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EARTHQUAKE MAGNITUDE, INTENSITY, ENERGY, AND ACCELERATION*

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THE MAGNITUDE of an earthquake was originally defined by the junior author (Richter, 1935), for shocks in southern California, as the logarithm of the maximum trace amplitude expressed in thousandths of a millimeter with which the standard short-period torsion seismometer (free period 0.8 sec., static magnification 2800, damping nearly critical) would register that earthquake at an epicentral distance of 100 kilometers. Gutenberg and Richter (1936) extended the scale to apply to earthquakes occurring elsewhere and recorded on other types of instruments.

Application of the scale involves tables of the logarithm of the maximum trace amplitude for a shock of magnitude zero as a function of epicentral distance. These tables, given in the papers referred to, are conveniently represented by a nomogram (fig. 1) designed by Mr. John M. Nordquist, who has drafted all the figures. The magnitude can then be found for shocks of "normal" depth (about 20 km.). For slightly different depths a correction can be determined by the methods of the present paper. For shocks deeper than about 40 km. no reliable method for assigning magnitudes has been developed.

The magnitude scale has been applied with success to the local earthquakes of New Zealand, where standard torsion seismometers are in operation (Hayes, 1941); its extended form has been used by Ramanathan and Mukherji (1938) and by Mukherjee and Rangaswami (1941).

The purpose of the present paper is primarily to develop and investigate the relation of the magnitude, thus defined, to the energy released in an earthquake; also the relation of intensity on the Modified Mercalli Scale of 1931 to instrumentally determined acceleration. The connection of both magnitude and intensity with other physical elements of an earthquake is also investigated, largely with the help of the empirical equation (eq. 20, below) connecting magnitude with acceleration at the epicenter. The effect of focal depth on all the quantities is discussed.

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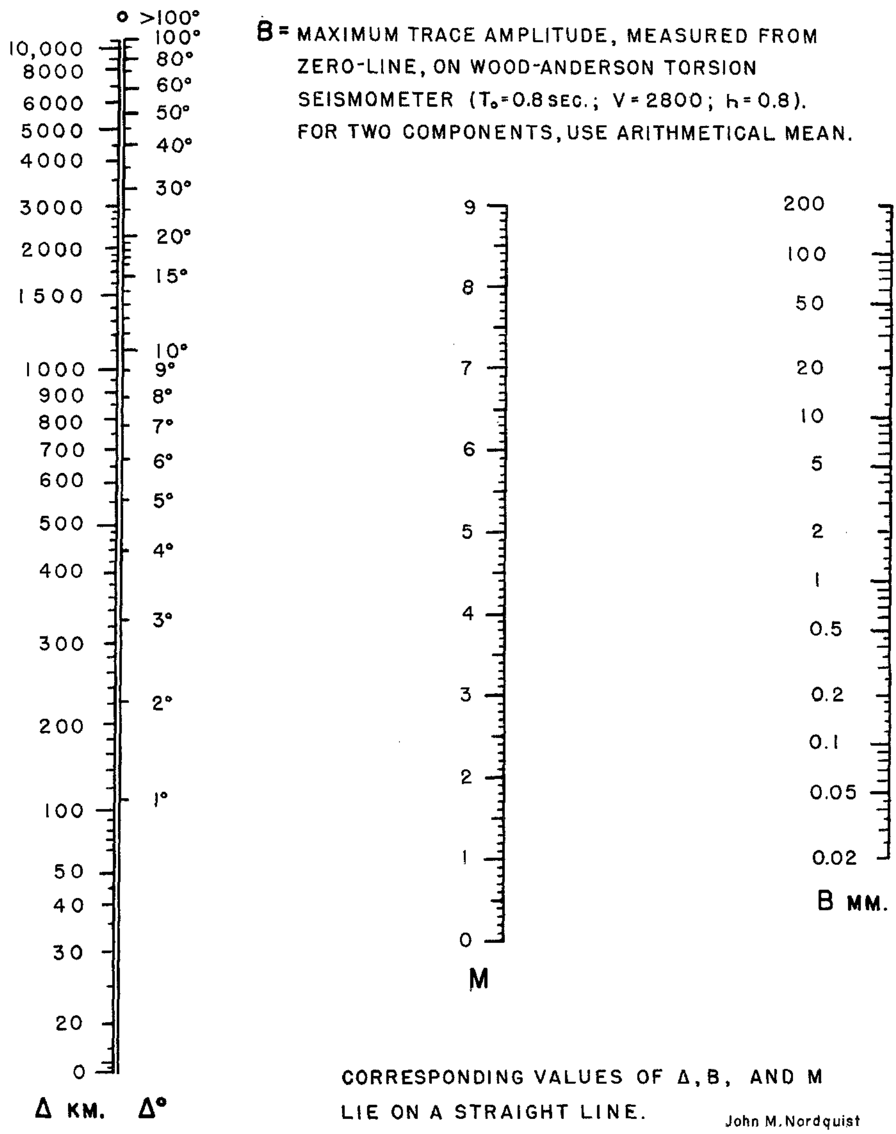

Fig. 1. Nomogram for determining earthquake magnitudes from trace amplitudes in millimeters of a standard-torsion seismogram. For $\Delta > 10°$ only, the ground amplitude in microns may be substituted, if 2.5 is subtracted from the result for M.

NOTATION

A = maximum ground amplitude (cm.)
a = maximum ground acceleration (cm/sec.^{2} = gals)
a_r = acceleration at limit of perceptibility (cm/sec.^{2} = gals)
B = seismographic trace amplitude (mm.)
b = value of B for a shock of magnitude zero
D = hypocentral distance (km.)
Δ = epicentral distance (km.)
E = energy of the shock (ergs)
h = hypocentral depth (km.)
H = usual depth of shocks in southern California (18± km.)
I = seismic intensity on the Modified Mercalli Scale of 1931 (Wood and Neumann, 1931)
i = angle of incidence of seismic ray
λ = wave length (km.)
M = earthquake magnitude
n = number of waves in maximum group
R = value of D at limit of perceptibility (km.)
r = value of Δ at limit of perceptibility (km.)
ρ = density (gm/cm.^{3})
T = period of vibration (sec.)
t = duration of maximum wave group (sec.)
V = instrumental magnification
v = wave velocity (km/sec.)
The zero subscript ($_0$) refers to the value of the respective quantity at the epicenter.

Materials used.—The major part of the data of this paper refers to recent shocks in southern California. Seismograms used were those of the eight stations of the local group, those of Berkeley and its associated stations (by courtesy of Dr. Perry Byerly of the University of California), those of the Lake Mead group (by courtesy of Dr. Dean Carder, Bureau of Reclamation, National Park Service and Coast and Geodetic Survey), and of Tucson (by courtesy of the Director, U. S. Coast and Geodetic Survey). Epicenters were carefully determined in the course of an investigation to be reported separately. Magnitudes were based on amplitudes recorded by standard torsion seismometers at all stations where these were available. The mean correction to be applied for each individual station, representing the effects of ground and instrumental idiosyncrasies, has been determined as follows:

Pasadena	+0.2	Berkeley	+0.2
Mount Wilson	+0.0	Lick	+0.1
Riverside	+0.2	San Francisco	+0.1
Santa Barbara	−0.1	Stanford (Branner)	−0.1
La Jolla	−0.0	Boulder City	+0.0
Tinemaha	−0.2	Fresno	−0.2
Haiwee	−0.0		

Those for the southern California stations agree closely with those determined previously (Richter, 1935), although there have been some instrumental changes. When these corrections are applied, the determination of magnitude is very consistent; only rarely does any station deviate from the mean by so much as 0.3. This indicates that the values of b (Richter, 1935, p. 6) require no significant modification.

At Pasadena the larger shocks are recorded by a pair of special torsion instruments with free period 10 sec., static magnification 4, and nearly critical damping. These will be referred to as the Pasadena strong-motion instruments.

