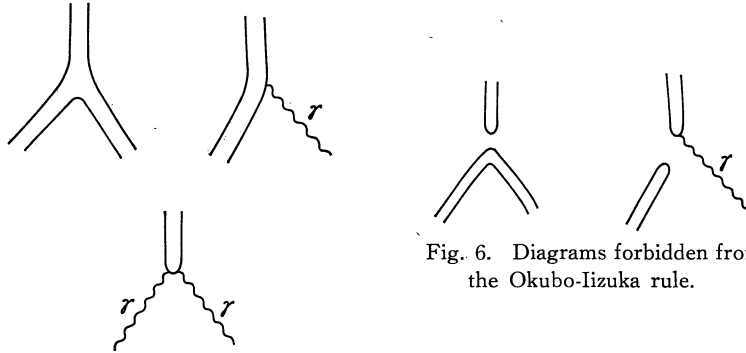


Fig. 5. Urbaryon diagrams for resonance decays counted by unity for  $8 \rightarrow 8+8$  and counted by  $\xi$  for  $8 \rightarrow 1+8$  ( $1 \rightarrow 8+8$ ).

Fig. 6. Diagrams forbidden from the Okubo-Iizuka rule.

Table II. Ratios of reduced decay widths predicted from the counting rule with overlapping  $\xi$ .

	Q.M.F. ( $\theta = -11^\circ$ ) $\xi = 0.54$ $\xi = 1$		L.M.F. ( $\theta = -24^\circ$ ) $\xi = 0.45$ $\xi = 1$		Experiments
$\tilde{\Gamma}(\phi \rightarrow \eta\gamma) / \tilde{\Gamma}(\omega \rightarrow \pi^0\gamma)$	0.24	0.21	0.18	0.11	$0.16 \pm 0.09$
$\tilde{\Gamma}(\omega \rightarrow \eta\gamma) / \tilde{\Gamma}(\omega \rightarrow \pi^0\gamma)$	0.05	0.06	0.05	0.08	$0.08 \pm 0.35$
$\tilde{\Gamma}(\eta \rightarrow 2\gamma) / \tilde{\Gamma}(\pi^0 \rightarrow 2\gamma)$	0.54	0.78	0.69	1.44	$0.58 \pm 0.13$ $(1.44 \pm 0.45)^a$
$\tilde{\Gamma}(\eta' \rightarrow 2\gamma) / \tilde{\Gamma}(\pi^0 \rightarrow 2\gamma)$	0.56	2.22	0.19	1.56	$< 13.7 \pm 2.2$
$\tilde{\Gamma}(A_2 \rightarrow \eta\pi) / \tilde{\Gamma}(A_2 \rightarrow K\bar{K})$	0.85	1.05	0.92	1.49	$1.15 \pm 0.24$
$\tilde{\Gamma}(A_2 \rightarrow \eta'\pi) / \tilde{\Gamma}(A_2 \rightarrow K\bar{K})$	0.20	0.95	0.02	0.51	$< 1.8$

a) The ratio calculated from the old experimental value of  $\Gamma(\eta \rightarrow 2\gamma)$ .

$V \rightarrow P\gamma$ ,  $k_f^5/M_T^2$  for  $T \rightarrow PP$  and  $M_P^3$  for  $P \rightarrow 2\gamma$ . Here  $P$ ,  $V$  and  $T$  denote pseudoscalar, vector and tensor mesons and  $k_f$  is the three-momentum of the final particles in c.m. frame.

The calculated values of Eqs. (3.1) are consistent with available experimental data<sup>10)</sup> for both cases at present. Results calculated from  $\xi=1$  are also listed in Table II for the sake of comparison. Gault et al. concluded that the experimental  $\Gamma(\eta-2\gamma)/\Gamma(\pi-2\gamma)$  was in favour of L.M.F. Recently reported values<sup>10)</sup> of  $\Gamma(\eta-2\gamma)$  is, however, about 1/3 as large as the old one, and it is consistent with Q.M.F. ( $\xi=0.54$ ) as well as L.M.F. ( $\xi=0.45$ ).

