

E-learning Investment Risk Management

Georgios N. Angelou & Anastasios A. Economides

UNIVERSITY OF MACEDONIA
Information Systems Department
Egnatia 156
Thessaloniki 54006, GREECE

ABSTRACT

E-learning markets have been expanding very rapidly. As a result, the involved senior managers are increasingly being confronted with the need to make significant investment decisions related to the e-learning business activities.

Real options applications to risk management and investment evaluation of Information and Communication Technologies (ICT) have mainly focused on a single and a-priori known option. However, these options are not inherent in any ICT investment. Actually, they must be carefully planned and intentionally embedded in the ICT investment in order to mitigate its risks and increase its return. Moreover, when an ICT investment involves multiple risks, by adopting different series of cascading options we may achieve risk mitigation and enhance investment performance. In this paper, we apply real options to the e-learning investments evaluation. Given the investment's requirements, assumptions and risks, the goal is to maximize the investment's value by identifying a good way to structure it using carefully chosen real options.

Key words: Decision Strategies, Electronic Learning (E-Learning), Distance Learning, Decision Models, Technology Infrastructures, Capital Investments, Decision Support, Information and Communications Technologies, Project Management, Real Options, Risk Management

INTRODUCTION

E-learning is the delivery and management of learning by electronic means. Various devices (workstations, portable computers, handheld devices, smart phones, etc.), networks (wireline, wireless, satellite, etc.) can be used to support e-learning (Wentling et. all., 2000). E-learning may incorporate synchronous or asynchronous communication, multiple senders and receivers (one-to-one, one-to-many, many –to many, etc.), multiple media and format independently of space and time.

Recently the e-learning markets have been expanding very rapidly and led to an unexpected revelation: the forces affecting higher education around the world are strikingly similar. This is true in at least four important areas: expanding enrollments; the growth of new competitors, virtual education and consortia; the global activity of many institutions; and the tendency for policy makers to use market forces as levers for change in higher education. Expansion of enrollments, accompanied by shifts in student demands and expectations, is a global phenomenon. The number of tertiary students worldwide doubled in size in just twenty years, growing from 40.3 million students in 1975 to 80.5 million students in 1995 (Newman and Couturier, 2002).

Previous research on e-learning cost analysis and investment evaluation does not consider the risk inherent in the business activity (Whalen and Wright, 1999; Downes, 1998; Morgan, 2000). In this work we apply a real option model to identify and control the e-learning investments risks in order to achieve a balance between reward and risk.

The real options approach applies methods of financial planning in investment valuation problems. An investment project embeds a real option when it offers to management the opportunity to take some future action (such as abandoning, deferring, or scaling up the project) in response to events occurring within the firm and its business environment (Trigeorgis, 1996). For example, by taking advantage of the option to defer the investment for some time the management can learn whether

there are better alternative technologies (Li and Johnson, 2002). This management's flexibility (called active management) to adapt its future actions in response to altered future business conditions expands an investment opportunity's value by improving upside potential and limiting downside losses (Trigeorgis, 1999). Business condition either refers to market conditions or firm conditions depending on where the investment is focusing. For example, an investment of an e-learning infrastructure for providing educational services only inside the premises of a big organization mainly refers to firm conditions. On the other hand, an e-learning application, which mainly focuses on providing services in the market (by a university or other institution), refers to market conditions. Figure 1 is a schematic diagram showing the probability distribution of cash flows for a passively versus actively managed project.

----- Figure 1 -----

By adopting the active management philosophy we decrease the possibility of experiencing losses while increase the possibility of gaining. This is achieved by deferring the investment's implementation, learning about the changing business conditions, and generally resolving over time part of the overall investment's risk.

Most previous research considers only ICT investment that embeds a single and a-priori known option. However, real options are not inherent in any ICT investment (Benaroch, 2002), and in any case they are not always easily recognizable (Bräutigam and Esche, 2003). In order to optimally configure an ICT investment it may require considering a series of cascading (compound) options that will help to mitigate risk and enhance economic or strategic performance.

Previous research on investment evaluation has applied real options to ICT, pharmaceuticals and petroleum fields (Iatropoulos et. al, 2004; Mun, 2002). In this paper, we apply real options to the e-learning investments risk management and evaluation adopting a framework, which is presented by Benaroch (2002). The target is to configure the investment using real options analysis in such way that risk is minimized while economic performance is maximized.

For valuating series of cascading options, we start with the log-transformed binomial model (LTBM) finding it easy to use for investments plans that contain more than one options. In this model only the revenues uncertainty is considered, while the cost is certain. In addition, we apply for the first time in the ICT literature (to our knowledge) the extended log-transformed binomial model (ELTBM), presented by Gamba and Trigeorgis (2001) for more complex investments involving both stochastic payoffs and stochastic cost and compound options. We investigate the impact of cost's uncertainty in investment's profitability. We perform sensitivity analysis for revenues' and cost's variance and correlation and examine their influence in investment's performance. In appendix A, we briefly introduce the two option pricing models used in our analysis.

We apply the methodology in a case study based on Mantzari and Economides (2004) work. They present a cost model for an e-learning platform investment and analyze the required number of students (customers) in order to start being profitable (break even point analysis).

The remainder of the paper is organized as follows. In Section 2, we offer background material on real options and how they relate to the e-learning business field. In addition, we present the option-based methodology for managing ICT investments' risks as well as explain the concept underlying the methodology. In Section 3 we apply the ROs methodology to justify and extract the optimum deployment strategy for a specific e-learning infrastructure investment. In Section 4, we examine the influence of the cost uncertainty on the options values as well as on the overall investment's profitability. In Section 5, we discuss about the overall applicability of the methodology as well as present key issues for future research. Finally, in Section 6 we offer some concluding remarks.

REAL OPTIONS IN CONTROLLING ICT INVESTMENT RISK

Real Options Review

An option gives its holder the right, but not the obligation, to buy (call option) or sell (put option) an underlying asset in the future. Financial options are options on financial assets (e.g. an option to

buy 100 shares of Nokia at 90€ per share on January 2007). Real options approach is the extension of the options concept to real assets. A real option is defined as the right, but not the obligation, to take an investment action on a real asset at a predetermined cost for a predetermined period of time. The real option approach to capital investment has the advantage to capture the value of managerial flexibility, which traditional discount cash flow (DCF) cannot properly address. This value is manifest as a collection of call or put options embedded in capital investment opportunities. These options typically include: option to defer, time-to-build option, option to alter operating scale (expand or contract), option to abandon, option to switch, growth option and multiple interacting options.

Spending money to exploit a business opportunity is analogous to exercising an option on, for example, a share of stock. It gives the right to make an investment's expenditure and receive an investment's asset. Real options' thinking considers that investment's asset fluctuates stochastically. The amount of money spent for investment corresponds to the option's exercise price (X). The present value of the project's asset (total gain of investment) corresponds to the stock price (V). The length of time the company can defer the investment decision without losing the opportunity corresponds to the option's time to expiration (T). The uncertainty about the future value of the project's cash flows (the risk of the project) corresponds to the standard deviation of returns on the stock (σ). Finally, the time value of money is given by the risk-free rate of return (r_f). The project's value as calculated by the real option methodology is the same with the value calculated by the Net Present Value (NPV) methodology when a final decision on the project can no longer be deferred (expiration date of the option). Table 1 summarizes the parameters' correspondence between a call option and an investment project.

