

ECG Signal Reconstruction from Undersampled Measurement Using A Trained Overcomplete Dictionary

Seung Jae Lee, Jun Luan and Pai H. Chou

Department of Electrical Engineering and Computer Science
University of California, Irvine, CA 92697, USA

Copyright © 2014 Seung Jae Lee, Jun Luan, and Pai H. Chou. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

We propose a new approach to reconstructing ECG signal from undersampled data based on constructing a combined overcomplete dictionary. The dictionary is obtained by combining the trained dictionary by K-SVD dictionary learning algorithm with universal types of dictionary such as DCT or wavelet basis. Using the trained overcomplete dictionary, the proposed method can find sparse approximation by compressive sensing. Experimental results on MIT-BIH arrhythmia database confirm that our proposed algorithm has high reconstruction performance while maintaining low distortion.

Keywords: ECG signal reconstruction, K-SVD, Compressive Sensing

1 Introduction

Electrocardiogram (ECG) is one of the most significant types of signals for cardiac analyses and diagnostics. Proper analyses of ECG signals requires sizable data which demand more storage and bandwidth to transmit for wearable and mobile types of health monitoring devices, The amount of signal data has seen explosive growth while pressures on energy efficiency increase.

Traditional compression is performed after the whole data is obtained. This offline approach can be efficient in analyzing the data to reduce redundancies in the signal,

but storing data can incur high power. Compressive Sensing (CS) [1] - [4] shows the underlying signal can be reconstructed precisely from fewer samples than Shannon's so long as the signal is sparse or compressible in a certain domain, though the reconstructed signal will contain small distortions. In this paper, we propose a new compressive sensing and reconstruction technique for ECG signals by using a trained overcomplete dictionary to be constructed by K-SVD [5]. A comparison is presented with conventional compressive sensing signal reconstruction method using MIT-BIH arrhythmia database [6].

2 Background

2.1. Compressive Sensing Framework

Compressive sensing (CS) uses a linear transform to compress a signal and reconstructs it by exploiting its sparsity, which means that most of the components are zero or almost zero to represent the signal. Mathematically speaking, it assumes an N -dimensional signal \mathbf{x} can be represented as

$$\mathbf{x} = \sum_{i=1}^k \alpha_i \psi_i = \Psi \boldsymbol{\alpha} \quad (1)$$

where $\boldsymbol{\alpha} \in \mathbb{R}^N$, and $\Psi \in \mathbb{R}^{N \times N}$. By sensing system, the observation \mathbf{y} can be represented as

$$\mathbf{y} = \Phi \mathbf{x} = \Phi \Psi \boldsymbol{\alpha} \quad (2)$$

where $\mathbf{y} \in \mathbb{R}^m$, and $\Phi \in \mathbb{R}^{m \times N}$, which is called the sensing matrix. If $m \ll N$, the signal sensed by the system is undersampled. That means Eq. (2) is underdetermined, where \mathbf{y} is the observed signal from the sensing system and \mathbf{x} is the original signal to be reconstructed. There is no unique solution due to the ill-posed condition. However, if it is assumed that the original vector \mathbf{x} is a sparse or compressible signal, which has a few coefficients in the transformed domain by a basis matrix Ψ , we can find sparse approximation by adding sparsity constraint to the ill-posed problem, which is represented as

$$\min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_1 \quad \text{s.t.} \quad \|\mathbf{y} - \Phi \Psi \boldsymbol{\alpha}\|_2 \quad (3)$$

Algorithms for finding sparse approximation include orthogonal matching pursuit (OMP) [7], and least absolute shrinkage and selection operator (LASSO) [8].

