Economic Dispatch Model With Fuzzy Wind Constraints and Attitudes of Dispatchers

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***Abstract* – The paper describes a new economic dispatch algorithm for systems with uncertain wind generation prediction, similar to the classical thermal dispatch model with load on a single bus. The optimization is achieved in a compromise between fuzzy constraints in the magnitude of wind penetration and the variation of running costs. The model includes also the attitudes of the dispatcher towards risk (security) and cost.**

***Index Terms* – Fuzzy systems, generation dispatch, wind.**

I. INTRODUCTION

THE European Union has been supporting, through its R&D Framework Programmes such as JOULE, THERMIE and ENERGY, the development of projects that deal with a high penetration of wind generation [1][2]. One of the concerns is the dispatch of thermal generation in the presence of uncertainty on wind generation. This has implications not only in the scheduling of units, in vertically organized systems, but also in the market price if the system is organized in a pool. This paper presents a model with uncertainty on wind prediction and with representation of dispatcher's attitudes towards risk by fuzzy descriptions [3]. It is extremely didactic and may be included in more complex models for unit commitment or for system planning.

II. A BASIC FUZZY MODEL FOR DISPATCH

The basic mathematical model for a time step is:

$$\min \Phi = \sum_i C_i(P_i) + C_w(W_{av} - W)$$

$$\text{subject to} \quad \begin{array}{l} \sum_i P_i + W = L \\ W \tilde{\leq} \Pi \end{array} \quad \text{and} \quad \begin{array}{l} P_i^{min} \leq P_i \leq P_i^{Max} \\ 0 \leq W \leq W_{av} \end{array}$$

Φ - Operation cost for the time step
$C_i(P_i)$ - generation cost function of unit i ($)
P_i - Constant power (MW) generated by generator i,
W - Wind power (MW)
W_{av} - Maximum wind power available (prediction, MW)
C_w - Penalty cost for not using all available wind
L, Π - Load (prediction, MW), Wind penetration (MW)
P_i^{min}, P_i^{Max} - technical limits for each thermal generator

The dispatcher may be forced to pay some compensation to private owners of wind parks if all available wind power is not utilized. This justifies penalty factor $C_{w.}$

The wind power used W defines a fuzzy border. One can calculate, in each time step, values in MW for W = Π_{min}, below which the operation is considered secure, relative to wind perturbations, and W= Π_{Max}, above which operation is not accepted due, for instance, to dynamic stability problems in case of wind perturbations [4]. A linear fuzzy constraint is defined in terms of the membership value μ. (Figure 1)

$$\mu = \begin{cases} 1 & \text{if} \quad W \leq \Pi^{min} \\ \dfrac{\Pi^{Max} - W}{\Delta\Pi} & \text{if} \quad \Pi^{min} \leq W \leq \Pi^{Max} \\ 0 & \text{if} \quad W \geq \Pi^{Max} \end{cases} \quad , \quad \Delta\Pi = \Pi^{Max} - \Pi^{min}.$$

One may transform the objective function into a linear fuzzy constraint, after determining Φ^{Max} and Φ^{min} from replacing the fuzzy constraint by W≤Π^{min} and by W≥Π^{Max}. These are just two classical dispatch problems. The basic model has a linear representation such that $\Delta\Pi = \Phi^{Max} - \Phi^{min}$:

$$\mu = \begin{cases} 1 & \text{if} \quad \Phi \leq \Phi^{min} \\ \dfrac{\Phi^{Max} - \Phi}{\Delta\Phi} & \text{if} \quad \Phi^{min} \leq \Phi \leq \Phi^{Max} \\ 0 & \text{if} \quad \Phi \geq \Phi^{Max} \end{cases}$$