The total value of a project that owns one or more options is given by Trigeorgis (1999):

$$\text{Expanded (Strategic) NPV} = \text{Static (Passive) NPV} + \text{Value of Options from Active Management (I)}$$

The flexibility value named as option premium is the difference between the NPV value of the project as estimated by the Static or Passive Net Present Value (PNPV) method and the Strategic or

Expanded NPV (ENPV) value estimated by the Real Options method. The higher the level of uncertainty, the higher the option value because the flexibility allows for gains in the upside and minimizes the downside potential.

-----Table 1-----

Option valuation models can be categorized in continuous time and discrete time domains. The most widely applied model in continuous time domain is the Black-Scholes formula, while in discrete time domain the Binomial one. However, continuous time models are not readily applicable for practical valuation purposes or integration with the models in strategic management theory, for example in combining game theory and real options (Smit and Trigeorgis, 2004). For a general overview of real option, Trigeorgis (1996) provides an in-depth review and examples on different real options. For more practical issues the reader is referred to Mun (2002 and 2003). Finally, Angelou and Economides (2004) present an extended survey of real options applications in real life ICT investment analysis.

Risk management with Real Options in E-learning business field

Virtual learning environments are providing teachers with new tools to manage courses and curricular resources, to communicate with students and to coordinate discussions and assessment tasks. Traditional support services such as libraries are changing dramatically; digital collections are overtaking physical collections with students being able to access their services at any time and from almost anywhere. Administrative systems such as student records are being linked to virtual learning environments making for a seamless linkage across administrative and teaching functions. Wiring and internet connectivity have become business critical to the modern university.

New pedagogical approaches are being developed to capitalise on the opportunities afforded by virtual environments and this is necessitating new forms of preparation and support for students and staff. The scope of these developments are extensive, they cut across all areas of institutional functioning and pose significant challenges to senior managers. How are they to make sense of the

range of influence of e-learning developments within their institution and assess the risks associated with these developments? What information will help decision-makers to make strategic choices about where to invest, what to invest and how much to invest? While some institutions have invested heavily in technologies to support learning others have adopted a more cautious approach. These differences in levels of investment depend on a complex mix of internal and external factors – institution's mission, strategic plan, level of technological expertise, staff and student skills in ICT, awareness of the benefits of e-learning and beliefs about what is possible, available funding, attitudes to risk, government policy and funding council initiatives.

The valuation of e-learning business activities is a challenging task since it is characterized by rapidly changing business and technology conditions. Traditional finance theory suggests that firms should use a Discounted Cash Flow (DCF) methodology to analyze capital allocation requests. However, DCF does not properly account the flexibility inherent in most e-learning investment decisions. For example, an e-learning infrastructure project may have a negative Net Present Value (NPV) when evaluated on a stand-alone basis, but it may also provide the option to launch future value-added services if business conditions are favorable. Real options analysis presents an alternative method since it takes into account the managerial flexibility of responding to a change or new situation in business conditions (Trigeorgis, 1996).

ICT investment risks include firm-specific risks, competition risks, market risks, and environmental and technological risks. Firm-specific risks are determined by endogenous factors such as a firm's ability to align its ICT projects portfolio to business strategy and the skill level of its ICT staff.

Competition risks include risks posed by competitors who may make preemptive moves to capture market share or make similar investments that may dilute the value of a firm's current ICT project portfolio. Market risks include uncertainty about customer demand for services that are enabled by a firm's ICT projects (Benaroch, 2002). We adopt this analysis for the e-learning investments too.

For example, an e-learning project may experience more market risk characteristics while another one more firm risk characteristics. Actually, if a project is focusing more on the open market, for

example e-learning services provided by a university, the risks are mainly coming from the market and competition field. On the other hand, when the e-learning service/product is focusing more on internal use by an organization, the risk is more firm specific.

ICT research on real options recognizes that ICT investments can embed various types of real options, including: defer, stage, explore, alter operating scale, abandon, lease, outsource, and growth (Trigeorgis, 1996). Each type of real option essentially enables the deployment of specific responses to threats and/or enhancement steps, under one of three investment modes. *Defer* investment to learn about risk in the investment recognition stage. Such learning-by-waiting helps to resolve market risk, competition risk, and firm risk. *Partial investment* with active risk exploration in the building stage. If we don't know how serious some risk is, investing on a smaller scale permits to actively explore it. *Dis-investment/Re-investment* with risk avoidance in the operation stage. If we accept the fact that some risk cannot be actively controlled, two options offer contingency plans for the case it will occur. The option to abandon operations allows redirecting resources if competition, market or organizational risks materialize. The option to contract (partially disinvest) or expand (reinvest) the operational investment in response to unfolding market and firm uncertainties. In general, the greater the risk, the more learning can take place, and the more valuable the option value is (Benaroch, 2001).

Option based methodology to control market and competition risk

The methodology we present next helps to address the question: What are the real options potentially embedded in an ICT investment that can and ought to be exercised in order to maximize the investment's value?

The methodology involves 4 main steps that must be repeated over time. In what follows, we explain these steps and illustrate them in the context of an e-learning investment case study (Matzari and Economides (2004).

1. *Define the investment plan and its risk.* State the investment goals, requirements and assumptions (technological, organizational, economic, etc.), and then identify the risks involved in the investment. After the definition of the business content by the management the specific risk issues and the analysis of the relationship between those issues should be taken place by the evaluation team. The lifecycle of an investment includes 5 stages. It starts at the inception stage, where the investment exists as an implicit opportunity that was probably facilitated by earlier investments. We call investment during this stage as shadow option. At the recognition stage the investment is seen to be a viable opportunity and we call it real option. The building stage follows upon a decision to undertake the investment opportunity. In the operation stage, the investment produces direct, measurable payoffs. Upon retirement, the investment continues to produce indirect payoffs, in the form of spawned investment opportunities that build on the technological assets and capabilities it has yielded. When these assets and capabilities can no longer be reused, the investment reaches the obsolescence stage, (Benaroch, 2001).
2. *Recognize shadow embedded options based on risk characteristics.* Start by mapping each of the identified investment risks to shadow embedded options that can control them. It may be necessary to reiterate this step to gradually identify compound options, because some options can be the prerequisites or the payoff of some other options.
3. *Choose alternative investment's configurations based on options exercise strategy.* Upon recognizing the shadow embedded options, use different subsets of these options to generate alternative ways to restructure the investment.
4. *Evaluate investment-structuring alternatives to find a subset of recognized options that maximally contribute to the investment's value.* To choose which of the recognized shadow options to create in order to increase the investment value, assess the value of each shadow option in relation to how it interact with other options, in relation to the risks it controls, and in relation to the cost of converting it into a real options. The

project's characteristics are mapped into the option variables. In practice, the DCF projection is rearranged in phases so that the options input values can be isolated. Determine the initial values of the five input variables (V, X, σ, T, r_f), where the variance has to be calculated or estimated. In particular, the variance estimation could be the most difficult task in the overall process. Its estimation can be done either by historical data from other similar projects or by technical estimation such as monte-carlo simulation (Herath and Park, 2002). Investment revenues strongly related to customer demand and product/service price may be results of detailed market survey before final decision. We do not focus on this part of the business analysis and assume that our analysis is starting after obtaining at least partially this information. Starting from the end and going backwards we estimate the option values at each investment stage. We adopt compound option analysis. Finally, we estimate the overall ENPV value, which includes all the embedded options in the selected deployment strategy (selected real options).

