

Economic fluctuations and statistical physics: Quantifying extremely rare and less rare events in finance

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Abstract

One challenge of economics is that the systems treated by these sciences have no perfect metronome in time and no perfect spatial architecture—crystalline or otherwise. Nonetheless, as if by magic, out of nothing but *randomness* one finds remarkably fine-tuned processes in time. We present an overview of recent research joining practitioners of economic theory and statistical physics to try to better understand puzzles regarding economic fluctuations. One of these puzzles is how to describe outliers, phenomena that lie outside of patterns of statistical regularity. We review evidence consistent with the possibility that such outliers may not exist. This possibility is supported by recent analysis of databases containing information about each trade of every stock.

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1. Introduction

In this talk, I will discuss recent applications of correlated randomness to one area of science for which statistical physics is proving to be particularly useful: economics. Our overall “take-home” message today sounds pretty general. In general, systems that display correlated randomness cannot be solved exactly. Not even the simple self-avoiding random walk can be solved! Nonetheless, there are two unifying principles that have organized many of the results we will be presenting today—scale invariance and universality. The key idea is that scale invariance is a statement not about algebraic equations of the form $x^{-3} = 1/8$ with a numerical solution ($x = 2$), but rather about functional equations of the form

$$f(\lambda x) \sim \lambda^{-\alpha} f(x) \quad (1)$$

and its relevant generalizations. These functional equations have as their solutions functional forms, and the solution to this homogeneous functional equation is a power-law form,

$$f(x) \sim x^{-\alpha}. \quad (2)$$

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2. The phenomenon of outliers

One quarter of any newspaper with a financial section is filled with economic fluctuation data. Most economic graphs look approximately like the one we get when we plot the S&P 500 stock index as a function of time. We can compare this empirical data with that generated by a simple uncorrelated biased random walk, a model first used over 100 years ago by Bachelier. At first it seems that there is little difference, but looking more closely we see events in the real data that do not have counterparts in the random walk. Black Monday in October 1987 is reflected in the real data, which shows a loss of 30% of the total value of the market in just one day. In the random walk we do not see fluctuations anywhere near this magnitude because the probability of taking n steps in the same direction of a random walk is $(1/2)^n$ —it decreases exponentially with n .

Economists nevertheless have traditionally used this uncorrelated biased Gaussian random walk to describe real economic data, relegating events such as Black Monday to the dustbin category of “outliers” [1–5].

Traditional economic theory does not predict outliers, but recent analysis of truly huge quantities of empirical data suggests that classic theories not only fail for a few outliers, but that there occur similar outliers of every possible size. In fact, if one analyzes only a small data set (say 10^4 data points), then outliers appear to occur as “rare events.” However, when orders of magnitude more data (10^8 data points) are analyzed, one finds orders of magnitude more outliers—so ignoring them is not a responsible option, and studying their properties becomes a realistic goal. One finds that the statistical properties of these “outliers” are identical to the statistical properties of everyday fluctuations. For example, a histogram giving the number of fluctuations of a given magnitude x for fluctuations ranging in magnitude from everyday fluctuations to extremely rare fluctuations (“financial earthquakes”) that occur with a probability of only 10^{-8} is a perfect straight line in a double-log plot.

An analogy with earthquake research is perhaps appropriate. If one studies limited data sets, a paradigm arises in which there are everyday (unnoticeable except by sensitive seismometer) “tremors,” punctuated from time to time by rare events (“earthquakes”). Thanks to the empirical work, we now know that the partition of shocks into “tremors” and “earthquakes” is not valid. Rather, if one examines enough data, one sees that the shocks occur for all possible magnitudes. The law named after Gutenberg and Richter refers to a statistical formula that gives all the data from the smallest tremors to the “big ones.” This law is that the histogram giving the number of shocks of a given size is a straight line in a log–log plot [6–8]—there are no outliers.

Thus, an erroneous paradigm (“normal” size earthquakes occasionally punctuated by “huge” earthquakes) can arise when a limited quantity of data are considered in which data are partitioned into everyday events (often describable by one statistical law) and rare events which, since they are not described by the law are terms outliers. Has an erroneous paradigm arisen in economic research? In economic research, there are fluctuations in stock prices, number of shares trading hands, and total number of fluctuations. Recent empirical studies calculating histograms for all three quantities are linear on log–log plots (albeit with different slopes). In mathematical language, the occurrence probability of such a quantity’s fluctuations appears to be described by a power law.

In economics, neither the existence of power laws nor the exact exponents have any accepted theoretical basis. To a professional economist, empirical laws such as the aforementioned power laws are called “stylized facts”. Accordingly, some theoretical understanding is urgently needed or else these laws will continue to be largely irrelevant. Of course, facts, even “stylized facts” without any theory, may have some practical value. For example, the Gutenberg–Richter law enables one to calculate the risk of a shock (tremor or earthquake) of a given magnitude, and hence informs the building codes of Los Angeles and Tokyo. Similarly, the empirical laws governing economic fluctuations enable one to calculate the risk of an economic shock of a given magnitude.

The lack of a coherent theory is unfortunate, especially in economics where facts without theoretical foundation is considered a deplorable situation. Accordingly, my collaborators and I have been seeking to develop a theoretical framework within which to interpret these new empirical facts, and recently some progress is beginning to occur [9–13]. This work is of interest since it provides a theoretical framework within which to interpret the new empirical laws. Specifically, the model fulfills these requirements for such a basic “microscopic” model of the stock market. It is founded on realistic features of the stock market, and reflects

the view that market participants have of the functioning of the market, as well as the main determinants of their trading behavior.