TABLE 1

DATA FROM STRONG-MOTION SEISMOGRAMS AT PASADENA

Date of shock	M	Δ	B	$\log B - M$	$\log B - M + 2 \log D$
1933, Oct. 24	3.9	22	0.4	−4.3	−1.4
1941, Jan. 29	4.1	23	0.5	−4.4	−1.4
1941, Oct. 21	4.8	39	0.8	−4.9	−1.6
1941, Nov. 14	5.4	41	1.3	−5.3	−2.0
1933, Oct. 2	5.3	41	2.6	−4.9	−1.6
1933, Mar. 10	6.2	65	2½	−5.8	−2.2
1938, May 31	5.3	80	0.6	−5.5	−1.7
1941, Sept. 21	5.2	103	0.5	−5.5	−1.5
1940, Feb. 19	4.6	107	0.15	−5.4	−1.3
1941, June 30	5.9	131	1.2	−5.8	−1.6
1940, May 17, 21^h	5.4	171	0.9	−5.5	−1.0
1940, May 17, 22^h	5.2	171	0.5	−5.5	−1.0
1934, June 7	6.0	288	1½	−5.8	−0.9
1940, May 18	6.7	300	2½	−6.3	−1.3
1941, Sept. 14, 8^h	5.8	390	¼	−6.4	−1.2
1941, Sept. 14, 10^h	6.0	390	0.3	−6.5	−1.3
1934, Jan. 30	6.5	450	2	−6.2	−0.9
1932, Dec. 20	7.3	525	4	−6.7	−1.3

Extensive use has been made of maximum accelerations for the larger shocks as computed at Washington from strong-motion records obtained by the U. S. Coast and Geodetic Survey (Neumann, 1935–1940, 1941; Bodle, 1941; other preliminary reports).

Data on intensities and radius of perceptibility for American shocks have been taken from reports collected and summarized by the U. S. Coast and Geodetic Survey (Heck and Bodle, 1931; Neumann and Bodle, 1932; Neumann, 1932–1940; Bodle, 1941), from numerous special papers in the *Bulletin of the Seismological Society of America*, and from press notices and other information locally available at Pasadena. Similar data on British earthquakes have been selected from Davison (1924). For Germany the data are from Sieberg (1940) and Sieberg and Lais (1925), and from reports by W. Hiller in the bulletin of the seismological station at Stuttgart. (See also Hiller, 1935.)

Magnitude scale for short distances.—The maximum amplitudes recorded by the torsion seismometer, which are the basis of the magnitude scale, represent seismic waves of different type at different distances. At short distances, up to about 100 kilometers, these are usually $\overline{S}$, the direct transverse wave through the upper crustal layers. From 100 to 1000 kilometers they are various transverse waves refracted through the deeper crustal layers. Beyond 1000 kilometers the maximum trace amplitude is that of a surface wave. All these remarks apply only to shocks at normal depth.

The calculated trace amplitude b, for a shock of magnitude zero, consequently need not be a continuous function of Δ, although it is presented as such. It should also be affected by focal depth.

At short distances the standard torsion seismometer records unmanageably large amplitudes for shocks large enough to be recorded at distant stations.

TABLE 2

CALIBRATION DATA: LOG b FOR STRONG-MOTION SEISMOMETER, PASADENA

Δ	0	10	20	30	40	50	60	80	100	200	300	400	500
−Log b	4.2	4.3	4.5	4.8	5.0	5.2	5.3	5.5	5.6	5.8	6.1	6.4	6.6

Accordingly, it cannot be used to construct a curve for b at distances under about 25 km.; and even the value of b at this distance given by Richter (1935) is very uncertain. This difficulty is now partly overcome by applying the data from the Pasadena strong-motion instruments, for shocks of known magnitude (table 1). The tabulated values of B are trace amplitudes for the largest recorded waves of short period. These are not the largest amplitudes on the strong-motion seismogram, which generally shows larger waves with periods of several seconds. However, these greater amplitudes correspond to smaller accelerations; and it is the shorter-period waves, with higher accelerations, which correspond to the maximum waves as recorded by the standard torsion seismometer.

The column $\log B - M$ in table 1 provides data for a magnitude calibration of the strong-motion instrument. The smoothed results of this calibration are presented in table 2. Because of the definition of magnitude, we have

$$\log B - M = \log b \qquad (1)$$

Table 2 is thus in effect an amplitude-distance table for the zero shock as recorded on the strong-motion seismometer. The entry for $\Delta = 0$ has been established with the help of a relation exhibited in the last column of table 1, which gives values of $\log B - M + 2 \log D$. Here h has been taken as 18 km. for all shocks except that of May 31, 1938, which has been assigned a depth of 25 km. from the observed travel times.

The quantities in this column are nearly constant, with a mean of −1.7, up to at least Δ = 100 km.; with increasing distance they gradually become smaller. This is probably associated with a transition of the maximum from $\bar{S}$ to some other S phase at about 100 km., and may also be affected by increase of period with distance. For Δ = 0 there results

$$\log b_0 = -1.7 - 2 \log h = -4.2 \tag{2}$$

For the short-period maximum waves at short distances, the magnifications of the strong-motion and standard torsion seismometers, assuming continuous sinusoidal wave trains, are close to their static magnifications, which are respectively 4 and 2800. To find $\log b_0$ as used in the magnitude scale, add log 700 = 2.8 to the result in (2), giving

$$\log b_0 = -1.4 \tag{3}$$

for the standard torsion seismometer.

TABLE 3

CALIBRATION DATA: LOG b FOR STANDARD TORSION SEISMOMETER

(Short distances, h = 18 km.)

Δ	0	5	10	15	20	25	30
−Log b....	1.4	1.4	1.5	1.6	1.7	1.9	2.1

The same process applied up to Δ = 25 km. gives the results of table 3, which constitute the extension of the original calibration table (Richter, 1935, p. 6) to short distances. The value 1.9 at 25 km. revises and replaces the former figure 1.65. Calculation gives 2.0 for Δ = 30; the former value 2.10 has been retained. For greater distances the change in period affects the magnification of the torsion seismometer, so that the process described cannot be used beyond 30 km.

Change of ground amplitude with distance and depth.—The foregoing discussion shows that the strong-motion data are represented by BD^2 = const. Since the magnification of these instruments is uniform for the periods involved, we may also write AD^2 = const. This holds for short distances; to investigate the conditions at greater distances data from the torsion seismometers are available (table 4).

The trace amplitudes tabulated as B have already been corrected, being multiplied by the antilogarithms of the station corrections used in the magnitude scale. The numerical factors in the column headings involve the magnification (2800) of the torsion seismometer, not used in calculation.

While the quantity AD^2 remains of the same order of magnitude through the range of distance considered, AD^2/T^2 is more nearly constant. It appears

TABLE 4

AMPLITUDES AND PERIODS AS FUNCTIONS OF DISTANCE FOR SELECTED CALIFORNIA SHOCKS

Station	Δ	T	B	28,000 A	0.028 ×		T	B	28,000 A	0.028 ×	
					AD^2	AD^2/T^2				AD^2	AD^2/T^2
	1938, May 31						1938, July 5				
Mount Wilson	77	0.3					0.2	25	25	0.17	0.4
Pasadena	80	0.3	190	210	1.5	15	0.2	27	30	0.21	0.5
La Jolla	101	0.3	110	120	1.3	13	0.3	35	40	0.43	0.5
Santa Barbara	223	0.4	42	55	2.8	14	0.6	4	5¼	0.26	0.7
Haiwee	267	0.8	40	80	5.7	9	1.0	4½	9	0.64	0.6
Boulder	345	0.7	15	27	3.2	6	0.9	1½	2½	0.30	0.4
Tinemaha	375	0.5	11	15	2.1	9	0.9	1½	2½	0.35	0.5
	1940, May 17, 21^h						1940, May 17, 22^h				
Riverside	87	0.2	160	160	1.3	32	0.3	120	120	1.0	11
La Jolla	150	0.3	130	140	3.2	35	0.3	46	50	1.1	13
Mount Wilson	152	0.6	100	150	3.5	10	0.6	90	135	3.2	9
Pasadena	163	0.5	150	200	5.3	21	0.6	76	115	3.1	9
Boulder	259	0.6	23	35	2.3	7	0.6	18	26	1.7	5
Haiwee	267	0.8	62	120	8.6	13	1.0	30	84	6.0	6
Santa Barbara	306	1	20	100	9.4	9	1¼	15	60	5.6	4
Tinemaha	372	0.7	19	35	4.8	10	1.1	10	30	4.1	4
	1933, October 2										
La Jolla	128	0.4	70	80	1.3	8					
Santa Barbara	164	0.4	70	80	2.2	14					
Haiwee	261	1	50	100	6.8	7					
Tinemaha	367	1.1	25	75	10.0	8					
Lick	510	1.2	5	20	5.2	3					
Stanford	543	1.7	5½	30	8.8	3					
Berkeley	590	1.7	3	18	6.3	2					
San Francisco	593	1.9	4½	32	11	3					

that AD^2/T = const. would fit the data still more closely, especially at the greater distances; however, the distinction is well within the limits of accuracy. This is also true for the readings for the strong-motion instrument (table 1); here the periods are determined with less accuracy, and have not been tabulated.