The aforementioned steps must be re-applied every new information set arrival when some risks get resolved or new risks surface. Real options analysis assumes that the future is uncertain and the management has the right to change decisions concerning investment deployment strategy when uncertainties become resolved or risks become known. Actually, when some of these risks become known, for example incoming results from a market survey, the analysis should be revisited to incorporate the decisions made or revisiting any input assumptions such as investment variance.

A SPECIFIC E-LEARNING BUSINESS ACTIVITY

Description of a specific E-learning Business Activity and NPV analysis

We examine a business activity to establish an enterprise, which will offer services for learning foreign languages through the World Wide Web (Mantzari and Economides, 2004). The users of our services will be students and adults having access to the Internet. The base scale

investment concerns learning English. It is matter of further growth investment opportunity to provide services for other foreign languages. The courses are developed digitally on a special educational software platform that is purchased to cover the needs of our company and it is installed on the collocated server. Afterwards the users of our services submit their own personal passwords and ID's in order to get connected to the server and attend the lessons through the Internet.

Competitive advantages of such business model for providing distance-learning services comparing to the conventional syllabus are: i) the absence of traditional classrooms which leads to reduced Operating Costs, ii) the absence of traditional way of teaching which reinforces autonomous learning, iii) offering services 24h a day, 7days a week that leads to maximum exploitation while at the same time it is more convenient for the users, iv) flexible pace of attending the lessons, and v) reduced fees due to the continuous functioning and the reduced operating costs.

Some investments assumptions

We examine the investment performance assuming a 11 years period of analysis and assume that all cash inflows and outflows are discounted at the risk-free rate $r_f=5\%$. We consider a risk free rate 5% according to the rate of return on Greeks's Treasury Bills. In addition, we separate the investment's costs, as seen in appendix B in two phases: a) in the initial phase of establishing an e-learning organization, the costs depend mainly on the number of courses (considering a large number of students), b) in the latter phase of operating it, the costs depend on the time duration, on the number of courses and on the number of students, which in addition is divided in fixed and variable cost. We consider as entry time to the market (to implement the investment) when customers (students) demand is such that the operating revenues are equal to the operating costs (Mantzari and Economides 2004).

In appendix C, we present the Cash Flows analysis for the base scale investment. In particular, the base scale investment further to the initial e-learning service provision it mainly contains the infrastructure investment that is able to support up to 1000 students per year. At the entry time in the market to total operating costs are equal to the investment revenues. The computed

present value of payoffs expected from the base scale investment becoming operational in time period $T=3$ is $V_{\text{base scale}}$, which includes (investment revenues – operating costs). As seen in appendix C the NPV of the e-learning infrastructure investment at $T=3$ is -3.000 k€, indicating so the non-profitability of the investment.

Methodology application for an e-learning business activity

Our target is to justify economically the investment of launching e-learning activities in the Greek market. Among others, we have to decide:

1. What is the entry time into the market?
2. What is the scale to enter?
3. What is the optimum way to configure investment in order to minimize risk and maximize profitability (ENPV)?

We follow the aforementioned four steps:

Step1. To define the investment plan and its risk

Here we define the investment content, goals and requirements. We start with an initial ICT solution, stating investment assumptions (economic, technological, organizational, etc.), and revealing the investment risks in light of these assumptions. These activities should be carried out relative to each of the stages in the investment lifecycle. In our case, we consider the recognition, building, and operation stages, and the involved risks that fall into these stages, Table 2.

-----Table 2 -----

One is environmental risk. There is much uncertainty about the customer demand. Low customer demand can change investment profitability from positive to negative. Another is firm-specific capability risk. There is uncertainty about the firm's capability to integrate efficiently the initially planned scale of the ICT infrastructure with the required applications as well as with the content of them. Finally, the last area is competition risk as a competitor could react by launching improved applications that will erode revenues from future customers.

We initially assume that all these risks affect only the expected revenues and not cost. Actually, cost influences directly the revenues too. Afterwards, we examine the impact of the cost's uncertainty on the investment's profitability.

Step2. Recognize shadow options based on risk characteristics

In the next step, we recognize shadow options that the investment could embed based on the aforementioned investment risks. The target is to configure the investment plan by using these options in a way that risks are mitigated while overall profitability is maximized. Actually, investments risks can be, at least partially, handled by adopting managerial flexibility, through option analysis. Table 3 shows the main sources of the risks of the e-learning investment that we examine in this paper and the shadow options that we adopt in order to control them.

-----Table 3-----

Step3. Choose alternative investment configurations based on options exercise strategy

In the next step we identify alternative ways to configure the e-learning investment using different subsets of the recognized shadow options. Although, it may seem that the number of possible configurations could be large, only configurations involving maximal subsets of shadow (viable) options are worth considering (Benaroch, 2002). We next illustrate plausible investment configuration that considers five of the recognized shadow options, Figure 2.

Business assumptions

We assume that market entry takes place when demand level reaches the critical number of students and the *Investment Operating Revenues are equal to Operating Costs (we assume that this is reached at year T)*.

We start our analysis considering that T is up to 3 years. We also consider that the construction phase of our platform is 1 year. Finally, we consider that critical mass for customers is reached at T=3. At the beginning, recognition phase, we face the option to defer investment up to time T in order to resolve market uncertainty concerning customers demand as well as competition threat.

The smaller the T the sooner we should perform investment and the smaller the option value to defer will be, since less amount of uncertainty is resolved. During time period T the firm is facing market uncertainty “clearness” and decides to enter the market when investment starts to become profitable.

-----Figure 2-----

Options Presentation

Our configuration considers five of the recognized shadow options (see Figure 2). In this work we consider only this deployment path. Additionally, we could consider other alternatives too, such as the deployment path that includes only the option to explore business activity. The option to explore would facilitate learning-by-doing, through a pilot effort that supports a part of the E-learning services while in case of favorable demand the full scale of the the business activity takes place. In our analysis we consider a more complicated deployment path in order to control high number of risks and provide more realistically the applicability of our methodology. Finally, the high number of shadow options that transformed to real ones does not necessarily indicate the maximum investment value since many of the options can control the same type of risks. In this case options are supplementary to the contribution of the overall investment value (Trigeorgis, 1996).

During the recognition stage

The first option is to defer the first cost outlay for up to three time periods (assuming that longer deferral would significantly increase the risk of competitive preemption). Deferral permits learning about the levels of demand experienced by other firms with comparable e-learning services, in support of resolving risks E1, E2 and E3. Deferral could also provide the time to get the cooperation of all parties so as to reduce risks F1 and P1. During the deferral period the firm faces the market uncertainty “clearing” especially concerning demand considering the trigger point to start investing when expected revenues becomes equal to investment’s operating costs. Finally, competition threat, risk C1, from another firm can be at least partially resolved during deferral period.

During the building stage

The building firm's staff may lack experience with linking ICT technologies to content applications such as educational issues. In addition, the firm may build the application right according to the required specifications, but still fail to realize the anticipated benefits because the requirements are wrong to begin with. This could result in poor application functionality, risks F1, P1. In order to control these risks we consider the option to contract, the initially planned, investment scale during the building stage. In addition, competition risk, C1, e.g. a competitor's response eliminates the firm's advantage, is reduced through option to contract the initial planned investment scale. Moreover, customers' uncertainty E1 during the building stage can be mitigated by adopting the contract option. Finally, the aforementioned option to defer enhances possibilities of mitigating such kind of risk during this stage, too.