3. First discovery of scaling and universality

That at least *some* economic phenomena are described by power law tails has been recognized for over 100 years since the University of Torino Engineering Professor Vilfredo Pareto investigated the statistical character of the wealth of individuals by modeling them using the *scale-invariant* distribution (2), where $f(x)$ denotes the number of people having income x or greater than x , and α is an exponent that Pareto estimated to be 1.5 [11,12]. Pareto noticed that his result was *universal* in the sense that it applied to nations—“*as different as those of England, of Ireland, of Germany, of the Italian cities, and even of Peru*”. A physicist would say that the universality class of the scale invariance functional equation (1) includes all the aforementioned countries as well as Italian cities, since by definition two systems belong to the same universality class if they are characterized by the same exponents. However, the exponent for income does not seem stable across time [13].

In the century following Pareto’s discovery, the twin concepts of scaling and universality have proved to be important in a number of scientific fields [14–16]. A striking example was the elucidation of the puzzling behavior of systems near their critical points. Over the past few decades it has come to be appreciated that the scale-free nature of fluctuations near critical points also characterizes a huge number of diverse systems also characterized by strong fluctuations. This set of systems includes examples that at first sight are as far removed from physics as is economics. For example, consider the percolation problem, which in its simplest form consists of placing pixels on a fraction p of randomly chosen plaquettes of a computer screen. A remarkable fact is that the largest connected component of pixels magically spans the screen at a threshold value p_c . This purely geometrical problem has nothing to do, at first sight, with critical point phenomena. Nonetheless, the fluctuations that occur near $p = p_c$ are scale free and functions describing various aspects of the incipient spanning cluster that appears at $p = p_c$ are described by power laws characterized by exponent values that are universal in the sense that they are independent of the details of the computer screen’s lattice (square, triangle, honeycomb). Nowadays, the concepts of scaling and universality provide the conceptual framework for understanding the geometric problem of percolation.

It is becoming clear that almost any system comprised of a large number of interacting units has the potential of displaying power-law behavior. Since economic systems are, in fact, comprised of a large number of interacting units has the potential of displaying power-law behavior, it is perhaps not unreasonable to examine economic phenomena within the conceptual framework of scaling and universality [1,3,4,14–22]. We will discuss this topic in detail below.

4. Inverse cubic law of stock and commodity price fluctuations

We return to the S&P 500 stock index as a function of time and the simple uncorrelated biased random walk, and plot not the absolute value of the index but instead the *change* in the index (the numerical derivative, the “return”). We normalize that by the standard deviation. We look over a 13-year period (Fig. 1) and see, e.g., that on Black Monday the fluctuations were more than 30 standard deviations (both positive and negative) for the day, and we also see a very noisy signal. The striking thing is to look at the other curve, the uncorrelated random walk, and see the Gaussian distribution for the fluctuations—which rarely display fluctuations greater than five standard deviations. The “outliers” that the economists are content to live with are any fluctuations of the actual data that are greater than five standard deviations. In this 13-year period there are exactly 64, i.e., 2^6 . If we count only those fluctuations of the actual data that are greater than ten standard deviations, we get exactly 8, i.e., 2^3 . If we count only those that are greater than 20, we get one, i.e., 2^0 : Black Monday. Each time we double the x -axis we change the y -axis by a power of 2^3 . At the top of this presentation we made reference to a power law of the form (2) with $\alpha = 3$, which corresponds to a functional equation, Eq. (1). The possibility that economic data obey such a scaling was pointed out in 1963 by Mandelbrot in his study of cotton price fluctuations [14].

If we replace our visual examination of these two graphs with a close computer analysis of not just the S&P 500 stock index but every stock transaction over an extended time period (approximately 10 GB of data), we

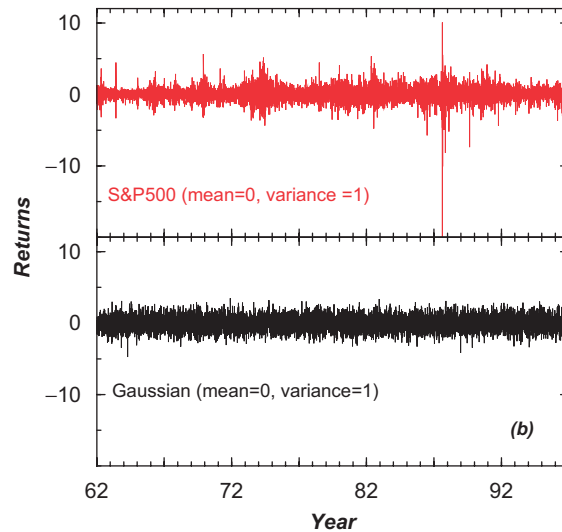


Fig. 1. The S&P 500 index is the sum of the market capitalizations of 500 companies. Shown is the sequence of 10-min returns for the S&P 500, normalized to unit variance, compared with sequence of i.i.d. Gaussian random variables with unit variance, which was proposed by Bachelier as a model for stock returns. The large fluctuations seen in 1987 are the market crash of October 19. Note that, in contrast to the top curve, there are no “extreme” events predicted by the bottom curve.

