Ecological Intuition versus Economic "Reason"

OLIVIER GUÉANT
Université Paris-Dauphine

ROGER GUESNERIE
Collège de France and Paris School of Economics

JEAN-MICHEL LASRY
CEREMADE, Université Paris-Dauphine

Abstract

This paper discusses the discount rate to be used in projects aimed at preserving the environment. The model has two different goods: one is the usual consumption good whose production may increase exponentially, and the other is an environmental good whose quality remains limited. The stylized world we describe is fully determined by four parameters, reflecting basic preferences, "ecological" and intergenerational concerns, and feasibility constraints. We define an ecological discount rate and examine its connections with the usual interest rate and the optimized growth rate. We discuss, in this simple world, different forms of the precautionary principle.

1. Introduction

Environmentalists have often dismissed the economists' approach of environmental problems, more especially when long-term issues are at stake. On the one hand, what may be called "ecological intuition" puts high priority on the long-run preservation of the environment. On the other hand, the

Olivier Guéant, Chaire Finance et Développement Durable, Université Paris-Dauphine, Place du Maréchal de Lattre de Tassigny, 75775 Paris, France. Roger Guesnerie, Collège de France, 3 rue d'Ulm, 75005 Paris, France (roger.guesnerie@college-de-france.fr). Jean-Michel Lasry, CEREMADE, Université Paris-Dauphine, Place du Maréchal de Lattre de Tassigny, 75775 Paris, France.

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cost-benefit analysis promoted from economic reasoning calls for the use of discount rates that apparently lead to dismiss the long-run concerns (see Arrow, Chenery, Minhas, and Solow 1961). The climate issue is the most recent avatar of the clash between "ecological intuition" and "economic reason" (see, e.g., Dasgupta 2004; Heal 2003; Lecocq and Hourcade 2001): in sharp contrast with most environmentalists and many climatologists' sensitivity, the computations based on Nordhaus (1993) and Nordhaus and Boyer (2000) suggest lenient climate policies. And although Nordhaus has been cautious in warning against misinterpretations, some of his less cautious readers (Lomborg 2001) claim that their fight against climate policies proceeds from "economic reason." The Stern review (Stern *et al.* 2006) has changed the tone of the debate significantly. It is clear that Stern's views of "economic reason" and the subsequent cost-benefit analysis were initially not broadly accepted in the economic profession, but it seems that they are now gaining some acceptance (for comments and response to comments see Weitzman 2007 and Stern 2008).

This paper attempts to retackle the clear antagonism between the two sides from a simple model that has been recurrently used in the economists' debate (see Heal 1998), but the relevance of which in the present debate has been recently more systematically stressed by Guesnerie (2004), Traeger (2012), Hoel and Sterner (2007), and Sterner and Persson (2008). The model assumes that there are two goods at each period: the environment (a nonmarket good) and standard aggregate consumption. The first one is supposed to be available in finite quantity when the second one is allowed to grow forever. The opposition between a finite level of environmental good and an increasing level of consumption good echoes a core determinant of the "ecological" sensitivity: sites, lands, seashores, and species are finitely available on the planet. On the contrary, modern optimism, based on the "economics" of past growth performance, leads to believe that consumption of the so-called private goods may be multiplied without limit (see Aghion, Howitt, Brant-Collett, and Garcia-Peñalosa 1998; Barro and Sala-i-Martín 2003).

We discuss the long-run cost-benefit analysis issues that arise within a model that has indeed two goods, with the respective interpretations of aggregate consumption and aggregate environmental quality that have just been introduced. As emphasized in Guesnerie (2004), in such a setting, cost-benefit analysis has to stress, not only the standard discount rates but also, *the "ecological" discount rate*, the evolution of which reflects the relative price of environment vis-à-vis the standard private good.[1]

The simple infinite horizon world under scrutiny is entirely described by *four parameters*. The first parameter describes how substitutable are the standard and environmental goods in producing welfare. Opinions on the

[1] It is well known that in an n-commodity world, there are as many discount rates as there are goods (see Malinvaud 1953 for a general appraisal of this question and Milleron, Guesnerie, and Crémieux 1978 for applications to standard problems of cost-benefit analysis).

value of this parameter may differ and lead to oppose a "moderate" environmentalist and a "radical" environmentalist. The second parameter is the classical elasticity of marginal utility which reflects the extent to which welfare is subject to saturation, and which classically determines the intertemporal "resistance to substitution." The third parameter is a pure rate of time preference which, in this setting, measures the degree of intergenerational altruism of the agents. The last parameter will be an interest rate which, in the logic of a simple endogenous growth context (of the AK type), indicates to which extent one can transfer consumption between periods and generations.

Within this model, the research agenda is clear: we have to understand how the parameters under consideration affect the trade-off between present and future consumption, both for standard or "environmental" consumption. As argued above, key dimensions of such trade-offs are captured through the *"ecological" discount rates*. Indeed, such rates provide central ingredients to the cost-benefit analysis of actions aiming at preserving the future environment. Our analysis can then focus on the assessment of what we call *environmental perpetuities*, which provide a key information for the cost-benefit analysis of actions aiming at avoiding "irreversible damage to the environment." This leads us to examine and assess the logic of the *precautionary principle*, which focuses attention on irreversible damage to the environment in case of "scientific uncertainty."

The paper proceeds as follows. Section 2 of the paper presents the setting of the model and the role of the different parameters. We present the basic concepts and introduce the "ecological discount rate" independently of the growth model.

In Section 3, we introduce a two-good growth model *à la Ramsey* in which the environmental good quality remains constant over time. Along with the derivation of asymptotic results, the analysis allows to exhibit the time pattern of both the optimal growth rates of private consumption and the "ecological discount rates." We are able to characterize yield curves in a way that allows us to single out a simple lower bound for the social loss due to an "irreversible damage to the environment," or to put it in another way to help us to price the so-called "environmental perpetuity."

Section 4 focuses on various forms of precautionary principles. (Earlier literature on the subject includes Gollier, Jullien, and Treich 2000.) We consider an irreversible damage to the environment that will take place at some later date and the effect of which on (present and future) welfare is uncertain. We raise the question of the willingness to pay of the present generation in order to avoid it. Indeed, the analysis in Section 2 provides an answer to the same question, when there is no uncertainty on the welfare effect of the damage. When, as considered in this section, the damage has an uncertain impact on welfare, we stress first a "weak precautionary principle": it is reminiscent, for ecological discount rates, of Weitzman's classical argument (2001) on long-run standard discount rates. Second, we exhibit a "strong precautionary principle," which we view as the most striking result of this paper. It tells us that the effort of the present generation should be based on a cost-benefit analysis that overweighs in a spectacular way the probabilities of the events associated with bad environmental outcomes.

The connections of the paper with the literature are as follows. Models with two goods include Heal (1998).[2] The model of the paper is the one considered in Guesnerie (2004), and the argument exploits the findings of this paper. It also refers to some of the insights of Hoel and Sterner (2007), Guéant, Lasry, and Zerbib (2007), and Sterner and Persson (2008), and the extension of Troeger (2012). All these papers refer to the concept of "ecological discount rates" emphasized in Guesnerie (2004), a concept that has also been stressed in a somewhat more complex setting than ours, and with a different focus, by Gollier (2010). Also, note that the importance of substitutability, which we emphasize here, has been stressed earlier in Gerlagh and Van der Zwann (2002).

Note that the views presented here on discounting and precaution have a motivation closely connected to the one of Weitzman (2009). However, our emphasis is on relative prices effects: even if we put emphasis on the uncertainty that surrounds the long-run environmental issues and on the weight to be put on the bad case, we do not stress "fat tails."

2. Model and Preliminary Insights

2.1. Goods and Preferences

We are considering a world with two goods. Each of them has to be viewed as an aggregate. The first one is the standard aggregate private consumption of growth models. The second one is called the environmental good. Its "quantity" provides an aggregate measure of "environmental quality" at a given time. It may be viewed as an index reflecting biodiversity, the quality of landscapes, nature and recreational spaces, the quality of climate, and the availability of water.

We call x_t the quantity of private goods available at period t, and y_t the level of environmental quality at the same period. Generation t, which lives at period t only, has ordinal preferences, represented by a Constant Elasticity of Substitution (CES; see Arrow *et al.* 1961) utility function: $v(x_t, y_t) = [x_t^{\frac{\sigma-1}{\sigma}} + y_t^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$.

However, the measurement of cardinal utility, on which intertemporal judgments of welfare will be made, involves an isoelastic function: $V(x_t, y_t) = \frac{1}{1-\eta} v(x_t, y_t)^{1-\eta}$.