During the Operating stage

The next option is the option to expand operations scale by 30% in year T+2 in case of favorable demand and risk mitigation E2 and E3. The last option is actually a combination of one call option and one put option having the same time to maturity. In time T+4 the firm possesses the option to choose between to expand or contract operation scale according to market conditions. Actually, the second option is to contract operations of the investment, by 25%, at time period T+4, in support of hedging risks E1 and E3. At the same time there is the call option to expand operations in case of high demand by making a fourth cost outlay. This option could control demand risk E2. In general a call option is optimally exercised when circumstances become favorable and a put option is exercised when circumstances become unfavorable. Finally, competition risk C2 can be hedged through the option to choose between contracting or expanding the investment scale according to the competitors actions that could either eliminate firm's market share or just influence the overall market demand for such kind of applications.

Step4. Options evaluation and Investments configurations alternatives profitability

In the final step, we evaluate embedded options included in the configuration alternatives. We initially assume that only revenues V are uncertain. We adopt the LTBM because it simplifies the valuation of compound options, Trigeorgis (1996). In addition, we apply for the first time in the ICT literature (to our knowledge), the ELTBM. The ELTBM presented by Gamba and Trigeorgis (2001) is suitable for complex investments involving both stochastic payoffs and stochastic costs and compound options. Actually, we examine the influence of both cost and revenues uncertainty on the overall investment's profitability for base scale e-learning infrastructure investment. We investigate the impact of cost uncertainty in the investment profitability, making sensitivity analysis for Revenues and Costs Variance and Correlation.

Option analysis and specific investments characteristics map

For the valuation of options we use the Log-Transformed Binomial Model (LTBM) with 50 steps time resolution (Gamba and Trigeorgis, 2001). Also, the variance of payoffs is considered at $\sigma = 50\%$ adopting similar to the literature values (Oslington, 2004; Angelou and Economides, 2005). The valuation of separate options is given below (Trigeorgis, 1996).

Option to Defer up to T

The option to defer is basically valued as an American call option on the project. The time T for entry in the market is defined as the time when the investment operating cost is equal to the investment revenues. Option Value to Defer, $OV(D)$, is given by:

$$OV(D) = \max (V - X_t, 0) \quad (2)$$

As seen in Table 4 the managerial flexibility to defer the investment up to three years, in order to wait for the resolvance of the customers demand, is 53,6 k€.

Option to Contract at T+1

During the implementation phase the firm possesses the option to contract the initially planned operation by 20% when the market conditions become unfavorable or the firm's capability to develop the project is inferior to the expected one. The option is valued as European Put option.

Option Value to Contract operations, $OV(C)$ is given by:

$$OV(C) = \max (X_c - c * V, 0) \text{ where } c = 20\% \quad (3)$$

We consider that contracting the operation by 20% will result in $X_c = 30$ k€ of operation savings and its option value will be 3,1 k€.

Option to Expand at $T+2$

In case of favorable demand the firm can expand its operation by 30%. Here, for simplicity we assume that the expected revenues will also increase by 30%. The option value to expand, $OV(E)$, is valued analogously to an European Call option to acquire part of the project by paying an extra outlay as exercise price X_3 . It is given by:

$$OV(E) = \max (e * V - X_3, 0) \text{ where } e = 30\% \quad (4)$$

while its value is 12,6 k€.

Option to choose between to expand and contract operations at $T+4$

The option to expand (alone) (scale up operations) by e' , the initially planned scale of project, is valued analogously to a European call option to acquire part of the project by paying the extra outlay X_4 as an exercise price.

The option to contract (alone) (scale down operation) by c' is valued as an European put on part of the project, with exercise price equal to the potential cost savings X_5 due to this operation contraction. The option value to choose between contraction and expansion, $OV(CH)$, is given by

$$OV(CH) = \max (e' * V - X_4, X_5 - c' * V, 0) \quad (5)$$

and its value is 34,8 k€.

Value of option Combinations with Interactions between each other

The value of an option in the presence of other options may differ from its value in isolation because of its strong interaction with these options. Trigeorgis (1993) offers a formal discussion of the factors affecting the non-additivity of options. We follow the compound options valuation process as presented by Herath and Park (2002). However, in our case, only the option to defer is prerequisite for the availability of the following options. This means that in order to possess one or

all of the following options we should first adopt the option to defer, fact that is very well understood since this indicates the initiation of the base scale investment. For the rest of the options, each option exercise is not a prerequisite for the possession of the next option.

The valuation of complex options remains a difficult endeavor. Since e-learning investments could be exposed to multiple risks, they may need to be configured using a series of cascading (compound) options. Standard valuation models (e.g., Black-Scholes model) ignore the fact that the value of individual options in a series of cascading options may be lowered or enhanced by interactions with other options. Here, we use the LTBM to simplify the valuation of cascading options. In addition we test the ELTBM in order to consider both revenues and cost uncertainty. Table 4 shows the value of the project with different combinations of the shadow options. In particular, the higher option value, in isolation, is the option to defer, which its value is 53,6 k€. We give for comparison the values of the rest of the options, in isolation, at time where the operation stage is starting. The option to choose the strategy between contraction and expansion in year 7 presents the highest value, 34,8 k€ from the rest of the options. Actually, the option to choose between expansion and contraction is the sum of the two separate options, the call option to expand and the put option to contract with the same expiration dates.

In our multioption analysis we consider that the option to Defer is prerequisite for the rest of the options. This means that the option to defer should be included in any of the combinations of the embedded options that we analyze.

Adopting the two-embedded options analysis in the investment plan we can see that the option to Contract contributes negatively to its predecessor option to Defer since their combined value is 38,3 k€. This happens due to the fact that in case of exercising the contract option, the revenues V that correspond to the initially planned base scale investment, will be decreased. We consider that by contracting operation by 20% we have 10% decrease of the initial infrastructure cost, since the infrastructure is the basis and prerequisite for a range of future operating capabilities. On the

contrary, the contribution of the option to Choose to the option to Defer is higher giving a value close to 64,2 k€.

In case of three options analysis for more efficient risk handling, the combination of options to Defer, Expand and Choose gives the higher value of about 67 k€. Finally, taking into account the total number of options the overall value is just 50 k€, since the option to contract operations contributes negatively to revenues V base scale of the option to Defer as in the two options analysis. Hence, the most promising configuration deployment strategy is the combination of the options to DECH that presents the highest value for the investment profitability.

-----Table 4-----

COST & REVENUES UNCERTAINTIES CONSIDERATION

In the following, we apply the ELTBM for more complex investments involving both stochastic payoffs and stochastic costs. We base our analysis on compound options in order to evaluate the highest pay off scenario. We investigate the impact of cost uncertainty in the investment profitability, making sensitivity analysis for various values of revenues' and cost's variance and correlation. It is the first time in ICT literature where both costs and revenues uncertainties are considered in compound ROs analysis. However, the complexity of the model is increasing as the number of steps is increasing. For this reason we examine the case for one time step, as our purpose is to show intuitively the influence of cost uncertainty in investment's performance, Figure 3. Though the complexity of our model is increasing the always increasing computing power can handle this complexity efficiently (Trigeorgis, 1996). In practice, the single-step analysis is appropriate for investments where management has limited opportunity to influence the outcome of the investment and reviews investment status per half or year. On the opposite, in case of large enterprise projects where there is a significant opportunity during the life of the project for management to influence the expected value of the project cash flows, a more realistic solution

would use a multiple steps analysis. In this case, management reviews quarterly and even weekly, and risk events will impact the project with a random periodicity. In conclusion, the frequency of management review for the investment status, such as customers demand, indicates the number of steps to be taken into account.