2.2. Dictionary Learning

The approximation performance to find sparse approximation in terms of the approximation quality and the sparsity of the sparse vector $\boldsymbol{\alpha}$ depend on the dictionary as well as the signal itself. If the dictionary contains well-estimated atoms or basis vectors, then the sparse vector will have a small number of non-zero

elements. Universally used dictionaries, such as those for discrete cosine transform, Fourier transform, and wavelet transform, are orthogonal dictionaries. They have mathematical simplicity and few redundancies to represent dictionaries themselves, but they are not suitable to represent signals with few redundancies. Due to the limitation of the orthogonal dictionaries, researchers proposed overcomplete dictionaries by data-driven learning. K-SVD is one of the popular algorithms for constructing overcomplete dictionaries by learning. Given a set of n -dimensional signals $\mathbf{Y} = [y_1 y_2 \dots y_N] \in \mathbb{R}^{n \times N}$, the goal of K-SVD is to find the optimal dictionary $\mathbf{A} \in \mathbb{R}^{n \times K}$, where \mathbf{A} is an overcomplete dictionary ($K > n$). T_0 is sparsity, which means the maximum number of non-zero coefficients to represent given signal.

$$\min_{\mathbf{A}, \alpha} \|\mathbf{y} - \mathbf{A}\alpha\|_F^2, \text{ subject to } \|\alpha\|_0 \leq T_0, \forall i \tag{4}$$

3 Proposed Algorithm

Our proposed algorithm is a new approach to reconstructing a signal from compressive sensing system. Our proposed algorithm has a preprocessing stage for constructing a dictionary, which can make the signal sparse or compressible in the transformed domain. Fig.1 shows the full procedure of the proposed algorithm.

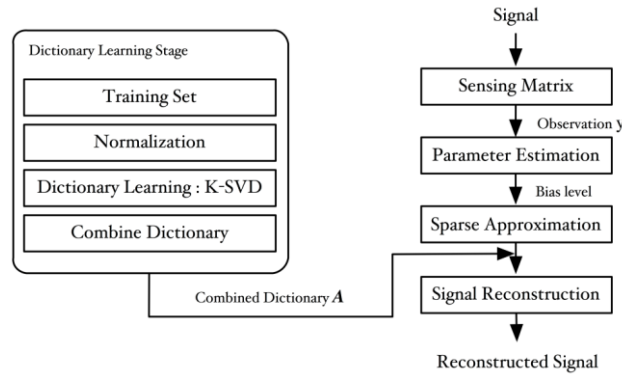


Fig. 1 Procedure of the proposed algorithm

3.1. Dictionary Learning Stage

Fig.1 shows the flow of our dictionary construction procedure for the overcomplete dictionary from the given dataset. For constructing a set of training data, our algorithm performs normalization. To remove the effects of the bias term, we also make the training data zero-mean. After normalization and removal of the bias term, a set of normalized signal vectors can be presented to $\mathbf{Y} = [y_1 y_2 \dots y_k] \in \mathbb{R}^{n \times k}$ as a data set for training. Using the training dataset \mathbf{Y} ,

the optimized dictionary to represent the data under the sparsity constraint is obtained by K-SVD dictionary learning.

The trained dictionary performs well when representing signals with a few coefficients, but not signals with generality due to reflecting the characteristics of the signal used at the training stage. To compensate for this shortcoming, our proposed method uses the combined dictionary, which is made up of general basis matrices, such as DCT or wavelet basis matrix, as atoms of the dictionary.

3.2. Reconstruction Strategy

Our proposed algorithm assumes that the signal would be obtained by random sampling, which means that the signal is selectively digitized in a time window. The sensing mechanism can be represented as a sensing matrix, $\Phi \in \mathbb{R}^{m \times n}$ ($m < n$), which can select random samples. The observation can be presented using the sensing matrix Φ . The key point of our proposed algorithm is the signal model. We assume that the original signal can be presented as

$$x = A\alpha + c \quad (5)$$

The signal is composed of two parts: the signal shape and the bias term, which defines vector c . Our target ECG signal has a changeable baseline, so it needs to separate the bias term to estimate the shape part more accurately. We can take the sensing matrix and our proposed signal model to derive the equation of observation y , thereby obtaining the following equation:

$$y = \Phi(A\alpha + c) = \Phi A\alpha + \Phi c \quad (6)$$

The observation y also has two parts: the shape (i.e., the zero-mean signal part) and the bias term. Since the sensing matrix is a simply random selection matrix, we can easily estimate the bias term by taking the average of the observation y for the parameter estimation step in Fig.1. The bias term, called vector c , is

$$c = c \cdot \mathbf{1} \simeq E(y) \cdot \mathbf{1} \quad (7)$$

where $\mathbf{1}$ means a vector whose elements are all 1. After estimating the parameter and subtracting it from observation, the simply zero-mean signal part remains.

$$\hat{y} = y - E(y) \cdot \mathbf{1} = \Phi A\alpha \quad (8)$$

The zero-mean observation part can be a simple linear equation with ill-posed conditions. In this paper, LASSO (Least Absolute Shrinkage and Selection Operator) is used as a solver for the convex problem. In this paper, we use MATLAB toolbox provided by sparselab [10] as a solver.