The above modeling calls for the following comments:

- Concerning v, we have to stress two points:
  - The reader has noted that x_t and y_t appear with the same coefficient in the function v. However, for a given generation, this is without

[2] There is also a related literature, which considers, as second good, an exhaustible resource. For recent developments of this literature, see d'Autume and Schubert (2008).

loss of generality since it is only a matter of unit in the measurement of y_t. Hence, giving the same weight to the private good index and the environmental quality index is a matter of notational convenience. However, leaving these weights constant across time or at least bounding them to be *nonvanishing* is a substantive assumption. It implies, in particular, that the concern for any of the two goods does not shrink. The present assumption on the symmetric role of x and y is intended to reflect the fact that we "only have one planet," the preservation of which is not, and will never be, a point of minor concern for its inhabitants, whatever their ability to produce large quantities of new private goods. Even, if the specific modeling is crude, this point seems well taken for our purpose in the sense that we do not deny *a priori* the soundness of "ecological intuition."

– v is a CES utility function, where σ is the elasticity of substitution between the two goods.[3] It describes a specific pattern of substitution: when the ratio environmental quantity (here quality) over private good quantity decreases by 1% the marginal willingness to pay for the environmental good, or its implicit price, increases by $1/\sigma\%$.

This setting with constant elasticity of substitution allows to write easily what may be called the Green NDP. If we, indeed, regard the consumption good as the *numéraire*, then the number $y(\frac{x}{y})^{\frac{1}{\sigma}}$ is what we call Green NDP. Note that it grows indefinitely whenever x grows indefinitely, if, as we suppose here, y remains finite. Also, note that the ratio of Green NDP over standard NDP is (independent of any *numéraire*) $\rho = (\frac{y}{x})^{1-\frac{1}{\sigma}}$ and the ratio of green NDP to total NDP is $\lambda = \frac{\rho}{1+\rho}$.

- Let us come to V. The marginal utility of a "util" of v takes the form $v^{-\eta}$: when v increases by 1%, marginal cardinal utility decreases by $\eta\%$. This coefficient $\frac{1}{\eta}$ has the standard interpretation of an intertemporal elasticity of substitution.

2.2. Social Welfare

Social welfare is evaluated as the sum of generational utilities. In line with the argument of Koopmans, we adopt the standard utilitarian criterion:

$$\frac{1}{1-\eta}\sum_{t=0}^{+\infty} e^{-\delta t} v(x_t, y_t)^{1-\eta}.$$

Two comments can be made:

- The coefficient δ is a rate of pure time preference. Within the normative viewpoint which we mainly stress here, the fact that this coefficient

[3] In what follows, we will ignore the Cobb-Douglas $\sigma = 1$ because it provides specific results.

is positive has been criticized, for example, by Ramsey who claims that it is "ethically indefensible and arises merely from the weakness of the imagination" or Harrod who views it as a "polite expression for rapacity and the conquest of reason by passion." Reconciling these feelings with Koopmanns's argument leads us, however, to accept a positive and small δ. The smaller the δ, the more "ethical considerations become preponderant." Along the "ethical" line of argument, it has been argued that the number might be viewed as the probability of survival of the planet.

- We may view the coefficient η as a purely descriptive one, reflecting intertemporal substitution, or as a partly normative coefficient, reflecting the desirability of income redistribution across generations. This is the more frequent interpretation we stress in the paper: a low (respectively, high) η reflects little (respectively, a lot of) concern for intergenerational equality.

At this stage, something more can be said on the philosophy of the approach taken here. We have adopted a stylized description of the trade-off between environmental quality and private consumption. We recognize that the modeling of the trade-off is crude. However, if the degree of substitutability between standard consumption good and environment is fixed, we leave its value open. At this stage, we do not decide whether σ is smaller, a plausible short-run hypothesis,[4] or greater than 1, and we leave it fixed. We associate a high σ (respectively, low σ) to a moderate (respectively, radical) environmentalist's viewpoint, the dividing line being obviously $\sigma = 1$.

At this stage, one should give some insights on the qualitative differences between the cases $\sigma > 1$ and $\sigma < 1$, i.e., between the opinions we attribute, respectively, to the "moderate" and the "radical" environmentalists. These differences echo the views that shape the understanding of the future long-run usefulness of environmental quality when compared to private consumption.

First, let us consider $\sigma > 1$. We have $v(x_t, y_t) = x_t[1 + (\frac{y_t}{x_t})^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$, and hence, v grows as x_t whenever $\frac{y_t}{x_t}$ tends to zero. The asymptotic relative contribution of environment to welfare is vanishing, and similarly, the Green NDP becomes small when compared to standard NDP. As we shall see later, the moderate environmentalist is very moderate in the long run.

On the contrary, in the case where $\sigma < 1$, it is useful to write $v(x_t, y_t) = y_t[1 + (\frac{y_t}{x_t})^{\frac{1-\sigma}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$. In that case, v does not grow any longer indefinitely with x_t, but tends to $\overline{y}$ (if $y_t = \overline{y}$ for $t \geq 0$). The increase in the consumption of private goods still contributes to welfare but with an asymptotic limit associated with the level of environmental quality. Standard NDP becomes small with respect to Green NDP.

[4] The marginal willingness for environmental amenities seems to grow faster than private wealth (see Krutilla and Cicchetti 1972).

Before turning to the intertemporal social optimum in a Ramsey growth model, we need to introduce the main concept of this paper, namely, the ecological discount rate.

2.3. Ecological Discount Rate

2.3.1. Definitions

In order to give some intuition on the question of discount rates, we shall consider a trajectory of the economy where environmental quality is fixed at the level $\overline{y}$ and where the sequence of private goods consumption denoted by x_t is also given (we will note g_t the growth rate implicitly defined by $x_{t+1} = e^{g_t} x_t$).

We shall investigate the implicit discount factors at the margin of our reference trajectory, which is the discount rates that make the reference trajectory locally optimal.

DEFINITION 1: *The implicit discount rate for private good between periods t and $t+1$ is r_t such that*

$$e^{-r_t} = e^{-\delta} \frac{\partial_x V(x_{t+1}, \overline{y})}{\partial_x V(x_t, \overline{y})}.$$

The discount rate between periods 0 and T is then classically defined as

$$R(T) = \frac{1}{T} \sum_{t=0}^{T-1} r_t.$$

The discount rate $R(T)$ tells us, as is standard, that one unit of consumption at period T is (socially) equivalent to $e^{-R(T)T}$ today.

We then introduce the ecological discount rate, which, as stressed in Guesnerie (2004), is the discount rate specific to the environmental good.[5]

DEFINITION 2: *The ecological implicit discount rate between two consecutive periods is β_t defined by:*

$$e^{-\beta_t} = e^{-\delta} \frac{\partial_y V(x_{t+1}, \overline{y})}{\partial_y V(x_t, \overline{y})}.$$

The discount rate between periods 0 and T is

$$B(T) = \frac{1}{T} \sum_{t=0}^{T-1} \beta_t.$$

[5] Hoel and Sterner (2007) consider the same model as here or as in Guesnerie (2004), without referring explicitly to the "ecological discount rate."

The ecological discount rate tells us that one marginal improvement of environment at period T is socially equivalent to $e^{-B(T)T}$ of the same improvement occurring today. It implies that the present generation, when viewing an improvement of environment occurring at period T (the improvement being, for example, triggered by some present spending), should compare the present cost with the discounted value (discounted with the ecological discount rate) of the *present* marginal willingness to pay for the same improvement today. (This is what is called "standard" ecological cost-benefit analysis by Guesnerie 2004).

2.3.2. General Properties

We now provide explicit formulas for the implicit discount rates along any given trajectory.

PROPOSITION 1: *The implicit private discount rate for the private good between periods t and $t+1$ can be equivalently written as either:*

$$r_t = \delta + g_t\eta + \frac{1-\sigma\eta}{\sigma-1}\ln\left(\frac{1+\rho_t}{1+\rho_{t+1}}\right)$$

or

$$r_t = \delta + g_t/\sigma + \frac{1-\sigma\eta}{\sigma-1}\ln\left(\frac{1+\rho_t^{-1}}{1+\rho_{t+1}^{-1}}\right),$$

where $\rho_t = \frac{y_t\partial_y V}{x_t\partial_x V} = (\frac{x_t}{y_t})^{\frac{1}{\sigma}-1}$ is the ratio of Green NDP over standard NDP.

The first formula shows how the standard logic of discount rates ($r_t = \delta + g_t\eta$) is affected by the environmental concern. The correction depends upon the evolution of the ratio ρ_t of Green NDP over standard NDP. The second formula looks strikingly different from the first one, although it is equivalent, but it puts emphasis on factors that will turn out to be dominant when $\sigma < 1$.

Now, we can finally relate the ecological discount rate to the interest rate:

PROPOSITION 2: *The ecological discount rate between periods t and $t+1$ is related to the interest rate by*

$$\beta_t = r_t - g_t/\sigma.$$

This last formula stresses the effect of the growth of private consumption on the ecological discount rate: it is qualitatively unsurprising that it is connected to the standard discount rate with a negative correction that increases with the growth rate and decreases when the elasticity of substitution increases. This formula, which *captures the relative price effect* that we are stressing here, is particularly simple and intuitively appealing. We can think about it as follows: it would be equivalent to give up one unit of environmental quality at the present period t, in order to provide e^{β_t} of environmental quality tomorrow, but the suggested move is equivalent, from the viewpoint of both generations, to give up ω_t units of private goods (where ω_t is the willingness to pay for environmental amenities) and to provide $\omega_t e^{r_t}$ units, as soon as $\omega_t e^{r_t}$ compensate for one unit of environmental quality at time $t+1$, which is the case, if and only if $\omega_t e^{r_t} = \omega_{t+1} e^{\beta_t}$. It is straightforward that $\omega_{t+1} = \omega_t e^{g_t/\sigma}$ so that $\omega_t e^{r_t} = \omega_t e^{g_t/\sigma} e^{\beta_t}$. The conclusion follows and stresses a key ingredient for the understanding of the argument of the present paper.