The result AD^2/T^2 = const. lends itself particularly well to theoretical interpretation. It can be written

$$\frac{AD^2}{T^2} = \frac{A_0 h^2}{T_0{}^2} \tag{4}$$

This can be used to derive the effect of depth. Neglecting absorption, and applying the inverse square law of radiation, hA_0/T_0 should be proportional to the square root of the radiated energy. See equation (24). From (4)

$$\frac{AD^2}{hT^2} = \text{const.} \times \frac{\sqrt{E}}{T_0} \tag{5}$$

For a shock of given energy E, the period at the epicenter, T_0, will not vary appreciably over wide limits of the depth h. Since

$$A = \frac{aT^2}{4\pi^2} \tag{6}$$

$$\frac{aD^2}{h} = \text{const.} \sqrt{E} \tag{7}$$

For straight rays $h = D \cos i$, hence

$$aD = \text{const.} \cos i \sqrt{E} \tag{8}$$

This result is equivalent to a suggestion by Blake (1941); the factor $\cos i$ was introduced in a similar discussion by Oldham (1926). Both authors point out a considerable degree of arbitrariness in the introduction of this factor. As shown above, the observations lead to it very naturally. Equation (8) implies that the inverse square law of radiation is satisfied along any given straight ray, but that the disturbances at different points of the surface are not in accordance with the simple law. This can only mean that the surface disturbance is not proportional to the amplitude or acceleration of the arriving wave; the effect is represented by the factor $\cos i$, which is a simplification standing in the place of a much more complicated expression. (See Wiechert, 1907, pp. 40–47.) There should also be involved transition of the maximum from one S phase to another, effects of the nature of ground at the surface, and other complicating circumstances.

Intensity and acceleration.—Data for setting up an empirical functional relation between intensity and acceleration in the California region are given in tables 5, 6, and 7. In table 5 the accelerations are those computed by the U. S. Coast and Geodetic Survey (especially Neumann, 1941, p. 17) from the strong-motion records obtained at the localities named; the intensities, which are given in accordance with the Modified Mercalli Scale of 1931, are assigned on the basis of reports collected by the U. S. Coast and Geodetic Survey as well as press reports and other local information. These intensities refer to the localities at which the accelerations were measured. In table 6 the accelerations a_0 are extrapolated to the epicenter by using the Pasadena strong-motion calibration data (table 2); because of the long period of the instrument, this applies equally well to the acceleration and to the amplitude. The corresponding intensities I_0, for the epicenter, are estimated from all available data. Table 7 corresponds to table 5, using strong-motion readings and local intensities at Pasadena.

Plotting I as a function of a, and I_0 as a function of a_0, gives a smooth curve which becomes a straight line if the logarithm of the acceleration is used. The data are very well represented by the resulting purely empirical equation

$$\log a = \frac{I}{3} - \tfrac{1}{2} \qquad (9)$$

TABLE 5

LOCAL ACCELERATION AND INTENSITY, CALIFORNIA SHOCKS

Date	Location	a	I	$\frac{I}{3} - \log a$
1939, May 4	Boulder Dam	40	6	0.4
1941, June 30	Santa Barbara	170	7	0.1
1940, Oct. 10	Vernon	15	5	0.5
1939, Dec. 27	Long Beach	14	5	0.6
1933, Mar. 10	Vernon	110	7½	0.5
1933, Mar. 10	Los Angeles (S.T.)	30	6½	0.7
1940, May 18	El Centro	170*	7½	0.3
1934, Dec. 30	El Centro	50	6	0.3
1937, Mar. 25	Colton	12	4	0.2
1937, Mar. 25	El Centro	5	4½	0.8
1941, Sept. 14	Bishop	13	6½	1.1
1937, July 7	Santa Ana	5	5	1.0
1939, Mar. 21	El Centro	25	4½	0.1
1939, Mar. 24	El Centro	40	4½	−0.1
1940, Jan. 12	Vernon	2	4	1.0

* One single oscillation exceeds 300 gals.

This can be seen from the columns which contain the values of $\frac{I}{3} - \log a$ and $\frac{I_0}{3} - \log a_0$. These quantities are very nearly constant, with a mean close to 0.5. See also figure 2 (a).

The term $\frac{I}{3}$ implies that the acceleration increases tenfold for every increase of three units in the intensity. This is identical with the result obtained by Cancani (1904) from the data of Omori and Milne, which he made the basis of an assignment of intensities to the degrees of the Mercalli Scale. However, the constant term differs; Cancani's conclusion is equivalent to $\log a = \frac{I}{3} - 1$, for the upper limit of each intensity grade. (See also Gassmann, 1927.) The reason for the significantly higher values found from the newer instrumental

TABLE 6

ELEMENTS OF CALIFORNIA SHOCKS

Date	M	a_0	I_0	r	$\frac{1}{3} I_0 - \log a_0$	$M - 0.6 I_0$	$I_0 - 6 \log \frac{R}{h}$	$M - 1.8 \log a_0$	a_r
1938, Dec. 3....	5.5		7?	180		1.3?	1.0?		
1939, May 4....	5.0	40	6	100	0.4	1.4	1.6	2.1	1.3
1939, Feb. 23....	4.8		6?	100		1.2?	1.6?		
1941, June 30....	5.9	200	7½?	230	0.2	1.4?	0.9?	1.8	1.2
1939, May 7....	4.4		5?	50		1.4?	2.1?		
1935, July 13....	4.7		5?	130		1.7?	0.0?		
1940, Oct. 10....	4.7	30	6?	100	0.5	1.1?	1.6?	2.0	0.9
1933, Oct. 2....	5.3	60	7	140	0.5	1.1	1.6	2.1	0.9
1939, Dec. 27....	4.7	20	5	60	0.4	1.7	1.6	2.3	1.3
1933, Mar. 10....	6.2	350	8½	300	0.3	1.1	1.1	1.7	1.3
1941, Jan. 29....	4.1	16	6?	160	0.8	0.5?	0.5?	1.9	1.8
1940, Apr. 18....	4.4		5?	100		1.4?	0.6?		
1939, Nov. 7....	4.7		5½	80		1.4	1.5		
1940, Feb. 19....	4.6		5	80		1.6	1.1		
1940, Feb. 25....	3.4		4	30		1.0	1.2		
1940, May 17....	5.4		6	170		0.8	0.2		
1937, Mar. 25....	6.0	200?	7?	250	0.0?	1.8?	0.3?	1.9?	0.9
1940, May 18....	6.7	240	10	350	0.9	0.7	2.3	2.4	0.6
1941, Sept. 14....	6.0	50	7?	220	0.6	1.8?	0.4?	2.9	0.3
1941, Sept. 21....	5.2		6	160		1.6	0.5		
1941, Oct. 21....	4.8	50	6½	90	0.5	0.9	2.5	1.8	2.0
1941, Nov. 14....	5.4	60	7	130	0.5	1.2	1.8	2.2	1.1
1937, July 7....	3.9	12	5	60	0.6	0.9	1.6	2.0	0.8
1940, Jan. 12....	4.0	5	5	50	1.0	1.0	2.0	2.7	0.5
1940, Oct. 10....	4.8	12	6	100	0.9	1.2	1.8	2.8	0.4

TABLE 7

ACCELERATION CALCULATED FROM STRONG-MOTION INSTRUMENTS, AND INTENSITY AT PASADENA (CALIFORNIA SHOCKS)