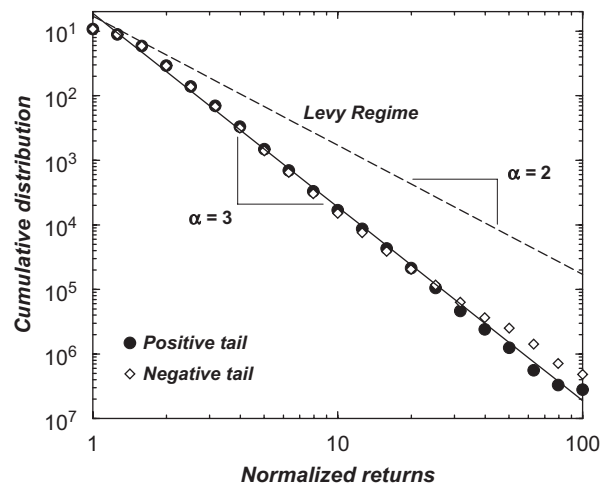


Fig. 2. Cumulative distributions of the positive and negative tails of the normalized returns of the 1000 largest companies in the TAQ database for the 2-year period 1994–1995. The solid line is a power-law regression fit in the region $2 \leq x \leq 80$.

find [23–26] that the actual graph giving the number of times a fluctuation exceeds a given amount as a function of that amount is perfectly straight on log–log paper *out to 100 standard deviations* (Fig. 2). The slope of the line, α , is indistinguishable from the value $\alpha = 3$ that we deduced from visual inspection. Note that this slope is significantly larger (by almost a factor of two) than the slope found by Mandelbrot in his research on cotton prices. Note also that our slope is outside the Lévy stable regime [27].

This is how we find laws in statistical physics, but finding them is only the first part—the empirical part—of our task. The second part—the theoretical part—is understanding them.

When we studied critical phenomena, the empirical part was a very important contributor toward our ultimate understanding of phase transitions and critical phenomena. The massing of empirical facts led to the recognition of regularities to which certain approaches could be applied, e.g., the Widom scaling hypothesis and the Wilson renormalization group. So also in economics we can perhaps first discover empirical

regularities—e.g., the inverse cubic law—that will prove useful in ultimately understanding the economy. I wish I could say that we already have an explanation for this inverse cubic law, but I cannot. We have the beginning of an explanation, but it is only the beginning since the current theory explains the inverse cubic law of price changes, as well as the “half cubic law” of trade volume [9,10] but does not explain the strange nature of the temporal correlations. The autocorrelation function of price changes decays exponentially in time so rapidly that after 20 min it is in the level of “noise.” However, the autocorrelation function of changes in the *absolute value* of the price (called the “volatility”) decays with a power law of exponent approximately 0.3 [28].

So having embarked on a path guided by these two theoretical concepts, what does one do? Initially, critical phenomena research—guided by Pareto’s principles of scaling and universality—was focused finding which systems display scaling phenomena, and on discovering the actual values of the relevant exponents. This initial empirical phase of critical phenomena research proved vital, for only by carefully obtaining empirical values of exponents such as α scientists could learn which systems have the same exponents (and hence belong to the same *universality class*). The fashion in which physical systems partition into disjoint universality classes proved essential to later theoretical developments such as the renormalization group [16]—which offered some insight into the reasons why scaling and universality seem to hold; ultimately it led to a better understanding of the critical point.

Similarly, our group’s initial research in economics—also guided by Pareto’s principles—has largely been concerned with establishing which systems display scaling phenomena, and with measuring the numerical values of the exponents with sufficient accuracy that one can begin to identify universality classes if they exist. Economic systems differ from often-studied physical systems in that the number of subunits are considerably smaller in contrast to macroscopic samples in physical systems that contain a huge number of interacting subunits, as many as Avogadro’s number 6×10^{23} . In contrast, in an economic system, one initial work was limited to analyzing time series comprising of order of magnitude 10^3 terms, and nowadays with high frequency data the standard, one may have 10^8 terms. Scaling laws of the form of (1) are found that hold over a range of a factor of $\approx 10^6$ on the x -axis [29–33]. Moreover, these scaling laws appear to be universal in that they, like the Pareto scaling law, hold for different countries [34], for cities [35–40], other social organizations [41–45], and even for bird populations [46].

Recent attempts to make models that reproduce the empirical scaling relationships suggest that significant progress on understanding firm growth may be well underway [47–52], leading to the hope of ultimately developing a clear and coherent “theory of the firm.” One utility of the recent empirical work is that now any acceptable theory must respect the fact that power laws hold over typically six orders of magnitude; as Axtell put the matter rather graphically: “*the power law distribution is an unambiguous target that any empirically accurate theory of the firm must hit*” [29].

With this background on power laws and scale invariance in geometry and in economics, we turn now to the well-studied problem of finance fluctuations, where a consistent set of empirical facts is beginning to emerge. One fact that has been confirmed by numerous, mostly independent, studies is that stock price fluctuations are characterized by a scale-invariant cumulative distribution function of the power-law form (1) with $\alpha \approx 3$ [23,24,53]. This result is also universal, in the sense that this inverse cubic law exponent is within the error bars of results for different segments of the economy, different time periods, and different countries—and is the same for stock averages as different as the S&P and the Hang Seng [26].

This “inverse cubic law” disagrees with the classic work of Ref. [14] on price fluctuations of cotton, which appear to display scale free behavior (“no outliers”) but with much fatter tails characterized by $\alpha \approx 1.7$; this work is of interest because if $\alpha < 2$, then the distribution is of the Lévy form. To understand this discrepancy, Matia and collaborators have wondered if the reason for the fatter tails of cotton is that cotton is a commodity, and commodities exist in limited supply so that when a commodity is needed one must sometimes pay exorbitant prices (e.g., electricity in California). Accordingly, they analyzed a large number of commodities, but they found that these commodities have tails described not by $\alpha < 2$ but rather by $\alpha \approx 3$ [27,44,54,55].