3. Optimized Growth: Private Consumption, Ecological Discount Rates, and Their Evolution

3.1. Introduction

In Guesnerie (2004), asymptotic results for the ecological discount rate were derived at the margin of any trajectory whether the considered trajectory was nonoptimal, or optimal either in a first best sense or in a second best sense. Here, the evolution of ecological discount rates is going to be studied at the margin of an optimal trajectory that depends on the value of the parameters. There is indeed *a priori* no reason to refer to the same growth rate of consumption under different assumptions on σ, since these assumptions reflect different views (moderate or radical) of the contribution of the environment to welfare, and then potentially very different views on desirable growth.

In our model, we stick to the option of a fixed environmental quality, but put emphasis on the endogeneity of private consumption, and we choose the simplistic endogenous setting of the AK type, where the interest rate r is exogenous, being then a one-dimensional sufficient statistics of the intertemporal production possibilities.[6] Hence, as announced in the introduction, our discussion within the model will focus on four parameters only. A first one, σ, associated with the ecological concern, a second one, η, linked to the intertemporal structure of preferences, the third one, δ, associated with "ethical" considerations, and the last one, r, describing economic constraints.

3.2. Optimized Growth and Asymptotic Results

Our viewpoint is normative, and we refer to the intertemporal social welfare function introduced above. The "social Planner" maximizes:

$$\sum_{t=0}^{\infty} e^{-\delta t} V(x_t, y_t).$$

[6] Note that such an interest rate r can be extracted from a *research arbitrage equation* (as in Aghion *et al.* (1998)), partly disconnected from the core model.

Our modeling choice leads the following economic and environmental constraints:

Economic constraints: $\alpha_{t+1} = e^r(\alpha_t - x_t)$, where α_t stands for the wealth at date t.[7]

Environmental constraints: The environmental quality is limited to $\overline{y}$, that is: $y_t \leq \overline{y}$.

We naturally assume that $r > \delta$. Furthermore, in this model, it is easy to check that optimization would lead to an infinite postponement of consumption if $r(1-\eta) > \delta$. We rule this out and assume that $\eta > 1 - \frac{\delta}{r}$. This means, given the order of magnitude that we have in mind for δ, that we will consider that η is essentially greater than 1.

This hypothesis on the elasticity of intertemporal substitution goes with another one that is going to be made in the remaining of this paper, namely, $\eta\sigma > 1$. Because we suppose that $\eta > 1$, this is simply a hypothesis on σ, which is supposed not to be too small.[8]

The next proposition gathers all the asymptotic results of social optimization. The first part stresses that optimality requires asymptotically constant growth whatever the parameters under scrutiny. However, both the asymptotic economic growth rates and the long-run ecological discount rates crucially depend on the value of σ and η:

PROPOSITION 3: *At the optimum, the private goods consumption grows asymptotically.*

The optimal asymptotic growth rate for the private good x_t^* depends on σ and is given by the following formulas:

- *If $\sigma > 1$ then $g_\infty^* = \frac{r-\delta}{\eta}$,*
- *If $\sigma < 1$ then $g_\infty^* = \sigma(r-\delta)$.*

The asymptotic ecological discount rate, associated with the socially optimal trajectory, is $B_\infty^* = \lim_{T\to+\infty} B^*(T)$ given by the following formulas:

- *If $\sigma > 1$, then $B_\infty^* = (1 - \frac{1}{\sigma\eta})r + \frac{1}{\sigma\eta}\delta$.*
- *If $\sigma < 1$, then $B_\infty^* = \delta$.*

For $\sigma > 1$, the asymptotic growth rate of consumption is $\frac{r-\delta}{\eta}$, fitting the standard formula of the one-good model: the presence of the environmental good has asymptotically no influence on the growth rate (although it does on

[7] A slightly more sophisticated version allows $\alpha_{t+1} = e^r[\alpha_t - x_t + w_t]$, where α_t stands for the wealth at date t and w_t is a possible exogenous production flow that introduces no binding constraint into the analysis.

[8] This last hypothesis is made to simplify and lighten this paper. However, since our conclusions are already in favor of voluntarist environmental policy, we believe that focusing on smaller σs would add little to the point.

the optimal trajectory). However, even in this case, the asymptotic ecological discount rate is always smaller than r.

The result for the other case ($\sigma < 1$) may be surprising for two reasons: first, it was *a priori* unclear that the "radical" environmentalist would choose a positive asymptotic growth. The second point is more surprising since the asymptotic ecological discount rate is totally disconnected from r and is very low[9] since we assume δ to be close to zero.

The opposition between the "radical" environmentalist and the "moderate" one is clearly stressed by the behavior of the ecological discount rate. The asymptotic difference is again spectacular, as shown if we plot the asymptotic ecological interest rate as a function of σ Figure 1.

The asymptotic results stress a discontinuity in the world around $\sigma = 1$. However, the significance of the discontinuity is clarified and qualified by the next result.

PROPOSITION 4: *At each period T, the optimal trajectory is a continuous function of the parameters σ. Subsequently, $\forall T < \infty, \sigma \mapsto B^*(T;\sigma)$ is continuous.*

In a sense, the discontinuity associated with $\sigma = 1$ is worrying and might be viewed as an objection[10] to our (admittedly crude) modeling choice. The above continuity result, which says that, at any given period, results are

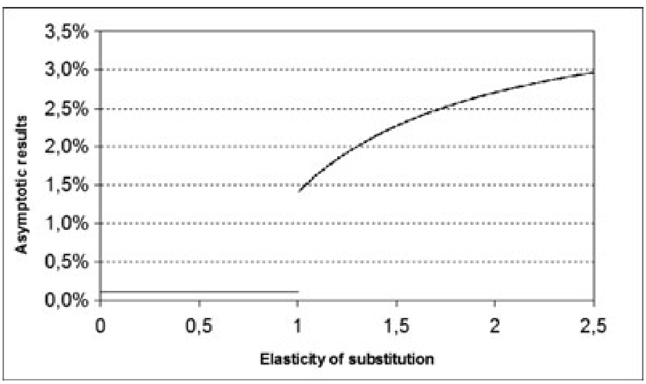

Figure 1: Dependence on σ of the variable B^*_∞ when $\eta = 1.5$, $r = 4\%$, and $\delta = 0.1\%$.

[9] In some sense, since we do not consider the negative impact of consumption on the environment, we end up with an upper bound on the ecological discount rate. For instance, if one considers an exogenous exhaustion of the environment at rate g', the asymptotic ecological discount rate is B^*_∞ given by $\delta - \eta g'$ if $\sigma < 1$ and by $(1 - \frac{1}{\sigma\eta})r + \frac{1}{\sigma\eta}\delta - \frac{g'}{\sigma}$ if $\sigma > 1$. Hence, the effect of the exhaustion is to decrease the ecological discount rate that can even be negative.

[10] Or an appropriate modeling option, since it suggests a possible catastrophic change of the system.

continuous functions of σ, weakens the objection: the discontinuity "in the limit" is compatible with continuity "at the limit": indeed, $B^*(T)$ is a continuous function of σ when T is fixed (and finite), as stated above.

All these results suggest to put the emphasis on the trajectory of discount rates.

3.3. The Dynamics of Ecological Discount Rates

Here, we are focusing attention on the evolution of ecological discount rates with time, and what can be called yield curves for ecological discount rates $B^*(T)$.

Since $B^*(T) = r - \frac{1}{\sigma}\frac{1}{T}\sum_{t=0}^{T-1} g_t^*$, the dynamics of the ecological discount rate is linked to the dynamics of growth. Indeed, the dynamics of optimal growth can be assessed here (we still suppose that $\sigma\eta > 1$).

PROPOSITION 5: *g_t^* converges monotonically toward its limit according to the following rules*:

- *If $\sigma < 1$ then, g_t^* is increasing.*
- *If $\sigma > 1$, then g_t^* is decreasing.*

COROLLARY 1: *The shape of the yield curve is the following*:

- *If $\sigma < 1$, then $T \mapsto B^*(T)$ is decreasing (respectively, increasing) and converges toward δ.*
- *If $\sigma > 1$, then $T \mapsto B^*(T)$ is increasing (respectively, decreasing) and converges toward $(1 - \frac{1}{\sigma\eta})r + \frac{1}{\sigma\eta}\delta$.*

To illustrate our proposition, we drew yield curves using a simulation of the growth path (Figures 2 and 3).