Date	M	Δ	a	I	$\frac{I}{3} - \log a$
1940, May 17........	5.4	171	0.6	3½	1.4
1933, Oct. 2........	5.3	41	10	4	0.3
1941, Oct. 21........	4.8	39	0.8	3	1.1
1941, Jan. 29........	4.1	23	5	4½	0.8
1941, Nov. 14........	5.2	40	1¼	3	0.9
1933, Mar. 10........	6.2	62	25±	5½	0.4
1941, June 30........	5.9	130	2½	2½	0.4
1941, Sept. 21........	5.2	101	10	3	0.0
1940, Feb. 19........	4.6	105	1	1	0.3
1932, Dec. 19........	7.3	525	½	2	1.0
1934, June 7........	6.0	288	0.3	2	1.2

data must be sought in the circumstance that the older instruments were chiefly of the long-period type, so that the maximum waves on their seismograms were not the waves of maximum acceleration, these latter usually being of short period and with relatively small amplitudes. As an example of this, $\frac{I}{3} - \log a$ has been calculated for the South German earthquake of 1911, the accelerations being calculated from the seismograms of the nearer stations (Gutenberg, 1915), and the intensities at the same locations being taken from the isoseismal map (Sieberg and Lais, 1925). The calculated accelerations

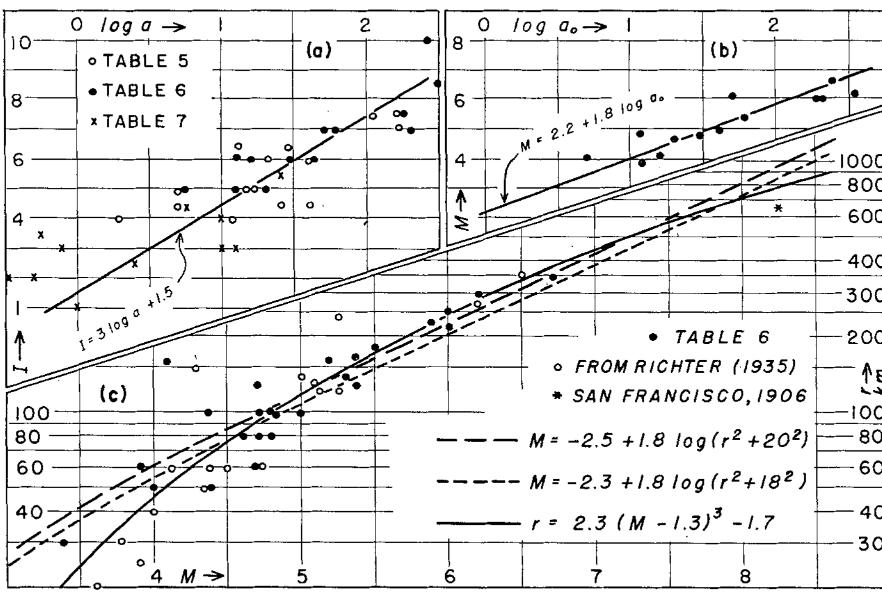

Fig. 2. Relations between elements of earthquakes in California.

scarcely exceed 1 gal, corresponding to an intensity of 6; the average for $\frac{I}{3} - \log a$ is about 2. The only short-period seismogram available, that of the 17-ton pendulum at Göttingen, strongly indicates that there were short-period waves with larger accelerations.

In the past, accelerations corresponding to given intensities have frequently been estimated from macroseismic effects. It is obvious that these must correspond only to that level of acceleration which persisted long enough to produce permanent effects, which must be less than the maximum acceleration recorded on a complete seismogram.

Intensity, depth, and radius of perceptibility.—From (7) and (9) it follows that, for any one given earthquake,

$$\frac{I}{3} + 2 \log D = \text{constant} \tag{10}$$

Expressing D in terms of Δ and h, and differentiating, it is found that $\frac{d^2I}{d\Delta^2} = 0$ when $\Delta = h$. That is, the intensity has a maximum rate of change with epicentral distance when that distance is equal to the hypocentral depth. At the epicenter $\frac{dI}{d\Delta} = 0$.

From (10) it directly follows that for any two distances and their corresponding intensities

$$I_1 - I_2 = 6 \log (D_2/D_1) \tag{11}$$

In particular, if $I_2 - I_1 = 1$,

$$D_1/D_2 = 10^{1/6} = 1.47$$

or, with obvious notation,

$$D_n = 1.47\, D_{n+1} \tag{12}$$

This implies that for shocks at a given depth successive isoseismals in a given range of distance have a definite spacing which is independent of the magnitude or energy of the shock or of the epicentral intensity. The isoseismals for $I = I_0 - 1, I_0 - 2$, etc., should always be at the same epicentral distance; if this spacing differs, it implies a difference in hypocentral depth.

At the outer boundary of the perceptibly shaken area (radius of perceptibility, r), take $I = 1.5$ (since by definition the shaking is felt when $I = 2$ and not felt when $I = 1$). In (11) substitute $I_1 = I_0, I_2 = 1.5, D_1 = h, D_2 = R$. Then

$$I_0 - 1.5 = 6 \log \frac{R}{h} \tag{13}$$

or

$$I_0 = 1.5 + 3 \log \left(\frac{r^2}{h^2} + 1 \right) \tag{14}$$

or

$$\frac{r}{h} = \sqrt{10^{\frac{I_0}{3} - \frac{1}{2}} - 1} \tag{15}$$

The last three equations should be good approximations only when r does not exceed 500 km.

A purely empirical relation which represents the results for California shocks is

$$r = \tfrac{1}{2} I_0^3 - 1.7 \tag{16}$$

for the usual depth ($h = H$), which may be generalized to

$$\frac{r}{h} = \frac{I_0^3 - 3.4}{2H} \tag{17}$$

Blake (1941, p. 227) has given an equation which is equivalent to

$$I_0 - I = s \log \frac{D}{h} \tag{18}$$

From observations he assigns to s the empirical value 5.35. Equation (11) of the present paper corresponds to $s = 6$. Either value agrees reasonably well with the data. See also the discussion of the parameters.

TABLE 8

CALCULATED VALUES OF r/h

I_0	3	4	5	6	7	8	9	10	11	12
r/h eq. (15)............	1.5	2.4	3.7	5.5	8.2	12	18	26	38	56
eq. (17)............	0.7	1.7	3.4	5.9	9.4	14	20	28	37	48
eq. (18)............	1.6	2.9	4.3	6.8	10.7	16	25	39	60	92

Table 8 gives values of r/h for integral values of I_0, calculated from equations (15), (17), and (18) with $s = 5.35$.

Table 9 gives r and I_0 for a number of representative earthquakes, with values of h calculated from (15). The agreement with depths found from microseismic data is generally good. Inspection of table 8 will show that depths calculated using the other formulas quoted will not differ significantly. Additional data on calculated depths will be found in later sections of this paper.

If we denote by a_r the value of the acceleration at the limit of perceptibility, (7) gives

$$a_0 h = a_r \frac{R^2}{h}, \quad \text{or} \quad a_0 h^2 = aD^2 = a_r R^2 \tag{19}$$

Since a_r corresponds to $I = 1.5$, equation (9) gives $a_r = 1$ gal, which agrees with most observations bearing on the point; see Ishimoto (1932), Ishimoto and Ootuka (1933). Values of a_r calculated from (19) appear in the last column of table 6; they agree well with the result given above.

Magnitude, acceleration, and intensity.—For shocks of given depth in any region there should be a functional relation between the magnitude and the maximum acceleration a_0. For the region of California this is shown in figure 2(b), plotted from the data of table 6. The data are well represented by

$$M = 2.2 + 1.8 \log a_0 \tag{20}$$

The values of $M - 1.8 \log a_0$ are tabulated in table 6.

