

ECG SYNTHESIS VIA DISCRETE COSINE TRANSFORM

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ABSTRACT

Results on modelling and synthesis of ECG from its component waves are presented after a suitable transformation. The transformed components add up to the transformation of the complete beat (CB) and thus enable synthesis of ECG. Pole-zero modelling of the transformed signals by Shanks method proved very successful.

I. INTRODUCTION

The electrocardiogram (ECG) is invariably used for monitoring the heart function in routine morphological studies as well as in critical situations for rhythm analysis in ICCUs. In spite of the fact that QRS complex is the most prominent feature in the ECG noise sources such as base line wander, power line interference, muscle tremor peaky tented T waves and small premature ventricular contractions make QRS detection inaccurate and ambiguous. Time and frequency domain methods or a combination of these two based on single lead and/or multilead analysis have been developed. More sophisticated detectors are based on syntactic methods, heuristic approaches involving nonlinear transformations, matched filters, etc. [1]. Because of the intricate problems, detection of P and T waves remained as a challenge till to-day.

Attempts have been made to model the time domain ECG signal directly, which required very high orders [2]. However no method has been suggested to delineate the QRS and other component waves by modelling. It is shown here that modelling with a lower order system function and delineation of component waves is feasible when the ECG cycle is subjected to a suitable transformation such as discrete cosine transform (DCT) or discrete sine transform (DST). The method has been extended to model several transformed ECG cycles in a single operation as well as delineation of component waves in a given ECG cycle [3]. Because of its simplicity Shanks method is used here.

II. MODELLING

A. Discrete Cosine Transform (DCT):
 The DCT of a discrete time signal $X[n]$, $n = 0, 1, 2, \dots, N-1$ is by definition

$$X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X[n] \cos \left[\frac{(2n+1)k}{2N} \right]$$

$$k = 0, 1, 2, \dots, N-1$$

The sequence $X[k]$ thus obtained is a minimum or near minimum phase signal. The DCT of a complete ECG cycle can be obtained as the sum of the transforms of the individual components and vice-versa. These transformed components are then modelled by the Shanks algorithm [4].

B. Shanks Algorithm [6]
 Aim is to design a system function $F(z)$ to approximate $X(z)$, the z-transform of the given sequence $X(k)$. That is

$$F(z) \approx X(z) = \sum_{k=-\infty}^{\infty} X(k) z^k$$

Let $F(z) = [A(z)/B(z)] = f_0 + f_1 z + f_2 z^2 + \dots$

and $A(z) = a_0 + a_1 z + a_2 z^2 + \dots + a_N z^N$

$B(z) = 1 + b_1 z + b_2 z^2 + \dots + b_M z^M$

N and M are arbitrary numbers and fix the coefficients a_i and b_j , $i=1, \dots, N$, $j=1, \dots, M$. For the impulse response $f[k]$ of the model to approximate $x[k]$ in the minimum mean square sense, b_i has to satisfy the simultaneous equations:

$$\sum_{i=1}^M b_i \phi_{ij} = \phi_i \quad j=1, \dots, M$$

where

$$\phi_{ij} = \sum_{k=N+1}^L x(k-j) \cdot x(k-i)$$

$$\phi_i = \sum_{k=N+1}^L x(k) \cdot x(k-i)$$

L being the signal length. Similarly, a_i has to satisfy the following equations:

$$\sum_{i=0}^N a_i \psi_{ij} = \psi_i \quad j = 0, 1, \dots, N$$

$$\psi_{ij} = \sum_{k=0}^L C(k-i) C(k-j)$$

$$\psi_i = \sum_{k=0}^L x(k) C(k-i)$$

where $C(k)$, $k = 0, 1, \dots, L$ are obtained from $1/B(z)$ by long division. That is

$$1/B(z) = C(0) + C(1)z + \dots + C(L)z^L$$

III. RESULTS AND CONCLUSIONS

A low pass filtered and digitized at 100 sps ECG signal and the component waves P, QRS and T are shown in Figs. 1a, b, c, d respectively. Their DCTs shown in Figs. 2a, b, c, d, are modelled using orders (6,6) for CB and (2,2) for each of the component waves. The time domain outputs are obtained by computing the IDCT of the impulse response of the model. The pole-zero plot, the model impulse response and its IDCT are shown in Figs. 3a, b and c. The fit of the component waves as well as the CB obtained from the models are in close agreement with the original signals.

It is shown that the ECG signal when transformed can be pole-zero modelled using any algorithm such as Shanks method. The transforms of the component waves add up to the transform of the complete beat and thus enable synthesis from the components. The sum of model orders of individual components add up to that of the entire cycle.

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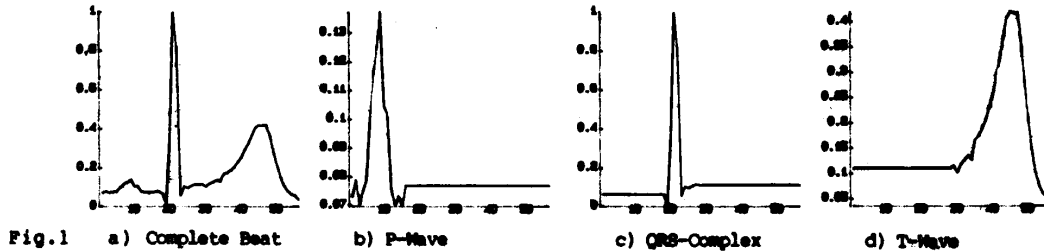


Fig.1 a) Complete Beat b) P-Wave c) QRS-Complex d) T-Wave

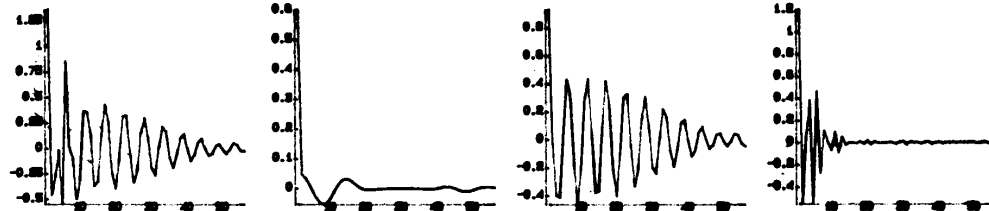


Fig.2 a) CB DCT b) P-Wave DCT c) QRS Complex DCT d) T-Wave DCT

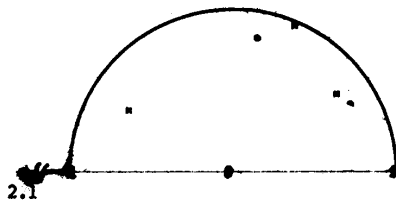
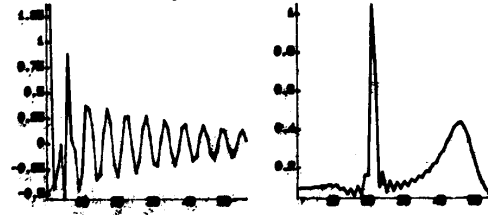


Fig.3 a) Pole-Zero Plot



b) Model Impulse Response c) IDCT of Imp. Resp.