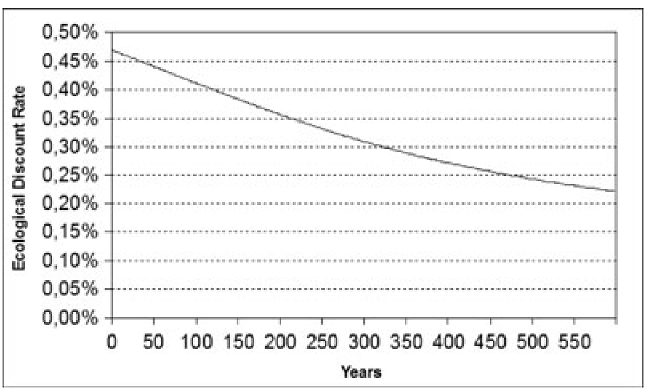

Figure 2: Yield curve example ($\sigma = 0.8$, $\eta = 1.5$, $r = 4\%$, and $\delta = 0.1\%$).

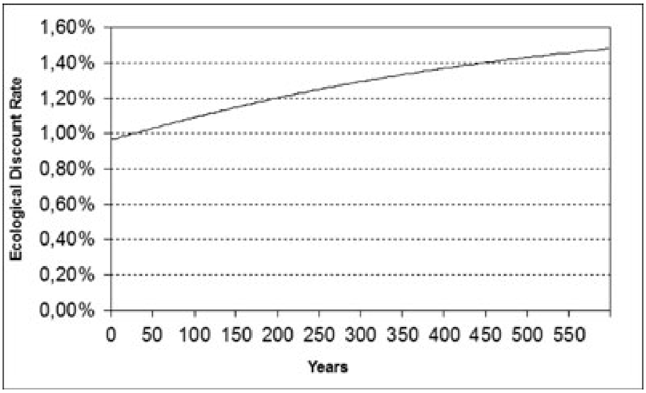

Figure 3: Yield curve example ($\sigma = 1.2$, $\eta = 1.5$, $r = 4\%$, and $\delta = 0.1\%$).

Two examples are given below:

As it comes from the previous statements, in the first case ($\sigma < 1$), the yield curve is decreasing and converges toward δ. In the second case ($\sigma > 1$), the yield curve is increasing and converges toward $(1 - \frac{1}{\sigma\eta})r + \frac{1}{\sigma\eta}\delta$.

The figures suggest that ecological discount rates converge slowly to their asymptotic value. Another interesting and related visual insight is that when σ is low, the rate is low, but, even when σ is high, because the curve is increasing, the environmental rate is still low in the medium run. Hence, what the figures show is that *for a time period between one and two centuries from now, the disagreement between the radical environmentalist and the moderate environmentalist* is not huge: the first one is between 0.45% and 0.35% and the second one is between 0.95% and 1.2%. Their willingness to pay, for say a generation living at date 150 equals the discounted value, with the ecological discount rate, respectively, roughly 2/3 and 1/3, multiplied by their own marginal willingness to pay, which itself depends on their wealth and their "ecological" views or intuition.

3.4. Environmental Perpetuity

Yield curves provide a key information about the dynamics of ecological discount rates. It should be noted that the conceptually important information conveyed in Proposition 3 on the limit behavior of discount rates has no clear operational consequence for cost-benefit analysis (we do not know how long is the long run). On the contrary, the understanding of the path of convergence stressed in Corollary 1 has an evident bite on the conclusions of cost-benefit analysis. In what follows, we are going to consider a simple problem that brings a necessary brick to the understanding of the (less simple) issues associated with the worldwide debate on the so-called precautionary principle.

The problem under scrutiny is the following: consider a damage to the environment that would take place today and say that in order to avoid this damage for itself, the present generation is willing to pay x. How much should it be willing to pay if this damage not only occurs now but is irreversible, i.e., if it deteriorates the well-being of all future generations? Let us call mx the willingness to pay to avoid this damage for all future generations, instead of x, the willingness to pay when the damage is temporary[11]) and only concerns the present generation.

In a sense, avoiding the damage can then be viewed as providing x environmental perpetuities (a perpetuity being an infinitely lived environmental service giving one unit of environmental good at each period). Hence, m is the "price" to be given to each of these perpetuities.

We provide here a lower bound on m.

PROPOSITION 6: *Let us introduce* $a = r(1 - \frac{1}{\sigma\eta}) + \delta\frac{1}{\sigma\eta}$.
In the present deterministic context, if the initial generation is willing to pay x in order to avoid a temporary (here one year) damage, it is willing to pay mx to avoid making it irreversible, where the number m is greater than $\frac{1}{a}$.

The reader will notice that in our admittedly simple world, the result has a striking simplicity and robustness. First, the lower bound to m is valid both[12] for $\sigma > 1$ and for $\sigma < 1$. Second, it is also remarkable that the bound on m does not depend on initial wealth.

Let us note that if the planner neglected the relative price effect associated with the increase in relative desirability of the environmental good, the discount rate would be r and m would be approximately $\frac{1}{r}$ (approximately because we use an exponential discounting) as for a classical perpetuity. Hence, the introduction of the environmental good can drastically change the willingness to pay of the present generation for an environmental perpetuity that protects all future generations from an irreversible damage. For instance, if we consider that $\delta \simeq 0$, then m is, in our deterministic study with $\sigma\eta > 1$, greater than the "naive" assessment $\frac{1}{r}$, the multiplier being $\frac{1}{1-\frac{1}{\sigma\eta}}$. If we consider the parameters values associated with the above graphs ($\eta = 1.5$), instead of having $m \simeq 25$ (respectively, $m \simeq 50$) for $r = 4\%$ (respectively, $r = 2\%$), we get when $\sigma = 0.8$, $m \geq 6 \times 25 = 150$ (respectively, $m \geq 300$) and with $\sigma = 1.2$, $m \geq 2.25 \times 25 \simeq 56$ (respectively, $m \geq 112.5$).

Let us now consider the case where the irreversible damage will occur later in period τ, possibly far away from now. Again, the above question is meaningful, although m is no longer *a priori* necessarily greater than one.

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[11] All these reasonings can easily be adapted to settings in which the life duration of each generation is T periods.

[12] However, the result depends on our hypothesis $\sigma\eta > 1$. If $\sigma\eta < 1$, then the lower bound is nothing but $\frac{1}{\delta}$ which is very high.

PROPOSITION 7: $m > e^{-a\tau}\frac{1}{a}$.

The previous proposition told us that a may be viewed as an upper bound for the discount rate to be used for evaluating "environmental perpetuities." It is remarkable that the present proposition tells us that the same is true, i.e., a can consistently be used to provide a lower bound of the value of what might be called an "environmental forward perpetuity."

4. Precautionary Principle: How to Tackle the Uncertainty about the Elasticity of Substitution σ?

4.1. An Unusual Form of Uncertainty

4.1.1. Introduction

In the preceding paragraphs on environmental perpetuities, we focus on the desirable action to be taken in order to avoid an "irreversible damage to the environment." The so-called precautionary principle, in its most standard formulations, stresses the uncertainty surrounding a damage: *"Where there are threats of serious or irreversible damage, lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation."* This leaves somewhat open the question of the right intensity of action. This is the question tackled in this section. It suggests cost-benefit analysis tools, aimed at evaluating the desirability of precaution in a situation where uncertainty plays a major role.

In the present framework, we focus attention on an irreversible damage that will take place in the future, and whose harmfulness is now unclear but will be fully revealed when the damage occurs. Noteworthy, we do not consider that the damage itself has an uncertain intensity, although this is clearly the case in reality. Rather, our focus is on its harmfulness. In other words, we will focus on the uncertain impact of the damage in terms of welfare. Indeed, we believe that, as far as the environmental protection of the planet and climate change in particular are concerned, an important part of the uncertainty originates in the extent of the welfare impact of "ecological accidents" and not only on their intensity.

Formally, we assume that the uncertainty bears on the welfare function and more precisely on σ: in the first periods, this uncertainty is not resolved and σ can take two values: σ_l or σ_h ($\sigma_l < 1 < \sigma_h$)—and we attribute probabilities p and $1-p$ to the respective cases. The two values reflect the *a priori* viewpoints of what we have called the radical and the moderate environmentalists. At time τ, an irreversible damage to the environment will take place (it consists here of a small decrease of $\overline{y}$) and the social cost of the damage will be revealed, i.e., the true value of σ will be known (either σ_l or σ_h). In a sense, the occurrence of the environmental "accident" at time τ provides an experiment that allows to assess exactly the value of σ. The fact that nothing will be learnt between now and τ remains extreme. This assumption simplifies the analysis, an analysis which remains an unavoidable reference and a prerequisite to the consideration of progressive accrual of the information.