Combining (9) and (20), there results

$$M = 1.3 + 0.6\, I_0 \tag{21}$$

As is to be expected, this also fits the observations when compared with them directly. No graph has been plotted, but the values of the quantity

TABLE 9

CALCULATED DEPTH OF EARTHQUAKES

Date	Region	r	I_0	h from micros. data	h from eq. (15)
1933, June 4	South Germany	6–8	4	5–10	3
1939, May 4	Boulder Dam	100	6	15	18
1906, Apr. 18	San Francisco	650	11	normal	17
1915, Oct. 2	Nevada	600	10	normal	23
1930, Aug. 31	Santa Monica Bay	150	7	normal	18
1933, Mar. 11	Long Beach	300	8½	normal	20
1938, May 31	Elsinore, Calif.	180	6	25±	33
1911, Nov. 16	South Germany	450	8	35	38
1926, June 26	Aegean Sea	1600	10	70	62
1935, Nov. 1	Canada	1000	8½	80±	67
1927, Apr. 19	Luzon	700	5½	100	160
1933, Nov. 14	Chile	600	7	110	76
1934, Mar. 1	Chile	900	8	120	75
1927, Apr. 13	Luzon	500	4½	140	170
1940, Nov. 10	Roumania	2000±	9	150	110
1933, Oct. 25	Argentina-Chile	1000	6	220	180
1926, July 26	Japan	1000±	4±	360	420±

$M - 0.6\, I_0$ are tabulated in table 6. Equation (21) also closely represents the majority of observations reported for New Zealand by Hayes (1941), who states that most of the exceptional instances are probably due to abnormal focal depth.

From equations (19), (20), and (21) table 10 has been constructed, showing values of I_0, a_0, and r for shocks of given magnitude at the usual depth in the California region. Values of $\log E$ are from equation (35), to be developed later.

The shock of magnitude 2.2 here appears as the minimum felt earthquake. Shocks of magnitude as low as 1.5 have occasionally been reported felt by unusually alert or sensitive observers. Even these instances may simply be due to exceptionally unstable ground; or the shocks may have originated at slightly smaller depth than usual.

The lower limit of damage (intensity 6) corresponds to a magnitude slightly below 5, with an acceleration of about one-thirtieth of gravity. Small damage at scattered points is not infrequently reported in shocks of magnitude as low as 4.5.

Acceleration of one-tenth gravity, corresponding to intensity 7.5, occurs in shocks of magnitude 5.8. On the average, between one and two shocks of this magnitude or greater occur annually in the California region.

TABLE 10

ELEMENTS OF SHOCKS OF GIVEN MAGNITUDE IN CALIFORNIA

(h = 18± km.)

M	2.2	3	4	5	6	7	8	8½
I_0	1.5	2.8	4.5	6.2	7.8	9.5	11.2	12.0
a_0	1	3	10	36	130	460	1670	3160
r	0	24	54	107	204	387	736	1012
$\log E$	15.3	16.7	18.5	20.3	22.1	23.9	25.7	26.6

Shocks of magnitude 7 represent the lower limit of major earthquakes, with intensity exceeding 9, maximum accelerations of nearly one-half gravity, and perceptibility extending to distances of nearly 400 kilometers. As the magnitude approaches 8 the acceleration transcends gravity intensity 10.5, and in the greatest shocks it significantly exceeds it, perhaps for an appreciable duration as in the Indian earthquake of 1897 (Oldham, 1899; see esp. pp. 79 and 353).

Equations (19) and (20) give

$$M = 2.2 + 3.6 \log \frac{R}{h} \tag{22}$$

If instead we combine the empirical equation (16) with (21), the result is

$$r = 2.3\,(M - 1.3)^3 - 1.7 \tag{23}$$

In figure 2 (c) r is plotted on a logarithmic scale as a function of M. The data are taken from table 6 and from Richter (1935). Curves are drawn for equation (22) assuming $h = 18$ and $h = 20$, and also for equation (23). Note the relatively large differences in the curves for the slightly different depths. Some of the high points on the plot may belong to somewhat deeper shocks.

Theoretical calculation of energy.—In the immediately following discussion all lengths are at first taken to be measured in centimeters; units as used in the rest of the paper are introduced after equation (25).

Consider that at the epicenter the radiated energy arrives principally in a series of n equal sinusoidal waves of length λ, amplitude A_0, and period T_0. The kinetic energy per unit volume is $\frac{\rho}{4}\left(\frac{2\pi A_0}{T_0}\right)^2$ where the quantity in parentheses is the maximum velocity of a particle, and one factor ½ is due to averaging $\sin^2 \frac{2\pi\tau}{T}$ over a period. (τ = time.)

If the wave velocity v is constant, this is the mean energy in a spherical shell of volume $4\pi h^2 n\lambda$; hence putting $nT_0 = t_0$ and $\lambda = vT_0$, so that $n\lambda = vt_0$,

$$E = 4\pi^3 h^2 v t_0 \rho \left(\frac{A_0}{T_0}\right)^2 \tag{24}$$

Introducing the acceleration from (6)

$$E = \frac{1}{4\pi} h^2 v t_0 \rho \, a_0^2 \, T_0^2 \tag{25}$$

If h and v are measured in kilometers and kilometers per second, respectively, the other units remaining unchanged, a factor 10^{15} must be introduced on the right side of (25). Assuming v slightly greater than 3 km/sec., $\rho = 3$, we may take

$$\log 10^{15} v\rho - \log 4\pi = 14.9 \tag{26}$$

and

$$\log E = 14.9 + 2 \log h + \log t_0 + 2 \log T_0 + 2 \log a_0 \tag{27}$$

If absorption is negligible, this represents the original energy radiated from the hypocenter. Suppose absorption represented by a factor e^{-2kh}, where k is the coefficient of absorption. For surface waves k has been found to be of the order 10^{-4} km^{-1}. The fact that $P'P'$ and $P'P'P'$ are observed indicates that k is of about the same order of magnitude for longitudinal waves. The effect of so small an absorption is completely negligible for present purposes.

Energy and magnitude in California.—Equation (27) will yield a functional relation between energy and magnitude of shocks in California, if the quantities t_0, T_0, and a_0 are known in terms of magnitude. For a_0 this is accomplished by equation (20). For t_0 it may be assumed that

$$\log t_0 = -0.7 + \tfrac{1}{4}M \tag{28}$$

There are few observational data bearing on the value of t_0, which is a rather arbitrarily selected quantity related to the duration of strong shaking near the epicenter. Fortunately, precision is not required, as the value of t_0 only slightly affects the final result. The above assumption gives for $M = 0$, $t_0 = 0.2$ sec.

(which represents the seismograms of the smallest recorded shocks); for $M = 6$, $t_0 = 6$ sec. Note that in the Long Beach earthquake of 1933 ($M = 6.2$) and the Imperial Valley earthquake of 1940 ($M = 6.7$) the duration of the waves with maximum acceleration was of the order of 10 sec. (Neumann, 1941, p. 17). For $M = 8\frac{1}{4}$ (San Francisco earthquake), the equation gives $t_0 = 25$ sec.

It is not necessary to make any fresh assumption for T_0 as a function of M, since this relation can be derived from equations (1) and (2). If T_0 does not exceed a few seconds, the magnification of the Pasadena strong-motion instrument may be taken equal to its static magnification ($V = 4$); and we have for this instrument

$$B_0 = 40\left(\frac{T_0}{2\pi}\right)^2 a_0 = a_0 T_0^2 \tag{29}$$

The magnification factor (40, since B is measured in millimeters) practically cancels $4\pi^2$.

Writing equation (1) for $\Delta = 0$, and using the value of $\log b_0$ from (2),

$$\log B_0 = M - 4.2 \tag{30}$$

Taking the logarithmic form of (29), and using (30),

$$2 \log T_0 = M - \log a_0 - 4.2 \tag{31}$$

This is sufficient for deriving the relation between energy and magnitude. The explicit relation between T_0 and M is not needed, but may be derived by combining (31) with (20), which gives

$$\log T_0 = -1.5 + 0.22\, M \tag{32}$$

The following are corresponding values of T_0 and M:

T_0	0.1	0.5	1.0	2.0	sec.
M	2.2	5.4	6.7	8.1	

These periods are of the right order of magnitude, and incidentally justify the initial assumption of short periods made in applying the static magnification.