-----Figure 3-----

We use the 1 step LTBM to calculate the ENPV and compare it with the ELTBM where both revenues and investment cost uncertainty are considered. However, this is not a problem since, the ELTBM appears to be more stable for small number of steps (here 1 step) compared to the single LTBM and especially for large values of cost's and revenues' variances. In addition, Gamba and Trigeorgis (2001) verify that the correlation between costs and revenues change plays an important role in having positive up and down probabilities for cost's and revenues' assets diffusion process. Actually, if the revenues and costs are uncorrelated then the log-transformed up and down probabilities in the lattice analysis are strictly positive. In our case, we assume a variance for cost 30% and a correlation between revenues and cost, $\rho_{vx} = -0.2$. We consider the stochastic changes in the asset value to be correlated with the stochastic changes in the investment cost. In particular, a negative ρ_{vx} could represent, for instance, that the inability to control the cost of the development project are associated with lower revenues after the project is completed. In Table 5, we present the results of our analysis for the scenario that involves the options to Defer at $t=3$, to Expand at $t=5$ and to Choose between to expand and contract at $t=7$. As we can see the options values, either in isolation or in combination of the optimum scenario for investment deployment strategy, are higher in case of considering both cost's and revenues' uncertainties.

-----Table 5-----

In addition, we have evaluated the impact of higher or lower variance, higher or lower correlation with respect to the base case, to the option to Defer investment up to the moment where the demand uncertainty will be at least partially resolved. The results are shown in the following Table 6:

-----Table 6-----

As it can be seen, a negative correlation $\rho_{v,x}$ contributes to higher option value. In addition, for zero cost uncertainty, the base scale option to defer value, calculated by the one step ELTBM, approaches to the base scale option to defer value calculated by the 50 steps LTBM and no revenues uncertainty. This proves the 1 step ELTBM stability giving comparable results to the 50 steps LTBM. Finally, uncorrelated assets (V,X) give an option value equal to the base case for cost variance less than revenues variance. However, as the cost's variance increases above the revenues' variance the option value increases respectively.

DISCUSSION AND FUTURE RESEARCH

The methodology we presented enables management to optimally configure technology investments. It facilitates a systematic identification of investment's configurations by framing flexibility in terms of risks that real options can control. Otherwise, it supports a solid quantitative configuration valuation for the purpose of identifying the most valuable configuration. This does not mean that the methodology is perfect. One of the main difficulties is the way we estimate the variances of investment's revenues and cost.

The methodology has been applied in an e-learning case study. It can be quite easily extended to other ICT business fields. For example Angelou and Economides (2006) apply ROs analysis to find optimum investment deployment strategy in Broadband investments business field under competition threat that can eliminate part of the business value during deffering period.

In general, the method can be applied in business cases where investments contain wait and see components (deffering periods) as well as risk issues that can be controlled and partially resolved by real options analysis. In case of competition, it is matter of compensation between uncertainty control achieved by the real options analysis and competition threats from other competitors that can enter sooner into the market, while the firm under investigation is waiting, and eliminate the available investment value. Under this analysis, the competitors can arrive randomly following a Poisson distribution (Trigeorgis, 1996). This is more valid in case of high number of comptitors

(players) where exogenous competition modeling is more practical. In particular, it can be considered that there is an e-learning platform that can support a number of e-learning courses, to similar scientific fields, provided by the firm (institution) of interest. However, other organizations, universities, can also provide similar, courses causing a degradation to the investment opportunity, which is available to the organization of interest.

In case of limited number of competitors (oligopoly) endogenous competition modelling is required adopting the real options with game theory. In this case, each of the players (competitors) will choose their optimum investment deployment strategy. The game equilibrium will be the deployment strategies or real options implementation, which will maximize utility of each of the players. It is subject of further work to consider a real competitive environment and customize or enhance existing real options models evaluation based on compound options analysis under endogenous competition modeling.

Finally, the proposed methodology may also be compared with other previous studies, such as Scott-Morton's MIT90s framework (Scott Morton, 1991), which it was used to analyse the effects of developments in information technology on business organisations. It has been also used more recently to examine how higher education institutions in Australia were managing the introduction of technology to deliver and administer education (Yetton, 1997). Scott-Morton's MIT90s framework assumes that an institution's effectiveness in the use of ICT for teaching and learning is a function of six inter-related elements:

1. the *external environment* within which the institution is operating
2. the institutional *strategy* in relation to ICT in teaching and learning
3. the way human resources are prepared and deployed (*individuals and their roles*) to support the implementation of ICT in teaching and learning
4. the organisational *structures* that support the application of ICT to teaching and learning
5. the characteristics of the *technology* being applied.

6. the *management processes* that facilitate the initiation, sustainability and success of the application of ICT in teaching and learning.

Our model can be used for risk recognition and its control with real options in external environment concerning competition, customers demand and technology uncertainty. In addition, management processes, human resources allocation and organizational structure analysis may include real options analysis to optimally configure investment's deployment strategy and control firm's specific uncertainty.

CONCLUSION

In this work, we present a real options methodology for controlling risk and choosing the optimum ICT investment's deployment strategy. We apply it in e-learning infrastructure business field (Mantzari and Economides, 2004). The target is to find the optimal investment's configuration, to handle more efficiently the investment's risk and so to increase its overall performance.

The results of our analysis show that by adopting multioption analysis in a compound basis can enhance investment performance. The specific e-learning investment scenario appears to be more profitable when we adopt real options analysis instead of NPV analysis, taking into account the same business assumptions given by Mantzari and Economides (2004) case study.

In addition, we apply both revenues' and cost's uncertainties modeling estimating the impact of the investment's cost uncertainty to the options' value as well as to the overall economic performance.

The e-learning investment's profitability appears even higher. Actually, as the project uncertainty is increasing, the managerial flexibility achieved by adopting real options contributes more to the final economic performance.

Finally, it is the subject of further work to consider a real competitive environment and customize or enhance existing real options models evaluation based on compound options analysis.

APPENDIX A

LTBM

LTBM has been proposed to overcome problems of consistency, stability and efficiency encountered in standard binomial model (Gamba and Trigeorgis, 2001). Whereas the binomial model views the behavior of V (the underlying asset or investment value) as being governed by a multiplicative diffusion process, the log-transformed binomial model transforms this process into an additive one. Actually, instead of looking directly at V , the log-transformed binomial model looks at state variable $S = \log V$. The log-transformed binomial algorithm consists of four main steps: parameter value specification, preliminary sequential calculation, determination of terminal values, and backward iterative process.

First, the standard parameters affecting option values (i.e., V , r , σ^2 , T , and the set of exercise prices or investment cost outlays X) are specified along with the desired number of subintervals, N . The greater N is chosen, the smaller the number of subintervals and the more accurate the numeric approximation is although at the expense of more computer time (and potentially growing approximation errors).