Newcomers to the field of scale invariance often ask why a power law does not extend “forever” as it would for a mathematical power law of the form $f(x) = x^{-\alpha}$. This legitimate concern is put to rest by reflecting on the fact that power laws for natural phenomena are not equalities, but rather are asymptotic relations of the form $f(x) \sim x^{-\alpha}$. Here the tilde denotes *asymptotic equality*. Thus $f(x)$ is not “approximately equal to” a power law

so the notation $f(x) \approx x^{-\alpha}$ is erroneous. Similarly, $f(x)$ is not proportional to a power law, so the notation $f(x) \propto x^{-\alpha}$ is also erroneous. Rather, asymptotic equality means that $f(x)$ becomes increasingly like a power law as $x \rightarrow \infty$. Moreover, crossovers abound in financial data, such as the crossover from power-law behavior to simple Gaussian behavior as the time horizon Δt over which fluctuations are calculated increases beyond about a year i.e., the power-law behavior holds for time horizons up to a month or even a year, but for horizons exceeding a year there is a distinct crossover to Gaussian behavior. Such crossovers are characteristic also of other scale-free phenomena in the physical sciences [15,16], where the Yule distribution often proves quite useful.

For reasons of this sort, standard statistical fits to data are erroneous, and often give distinctly erroneous values of the exponent α . Rather, one reliable way of estimating the exponent α is to form successive slopes of pairs of points on a log–log plot, since these successive slopes will be monotonic and converge to the true asymptotic exponent α . One finds that successive slopes for the empirical data converge rapidly to a value $\alpha \approx 3$ while successive slopes for the model diverge. While it is clear that a simple three-factor model [56] cannot generate power-law behavior, it is less clear why the empirical data analyzed appear at first glance to be well approximated by the model. The first fact is that the region of linearity of the data is not so large as in typical modern studies because the total quantity of data analyzed is not that large, since only a low-frequency time series comprising daily data is used. Only 28,094 records are analyzed [56] (not 4×10^7 as in recent studies [24,26]) and the model simulations are presented for limited sample size. The second fact is that when one superposes a curved line (the model) on a straight line (the data), the untrained eye is easily tempted to find agreement where none exists—and closer inspection of Figs. 2–5 of Ref. [56] reveals actually a rather poor agreement between model and data due to the pronounced downward curvature of the model’s predictions [57].

5. Quantifying fluctuations in market liquidity: analysis of the bid–ask spread

The primary function of a market is to provide a venue where buyers and sellers can transact. The more the buyers and sellers at any time, the more efficient the market is in matching buyers and sellers, so a desirable feature of a competitive market is liquidity, i.e., the ability to transact quickly with small price impact. To this end, most exchanges have market makers (e.g., “specialists” in the NYSE) who provide liquidity by selling or buying according to the prevalent market demand. The market maker sells at the “ask” (offer) price A and buys at a lower “bid” price B ; the difference $s \equiv A - B$ is the bid–ask spread.

The ability to buy at a low price and sell at a high price is the main compensation to market makers for the risk they incur while providing liquidity. Therefore, the bid–ask spread must cover costs incurred by the market maker [58–64] such as: (i) order processing costs, e.g., costs incurred in setting up, fixed exchange fees, etc., (ii) risk of holding inventory, which is related to the volatility, and (iii) adverse information costs, i.e. the risk of trading with a counter-party with superior information. Since the first component is a fixed cost, the interesting dynamics of liquidity is reflected in (ii) and (iii). Analyzing the statistical features of the bid–ask spread thus also provides a way to understand information flow in the market.

The prevalent bid–ask spread reflects the underlying liquidity for a particular stock. Quantifying the fluctuations of the bid–ask spread thus offers a way of understanding the dynamics of market liquidity. Using quote data for the 116 most-frequently traded stocks on the New York Stock Exchange over the two-year period 1994–1995, Plerou et al. [65] have recently analyzed the fluctuations of the average bid–ask spread S over a time interval Δt . They find that S is characterized by a distribution that decays as a power law $P\{S > x\} \sim x^{-\zeta_S}$, with an exponent $\zeta_S \approx 3$ for all 116 stocks analyzed (Fig. 3). Their analysis of the autocorrelation function of S shows long-range power-law correlations, $\langle S(t)S(t + \tau) \rangle \sim \tau^{-\mu}$, similar to those previously found for the volatility. They also examine the relationship between the bid–ask spread and the volume Q , and find that $S \sim \log Q$. They find that a similar logarithmic relationship holds between the transaction-level bid–ask spread and the trade size. They also show that the bid–ask spread and the volatility are also related logarithmically. Finally, they study the relationship between S and other indicators of market liquidity such as the frequency of trades N and the frequency of quote updates U , and find $S \sim \log N$ and $S \sim \log U$.