Applying (28) and (31) to (27),

$$\log E = 10.0 + 2 \log h + \log a_0 + 1.25\, M \tag{33}$$

Introducing the value of $\log a_0$ in terms of M from (20),

$$\log E = 8.8 + 2 \log h + 1.8\, M \tag{34}$$

Since all these equations are valid only for California shocks, we may assign to h its usual value in the region, as determined from travel times; this is near 18 km., and we take $2 \log h = 2.5$. Hence

$$\log E = 11.3 + 1.8\,M \tag{35}$$

For calculated values see table 10. Equation (35) replaces the equation $\log E = 8 + 2\,M$ formerly used by the authors (Gutenberg and Richter, 1936, pp. 124–125), which neglected the variation of t_0, T_0, and other elements of the seismogram, with M. Equation (35) gives a larger and hence more acceptable value (2×10^{11} ergs) for the energy of the smallest recorded earthquake ($M = 0$). For large shocks it also gives reasonable results; $M = 8\frac{1}{4}$, which corresponds to the San Francisco earthquake, gives 10^{26} ergs. Replacing 2 by 1.8 as the coefficient of M has the effect of decreasing the energy ratio between two shocks differing by one unit of magnitude from 100 to about 60. Since the frequency of occurrence of earthquakes increases about tenfold when the magnitude is decreased by one unit, the mean annual release of energy in a given magnitude range remains about six times that for the range one unit lower. While this result here appears as a consequence of equation (35), which is established only for shocks in California, the fair success in extending the magnitude scale to apply in other regions shows that the corresponding relation for these other regions cannot differ greatly in form, so long as the hypocenter is within the continental crustal layers.

Calculation of energy for variable depth.—To apply (27) to shocks at other depths than 18 kilometers we must express t_0 and T_0 as functions of energy, instead of magnitude as given in (28) and (32). Using (35) to replace M in these equations by E, we find

$$\log t_0 = -2.3 + 0.14 \log E \tag{36}$$

and

$$2 \log T_0 = -5.76 + 0.24 \log E \tag{37}$$

These equations may reasonably be assumed to be independent of h, at least to the approximation needed at this point. Substitution in (27) gives

$$\log E = 11.1 + 3.2 \log h + 3.2 \log a_0 \tag{38}$$

Applying equation (9), which is independent of h,

$$\log E = 9.5 + 3.2 \log h + 1.1\, I_0 \tag{39}$$

Using equation (19), taking $a_r = 1$, (38) gives

$$\log E = 11.1 + 6.4 \log R - 3.2 \log h \tag{40}$$

The last three equations can be used to compute the energy if the depth is known. The results decrease in reliability as the depth increases; this is partly due to the character of the assumptions involved, but it must also be considered that values of I_0 and R are usually uncertain for deep-focus earthquakes.

Table 11 gives the energies of representative earthquakes at various depths, calculated from equations (39) and (40). For the shallower shocks the agreement with the energies calculated from the magnitude by using equation (35) is within the limits of error (usually less than one unit of log E). No complete

TABLE 11

CALCULATED ENERGY OF EARTHQUAKES

Date	Region	h	I_0	r approx.	Log E	
					Eq. (39)	Eq. (40)
1939, May 4	Boulder Dam	15	6	100	20	20
1933, Mar. 11	Long Beach	18	8½	300	23	23
1906, Apr. 18	San Francisco	18	11	650	26	25
1938, May 31	Elsinore, Calif.	25±	6	180	21	21
1927, Mar. 7	Tango, Japan	25±	10	520	25	24
1897, June 12	India	25?	12	1400	27	27
1911, Nov. 16	South Germany	35	8	450	23	23
1939, Jan. 24	Chillán, Chile	70	10	?	26	?
1926, June 26	Aegean Sea	70	10	1600	26	26
1935, Nov. 1	Canada	80±	8½	1000	25	24
1927, Apr. 19	Luzon	100	5½	700	22	23
1927, Apr. 14	Andes Mts.	110	8½	1500	25	25
1927, Apr. 13	Luzon	140	4½	500	21	22
1940, Nov. 10	Roumania	150	9	2000±	26	25
1933, Oct. 25	Andes Mts.	220	6	1000±	24	23
1906, Jan. 21	Off Japan	340	7±	?	25	?
1926, July 26	Japan	360	4	1000	22	22

data are available for shocks with depths exceeding 400 km. For some shocks at depths approaching 600 km. the reports indicate epicentral intensities of 4 or 5, which would correspond to log $E = 24$ approximately; some of these are reported felt at epicentral distances greater than 1000 km.

The energies calculated from I_0 and from r are generally consistent; table 11 thus tends to confirm the general impression that the energies of the larger deep-focus earthquakes are comparable with those of the greatest shallow shocks.

Intensity and radius of perceptibility in North America and Europe.—Data for I_0 and r in California have been given in table 6. These correspond, consequently, to the structural conditions in that locality. For some of these shocks the depth has been determined instrumentally as near 18 km. In previous work with these data, it has been assumed that this applies to the entire group of shocks. It remains further to justify this assumption by direct correlation of I_0 and r, and by comparison with the same data for shocks in other regions. In figure 3 (a), r is plotted on a logarithmic scale as a function of I_0, using the data of table 6 and a few other shocks as reported previously.

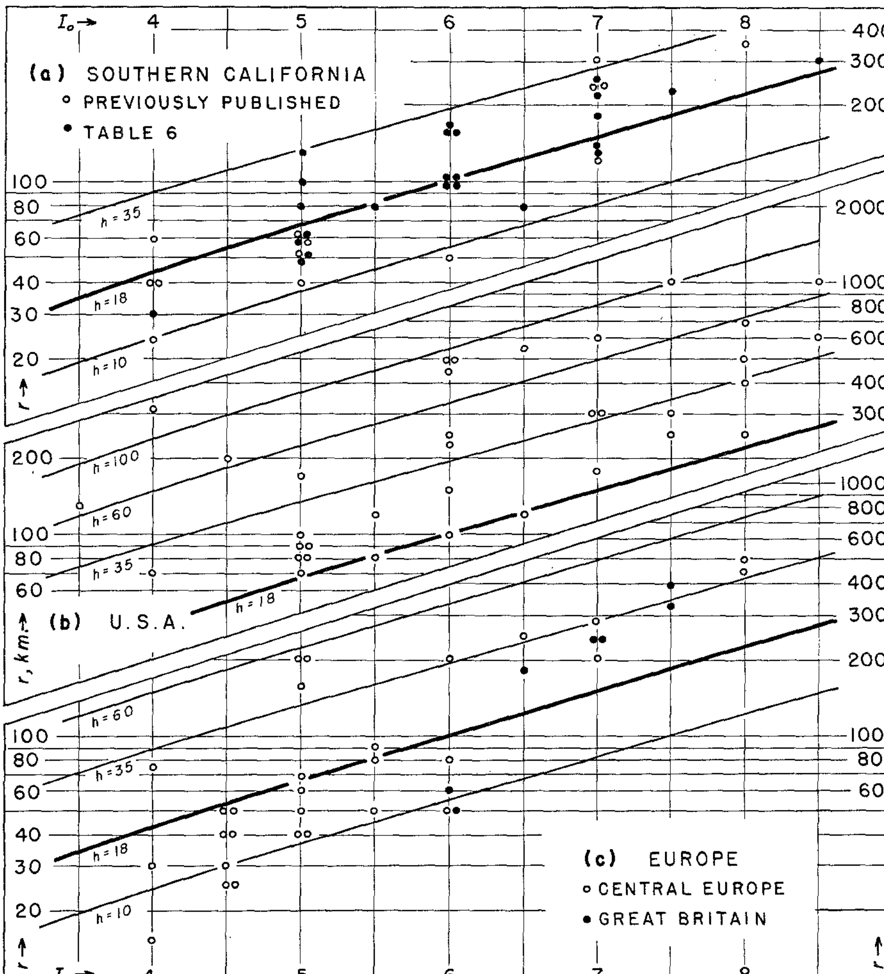

Fig. 3. Radius of perceptibility as function of epicentral intensity.

Table 12 gives I_0 and r for shocks in the United States and Europe. The column headed h' gives the value of the "depth" calculated for each shock from equation (15). It is likely that this quantity in many cases differs from the true depth h, as it must be much affected by differences in local structure and ground. It should be taken merely as a measure of the extent of the perceptibly disturbed area, relative to the epicentral intensity.