In conclusion, Q.M.F. ( $\xi=0.54$ ) supported in the previous section seems consistent also with the data of the resonance decays, while the case  $\xi=1$  is not excluded. In order to get a positive conclusion only from the decay data, it is necessary to determine the resonance widths more confirmly.

#### § 4. Conclusion and remarks

From the preceding discussion, it has been concluded that the urbaryon picture and the counting rule are also realized in hadronic reactions at high

energy and resonance decays as well as mass levels, even if  $\eta$  or  $\eta'$  meson is concerned. The effective suppression of the singlet octet coupling ratio  $S = \langle |\eta_1\rangle / \langle |\eta_8\rangle$  observed for the hadronic reactions is attributed to the overlapping between the spatial wave functions of the singlet and octet pseudoscalar mesons on the basis of  $U(6)$  symmetry. Furthermore, this is thought to show that the interpretation of the pseudoscalar mass relation from  $U(6)$  symmetry theory proposed by Dalitz and Sutherland<sup>5)</sup> is reliable one. Data of hadronic scattering cross sections at high energy are in favour of the quadratic mass formula, but they are inconsistent with the linear mass formula.

The singlet-octet coupling ratio of the vector or tensor mesons does not suppressed since the spatial wave functions of these mesons are the same in the first approximation. This is also consistent with available data.

It is interesting that the ratio of the **1-35** overlapping to **35-35** has the same value for the high energy reactions, the resonance decays and the mass relation of mesons. It has been recently shown<sup>11)</sup> that the  $F/D$  ratio has the same value for the mass relation of baryons, hyperon decays and hadron scattering at high energy. These facts supply suggestions to gain the insight into the urbaryon picture and the spatial wave functions or the constructive force.

In this paper discussion has been restricted in the triplet model. The possibility of the quartet model will be examined elsewhere. More confirm data on the radiative decay widths is desired, which will give an important clue to determine the model of hadrons.<sup>12)</sup>

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### References

- 1) T. Matsuoka, K. Ninomiya and S. Sawada, *Prog. Theor. Phys.* **41** (1969), 572, 1533.
- 2) S. Okubo, *Phys. Letters* **5** (1963), 165.  
J. Iizuka, K. Okada and O. Shito, *Prog. Theor. Phys.* **35** (1966), 1061.
- 3) A. D. Martin and C. Michael, *Phys. Letters* **37B** (1971), 513.  
T. Suzuki, *Soryusiron Kenkyu* (mimeographed circular in Japanese) **45** (1972), C86.
- 4) F. D. Gault, H. F. Jones and M. D. Scadron, *Nucl. Phys.* **B51** (1973), 353.
- 5) R. H. Dalitz and D. G. Sutherland, *Nuovo Cim.* **37** (1965), 1777.
- 6) H. O. Cohen et al., *Phys. Letters* **21** (1966), 347.  
E. H. Harvey et al., *Phys. Rev. Letters* **27** (1971), 885.  
W. D. Apel et al., *Phys. Letters* **46B** (1973), 459.  
Z. Carmel et al., *Proceedings of the XVI International Conference on High Energy Physics, Batavia* (1972).  
Barkley Group A, *Proceedings of the XVI International Conference on High Energy Physics, Batavia* (1972).
- 7) O. Guisan et al., *Phys. Letters* **18** (1965), 200.  
A. V. Stirling et al., *Phys. Rev. Letters* **14** (1965), 763.



- P. Astbury et al., Phys. Letters **23** (1966), 396.  
A. Firestone et al., Phys. Rev. Letters **25** (1970), 958.  
8) M. Abramovich et al., Nucl. Phys. **B27** (1971), 477.  
L. Moscoso et al., Nucl. Phys. **B36** (1972), 332.  
9) D. Cline et al., Phys. Rev. Letters **23** (1969), 1318.  
10) Particle Data Group, 1973, April.  
W. D. Apel et al., Phys. Letters **41B** (1972), 234.  
A. Browman et al., Cornell University Preprint CLNS 224.  
11) T. Matsuoka and S. Sawada. Prog. Theor. Phys. **49** (1973), 1251.  
12) T. Suzuki, Prog. Theor. Phys. **51** (1974), 1893.