The second step involves preliminary calculations needed for the rest of the algorithm. Using the values of variables calculated along the way from preceding steps, the algorithm sequentially determines the following key variables:

1. time-step: $k = \sigma^2 T / N$;
2. drift: $\mu = (r / \sigma^2) * 1/2$;
3. state-step: $H = \sqrt{k + (\mu k)^2}$;
4. probability: $P = (1/2) * (1 + \mu k / H)$.

The third step involves the determination of terminal boundary values (at $j = N$), where j denotes the integer number of time steps with length k . For each state i , the algorithm fills in the underlying asset (project) values from $V(i) = e^{S_0 + iH}$ (since $S \equiv \ln V = S_0 + iH$); and the total investment opportunity values (or expanded NPV) from the terminal condition $R(i) = \max(V(i), 0)$. The integer index i of the

stage variable S is corresponding to the net number of ups less downs. $R(i)$ denotes the total investment opportunity value (i.e., the combined value for the project and its embedded real options) at state i .

The fourth step follows a backward iterative process for the estimation of total investment value $R(i)$ at state i . Starting from the end ($j = N$) and working backward for each time-step j ($j = N-1, \dots, 1$) we calculate the total investment opportunity values. Between any two consecutive periods, the value of the opportunity in the earlier period (j) at state i , $R(i)$ is determined iteratively from its expected end-of-period values in the up and down states calculated in the previous time-step ($j + 1$), discounted back one period of length $\tau = k/\sigma^2$ at the risk-free interest rate r_f ,

$$R_{(i)} = \frac{PR_{(i+1)} + (1-P)R_{(i-1)}}{1 + r_f k / \sigma^2} \quad (A1)$$

Assuming one-step diffusion process the call option can be written as

$$\begin{aligned} C &= \frac{PC_u + (1-P)C_d}{1 + r_f k / \sigma^2} = \\ &= \frac{P \max[uV - X, 0] + (1-P) \max[dV - X, 0]}{1 + r_f k / \sigma^2} \end{aligned} \quad (A2)$$

where the state rise and fall parameters are $u = e^H$, $d = 1/u$

ELTBM

The ELTBM values real options whose payoffs depend on several state variables (i.e. cost and revenues diffusion processes). Actually, it is an extension of the LTBM taking into account multi-dimensional diffusion processes for investment's variables such as cost and revenues.

The methodology is similar to the previous one. However, in this case complexity is increasing depending on the number of the diffusion process. In particular, considering a two-dimensional diffusion process the respective parameters are given by:

1. $k_s = \sigma^2 T/N$;
2. $\mu_s = (r/\sigma^2)^{1/2}$;
3. $H_s = \sqrt{k + (\mu k)^2}$;

$$4. R_{s,s'} = k_s k_{s'} / (H_s H_{s'})$$

$$5. M_s = k_s^2 \mu_s / H_s$$

$$6. s, s' = 1, 2.$$

Note that states rise and fall parameters are $u_s = e^{H_s}$, $d_s = 1/u_s$

The respective probabilities P are given by the following expressions. We also present their meaning assuming revenues and costs state variables)

$$P_1 = P_{uu} = (1 + (R^* \rho + M_1 M_2) + M_1 + M_2) / 4 \text{ (revenues rise, cost rises)}$$

$$P_2 = P_{ud} = (1 - (R^* \rho + M_1 M_2) + M_1 - M_2) / 4 \text{ (revenues rise, cost falls)}$$

$$P_3 = P_{du} = (1 - (R^* \rho + M_1 M_2) - M_1 + M_2) / 4 \text{ (revenues fall, cost rises)}$$

$$P_4 = P_{dd} = (1 + (R^* \rho + M_1 M_2) - M_1 - M_2) / 4 \text{ (revenues fall, cost falls)}$$

where $\rho = \rho_{12}$ is the correlation between revenues and costs and $R = R_{12}$.

Finally, the value of the call option C can be written as:

$$C = \frac{P_{uu} C_{uu} + P_{ud} C_{ud} + P_{du} C_{du} + P_{dd} C_{dd}}{1 + r_f k / \sigma^2} = \frac{P_{uu} \max[uV - uX, 0] + P_{ud} \max[uV - dX, 0] + P_{du} \max[dV - uX, 0] + P_{dd} \max[dV - dX, 0]}{1 + r_f k / \sigma^2} \quad (A3)$$

APPENDIX B

-----Table 7-----

APPENDIX C

-----Table 8-----

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Figures

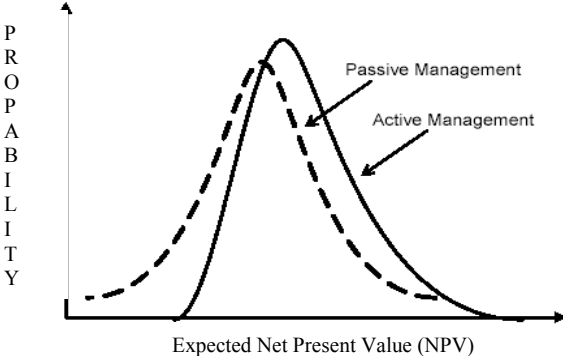


Figure1. Uncertainty under passive and active management of the investment project, (Trigeorgis, 1996).

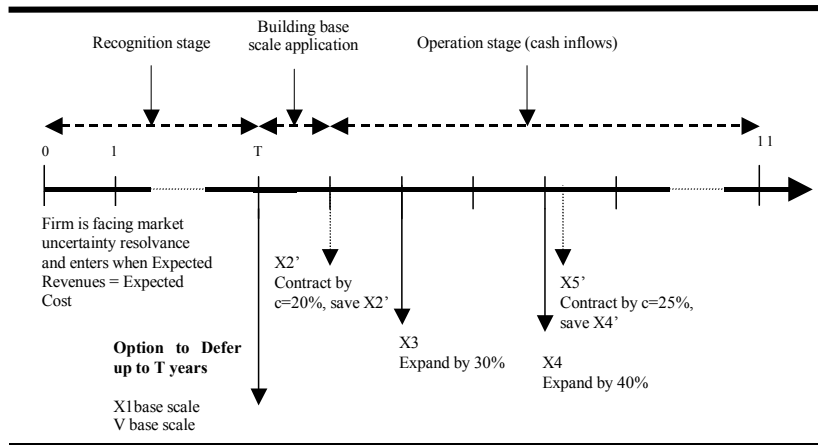


Figure 2: A configuration involving five of the shadow options that the e-learning investment can embed

1. The **base scale** option permits realizing the investment into one cost outlay $X1 = 190.000 \text{ €}$, which is **deferrable** for up to three years, in order to resolve market uncertainty.
 2. The option to **contract** the initially planned scope of operations by 20% saving so in cost operations $X2' = 30.000 \text{ €}$.
 3. The option to **expand** further operations in case of favorable customers demand by 30%, by making a third cost outlay, $X3 = 55.000 \text{ €}$ (it is one third of initial infrastructure investment)
 4. The options to choose between expand and contract operations.
 The **expand** option permits scaling up operations by 40%, by making a fourth cost outlay, $X4 = 75.000 \text{ €}$, for the base scale
 The option to **contract** scope of operations by 25% saving so in cost operations $X5' = 35.000 \text{ €}$.
- Option exercise costs and revenues to expand – Contract operation presented above concerns base scale operations and single options analysis. In case of compound option analysis expand and contract values as well as option revenues are changing according to predecessor option type.