The problem has the form of a Zadeh's classical Symmetrical Fuzzy Programming and can be formulated [5] as $\text{Max } \mu$, subject to:

$$g_1(\mathbf{P},\mu) = \sum_i C_i(P_i) + C_w\left(W_{av} - \left(L - \sum_i P_i\right)\right) + \mu\Delta\Phi - \Phi_{Max} = 0$$

$$g_2(\mathbf{P},\mu) = \sum_i P_i + \Pi_{Max} - \mu\Delta\Pi - L = 0$$

$$0 \leq L - \sum_i P_i \leq W_{av}$$

$$P_i^{min} \leq P_i \leq P_i^{Max} \quad \text{and} \quad 0 \leq \mu \leq 1$$

If we admit that the inequalities are satisfied, then a Lagrangean function Λ may be built

$$\Lambda = \mu + \alpha\, g_1(\mathbf{P},\mu) + \beta\, g_2(\mathbf{P},\mu)$$

and the necessary conditions for optimality derived:

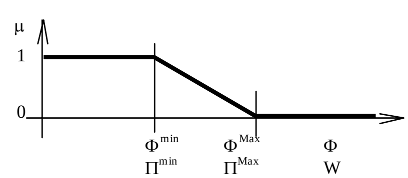

Figure 1 - Representation of fuzzy constraints in wind penetration and in operation cost. Above Π_{Max} the operation is not acceptable for security reasons; above Φ^{Max} it is not acceptable due to excessive cost.

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$$\frac{\partial \Lambda}{\partial P_i}=\alpha\frac{\partial C_i}{\partial P_i}+\beta=0\ ,\ \forall i \qquad \frac{\partial \Lambda}{\partial \alpha}=g_1(\mathbf{P},\mu)=0$$

$$\frac{\partial \Lambda}{\partial \mu}=1+\alpha\left(C_w\Delta\Pi+\Delta\Phi\right)-\beta\Delta\Pi=0 \qquad \frac{\partial \Lambda}{\partial \beta}=g_2(\mathbf{P},\mu)=0$$

The first condition stipulates the well know result that all generators (except those affected by inequalities) should be working at constant marginal cost, given now by

$$\frac{\partial C_i}{\partial P_i}=-\frac{\beta}{\alpha}\quad ,\quad \forall i$$

The last two conditions may be solved to give μ:

$$\mu'=\frac{\Phi^{Max}-\left[\sum_i C_i(P_i)+C_w\left(W_{av}-\left(L-\sum_i P_i\right)\right)\right]}{\Delta\Phi}$$

$$\mu''=\frac{\Pi^{Max}-\left(L-\sum_i P_i\right)}{\Delta\Pi}$$

This finally suggests an algorithm to determine μ:

Solve two classical dispatch problems, replacing the fuzzy constraint by $W \le \Pi^{min}$, to obtain Φ^{Max}, and by $W \le \Pi^{Max}$ to obtain Φ^{min}.
Start with a trial $\lambda=-\frac{\beta}{\alpha}$
REPEAT
  Set all generators with $\partial C/\partial P > \lambda$ to P^{min}.
  Set all generators with $\partial C/\partial P < \lambda$ to P^{Max}.
  Calculate $\sum P_i$ and $\sum C_i(P_i)$ for the P_i values obtained with the current value of λ.
  Calculate μ' and μ''.
  IF $\mu'<\mu''$ THEN decrease λ.
  IF $\mu'>\mu''$ THEN increase λ.
UNTIL $|\mu'-\mu''|$ < Tolerance specified.

The optimal μ establishes a compromise between increasing risk (with wind penetration) and reducing the generation cost of the classical units. One must also take in account that the third and fifth constraints of the problem lead to

$$1 \ge \mu'' \ge \frac{\Pi^{Max}-W_{av}}{\Delta\Pi}$$

If $W_{av} < \Pi^{min}$, there is no need to solve the fuzzy problem and the fuzzy constraint is replaced by $W = W_{av}$. And if $W_{av} > \Pi^{Max}$ then the above constraint will be dominated by the fifth constraint.