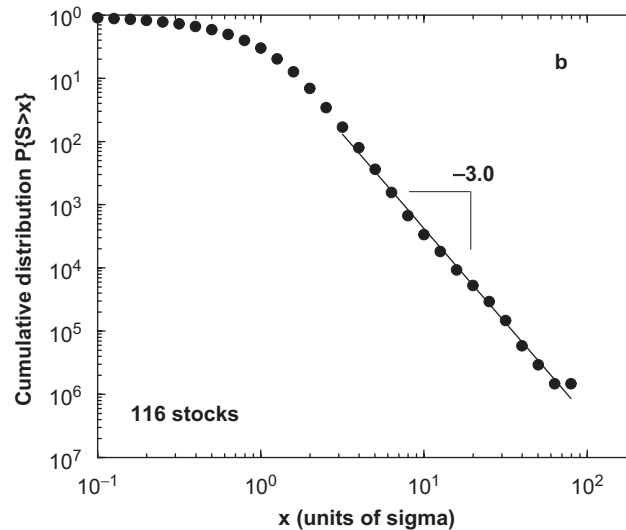


Fig. 3. Log–log plot of the cumulative distribution of the bid–ask spread $S_{\Delta t}$ which is normalized to have zero mean and unit variance, for all 116 stocks in our sample for the two-year period 1994–1995. A power-law fit in the region $x > 3$ gives a value for the exponent $\zeta_S = 3.0 \pm 0.1$. Fits to individual distributions give similar results for the exponent values.

6. Quantifying demand

One reason the economy is of interest to statistical physicists is that, like an Ising model, the economy is a system made up of many subunits. The subunits in an Ising model are called spins, and the subunits in the economy are buyers and sellers. During any unit of time these subunits of the economy may be either positive or negative as regards perceived market opportunities. People interact with each other, and this fact often produces what economists call “the herd effect.” The orientation of whether we buy or sell is influenced not only by our neighbors but also by news. If we hear bad news, we may be tempted to sell. So the state of any subunit is a function of the states of all the other subunits and of a field parameter.

On a qualitative level, economists often describe a price change as a hyperbolic-tangent-like function of the demand. The catch is that “demand” is not quantified. So one of the first things we had to do was quantify demand [66].

We did this by analyzing huge databases comprising every stock bought or sold—which gives not only the selling price and buying price, but also the asking price and the offer price. If we go to the open market to buy presents we will often be given an asking price we are not willing to pay, and we may counter with a much smaller offer. Ultimately when the sale is struck, the price may be above the midpoint between the asking price and the offer—and we assign a variable $a_i = +1$ to the sale—if below the midpoint, $a_i = -1$. If we sum all these indices a_i over a time interval Δt

$$H \equiv \sum_{i=1}^N a_i = \begin{cases} + & \text{[Big Demand]} \\ - & \text{[Small Demand]} \end{cases}$$

$$N = N_{\Delta t} = \text{Number of sales in } \Delta t,$$

then we can calculate the analog of a magnetic field, which provides a way of quantifying demand. If most of the a_i are positive, the field will be positive, and vice versa. A hint that this definition of magnetic field makes sense is the fact that a plot of price change as a function of the “magnetic field” variable defined above remarkably resembles a plot of the magnetization of a magnet as a function of the magnetic field [66] (Fig. 4). The full implications of the remarkable observation that a plot of price change as a function of the “magnetic field” resembles a plot of the magnetization in a magnet are a challenging problem.