4.1.2. The Optimization Problem

As suggested above, let us assume that $\sigma \in \{\sigma_l, \sigma_h\}$, where $\sigma_l < 1 < \sigma_h$ is learnt instantaneously at a time $\tau > 0$. The new optimization problem to determine the consumption path is the following:

$$\sum_{t=0}^{\tau-1} e^{-\delta t}[p\,V(\sigma_l; x_t, \overline{y}) + (1-p)\,V(\sigma_h; x_t, \overline{y})] + p\mathcal{U}(\alpha_\tau, \sigma_l) + (1-p)\mathcal{U}(\alpha_\tau, \sigma_h),$$

with α_0 given, $\alpha_{t+1} = e^r[\alpha_t - x_t]$ and where $\mathcal{U}(\alpha, \sigma) = \mathrm{Max}_{(x_t)_{t\geq\tau}} \sum_{t=\tau}^{\infty} e^{-\delta t} V(\sigma; x_t, \overline{y})$ is the Bellman function associated with the non-random problem after we learnt σ. At this time, the deterministic results provide the required information, given the initial condition which is the remaining wealth α_τ.

The next sections stress that the case $\sigma < 1$ should be weighted significantly in our present decisions, even if it is unlikely. We will present different forms of this result that clearly echo the just discussed precautionary principle.

4.1.3. A First Result: A Weak Precautionary Principle

The first version of this precautionary principle (the weak precautionary principle) is an asymptotic statement: the rate to be used to discount environmental good is asymptotically the ecological discount rate corresponding to the smallest σ (i.e., $\sigma = \sigma_l < 1$). The second and stronger form of precautionary principle bears on the way m depends on p.

The resolution of the above problem is similar to what we have done before in the deterministic case, at least for the asymptotics. After σ has been elicited, the two trajectories x_t^{*l} and x_t^{*h}, which are identical for $t < \tau$, diverge: if σ is equal to σ_l, the asymptotic growth rate of $x_t^* = x_t^{*l}$ is $g_\infty^* = \sigma_l(r - \delta)$ and if σ is equal to σ_h, the asymptotic growth rate of $x_t^* = x_t^{*h}$ is $g_\infty^* = \frac{r-\delta}{\eta}$.

Using these asymptotic results on growth and the formula defining the ecological discount rate in this context—namely, $e^{-B^*(T)T} = e^{-\delta T}[\frac{p\,\partial_y V(\sigma_l; x_T^{*l}, \overline{y}) + (1-p)\,\partial_y V(\sigma_h; x_T^{*h}, \overline{y})}{p\,\partial_y V(\sigma_l; x_0^*, \overline{y}) + (1-p)\,\partial_y V(\sigma_h; x_0^*, \overline{y})}]$—we can deduce the asymptotic value of the ecological discount rate.

PROPOSITION 8 (*Weak Precautionary Principle*): *Viewed from time zero, the asymptotic ecological discount rate B_∞^* does not depend on $p > 0$ and is equal to*

$$B_\infty^* = \delta.$$

Uncertainty leads us to consider asymptotically the smallest possible ecological rate. This is the counterpart for the "ecological discount rate" of the limit behavior of discount rates, stressed by Weitzman (2001). This is a precautionary principle, in the sense that it suggests putting emphasis on the long-run bad situations even if uncertain. It is weak, since, as argued above, its operational content for cost-benefit analysis is almost nil. The next section provides an operational precautionary principle.

4.2. Strong Precautionary Principle

4.2.1. The Question

The question raised here is similar to that raised in a deterministic context: how much is the present generation willing to pay in order to avoid an irreversible damage to the environment that would take place at time τ? However, and contrary to our deterministic case, the harmfulness of the (fixed) damage in terms of welfare is not well ascertained.

Our objective is to generalize the previous deterministic results on the multiplier m, which relates the willingness to pay of the present generation[13] to avoid the damage for itself, forgetting about its descendants or viewed as temporary, to its willingness to pay to avoid the irreversible damage at date τ.

We know the answer in the limit deterministic cases: m has a lower bound $e^{-a\tau}\frac{1}{a}$, where $a = a(l) = r(1 - \frac{1}{\sigma_l \eta}) + \delta\frac{1}{\sigma_l \eta}$ if σ is equal to σ_l , and similarly, $a = a(h) = r(1 - \frac{1}{\sigma_h \eta}) + \delta\frac{1}{\sigma_h \eta}$ if σ is equal to σ_h.

What are plausible conjectures on the bounds on the multiplier in the stochastic case? One may expect m to be bounded from below by an expression of the form

$$e^{-a\tau}\left[p\frac{1}{a(l)} + (1-p)\frac{1}{a(h)}\right],$$

where a would neither be $a(h)$ nor $a(l)$ and where the term between bracket is the expected value of the future damage to the environment as seen from period τ.

The following proposition shows that the intuitive conjecture is valid only once the probability of the bad case[14] is biased upward. Indeed, this upward bias is spectacular:

PROPOSITION 9 (*Strong Precautionary Principle, first version*):

Let us introduce, as in the deterministic case, $a(h) = r(1 - \frac{1}{\sigma_h \eta}) + \delta\frac{1}{\sigma_h \eta}$, *and similarly,* $a(l) = r(1 - \frac{1}{\sigma_l \eta}) + \delta\frac{1}{\sigma_l \eta}$.

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[13] Note that, naturally, the willingness to pay of the present generation depends on its wealth and the true value of σ. In the uncertain case under scrutiny, again, the willingness to pay of the present generation does depend on the plausibility of the two cases, as measured by p, the probability of being characterized by a low σ.

[14] The bad case here, and from now, refers to the case of a low σ ($\sigma = \sigma_l < 1$).

In the random case, if p *lies in* $(0, 1)$, *we have*

$$m > e^{-B^*(\tau)\tau}\left[\frac{1}{a(l)}\left(\frac{pN^*(\tau)}{pN^*(\tau)+(1-p)}\right)+\frac{1}{a(h)}\left(\frac{(1-p)}{pN^*(\tau)+(1-p)}\right)\right],$$

where $N^*(\tau) > 1$ *grows exponentially with* τ.

The above formula provides information on the bounds on m that, as desirable, do encompass the information obtained in the deterministic case. Note, however, that the bound we find here does not only depend, as in the deterministic case, on the four basic parameters of the models, but also on the characteristics of the initial situation (in particular, through $N^*(\tau)$).

The proof is given in the Appendix, but we may give some insights into it. The fact that we discount at time 0 the willingness to pay at τ with the ecological discount rate $B^*(\tau)$ is intuitively unsurprising, as well as the consequence of easy algebra. As suggested above, the fact that the (bounds on the) value of the irreversible damage to the environment, seen from period τ, when the bad case occurs (respectively, the good case), be proportional to $\frac{1}{a(l)}$ (respectively, $\frac{1}{a(h)}$), is, in view of our previous results, intuitive. Now, concerning the weights to associate to each case, it may be less intuitive that the ratio of the appropriate corrections to be made to p and $1-p$ has to be the ratio of the marginal utility of the environment in the bad and in the good cases, which is nothing else that $N^*(\tau)$. The fact that $N^*(\tau)$ increases with τ, and is unbounded, follows from the examination of the formulas, just in line with the intuition briefly presented in Section 1.

Hence, the expectation of the deterministic lower bounds stressed in Proposition 6 (which comes naturally into the picture, as suggested above) has to be measured with distorted probabilities. Indeed, the probability to attribute to the bad case with respect to the good case has to be severely distorted: the later the date, the more weight we put on the bad case, the weight becoming closer to its limit 1, counteracting the (weak) tendency of the (ecological) discount rate to dismiss precaution for late damages. Let us be more explicit on that by considering the following corollary:

COROLLARY 2 (*Strong Precautionary Principle, second version*): *There exists a function* $(p, \tau) \mapsto \phi(p, \tau)$, *concave with respect to* p, *verifying*:

$$\phi(0,\tau) = \frac{1}{a(h)}, \qquad \phi(1,\tau) = \frac{1}{a(l)},$$

$$\frac{d\phi}{dp}(p=0,\tau) = \left(\frac{1}{a(l)} - \frac{1}{a(h)}\right)N^*(\tau) \underset{\tau\to+\infty}{\longrightarrow} +\infty,$$

$$\frac{d\phi}{dp}(p=1,\tau) = \left(\frac{1}{a(l)} - \frac{1}{a(h)}\right)\frac{1}{N^*(\tau)} \underset{\tau\to+\infty}{\longrightarrow} 0,$$

$$\lim_{\tau \to +\infty} \phi(p, \tau) = \frac{1}{a(l)}, \forall p > 0$$

such that

$$m > e^{-B^*(\tau)\tau} \phi(p, \tau).$$

To well understand the last statements, let us come back to the "plausible" conjecture discussed at the beginning of the subsection. It was suggested that m might be the discounted value (with the appropriate discount rate) of $p\frac{1}{a(l)} + (1-p)\frac{1}{a(h)}$. What our analysis says is that if τ is large, and p large, the intuitive formula tends to be right; but that when p is small, the lower bound on the multiplier is far from the discounted value of $p\frac{1}{a(l)} + (1-p)\frac{1}{a(h)}$ and closer to the discounted value of $\frac{1}{a(l)}$. This is a clear and strong form of precautionary principle. If we do not know whether or not an environmental accident will lead to real downfall in welfare in the future, here at date 100, a key element of our computation is, in a sense, to proceed as if the bad case were to happen for sure.