4 Experimental Results

For comparing the proposed algorithm with a conventional CS-based signal reconstruction algorithm, we have chosen the MIT-BIH arrhythmia database as the test signals. It provides 48 sets of 2-lead ECG signals at 360 11-bit samples/sec. Our metrics for evaluating signal reconstruction are percentage root-mean-squared distortion (PRD) and compression ratio (CR). PRD [9] shows the reconstruction error as a percentage and is defined as

$$\mathbf{PRD} = \sqrt{\frac{\sum_{i=1}^n (x_i - \hat{x}_i)^2}{\sum_{i=1}^n x_i^2}} \times 100 \quad (9)$$

where n is the number of samples, and x_i and \hat{x}_i are respectively the original data and the reconstructed data from the proposed algorithm. CR is defined as

$$\mathbf{CR} = \frac{b_{\text{orig}} - b_{\text{comp}}}{b_{\text{orig}}} \times 100 \quad (10)$$

where b_{orig} and b_{comp} represent the number of bits required for the original and compressed signals, respectively.

4.1. Dictionary Construction

We have chosen 24 out of 48 signals from the MIT-BIH arrhythmia database for training our algorithm and constructing the dictionary. Another 24 signals except for the signals used in training are utilized to evaluate the performance of our proposed algorithm. Table 1 depicts the signals included in training dataset.

User-specific parameters on the procedures of dictionary learning include sparsity, the size of dictionary, and the size of minimum signal block. The proposed algorithm uses $\mathbf{T}_0 = \mathbf{10}$ as sparsity constraint for dictionary construction. It also uses 64 samples as a minimum signal block, and 5012 atoms as a dictionary size, which means that the trained dictionary is a 64×5012 matrix. After obtaining the trained dictionary, we combine it with 64×64 DCT basis and wavelet basis, and the final size of the combined dictionary is 64×5140 .

Table 1. Data set for training and testing from MIT-BIH Arrhythmia Database

Records for Training	Records for Testing
100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 118, 119, 121, 122, 123, 124, 200, 201	117, 202, 203, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 228, 230, 231, 232, 233, 234

4.2. Comparison of the Reconstruction Results

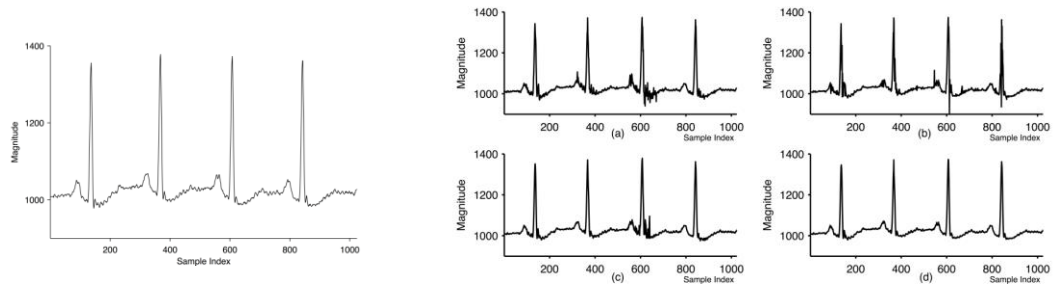


Fig. 2 Original Signal (Left) and Reconstruction Results (Right) (a) DCT basis (b) Wavelet basis (Daubechies4) (c) Trained Dictionary by K-SVD (d) Combined Dictionary

Figs. 2 (a) and 2 (b) show the results using conventional DCT and wavelet bases (daubechies4) with half of the randomly selected samples from original signal, i.e., CR=50%. Our method using the combined dictionary achieves PRD=1.34% when CR=50%. Fig. 3 depicts the performance over the MIT-BIH arrhythmia database. It shows high CR leads to high distortion. According to the dictionary or basis used into the reconstruction method, the performance is shown differently. Our proposed method shows less distortion than the other conventional methods do.