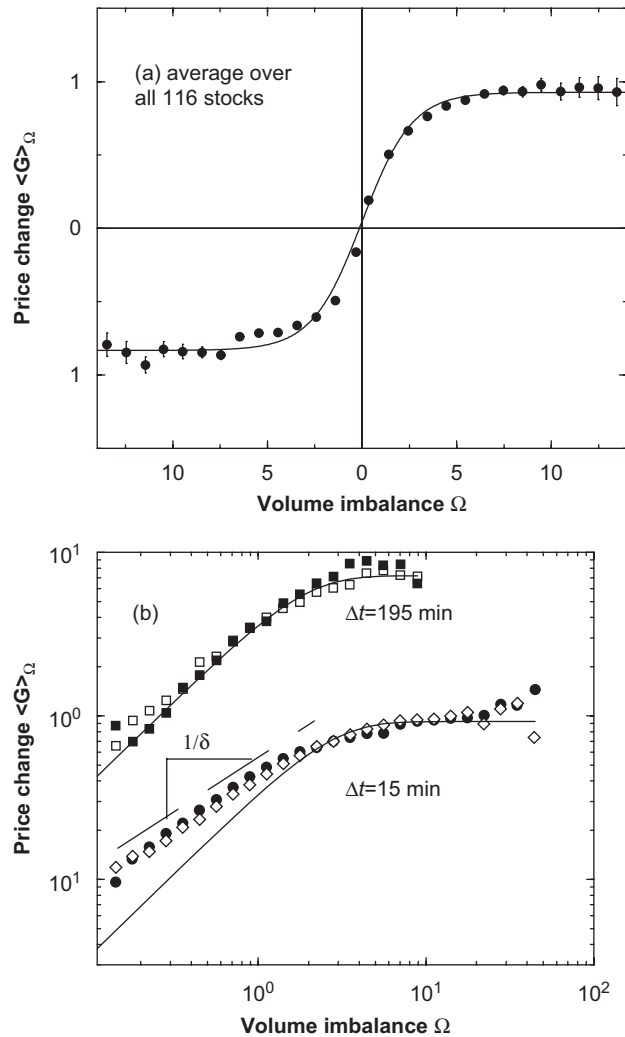


Fig. 4. (a) Conditional expectation $\langle G \rangle_{\Omega}$ averaged over all 116 stocks studied, over a time interval $\Delta t = 15$ min, where Ω is defined as the difference in number of shares traded in buyer and seller initiated trades. We normalize G to have zero mean and unit variance. Since Ω has a tail exponent $\zeta = 3/2$ which implies divergent variance, we normalize Ω by the first moment $\langle |\Omega - \langle \Omega \rangle| \rangle$. We calculate G and Ω for $\Delta t = 15$ min. The solid line shows a fit to the function $B_0 \tanh(B_1 \Omega)$. (b) $\langle G \rangle_{\Omega}$ on a log–log plot for different Δt . For small Ω , $\langle G \rangle_{\Omega} \simeq \Omega^{1/\delta}$. For $\Delta t = 15$ min find a mean value $1/\delta = 0.66 \pm 0.02$ by fitting $\langle G \rangle_{\Omega}$ for all 116 stocks individually. The same procedure yields $1/\delta = 0.34 \pm 0.03$ at $\Delta t = 5$ min (interestingly close to the value of the analogous critical exponent in mean field theory). The solid curve shows a fit to the function $B_0 \tanh(B_1 \Omega)$. For small Ω , $B_0 \tanh(B_1 \Omega) \sim \Omega$, and therefore disagrees with $\langle G \rangle_{\Omega}$, whereas for large Ω the fit shows good agreement. For $\Delta t = 195$ min (1/2 day) (squares), the hyperbolic tangent function shows good agreement.

7. Cross-correlations among fluctuations of different stocks

A useful software package could determine how the fluctuations of one stock price correlate with those of another. This question of cross-correlation is one we have been studying [24,25,67–73]. To quantify cross-correlations, we draw a circle corresponding to the stock price x and draw a second circle corresponding to the stock price x , say, 5 min later. If we make the difference in the radii proportional to G , the stock price change, then we can think of the market as thousands of circles, each growing and shrinking—a kind of pulsation that is a function of time. The key is that these correlations change in time. Car sales by Ford and GM may be anti-correlated during some time periods and positively correlated during others.

The standard approach to this problem is to calculate, by brute force, a huge square matrix that has as many rows as there are companies in the database. Each element of the matrix is the correlation between the

price change of company i and the price change of company j , but to find a genuine correlation we have to be able to distinguish between correlations from coincidences. In order to do that we draw on something developed by Wigner in his work in nuclear physics—random matrix theory. Random matrix theory compares the matrix calculated by brute force from stock market data with a random matrix that also has 1000 rows and 1000 columns—but with every number generated randomly. Somewhere hidden in the huge matrix calculated by brute force from stock market data are the true correlations. To uncover them, we first diagonalize the matrix in order to determine its eigenvalues, and then make a histogram that gives the number of times each given eigenvalue is found. The histogram curve of a random matrix, unlike this one from real data, can be predicted exactly. For a random matrix there is never an eigenvalue > 2.0 . The histogram of the empirical stock price data, on the other hand, contains a significant number of eigenvalues > 2.0 . Some are as big as 5.0. These eigenvalues of necessity must correspond to genuine correlations.

The eigenvalue of a matrix has a corresponding eigenvector—a column matrix of 1000 elements—each element of which is a different weight from each of the 1000 stocks. So we can look at the column vectors that correspond to these deviating, genuinely correlated eigenvalues and ask: what kind of stocks entered into each of these eigenvectors? What we found, fortunately, has implications for portfolios. If we restart the graph at 2.0—removing the distortions of the random values—and look at the 20 eigenvalues > 2.0 , we see that the stocks that make up most of the weights in the corresponding eigenvectors are almost entirely transportation stocks in the first case, almost entirely paper in the second, almost entirely pharmaceuticals in the third, and so on. In other words, the market *automatically* partitions itself into separate business sectors [72–74]. Thus a physicist who know nothing about the stock market can mathematically partition the economy into separate business sectors!

The sectors and the quantitative degree to which each constituent firm conforms to the sector can be monitored and updated as a function of time, e.g., every 15 min. Firms that belong to the same business sector can be monitored in a kind of rainbow spectrum. The “good” firms sticking to the business sector are assigned to the “violet” end of the spectrum, and the “bad” firms deviating from the sector are assigned to the “red.” When a firm first starts to move to the red end of the spectrum start to deviate, this alerts the trader to consider action.

8. Unifying the power laws: a first model

There has recently been progress in developing a theory for some of the power-law regularities discussed above. For example, based on a plausible set of assumptions, we proposed a model that provides an explanation for the empirical power laws of return, volume and number of trades [9]. In addition, our model explains certain striking empirical regularities that describe the relationship between large fluctuations in prices, trading volume, and the number of trades. In our model, large movements in stock market activity arise from the trades of the large participants. Starting from an empirical characterization of the size distribution of large market participants (mutual funds), we show that their trading behavior when performed in an optimal way, generates power laws observed in financial data.

Define p_t as the price of a given stock and the stock price “return” r_t as the change of the logarithm of stock price in a given time interval Δt , $r_t \equiv \ln p_t - \ln p_{t-\Delta t}$. The probability that a return is in absolute value larger than x is found empirically to be [26,53]

$$P(|r_t| > x) \sim x^{-\zeta_r} \quad \text{with } \zeta_r \approx 3. \quad (3)$$

Empirical studies also show that the distribution of trading volume V_t obeys a similar universal power law [75],

$$P(V_t > x) \sim x^{-\zeta_V} \quad \text{with } \zeta_V \approx 1.5, \quad (4)$$

while the number of trades N_t obeys [76]

$$P(N_t > x) \sim x^{-\zeta_N} \quad \text{with } \zeta_N \approx 3.4. \quad (5)$$

The “inverse cubic law” of Eq. (3) is “universal,” holding over as many as 80 standard deviations for some stock markets, with Δt ranging from one minute to one month, across different sizes of stocks, different time

periods, and also for different stock market indices [26,53]. Moreover, the most extreme events—including the 1929 and 1987 market crashes—conform to Eq. (3), demonstrating that crashes do not appear to be outliers of the distribution. We tested the universality of Eqs. (4) and (5) by analyzing the 35 million transactions of the 30 largest stocks on the Paris Bourse over a 5 yr period. Our analysis shows that the power laws (4) and (5) obtained for US stocks also hold for a distinctly different market, consistent with the possibility that Eqs. (4) and (5) are as universal as (3).