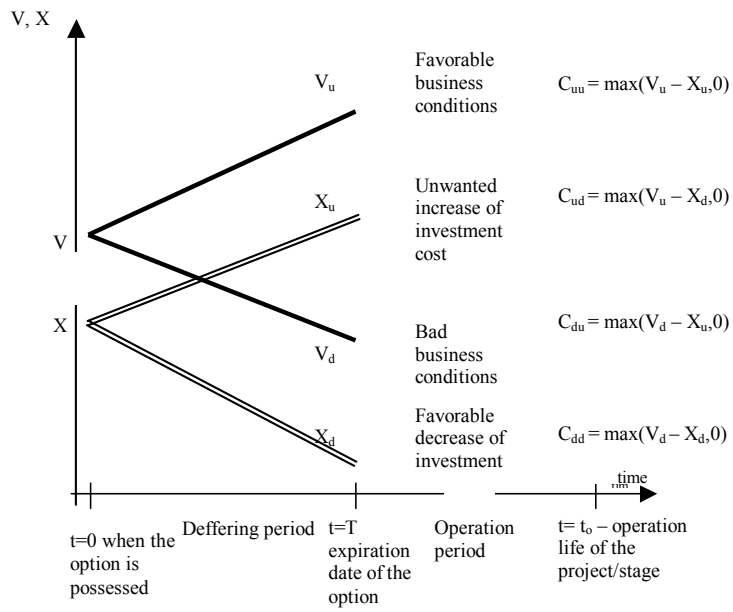


Figure 3. Revenues and Cost diffusion process, one time step
 Option value at t=0 is given in Appendix A (A3)

Tables

Investment Opportunity	Variable	Call option
Present value of a project's assets or Present Value of cash flows from investment (Revenues)	V	Stock price
The amount of money spent for the investment, Investment expenditure required to exercise the option (cost of converting the investment opportunity into the option's underlying asset, i.e., the operational project)	X	Agreed Exercise price of the Option
Length of time where the investment's decision may be deferred	T	Option's time to expiration (i.e., the maximum length of the deferral period).
Time value of money	r_f	Risk-free rate of return
Variance (Riskiness) of the investment's project assets (Costs, Revenues)	σ^2	Variance of returns on stock

Table 1. Parameters' analogy between a call option and an investment opportunity

Stage	Goals	Risks and Opportunities
Recognition	To establish an enterprise, which will offer services for learning foreign languages through the World Wide Web	Environmental (E1) - Low customer/student demand that might not be profitable to let investment pass from the Recognition to Building stage. Firm has to decide when to enter in the market and in what scale.
Building	The initial e-learning solution involves developing an infrastructure platform that will support languages distance learning services	Project (P1)/Organizational (O1) - Firm staff may lack experience with linking ICT technologies with content applications such as educational issues. Functionality (F1) - The firm may build the application right according to the required specifications, but still fail to realize the anticipated benefits because the requirements are wrong to begin with. This could result in poor application functionality
Operation	Support e-learning services for foreign languages	Environmental (E1) - low customer demand could make it non economical to let the investment live long. Environmental (E2) - demand exceeds expectations (follow-up opportunities exist) Environmental (E3) - too high customer demand could result in an inability of the back office of the firm to handle the extra processing load presented by customers/students Competition (C1) - competitors could react by launching an improved application, and thus erode the extra demand generated produced by the e-learning application

Table 2: First step of the approach applied to the e-learning investment

Risk Area		Risk Opportunity	Investment Lifecycle Stages - Shadow Options Allocations			
			Recognition	Building	Operation	
			Option to Defer	Option to Contract scale of Investment	Option to Expand	Option to Choose between further Expansion and Contraction
Firm Specific Risks	Project	P1 staff lacks needed technical skills to successfully intergate and operate ICT infrastructure-applications with content	+	+		
	Functionality	F1 wrong design (e.g., analysis failed to assess correct requirements)		+		
Market Specific Risks	Competition	C1 competition's response eliminates the firm's advantage	+	+		+
	Environmental	E1 low customer demand, with inability to pull out of market	+	+		+
		E2 demand exceeds expectations (follow-up opportunities exist)	+		+	+
		E3 too high customer response may overwhelm the application	+		+	+

Table 3: E-learning investment risks mapped to operating options that could mitigate them

Option Combination	Option Value LTBM 50 (values in k€)	ENPV (overall investment value)	Value at
Defer (D)	53,6	50,6	t=0
Contract (CN)	3,1	0,1	t=T
Expand (E)**	12,6	9,6	t=T+1
Option to Choose (CH)*** (expand/contract)	34,8	31,8	t=T+1
DCN 1	38,3	35,3	t=0
DE 2	57	54	t=0
DCH 3	64,2	61,2	t=0
DCNE 4	42	39	t=0
DCNCH 5	48	45	t=0
DECH 6	67	64	t=0
DCNECH 7	50	47	t=0

Option Name	Exercise Price (for base scale)	PV(V) base scale	Year to maturity	Option Type
Defer (D)	190	161 (187)	up to T=3	American Call
Contract (CN) (20%)	30	37	T+1	European Put
Expand (E) (30%)	55	56	T+2	European Call
Option to Choose (CH) (expand/contract) (40%/25%)	75/35	74/46	T+4	European Call/Put

In this case we consider that base scale investment results to $V'=0,8V$ while $X'=0,9X$

** Option to Expand at time T+2, value at t=T
 *** Option to Choose at time T+4, value at t=T+2
 **** Option to Switch use between T+5 and 11, at t=T+4
 ENPV = Option Value x 1.000 - 3.000

1 $\max(0,8V+CN-0,9X, 0)$
 2 $\max(V+E-X, 0)$
 3 $\max(V+CH-X, 0)$

4 $\max(0,8V+\max((Xc+E-0,2V,E)-0,9X\text{basescale}, 0), \dots$
 5 $\max(0,8V+\max((Xc+CH-0,2V,CH)-0,9X\text{basescale}, 0), \dots$
 6 $\max(V+\max((eV+CH-Xe,CH)-X\text{basescale}, 0) \text{ where } e=0,3 \dots$
 7 $\max(0,8V+\max((Xc+\max(eV+CH-Xe, CH)-cV,\max(eV+CH-Xe, CH))-0,9X\text{basescale}, 0) \dots$

To mention here that these expressions do not give the value of three options all together since all are exercised in different time moments.

With this we want to indicate the logical model that we follow based on nested option analysis as presented by Herath and Park 2002

Finally, to mention that values for nested options are at times where their predecessor option is exercised.

In our analysis only option to Defer exercise is prerequisite for the next options

We consider the Option to Defer and the Option to Contract
 We consider the Option to Defer and the Option to Contract
 We consider the Option to Defer and the option to Choose between E and CN

Table 4: comparative value contribution of options in the investment alternatives.