TABLE 12

MAXIMUM INTENSITY AND RADIUS OF PERCEPTIBILITY FOR SELECTED EARTHQUAKES

No.	Date	Region	I_0	r	h' *
		UNITED STATES OF AMERICA			
1	1904, Mar. 21	Maine	7	600	72
2	1935, Nov. 1	Canada	8½	1000	67
3	1934, Apr. 14	Adirondack Mts., N. Y.	5	80	22
4	1937, July 18	Long Island	4	70	29
5	1939, Nov. 14	New Jersey	5	90	24
6	1938, Aug. 22	New Jersey	5	90	24
7	1929, Aug. 12	Attica, N. Y.	5	80	22
8	1931, Sept. 20	Ohio	7	180	22
9	1937, Mar. 8	Ohio	7½	300	30
10	1928, Nov. 2	Southern Appalachians	6	250	45
11	1916, Feb. 21	Southern Appalachians	6½	550	80
12	1935, Jan. 1	Southern Appalachians	5	100	27
13	1924, Oct. 20	Southern Appalachians	4½	200	67
14	1913, Jan. 1	South Carolina	7½	250	25
15	1886, Aug. 31	Charleston	9	1600	90
16	1937, Nov. 17	Illinois	5	70	19
17	1917, Apr. 9	Missouri	6	500	90
18	1939, Nov. 23	Illinois-Missouri	6	450	80
19	1895, Oct. 31	Missouri	7½	1000	100
20	1934, Aug. 19	Missouri	6	150	27
21	1811/1812	New Madrid	12	1500	27
22	1937, May 16	Arkansas	3½	130	70
23	1938, Sept. 17	Arkansas	4	320	130
24	1931, Dec. 16	Mississippi	7	300	36
25	1931, Oct. 19	Louisiana	5½	120	26
26	1935, Mar. 1	Nebraska	6	230	41
27	1936, June 19	Texas	5	170	46
28	1925, July 30	Texas	6	500	90
29	1931, Aug. 16	Texas	8	700	58
30	1906, Nov. 15	New Mexico	8	250	21
31	1934, July 30	Nebraska	6	100	18
32	1925, Nov. 17	Wyoming	6½	120	18
33	1925, June 27	Montana	9	500	28
34	1935, Oct. 18	Montana	8	500	42
35	1935, Oct. 31	Montana	8	400	33
36	1934, Mar. 12	Utah	8½	600	41
37	1932, Aug. 6	Washington	5½	80	18
38	1939, Nov. 12	Washington	7½	300	30
39	1932, June 6	Eureka, Calif.	8	300	25
40	1932, Dec. 20	Nevada	10	550	21

* For h', see text, p. 182.

TABLE 12—*Continued*

No.	Date	Region	I_0	r	h' *
		UNITED STATES OF AMERICA—*Continued*			
41	1906, Apr. 18......	San Francisco....................	11	650	17
42	1937, Mar. 8......	Berkeley.........................	6½	110	17
43	1934, June 7......	Parkfield, Calif..................	8	280	23
44	1941, June 30......	Santa Barbara..................	8	230	19
45	1941, Nov. 14......	Torrance, Calif..................	7	130	16
46	1933, Oct. 2......	Long Beach, Calif...............	7	140	17
47	1933, Mar. 10......	Long Beach, Calif...............	8½	300	21
48	1938, May 31......	Elsinore, Calif....................	6	180	32
49	1940, May 18......	Imperial Valley..................	10	350	13
		GREAT BRITAIN			
50	1901, Sept. 18......	Scotland.........................	7	240	29
51	1905, Sept. 21......	Scotland.........................	6	50	9
52	1912, May 3......	Scotland.........................	6	60	11
53	1903, June 19......	Wales...........................	6½	180	27
54	1906, June 27......	Wales...........................	7	240	29
55	1884, Apr. 22......	England.........................	7½	320	32
56	1896, Dec. 17......	England.........................	7½	390	39
		CENTRAL EUROPE			
57	1938, June 11......	Belgium..........................	7	280	34
58	1937, Nov. 20......	Lower Rhine.....................	4½	40	13
59	1939, July 21......	Lower Rhine.....................	4	30	13
60	1928, Dec. 13......	Rhineland........................	5½	90	20
61	1933, Feb. 8......	Upper Rhine.....................	7	200	24
62	1935, Jan. 17......	Black Forest.....................	5	40	11
63	1935, Dec. 30......	Black Forest.....................	6½	250	38
64	1936, Apr. 19......	Schwäbische Alp.................	4½	50	17
65	1937, June 17......	Schwäbische Alp.................	5	70	19
66	1938, Aug. 2......	Schwäbische Alp.................	5½	50	11
67	1939, Mar. 1......	Schwäbische Alp.................	6	50	9
68	1911, Nov. 16......	Schwabische Alp.................	8	450	37
69	1913, July 20......	Schwäbische Alp.................	6	200	36
70	1933, Feb. 21......	Schwäbische Alp.................	5	160	43
71	1933, Feb. 26......	Schwäbische Alp.................	4	75	32
72	1933, June 4......	Schwäbische Alp.................	4	7	3
73	1933, Oct. 10......	Schwäbische Alp.................	4½	50	17
74	1933, Dec. 30......	Schwäbische Alp.................	4½	25	8

* For h', see text, p. 182.

TABLE 12—*Concluded*

No.	Date	Region	I_0	r	h' *
		CENTRAL EUROPE—*Continued*			
75	1934, Jan. 1......	Schwäbische Alp..................	5	40	11
76	1934, Mar. 17......	Schwäbische Alp..................	4	15	6
77	1934, Mar. 24......	Schwabische Alp..................	4½	25	8
78	1928, Aug. 3......	Schwabische Alp..................	5	50	14
79	1931, Dec. 12......	Schwäbische Alp..................	4½	40	13
80	1931, Dec. 22......	Schwäbische Alp..................	4½	30	10
81	1935, June 27......	Schwäbische Alp..................	8	500	42
82	1935, June 28......	Schwäbische Alp..................	5	200	54
83	1938, Apr. 11......	Upper Swabia.....................	6	80	14
84	1936, Mar. 15......	Lake Constance..................	5½	80	18
85	1936, July 1......	Lake Constance	5	60	16
86	1935, June 28......	Lake Constance..................	5	200	54

* For h', see text, p. 182.

The radius of perceptibility r is plotted on a logarithmic scale as a function of I_0 in figure 3 (b) for shocks numbers 1 to 38 of table 12, and in figure 3 (c) for shocks numbered 50 and over (Europe). Figure 3 includes curves drawn to represent equation (13) for various constant values of h.

In any one regional group the plotted points in figure 3 appear to fall systematically higher (greater r for given I_0) for larger shocks than for smaller ones. For Germany this agrees with the findings by Hiller (1935) that for many small shocks the microseismic evidence indicates shallow depth. Consequently there is no genuine systematic discrepancy between the observations and equation (13); the apparent effect is due to the occurrence of shocks at several different levels, at the deeper of which small shocks are less readily observed and may actually be rarer.

Some of the shocks in figure 2 (b) show unusually large apparent depth h'. A striking example is No. 19, the Missouri earthquake of 1895, which occasioned only moderate damage near its epicenter and yet was felt from the District of Columbia to New Mexico and from Canada to Louisiana (Heinrich, 1941, p. 197). Although an effect of local structure is possible, this and similar shocks must have had a significantly greater depth than ordinary. In the same general region shocks occur (No. 16) with normal relation of r to I_0.

The shocks listed in table 12 have been divided into four groups, representing four different ranges of h'. The epicenters are shown on the map (fig. 4) with distinct symbols for the four groups, each epicenter being marked with its serial number in table 12.

The shocks of greatest apparent depth h' fall into two geographically individualized groups. One of these (shocks numbered 1 and 2) is associated with the southern border of the Canadian Shield. For the shock of 1935 (No. 2) instrumental evidence independently suggests a depth of the order of 80 km. The earthquake of February 28, 1925, in the same region, also appears to belong to the same class; instrumental data are less satisfactory but are not at all inconsistent with depth of the order of 50 km. The New Hampshire earthquake of December 20, 1940, also had a large radius of perceptibility relative to its epicentral intensity.

The second group of shocks with large h' includes epicenters in the southern Appalachians, the central Mississippi Valley, and Texas. The fact that at St.