We develop a model that demonstrates how trading by large market participants explains the above power laws. We begin by noting that large market participants have large price impacts [77–80]. To see why this is the case, observe that a typical stock has a turnover (fraction of shares exchanged) of approximately 50% a year, which implies a daily turnover of approximately $50\%/250 = 0.2\%$ —i.e., on average 0.2% of outstanding shares change hands each day. The 30th largest mutual fund owns about 0.1% of such a stock [81]. If the manager of such a fund sells its holdings of this stock, the sale will represent half of the daily turnover, and so will impact both the price and the total volume [82–84]. Such a theory where large individual participants move the market is consistent with the evidence that stock market movements are hard to explain with changes in fundamental values [85].

Accordingly, we perform an empirical analysis of the distribution of the largest market participants—mutual funds. We find, for each year of the period 1961–1999, that for the top 10% of distribution of the mutual funds, the market value of the managed assets S obeys the power law

$$P(S > x) \sim x^{-\zeta_S} \quad \text{with } \zeta_S = 1.05 \pm 0.08. \tag{6}$$

Exponents of ≈ 1 have also been found for the cumulative distributions city size [86] and firm sizes [29,32,87], and the origins of this “Zipf” distribution are becoming better understood [39]. Based on the assumption that managers of large funds trade on their intuitions about the future direction of the market, and that they adjust their speed of trading to avoid moving the market too much, we will see that their trading activity leads to $\zeta_r = 3$ and $\zeta_V = 1.5$,

In order to proceed, we (A) present empirical evidence for the shape of the price impact, (B) propose an explanation for this shape, and (C) show how the resulting trading behavior generates power laws (3)–(5).

8.1. Empirical evidence for the square root price impact of trades

The price impact Δp of a trade of size V has been established to be increasing and concave [66,88]. We hypothesize that for large volumes V its functional form is

$$r = \Delta p \simeq kV^{1/2} \tag{7}$$

for some constant k .

A direct statistical test of this hypothesis can be performed by analyzing $E[r^2 | V]$. Performing this regression using r and V calculated over 15 min intervals, we find

$$E[r^2 | V] \sim V. \tag{8}$$

This regression is, however, not definitive evidence for Eq. (7). This regression is performed in fixed Δt so is exposed to the effect of fluctuations in the number of trades—i.e., if N denotes the number of trades in Δt , $r^2 \sim N$ and $V \sim N$ so Eq. (8) could be a consequence of this effect.

Since relation (7) implies $P(r > x) \sim P(kV^{1/2} > x) = P(V > x^2/k^2) \sim x^{-2\zeta_r}$, it follows that

$$\zeta_r = 2\zeta_V. \tag{9}$$

Thus, the power law of returns, Eq. (3), follows from the power law of volumes, Eq. (4), and the square root from price impact, Eq. (7).

Recent work by Farmer and Lillo [89] reports an exponent ≈ 0.3 for the price impact function. Apart from the quality of scaling and the limited data, one possible problem with this estimation is that large trades are usually executed in smaller tranches [90]. Ref. [89] also reports that the volume distribution is not a power law for the LSE. Further work by Ref. [90] has made it clear that the finding of Ref. [89] of a non-power-law

distribution of volume is an artifact arising from incomplete data due to the exclusion of upstairs market trades.

We next develop a framework for explaining Eqs. (4) and (7).

8.2. Explaining the square root price impact of trades, Eq. (7)

We consider the behavior of one stock, whose original price is, say 1. The mutual fund manager who desires to buy V shares offers a price increment Δp , so that the new price will become $1 + \Delta p$. Each seller i of size s_i who is offered a price increase Δp supplies the fund manager with q_i shares. Elementary considerations lead us to hypothesize $q_i \sim s_i \Delta p$. The number of sellers available after the fund manager has waited a time T is proportional to T . So, after a time T , the fund manager can on the average buy a quantity of shares equal to $kT \langle s \rangle \Delta p$ for some proportionality constant k . The search process stops (and the trades are executed simultaneously) when the desired quantity V is reached—i.e., when $kT \langle s \rangle \Delta p = V$, so the time needed to find the shares is

$$T = \frac{V}{\langle s \rangle k \Delta p} \sim \frac{V}{\Delta p}. \quad (10)$$

Hence there is a trade-off between cost Δp and the time to execution T ; if the fund manager desires to realize the trade in a short amount of time T , the manager must pay a large price impact $\Delta p \sim V/T$.

Let us consider the manager's decision problem. Managers trade on the assumption that a given stock is mispriced by an amount M , defined as the difference between the fair value of the stock and the traded price [80,91,92]. The manager wants to exploit this mispricing quickly, as he expects that the mispricing will be progressively corrected, i.e., expects that the price will increase at a rate μ . Hence, after a delay of T , the remaining mispricing is only $M - \mu T$. The total profit per share B/V is the realized excess return $M - \mu T$ minus the price concession Δp , which gives

$$B = V(M - \mu T - \Delta p). \quad (11)$$

The fund manager's goal is thus to maximize B , the perceived dollar benefit from trading. The optimal price impact Δp maximizes B subject to Eq. (10), $T = aV/\Delta p$, i.e., Δp maximizes $V(M - \mu aV/\Delta p - \Delta p)$, which we will see gives Eq. (7).