Let us illustrate the importance of the point with numbers. Suppose that we are in a world in which the present willingness to pay to avoid the irreversible damage from the viewpoint of the sole present generation welfare is, let us say, 0.1% of its NDP, if the harm is minor in terms of welfare, and 1% if the welfare harm is high. What bounds can we find on its willingness to pay for avoiding the irreversible accident occurring at period $\tau = 100$? With the data previously used ($r = 4\%$, $\delta = 0.1\%$, $\eta = 1.5$, $\sigma_l = 0.8$, $\sigma_h = 1.2$), we have $a(l) = 1/150$ and $a(h) \simeq 1/56$ so that $e^{-a(l)\tau} \simeq 1/2$ and $e^{-a(h)\tau} \simeq 1/5.9$. Hence, for a small p, say $p = 1/10$, the intuitive lower bound for the multiplier, applying broad linear approximations, is just below 14 (which means a willingness to pay to avoid the accident of 2.6% of NDP[15]). Now, with the bounds given in Proposition 9, the same calculations, assuming that τ is large enough to apply the approximation, give a lower bound for m equal to 31.5, i.e., a willingness to pay of around 6% of NDP (more than twice more), and this for a low probability of the occurrence of the accident... For a high probability accident, the lower bound on the multiplier is 75.

Although they are already large, we leave to the reader to view these numbers as applying metaphorically to the question of climate change, particularly in view of the fact that the computed multipliers are even far higher.

Indeed, although an exact solution of the optimization program is untractable analytically, the random case can easily be solved numerically for all ps. We illustrate our results with the preceding set of parameters in which, as above, before time $\tau = 100$ (σ is revealed at this time), the agent hesitates between $\sigma_h = 1.2$ and $\sigma_l = 0.8$ (with probabilities $1-p$ and p). In this situation, with $r = 4\%$, $\delta = 0.1\%$, $\eta = 1.5$, we find numerically the ecological discount rates and compute m for any possible p in $[0; 1]$.

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[15] Applying once again a rough linear approximation.

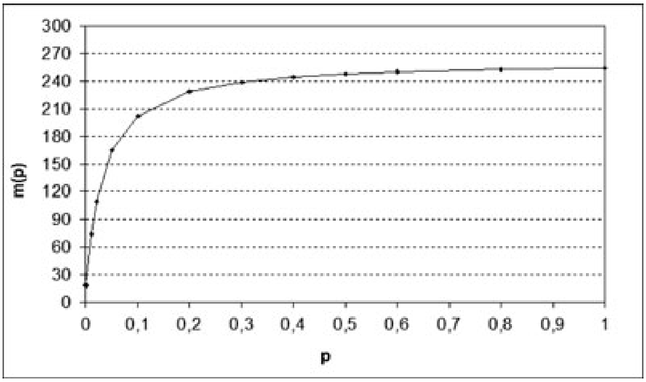

Figure 4: $p \mapsto m(p)$.

Figure 4 illustrates in a spectacular way our qualitative statement: the function $p \mapsto m(p)$ is quickly increasing (and concave). Hence, even for small p strictly greater than 0, m is far from $m(p = 0)$ and closer to $m(p = 1)$.

5. Conclusion

The paper proposes a simple model for discussing the long-run issues associated with environmental quality. The model describes a world with four parameters that respectively reflect ecological concern, resistance to intertemporal substitution, intergenerational altruism, and feasibility constraints. These parameters are supposed to remain constant over time, an assumption which makes the model tractable and simple, although it is certainly too extreme. Note that the paper takes a parsimonious defense of the environmentalist viewpoint in the sense that we rule out values of parameters too much favorable to his views: we assume that growth has no negative effect on the environment, etc.

The paper shows that long-run environmental policies are crucially affected by the "ecological view," in particular but not only, if the radical viewpoint is adopted. Also, the paper shows that the radical viewpoint on environment, even when it is unlikely to be true, has, however, bite on the determination of present policies, a fact that may be viewed as supporting some form of a precautionary principle. In a companion paper (work in progress), we will provide back of the envelope computations based on a variant of the present model to the global warming issue that suggests an upward reevaluation of the Stern estimates of the merits of action.

Let us repeat that our simple setting allows to focus both on the relative price effect and the uncertainty dimension of the economic appraisal of ecological intuition. To put it in a nutshell, the paper stresses that the "economic" argument, along which we should not sacrifice the present generations' welfare to the welfare of our descendants that will be wealthier than us, is valid here, but has to be strongly qualified. There is a most valuable gift that is worth transmitting to our descendants, because it may be very important for them, although this is not sure. That gift is a good environment.

Appendix

Proof of Proposition 1: The implicit discount rate r_t for private goods between periods t and $t+1$ is uniquely defined by

$$e^{-r_t} = e^{-\delta} \frac{\partial_x V(x_{t+1}, \overline{y})}{\partial_x V(x_t, \overline{y})} = e^{-\delta} \left(\frac{x_{t+1}}{x_t} \right)^{-\frac{1}{\sigma}} \left(\frac{x_{t+1}{}^{\frac{\sigma-1}{\sigma}} + \overline{y}^{\frac{\sigma-1}{\sigma}}}{x_t{}^{\frac{\sigma-1}{\sigma}} + \overline{y}^{\frac{\sigma-1}{\sigma}}} \right)^{\frac{1-\sigma\eta}{\sigma-1}}.$$

Taking logarithms, this gives

$$\begin{aligned} r_t &= \delta + g_t/\sigma - \frac{1-\sigma\eta}{\sigma-1} \ln \left(\frac{x_{t+1}{}^{\frac{\sigma-1}{\sigma}} + \overline{y}^{\frac{\sigma-1}{\sigma}}}{x_t{}^{\frac{\sigma-1}{\sigma}} + \overline{y}^{\frac{\sigma-1}{\sigma}}} \right) \\ &= \delta + g_t/\sigma + \frac{1-\sigma\eta}{\sigma-1} \ln \left(\frac{1+\rho_t^{-1}}{1+\rho_{t+1}^{-1}} \right). \end{aligned}$$

This is the second formula of Proposition 1. The first formula can be obtained by the same reasoning:

$$\begin{aligned} r_t &= \delta + g_t/\sigma - \frac{1-\sigma\eta}{\sigma-1} \ln \left[\left(\frac{x_{t+1}}{x_t} \right)^{\frac{\sigma-1}{\sigma}} \frac{1 + \left(\frac{\overline{y}}{x_{t+1}} \right)^{\frac{\sigma-1}{\sigma}}}{1 + \left(\frac{\overline{y}}{x_t} \right)^{\frac{\sigma-1}{\sigma}}} \right] \\ &= \delta + g_t\eta + \frac{1-\sigma\eta}{\sigma-1} \ln \left(\frac{1+\rho_t}{1+\rho_{t+1}} \right). \end{aligned}$$

■

Proof of Proposition 2: We have $e^{-\beta_t} = e^{-\delta} \frac{(\partial_y V)_{t+1}}{(\partial y V)_t} = e^{-\delta} \frac{(\partial_x V)_{t+1}}{(\partial_x V)_t} (\frac{x_{t+1}}{x_t})^{1/\sigma} = e^{-r_t} e^{g_t/\sigma}$, and hence, $\beta_t = r_t - g_t/\sigma$. ■

Proof of Proposition 3: We consider the Lagrangian of the problem $\mathcal{L} = \sum_{t=0}^{\infty} \exp(-\delta t) [V(x_t, y_t) + \lambda_t (e^r [\alpha_t - x_t] - \alpha_{t+1}) + \mu_t (\overline{y} - y_t)]$.

The first-order conditions are the following:

$$\begin{cases} \partial_{x_t} \mathcal{L} = 0 \iff \partial_x V(x_t^*, y_t^*) = e^r \lambda_t, \\ \partial_{\alpha_{t+1}} \mathcal{L} = 0 \iff \lambda_{t+1} \exp(r - \delta) = \lambda_t, \\ \partial_{y_t} \mathcal{L} = 0 \iff \partial_y V(x_t^*, y_t^*) = \mu_t. \end{cases}$$

The first thing to note is that $y_t^* = \overline{y}$. Then, since $r > \delta$, λ_t and $\partial_x V(x_t^*, \overline{y})$ are both decreasing and tend to zero. The natural consequence is that the consumption of the private good x_t^* grows and tends to $+\infty$.

The growth path x_t^* is then characterized by $x_t^{*-\frac{1}{\sigma}}[x_t^{*\frac{\sigma-1}{\sigma}} + \overline{y}^{\frac{\sigma-1}{\sigma}}]^{\frac{1-\sigma\eta}{\sigma-1}} = \partial_x V(x_t^*, \overline{y}) = e^r\lambda_t = \frac{\lambda_0 e^r}{\exp((r-\delta)t)}$.

- In the $\sigma > 1$ case, $x_t^{*-\eta} \sim_\infty \frac{\lambda_0 e^r}{\exp((r-\delta)t)}$. Hence, the asymptotic growth rate is the same as if there were no consideration of the environmental good $g_\infty^* = \frac{r-\delta}{\eta}$.
- In the $\sigma < 1$ case, $x_t^{*-\frac{1}{\sigma}} \sim_\infty \frac{\lambda_0 e^r}{\overline{y}^{\frac{1}{\sigma}-\eta}\exp((r-\delta)t)}$. Hence, the growth rate in that case is given by $g_\infty^* = \sigma(r-\delta)$.