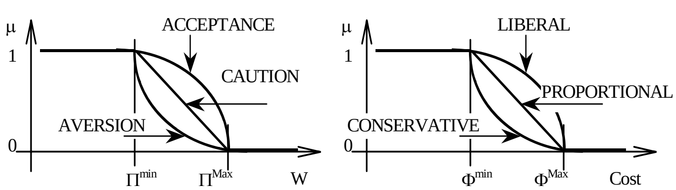

Figure 2 – Three attitudes towards risk (left) and running costs (right) defined by the shape of the membership function related with the fuzzy wind penetration constraint and with the best and worst obtainable costs

When $\Pi^{min} \le W_{av} \le \Pi^{Max}$, there is the possibility that the inequality above prevents the algorithm to converge to similar values of μ regarding the cost objective and the penetration constraint - and then the algorithm must stop at the best possible value such that one may obtain $\text{Max}\{\min \mu\}\ ,\ \mu \in \{\mu', \mu''\}$.

III. Attitudes Towards Cost and Risk

Instead of having linear fuzzy constraints, we can define quadratic expressions; for instance, for wind penetration,

$$\mu=\begin{cases}1 & \text{if} \quad W \le \Pi^{min}\\ f(W) & \text{if} \ \Pi^{min} \le W \le \Pi^{Max}\\ 0 & \text{if} \quad W \ge \Pi^{Max}\end{cases} \quad \text{with} \quad \begin{array}{l} f(W)=a_1W^2+b_1W+c_1\\ f(\Pi^{min})=1 \quad , \quad f(\Pi^{Max})=0\\ f(W)\ge 0 \ \text{ for } \ W\in\left[\Pi^{min},\Pi^{Max}\right]\end{array}$$

The values of a_1, b_1 and c_1 may be selected to give curves that represent distinct attitudes of the dispatcher – see Figure 2. Likewise, we can define a membership function for running costs, depending on constants a_2, b_2 and c_2. The expressions that need to be applied to obtain a common value of μ are now

$$\mu'=\frac{\left[2a_2\sum_i C_i(P_i)+2a_2C_w\left(W_{av}-\left(L-\sum_i P_i\right)+b_2\right)^2-b_2^2+4a_2c_2\right]}{4a_2}$$

$$\mu''=\frac{\left(2a_1L-2a_1\sum_i P_i+b_1\right)^2-b_1^2+4a_1c_1}{4a_1}$$

Therefore, given the attitude of the dispatcher towards the risk of having high wind penetration, and towards running costs, we can define a compromise solution that includes a definition of the wind penetration – controllable by the disconnection of the adequate number of wind generators.

IV. Conclusion

The attitude of dispatchers towards risk and cost, when facing uncertainty in wind prediction, can now be easily included in dispatch and unit commitment models. This paper presents no numerical examples but the algorithm is easily implemented and outputs easily analyzed.

V. References

[1] M. Matos et al., “Economic operation of isolated networks with wind power”, *Wind Engineering* (Multi-Science Publishing Co.), vol. 23, n. 2, pag. 89-105, 1999

[2] N. Hatziargyriou, Manuel Matos, J. A. Peças Lopes et al., "Preliminary Results From The More Advanced Control Advice Project For Secure Operation Of Isolated Power Systems With Increased Renewable Energy Penetration And Storage – More Care", *Proc. of IEEE Porto PowerTech'2001*, vol.4, pp.317-322, Porto, Portugal, Sep 2001.

[3] V. Miranda, Pun Sio Hang, “Unit Commitment module – main innovative characteristics”, *Proc. of CARE Project Workshop on Secure Wind Power Penetration in Isolated Systems*, Crete, Greece, Jul 1999

[4] M. Vinícius, J. A. Peças Lopes, U. Bezerra, H. Zürn, R. Almeida, “Influence of the Variable Speed Wind Generators in Transient Stability Margin of the Conventional Generators Integrated in Electrical Grids”, *IEEE Trans. Energy Conversion*, vol.19, no.4, pp.692-701, Dec 2004.

[5] H.-J. Zimmerman, *Fuzzy Set Theory and its Applications*, Kluwer, 1985