Options	Option Value LTBM 50 steps (values in k€)	Option Value LTBM 1 step (values in k€)	Option Value ELTBM 1 step (values in k€)
Defer (D)	54	65	65
Expand (E)	12,6		15,8
Option to Choose (CH) (expand/contract)	34,8		47
DECH	67	88	77,2

Base case parameters

Revenues Variance (Volatility) 50%

Costs Variance (Volatility) 30%

Correlation $\rho_{vx} = -0,2$

The rest of parameters are as before

Table 5: Option values comparison between Revenues uncertainty only and Cost-Revenues uncertainty consideration

Option to Defer base scale investment	ρ_{vx}	Vbase scale variance (volatility) σ_v (%)	Xbase scale variance (volatility) σ_x (%)	Comments
13	1	50	30	
35	0,5	50	30	
56,3	0	50	30	
65	-0,2	50	30	
78	-0,5	50	30	
99	-1	50	30	
55,8	-1	50	0	In approximates the LTBM with 50 steps where no cost uncertainty is considered
55,8	-0,2	50	0	
55,8	1	50	0	
55,8	0	50	0	
55,8	0	50	5	
56	0	50	15	
56	0	50	25	
56	0	50	40	
59	0	50	45	
64	0	50	50	
73	0	50	60	
86	0	50	70	
97,4	0	50	80	
57,2	-0,2	50	5	
62	-0,2	50	20	
98	-0,2	50	70	

Table 6: Option value to Defer base scale investment for various values of cost-revenues correlation and volatilities

Cost Description	Value	Cost Category	Comments	Out Flows Distribution*	Total value**	
Switch	385,65 €	LAN & INTERNET CONNECTION COSTS	Infrastructure	At once	385,65 €	
Router	154,74 €	LAN & INTERNET CONNECTION COSTS	Infrastructure	At once	154,74 €	
UPS	1.050,03 €	LAN & INTERNET CONNECTION COSTS	Infrastructure	At once	1.050,03 €	
Server	5.221,27 €	COLLOCATION HOSTING COSTS	Infrastructure	At once	5.221,27 €	
Operating System	1.356,90 €	COLLOCATION HOSTING COSTS	Infrastructure	At once	1.356,90 €	
Workstations	15.968,00 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	15.968,00 €	
LAN cards	561,28 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	561,28 €	
Ms Office Xp	4.418,56 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	4.418,56 €	
Printers	601,68 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	601,68 €	
Scanners	359,34 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	359,34 €	
Zip Drives	1.891,36 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	1.891,36 €	
Zip Disks	290,56 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	290,56 €	
Cd-R	37,50 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	37,50 €	
Cd-RW	16,00 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	16,00 €	
Office staff	300,00 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	300,00 €	
Laser Toners	153,00 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	153,00 €	
Paper for Printers & FAX	94,20 €	HEADQUARTERS OFFICES COSTS	Infrastructure	At once	94,20 €	
Desks	3.200,00 €	Fixed equipment costs	Infrastructure	At once	3.200,00 €	
Desk Chairs	3.200,00 €	Fixed equipment costs	Infrastructure	At once	3.200,00 €	
Chairs	1.600,00 €	Fixed equipment costs	Infrastructure	At once	1.600,00 €	
Bookshelves	600,00 €	Fixed equipment costs	Infrastructure	At once	600,00 €	
Fax, Copier	2.000,00 €	Fixed equipment costs	Infrastructure	At once	2.000,00 €	
Pedagogical & Administrative Training	4.500,00 €	PREPARATION COSTS	Infrastructure	At once	4.500,00 €	
Installation on a dedicated server	3.000,00 €	PREPARATION COSTS	Infrastructure	At once	3.000,00 €	
Technician's Training	2.000,00 €	PREPARATION COSTS	Infrastructure	At once	2.000,00 €	
*** Educational Software's licenses	55.000,00 €	COLLOCATION HOSTING COSTS	Variable Cost	At once	55.000,00 €	
Total (infrastructure investment cost)	107.960,07 €	Cost outlayed at once		At once	107.960,07 €	
ADSL (per year)	618,96 €	LAN & INTERNET CONNECTION COSTS	Operating Fixed	Yearly		
ADSL Internet (per year)	1.140,00 €	LAN & INTERNET CONNECTION COSTS	Operating Fixed			
Collocation Hosting (per year)	13.206,24 €	COLLOCATION HOSTING COSTS	Operating Fixed			
Rent	12.000,00 €	Operating Costs (annual)	Operating Fixed			
Electricity supply	840,00 €	Operating Costs (annual)	Operating Fixed			
Telephone	2.400,00 €	Operating Costs (annual)	Operating Fixed			
Water supply	360,00 €	Operating Costs (annual)	Operating Fixed			
Heating	1.200,00 €	Operating Costs (annual)	Operating Fixed			
Chairman	30.000,00 €	Salaries & wages	Operating Fixed			
Financial Manager	36.000,00 €	Salaries & wages	Operating Fixed			
Marketing Manager	30.000,00 €	Salaries & wages	Operating Fixed			
Technical Administrator	6.420,00 €	Salaries & wages	Operating Fixed			
Financial Services Employee	12.000,00 €	Salaries & wages	Operating Fixed			
Help Desk Employee	12.000,00 €	Salaries & wages	Operating Fixed			
Accountant	3.600,00 €	Salaries & wages	Operating Fixed			
Tutors (case 1)	98.380,80 €	Salaries & wages	Variable Cost			
Furniture & Fixed Equipment	2.760,00 €	Amortizations****	Operating Fixed			
Hardware	9.167,67 €	Amortizations****	Operating Fixed			
Total***** (Present Value)	272.093,67 €					1.700,00 €
Operating cost		Cost outlayed Annually			Yearly	
Total at investment time T	380.053,74 €					

Table 7. Cost Structure

Base scale Cash Flows Analysis

Year	0	1	2	3	4	5	6	7	8	9	10	11	
Infrastructure costs (building stage)	0	0	0	190.000	0	0	0	0	0	0	0	0	
Operating fixed costs	0	0	0	0	174.000	182.700	191.835	201.427	211.498	222.073	233.177	244.835	
No of students/Customers ²	0	0	0	0	585	644	708	779	856	942	950	950	
Operating variable costs/student	0	0	0	0	2,15	2,26	2,37	2,49	2,61	2,74	2,88	3,03	
Operating variable costs	0	0	0	0	1.258	1.453	1.678	1.938	2.238	2.585	2.737	2.874	
Total Costs Cash Flows	0	0	0	190.000	175.258	184.153	193.513	203.365	213.736	224.658	235.914	247.709	
Revenues (300€/student initially)	0	0	0	0	175.500	193.050	212.355	233.591	256.950	282.645	285.000	285.000	
Annual Operating Cash Flows Summary	0	0	0	0	242	8.897	18.842	30.226	43.213	57.986	49.086	37.291	
Total Costs PV		1.320.675 €											
Revenus PV		1.317.823 €											
NPV (Passive Analyses)		-2.853 €											
Xbase scale at t=T		190.000 €											
Vbase scale at t=0		161.276 €											

We consider three years maximum deferral period and 12 years analysis period

Base scale investment plan to support up to 1000 students and one language. We consider that the operation period is 7 years

Infrastructure investment includes further to costs described in the detailed table the 60% of the fixed operating cost (preparation cost to lunch activities) of the 1st year operation plus 30.000 € extra Marketing Expenses

We consider a 10% yearly increase of the customers for the base scale investment plan. The initial planning expects up to 1000 units. For each language there is 2,15 € per student per year. In addition, operating variable cost per student/language/year i

1 Operating fixed cost is increasing by 5% each year

2 Break Event Point Analysis performed by Mantzari and Economides 2004 indicates a no of users of about 590. We consider entry in the market when this threshold is reached

We assume that the Annual Expenses increase by 5% per year. On the opposite, we assume that the Student/customers fees do not change.

Table 8. Detailed Cash Flow Analysis for the base case investment (up to 1000 students/customers)