The results on the ecological discount rate then follow from Proposition 2. ■

Proof of Proposition 4: *(This proof can be omitted at first reading.)* It is very important here to consider $v(x, y) = [\frac{1}{2}x^{\frac{\sigma-1}{\sigma}} + \frac{1}{2}y^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$ with the weights $\frac{1}{2}$ to extend the function properly and also to remind that $V = \frac{v^{1-\sigma'}-1}{1-\sigma'}$. Obviously, it does not change anything to our preceding results since these changes only consist in additive or multiplicative scalar adjustment.

We have $\beta_t = r - \frac{g_t^*}{\sigma}$, and thus $B^*(T) = r - \frac{1}{\sigma}\frac{1}{T}\ln(\frac{x_T^*}{x_0^*})$.

Therefore, the only thing to prove is that $\forall t$, x_t^* is a continuous function of σ. But we know that the growth path is defined by the first-order condition $\partial_x V(x_t^*;\sigma) = \frac{\lambda_0 e^r}{\exp((r-\delta)t)}$, where we omitted the reference to $\overline{y}$ here since we focus on σ. Then, it is easy to see that the only two things we need to prove are that

- The Lagrange multiplier λ_0 is a continuous function of σ.
- The function $h(\xi, \sigma)$ implicitly defined by $\partial_x V(g(\xi, \sigma);\sigma) = \xi$ is continuous.

The second point is easy. Notice first that the function $(x, \sigma) \mapsto V(x;\sigma)$ can be extended to a C^2 function (the proof is easy). Then, by the implicit function theorem, $h(\xi, \sigma)$ is a C^1 function ($(\xi, \sigma) \in (\mathbb{R}^{+*})^2$). Therefore, the only thing to prove is that the first Lagrange multiplier λ_0 is a continuous function of σ. Let us recall that λ_0 is defined by the resources constraint $\sum_{t=0}^{\infty} x_t^* e^{-rt} = \sum_{t=0}^{\infty} h(\lambda_0 e^r \exp((\delta - r)t), \sigma)e^{-rt} = \alpha_0 + \sum_{t=0}^{\infty} w_t e^{-rt} (:= \Lambda_\infty$[16]$)$.

Here, we cannot apply directly the implicit function theorem to the left-hand side. However, if we consider the restricted optimization problem with a fixed time horizon T,[17] then the associated Lagrange multiplier (λ_0^T) is

[16] This quantity is supposed finite for the problem to have a solution.
[17] Max $\sum_{t=0}^{T} \exp(-\rho t)u(x_t, \overline{y})$ s.t. $\alpha_{t+1} = e^r[\alpha_t + w_t - x_t]$.

implicitly defined by

$$\sum_{t=0}^{T} h\big(\lambda_0^T \exp((\delta - r)t), \sigma\big)e^{-rt} = \alpha_0 + \sum_{t=0}^{T} w_t e^{-rt} (:= \Lambda_T),$$

and the implicit function theorem applies: λ_0^T is a C^1 function of σ.

Now, we can approximate λ_0 by λ_0^T and this gives: $|\lambda_0(\sigma) - \lambda_0(\tilde{\sigma})| \leq |\lambda_0(\sigma) - \lambda_0^T(\sigma)| + |\lambda_0^T(\sigma) - \lambda_0^T(\tilde{\sigma})| + |\lambda_0^T(\tilde{\sigma}) - \lambda_0(\tilde{\sigma})|$.

Hence, we see that the only thing to prove is a pointwise convergence in the sense that for σ fixed, we have a convergence of $\lambda_0^T(\sigma)$ toward $\lambda_0(\sigma)$ as $T \to \infty$. To prove that let us introduce $F_T : z \mapsto \sum_{t=0}^{T} h(ze^r \exp((\delta - r)t), \sigma)e^{-rt}$, and similarly, $F : z \mapsto \sum_{t=0}^{\infty} h(ze^r \exp((\delta - r)t), \sigma)e^{-rt}$. These two functions are positive and decreasing because h is a positive and decreasing function of ξ. Moreover, F_T is continuous and there is a pointwise convergence of F_T toward F. By monotony, F_T converges toward F uniformly on every compact set, and therefore, F is a continuous function and so is the inverse of the function F.

By the second Dini's theorem then, the inverse of the function F_T converges uniformly on every compact set toward the inverse of the function F.

But $\lambda_0^T - \lambda_0 = F_T^{-1}(\Lambda_T) - F^{-1}(\Lambda_\infty)$, and hence, since $\Lambda_T \to \Lambda_\infty$, we are done with the proof. ■

Proof of Proposition 5: Let us go back the first-order conditions that define the growth path. We have $\partial_x V(x_t^*, \overline{y}) = e^{r-\delta}\partial_x V(x_t^* e^{g_t^*}, \overline{y})$.

Therefore, the growth rate g, as a function of x, is defined implicitly by (we now omit the $\overline{y}$ terms) $V'(x)\exp(r - \delta) = V'(xe^{g(x)})$.

Taking logs and deriving, we get $\frac{V''(x)}{V'(x)} = \frac{V''(xe^{g(x)})}{V'(xe^{g(x)})} e^{g(x)}(1 + g'(x)x)$.

Hence, the sign of $g'(x)$ is the sign of $V'(x)V''(xe^{g(x)})e^{g(x)} - V'(xe^{g(x)})V''(x)$. This sign is simply the sign of $\frac{d}{dx}\frac{V'(xe^g)}{V'(x)}$, where g is now an independent variable. The latter expression can be written as $e^{-g/\sigma}\frac{d}{dx}[\frac{\overline{y}+(xe^g)^{\frac{\sigma}{\sigma-1}}}{\overline{y}+x^{\frac{\sigma}{\sigma-1}}}]^{\frac{1-\sigma\eta}{\sigma-1}}$.

The sign of this derivative is the sign of $\frac{1-\sigma\eta}{\sigma-1}\frac{\sigma-1}{\sigma}(e^{g\frac{\sigma-1}{\sigma}} - 1) = \frac{1-\sigma\eta}{\sigma}(e^{g\frac{\sigma-1}{\sigma}} - 1)$.

Since $g > 0$ in our context, this expression has the same sign as $1 - \sigma$ and this proves our result. ■

Proof of Proposition 6: By definition, m is equal to $\sum_{T=0}^{\infty} \exp(-B^*(T)T)$. Since we want to find a lower bound for m, we need to find an upper bound for $B^*(T)$.

The ecological rate $B^*(T)$ can be written as $B^*(T) = r - \frac{1}{\sigma T}\sum_{t=0}^{T-1} g_t^*$. Hence, the problem boils down to find a lower bound for g_t^*.

Now, from Proposition 1, we know that a lower bound to g_t^* is $\frac{r-\delta}{\eta}$ so that $B^*(T) \leq a$.

This gives $m = \sum_{T=0}^{\infty} \exp(-B^*(T)\,T) \geq \sum_{T=0}^{\infty} \exp(-aT) = \frac{1}{1-\exp(-a)} \geq \frac{1}{a}$. ■

Proof of Proposition 7: By definition, m is now equal to $\sum_{T=\tau}^{\infty} \exp(-B^*(T)\,T)$.

Using the same inequality as before, we have $m \geq \sum_{T=\tau}^{\infty} \exp(-aT) = \frac{\exp(-a\tau)}{1-\exp(-a)} \geq e^{-a\tau}\frac{1}{a}$. ■

Proof of Proposition 8: Let us consider $T > \tau$ and let us recall first the definition of $B^*(T)$ in this context:

$$B^*(T) = \delta - \frac{1}{T}\ln\left[\frac{p\,\partial_y V\big(\sigma_l;\, x_T^{*l}, \overline{y}\big) + (1-p)\,\partial_y V\big(\sigma_h;\, x_T^{*h}, \overline{y}\big)}{p\,\partial_y V\big(\sigma_l;\, x_0^{*l}, \overline{y}\big) + (1-p)\,\partial_y V\big(\sigma_h;\, x_0^{*h}, \overline{y}\big)}\right].$$

To prove our result, it is sufficient to prove that the expression in the logarithm remains bounded as T increases. Hence, we are going to prove that the following expression is bounded:

$$p\,\overline{y}^{-\frac{1}{\sigma_l}}\left[x_T^{*l\,\frac{\sigma_l-1}{\sigma_l}} + \overline{y}^{\frac{\sigma_l-1}{\sigma_l}}\right]^{\frac{1-\sigma_l\eta}{\sigma_l-1}} + (1-p)\,\overline{y}^{-\frac{1}{\sigma_h}}\left[x_T^{*h\,\frac{\sigma_h-1}{\sigma_h}} + \overline{y}^{\frac{\sigma_h-1}{\sigma_h}}\right]^{\frac{1-\sigma_h\eta}{\sigma_h-1}}.$$

The first part of the expression converges toward $p\,\overline{y}^{-\eta}$ and is therefore bounded.