The time to execution is $T \sim V/\Delta p \sim V^{1/2}$, and the number of "chunks" in which the block is divided is $N \sim T \sim V^{1/2}$. These effects have been qualitatively documented in [77,78,88]. The last relation gives

$$\zeta_N = 2\zeta_V, \quad (12)$$

which in turn predicts $\zeta_N = 3$, a value, approximately consistent with the empirical value of 3.4 [76].

Thus far, we have a theoretical framework for understanding the square root price impact of trades Eq. (7), which with Eq. (4) explains the cubic law of returns Eq. (3). We now focus on understanding Eq. (4).

8.3. Explaining the power laws

Next we show that returns and volumes are power law distributed with tail exponents

$$\zeta_r = 3, \quad \zeta_V = 3/2, \quad (13)$$

provided the following conditions hold: (i) the power-law exponent of mutual fund sizes is $\zeta_S = 1$ (Zipf's law); (ii) the price impact follows the square root law (7); (iii) funds trade in typical volumes $V \sim S^\delta$ with $\delta > 0$; (iv) funds adjust trading frequency and/or volume so as to pay transactions costs in such a way that defining

$$c(S) \equiv \frac{\text{Annual amount lost by the fund in price impact}}{\text{Value } S \text{ of the assets under management}}, \quad (14)$$

then $c(S)$ is independent of S for large S .

The empirical validity of conditions (i) and (ii) was shown above, while condition (iii) is a weak, largely technical, assumption. Condition (iv) means that funds in the upper tail of the distribution pay roughly similar annual price impact costs $c(S)$ reaches an asymptote for large sizes. We interpret this as an evolutionary

“survival constraint”. Funds that would have a very large $c(S)$ would have small returns and would be eliminated from the market. The average return $r(S)$ of funds of size S is independent of S [93]. Since small and large funds have similarly low ability to outperform the market, $c(S)$ is also independent of S .

For each block trade $V(S)$ a fund of size S incurs a price impact proportional to $V\Delta p$ which, from condition (ii), is $V^{3/2}$. If $F(S)$ is the fund’s annual frequency of trading, then the annual loss in transaction costs is $F(S) \cdot V^{3/2}$, so

$$c(S) = F(S) \cdot [V(S)]^{3/2}/S. \quad (15)$$

Condition (iv) implies that either $V(S)$ or $F(S)$ will adjust in order to satisfy

$$F(S) \sim S \cdot [V(S)]^{-3/2}. \quad (16)$$

Condition (i) implies that the probability density function for mutual funds of size S is $\rho(S) = -\partial G/\partial S \sim S^{-2}$. Since condition (iii) states that $V \sim S^\delta > x$, and since they trade with frequency given $F(S)$ in Eq. (16),

$$P(V > x) \sim \int_{S^\delta > x} F(S)\rho(S) dS \sim \int_{S > x^{1/\delta}} S^{1-3\delta/2} S^{-2} dS \sim x^{-3/2}. \quad (17)$$

which leads to a power-law distribution of volumes with exponent $\zeta_V = 3/2$. Moreover, from Eq. (9), it follows that $\zeta_r = 3$. In addition, the above result does not depend on details of the trading strategy, such as the specific value of δ .

9. Outlook

No one can predict future trends, but approximate inequalities are sometimes predictable. For example, if physicists collaborate with economists, the result is more probable to be useful, and responsible to appreciating the great body of prior work done by economists. Our econophysics work has benefitted from collaborations with top-quality energetic economists, including Gabaix of the MIT Economics Department, Pammolli and Riccaboni of the Lecco Economics Department [94,95], Canning of Harvard University Economics Department [42,43], Podobnik from Mathematical Finance in Zagreb [96–100] and Salinger of the Boston University Economics Department [30–33,47,101].

Second, we can learn from the work of the masters in science. In this talk, we mentioned Vilfredo Pareto, a “father” of both scaling and universality, and a professor in the university hosting this conference. Another great person is Omori, who discovered relations among aftershocks to an earthquake. The possibility that the Omori law governing such aftershocks has its analogs in financial “earthquakes” has been proposed by Lillo and Mantegna and developed in a series of papers by Havlin, Yamasaki, and collaborators [102–106].

Lastly, if physicists are humble and realize that in some cases what we can contribute of most utility in economics is novel ways of thinking about and analyzing data, especially since many methods from mathematical statistics are not focussed on handling the strange behavior of nonstationary functions that obey scale invariance, over a limited region of the range of variables. An example might be time series analysis, where Carbone and her collaborators have significantly extended the commonly used detrended fluctuation analysis (DFA) [107–111] to a new approach called the detrended moving average (DMA) [112–116].

Not only is it possible for methods and concepts of statistical physics (and nuclear physics [25,71,73]) to influence economic thought, but it is also possible that methods and concepts of economics can influence physics thought. In particular, there are notable “unsolved physics problems” such as turbulence, for which recent work on parallels reveals both qualitative similarities [117] and quantitative differences [118,119]. Similarly, problems of economics have stimulated new ideas in mathematics, such as the truncated Lévy walk [98,99,120–123]. Finally, physicists strive to find universality [15,16,124] in economic phenomena [125], and most recent work (but not all work [126]) supports this idea.

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