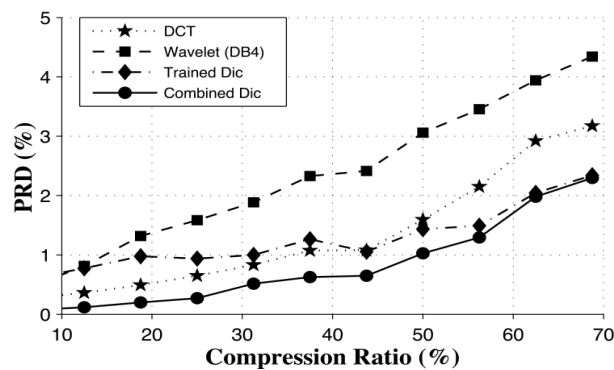


Fig. 3 Comparison of Compression Performance between Conventional Methods

5 Conclusion

We propose reconstructing ECG signals from undersampled data by compressive sensing. The signal reconstruction from undersampled data is performed by finding a sparse solution using a given dictionary, and the overall performance depends on the dictionary utilized in the approximation process. Dictionaries constructed by our method achieve good approximation performance by combining the trained dictionary by K-SVD with universal types of dictionaries such as DCT or wavelet transform. Our algorithm shows good performance with low distortion in terms of the CR and PRD over MIT-BIH arrhythmia database. Since the algorithm is learning-based method, the performance can be improved if the training dataset is organized well.

Acknowledgements. Research reported in this publication was supported by National Institutes of Health under award number R41HL112435. The content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institutes of Health.

References

- [1] E. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information", *IEEE Trans. Inform. Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006. <http://dx.doi.org/10.1109/tit.2005.862083>
- [2] D. L. Donoho, "Compressed sensing," *IEEE Trans. on Inform. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006. <http://dx.doi.org/10.1109/tit.2006.871582>
- [3] E. Candes and T. Tao, "Near optimal signal recovery from random projections: Universal encoding strategies", *IEEE Trans. Inform. Theory*, vol. 52, no. 12, pp. 5406–5425, Dec. 2006. <http://dx.doi.org/10.1109/tit.2006.885507>
- [4] E. Candes, and M. B. Wakin, "An introduction to compressive sampling", *Signal Processing Magazine*, IEEE, vol.25, no.2, pp.21-30, Mar.2008. <http://dx.doi.org/10.1109/msp.2007.914731>
- [5] M. Aharon, M. Elad, and A. Bruckstein, "K-SVD: An algorithm for designing overcomplete dictionaries for sparse representation", *IEEE Trans. Signal Processing*, vol. 54, no. 11, pp. 4311–4322, 2006. <http://dx.doi.org/10.1109/tsp.2006.881199>

- [6] A. L. Goldberger et al, “Physiobank, physiotookit, and physionet components of a new research resource for complex physiologic signals”, *Circulation*, vol. 101, no. 23, pp. e215–e220, 2000. <http://dx.doi.org/10.1161/01.cir.101.23.e215>
- [7] J. A. Tropp, “Greed is good: Algorithmic results for sparse approximation”, *IEEE Trans. Inform. Theory*, vol. 50, no. 10, pp. 2231–2242, 2004. <http://dx.doi.org/10.1109/tit.2004.834793>
- [8] R. Tibshirani, “Regression shrinkage and selection via the lasso”, *Journal of the Royal Statistical Society. Series B (Methodological)*, pp. 267–288, 1996.
- [9] S. M. S. Jalaliddine et al, “ECG data compression techniques: A unified approach”, *IEEE Trans. Biomed. Eng.*, vol. 37, no. 4, pp. 329–343, Apr. 1990. <http://dx.doi.org/10.1109/10.52340>
- [10] “MIT-BIH arrhythmia database” [Online]. Available: <http://www.physionet.org/physiobank/database/mitdb/>
- [11] “sparselab toolbox for MATLAB” [Online]. Available: <http://sparselab.stanford.edu>

Received: October 20, 2014; Published: December 2, 2014