For the second part of the expression, $x_T^{*h\,\frac{\sigma_h-1}{\sigma_h}} + \overline{y}^{\frac{\sigma_h-1}{\sigma_h}} \to \infty$ so that, since $\frac{1-\sigma_h\eta}{\sigma_h-1} < 0$ (we supposed $\sigma_h\eta > 1$), the second part of the expression tends toward 0 and this proves the result. ■

Proof of Proposition 9: For $T \geq \tau$, we have by definition $\exp(-B^*(T)\,T) = \exp(-\delta T)\,[\frac{p\,\partial_y V(\sigma_l;x_T^{*l},\overline{y}) + (1-p)\,\partial_y V(\sigma_h;x_T^{*h},\overline{y})}{p\,\partial_y V(\sigma_l;x_0^{*},\overline{y}) + (1-p)\,\partial_y V(\sigma_h;x_0^{*},\overline{y})}]$.

We are going to separate the reasoning into two parts to factor out what happens after time τ on the two different trajectories. We have

$$\begin{aligned}
\exp(-B^*(T)\,T) = {} & pe^{-\delta(T-\tau)}\left[\frac{\partial_y V\big(\sigma_l;\, x_T^{*l}, \overline{y}\big)}{\partial_y V\big(\sigma_l;\, x_\tau^{*l}, \overline{y}\big)}\right] \\
& \times e^{-\delta\tau}\left[\frac{\partial_y V\big(\sigma_l;\, x_\tau^{*l}, \overline{y}\big)}{p\,\partial_y V(\sigma_l;\, x_0^{*}, \overline{y}) + (1-p)\,\partial_y V(\sigma_h;\, x_0^{*}, \overline{y})}\right] \\
& + (1-p)\,e^{-\delta(T-\tau)}\left[\frac{\partial_y V\big(\sigma_h;\, x_T^{*h}, \overline{y}\big)}{\partial_y V\big(\sigma_h;\, x_\tau^{*h}, \overline{y}\big)}\right] \\
& \times e^{-\delta\tau}\left[\frac{\partial_y V\big(\sigma_h;\, x_\tau^{*h}, \overline{y}\big)}{p\,\partial_y V(\sigma_l;\, x_0^{*}, \overline{y}) + (1-p)\,\partial_y V(\sigma_h;\, x_0^{*}, \overline{y})}\right].
\end{aligned}$$

The terms $e^{-\delta(T-\tau)}[\frac{\partial_y V(\sigma_l; x_T^{*l}, \overline{y})}{\partial_y V(\sigma_l; x_\tau^{*l}, \overline{y})}]$ and $e^{-\delta(T-\tau)}[(1-p)\frac{\partial_y V(\sigma_h; x_T^{*h}, \overline{y})}{\partial_y V(\sigma_h; x_\tau^{*h}, \overline{y})}]$ can easily be controlled using what we know from the deterministic cases: they are, respectively, greater than $e^{-a(l)(T-\tau)}$ and $e^{-a(h)(T-\tau)}$.

The other terms correspond to what happens before time τ and we would like to link them to the ecological discount rate $B^*(\tau)$.

Let us take first the term corresponding to the "l-trajectory":

$$e^{-\delta\tau}\left[\frac{\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big)}{p\,\partial_y V(\sigma_l; x_0^*, \overline{y}) + (1-p)\,\partial_y V(\sigma_h; x_0^*, \overline{y})}\right]$$

$$= e^{-\delta\tau}\left(\frac{p\,\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big) + (1-p)\,\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)}{p\,\partial_y V(\sigma_l; x_0^*, \overline{y}) + (1-p)\,\partial_y V(\sigma_h; x_0^*, \overline{y})}\right)$$

$$\times\left(\frac{\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big)}{p\,\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big) + (1-p)\,\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)}\right)$$

$$= e^{-B^*(\tau)\tau}\left[\frac{\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big)}{p\,\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big) + (1-p)\,\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)}\right]$$

$$= e^{-B^*(\tau)\tau}\,\frac{\dfrac{\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big)}{\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)}}{p\,\dfrac{\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big)}{\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)} + (1-p)}.$$

Now, let us turn to the term corresponding to the "h-trajectory":

$$e^{-\delta\tau}\left[\frac{\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)}{p\,\partial_y V(\sigma_l; x_0^*, \overline{y}) + (1-p)\,\partial_y V(\sigma_h; x_0^*, \overline{y})}\right]$$

$$= \left(\frac{\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)}{p\,\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big) + (1-p)\,\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)}\right)$$

$$\times\, e^{-\delta\tau}\left(\frac{p\,\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big) + (1-p)\,\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)}{p\,\partial_y V\big(\sigma_l; x_0^*, \overline{y}\big) + (1-p)\,\partial_y V\big(\sigma_h; x_0^*, \overline{y}\big)}\right)$$

$$= \left[\frac{\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)}{p\,\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big) + (1-p)\,\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)} \right]$$

$$\times\, e^{-B^*(\tau)\tau} = e^{-B^*(\tau)\tau} \frac{1}{p\,\dfrac{\partial_y V\big(\sigma_l; x_\tau^{*l}, \overline{y}\big)}{\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)} + (1-p)}.$$

Now, if we compile all the inequalities, we obtain

$$e^{-B^*(T)T} > e^{-B^*(\tau)\tau} \left[p e^{-a(l)(T-\tau)} \left(\frac{N^*(\tau)}{pN^*(\tau) + (1-p)} \right) + (1-p)\, e^{-a(h)(T-\tau)} \left(\frac{1}{pN^*(\tau) + (1-p)} \right) \right],$$

where $N^*(\tau)$ stands for $\frac{\partial_y V(\sigma_l; x_\tau^{*l}, \overline{y})}{\partial_y V(\sigma_h; x_\tau^{*h}, \overline{y})}$.
If we sum everything, we get

$$m > e^{-B^*(\tau)\tau} \left[p \frac{1}{a(l)} \left(\frac{N^*(\tau)}{pN^*(\tau) + (1-p)} \right) + (1-p) \frac{1}{a(h)} \left(\frac{1}{pN^*(\tau) + (1-p)} \right) \right].$$

We see that one thing remains to be done: studying $N^*(\tau)$.
We can write

$$\begin{aligned}
N^*(\tau) &= \frac{\partial_y V(\sigma_l; x_\tau^{*l}, \overline{y})}{\partial_y V\big(\sigma_h; x_\tau^{*h}, \overline{y}\big)} \\
&= \left[\overline{y}^{-\frac{1}{\sigma_l}} \left[x_\tau^{*l\,\frac{\sigma_l - 1}{\sigma_l}} + \overline{y}^{\frac{\sigma_l - 1}{\sigma_l}} \right]^{\frac{1-\sigma_l \eta}{\sigma_l - 1}} \right] \Big/ \left[\overline{y}^{-\frac{1}{\sigma_h}} \left[x_\tau^{*h\,\frac{\sigma_h - 1}{\sigma_h}} + \overline{y}^{\frac{\sigma_h - 1}{\sigma_h}} \right]^{\frac{1-\sigma_h \eta}{\sigma_h - 1}} \right] \\
&= \left[\overline{y}^{-\eta} \left[1 + \left(\frac{x_\tau^{*l}}{\overline{y}} \right)^{\frac{\sigma_l - 1}{\sigma_l}} \right]^{\frac{1-\sigma_l \eta}{\sigma_l - 1}} \right] \Big/ \left[\overline{y}^{-\eta} \left[1 + \left(\frac{x_\tau^{*h}}{\overline{y}} \right)^{\frac{\sigma_h - 1}{\sigma_h}} \right]^{\frac{1-\sigma_h \eta}{\sigma_h - 1}} \right] \\
&= \left[1 + \left(\frac{x_\tau^{*l}}{\overline{y}} \right)^{\frac{\sigma_l - 1}{\sigma_l}} \right]^{\frac{1-\sigma_l \eta}{\sigma_l - 1}} \Big/ \left[1 + \left(\frac{x_\tau^{*h}}{\overline{y}} \right)^{\frac{\sigma_h - 1}{\sigma_h}} \right]^{\frac{1-\sigma_h \eta}{\sigma_h - 1}}.
\end{aligned}$$

It is clear that this expression grows exponentially with τ (since $\sigma_h \eta > 1$).
Also, under our hypotheses, this expression is always greater than 1 because we divide a term greater than 1 by a term smaller than 1. ■

Proof of Corollary 2: From Proposition 9, we see that the only thing to prove is that the function

$$p \mapsto p \frac{1}{a(l)} \left(\frac{N^*(\tau)}{pN^*(\tau) + (1-p)} \right) + (1-p) \frac{1}{a(h)} \left(\frac{1}{pN^*(\tau) + (1-p)} \right)$$

lies above its chord $[(p = 0, \frac{1}{a(h)}), (p = 1, \frac{1}{a(l)})]$.

This is guaranteed since $N^*(\tau)$ is greater than 1 (see Proposition 9). ■

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