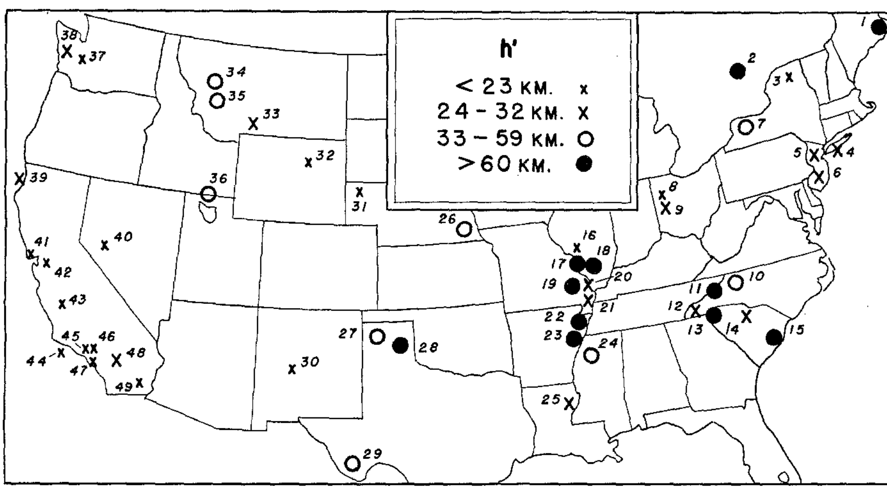

Fig. 4. Map of selected shocks, United States and Canada. The value of h' (apparent depth) indicates the extent of the disturbed area relative to the epicentral intensity. Serial numbers refer to table 12.

Louis, Little Rock, and Chicago the first longitudinal waves from certain distant shocks have been found to arrive as much as 4 seconds early (Lee, 1937; Gutenberg and Richter, 1938) suggests unusual structure of the deeper crustal layers and unusually high average wave velocities.

The values of h' (42 and 33 km.) for the two Montana shocks of 1935 follow unavoidably from the relatively large though strikingly different values of r; but these results conflict seriously with the depths of only a few kilometers determined from accelerograph records at Helena (Neumann, 1937).

The shocks in California have been included as representatives of the generally low value of h' in the Pacific Coast region, which corresponds to the instrumentally determined depth of about 18 km. for most of these shocks.

Slight changes in the estimated I_0 or r would shift several of the mapped shocks from one class to another. Increasing I_0 by one unit for given r will decrease h' by about one-third. Consequently, the method is not sufficiently accurate for deciding whether any of the shocks with largest h' actually belong to the class of intermediate earthquakes (defined as having actual hypocentral depth in excess of 60 km.).

Parameters of the equations.—Equations (9), (13), (20), and (21) form an interdependent system which is adjusted to observational data in several ways. They contain six independent constants. One of these, occurring only in (13), is Blake's constant s (Blake, 1941), which he takes to be 5.35 instead of the value 6 here used. If (9) is retained unmodified, a in equation (7) should have the exponent $6/s$ instead of 1. The effect of changing s from 5.35 to 6 is within the limits of error. The constant 1.5 in (13) is fixed by the slightly arbitrary choice of $I = 1.5$ at the limit of perceptibility.

The choice of constants in (9) is limited by data on the minimum perceptible acceleration a_r. This must agree with the choice of 1.5 in (13); (9) as here used then gives $a_r = 1$, which is satisfactory.

In (20) putting $a_0 = a_r$ should give the magnitude of the minimum perceptible shock. This restricts the constant term. The limits on the one remaining parameter appear better from (21), which should give the magnitude of the largest shocks for $I_0 = 12$. Since (21) follows from (9) and (20), this limits the coefficient of $\log a_0$ in (20).

It must be remembered that I and I_0 represent numbers on a partly arbitrary scale, assigned to the nearest unit, or occasionally to half units. This gives a peculiar appearance to those figures where values of I are plotted.

Another constant, $\log b_0$, is involved in the calculation of energies, but it is rather closely restricted by the observations. The same is true of the two constants in the equation for $\log t_0$ (28). In addition to these, the parameters in (20) are involved in the energy-magnitude relation (35), which imposes further limitations upon them. Calculation should not give absurdly small values for the energy of the smallest shocks (magnitude zero), nor too large values for the energies of great shocks.

Though each single equation of the group can be fitted to its pertinent data within a rather wide range of the parameters, this choice is much restricted by the consequent effect on the parameters of the related equations. The selection made here is that which seems to give the best general fit, so that each individual set of data is less perfectly represented than would appear to be the case if it were being handled separately without reference to other observations. It follows that any future improvement in any part of the data probably will necessitate revision of the entire system of equations, parameters, and constants.

The various parameters, which have been discussed from the point of view of practical seismology, belong to different categories from the point of view of pure physics. The fundamental physical equation must connect the acceleration a at a given point with the energy E radiated from the source. To the first approximation this involves only constants and the relative position of hypocenter and point of observation, specified by h and Δ. Such a general equation can be obtained by combining (19) and (38), which leads to

$$\log a = 0.31 \log E + \log h - 2 \log D - 3.5 \quad (41)$$

or, with a slight rounding off,

$$a = \frac{h \sqrt[3]{E}}{3000(\Delta^2 + h^2)} \quad (42)$$

Equation (41) contains four fundamental parameters as coefficients of the several terms. Since a is usually not directly accessible it is replaced for practical purpose by I, which is introduced through equation (9). E is accessible only by way of extensive computation, if at all; for California shocks at usual depth it is similarly replaced by the practical quantity M, which is related to it by (35). If greater precision were possible, (35) might be replaced by a more elaborate equation, which could then be used as a definition of magnitude in place of the practically convenient definition (1). Equation (9) and (35) thus introduce four new parameters, which are in the nature of arbitrary definitions of scale and of no physical importance. To reach such equations as (13), in which the radius of perceptibility enters, an additional "physiological parameter," $a_r = 1$ gal, is required.

All the important equations of the present paper can be derived from (41), (9), (35), and the datum $a_r = 1$.

Summary

The paper investigates the principal physical elements of earthquakes: the magnitude M, energy E, intensity I, acceleration a, and their relation to the depth h and radius of perceptibility r. ($r^2 + h^2 = R^2$. Subscript zero ($_0$) refers to the epicenter.) Equations

$$\log a = \frac{I}{3} - \tfrac{1}{2} \quad (9)$$

and

$$\frac{AD^2}{T^2} = \text{constant} \quad (4)$$

(A = ground amplitude, T = period, D = hypocentral distance for a given shock) are established empirically for California shocks. Equation (9) holds very generally, and offers a basis for a more accurate definition of I, like that suggested by Cancani. Equation (4) is here used very generally at short distances; but it is approximate only, may differ regionally, and bridges over the probably discontinuous transition of the maximum acceleration from $\overline{\mathrm{S}}$ to

some other transverse wave, with increasing distance. However, consequences derived from (4) nowhere conflict seriously with observation.

The instrumental earthquake-magnitude scale has been extended to cover short distances. The results enter into an empirical relation

$$M = 2.2 + 1.8 \log a_0 \tag{20}$$

from which and (9) follows

$$M = 1.3 + 0.6\, I_0 \tag{21}$$

These two equations are established and verified for the California region; they should also hold in other regions of similar structure for earthquakes originating at about the same depth (which is roughly 18 km.).

The simplest possible assumptions (constant velocity, negligible absorption, sinusoidal waves) lead to the general equation

$$\log E = 14.9 + 2 \log h + \log t_0 + 2 \log T_0 + 2 \log a_0 \tag{27}$$

(t_0 = duration, T_0 = period, of sinusoidal wave train at the epicenter).

Equations (27), (9), and (4) give the generally applicable results

$$a_0 h^2 = a D^2 = a_r R^2 \tag{19}$$

$$I_1 - I_2 = 6 \log \frac{D_2}{D_1} \tag{11}$$

$$I_0 - 1.5 = 6 \log \frac{R}{h} \tag{13}$$

a_r, the minimum perceptible acceleration, is approximately 1 gal.

For shocks at the usual depth in California

$$\log E = 11.3 + 1.8M \tag{35}$$

For other depths, and probably for other regions,

$$\log E = 9.5 + 3.2 \log h + 1.1\, I_0 \tag{39}$$

$$\log E = 11.1 + 6.4 \log R - 3.2 \log h \tag{40}$$

A summary of the physical elements for shocks in California is given in table 10.

Equation (13) is used to calculate apparent depths for earthquakes in the United States and Europe. The results tend to confirm the relatively shallow origin of shocks on the Pacific Coast compared with those occurring elsewhere, particularly under the Canadian Shield, the central Mississippi Valley, and the southern Appalachians.

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