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ECONOMICO-MATHEMATICAL ANALYSIS OF TRANSITION FROM OPEN-PIT TO UNDERGROUND MINING

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ABSTRACT

Shallow ore deposits are mined by surface methods but a depth is reached in the case of most deposits after which underground methods are applied for the extraction of the remaining ore. The determination of this depth and its analysis is the subject of this paper. In order to determine transition depth from open-pit to underground mining on the basis of the allowable and economically feasible overall stripping ratios, an economic-mathematical equation is first introduced. Using the achieved equation and an analytical procedure on a particular two-dimensional hypothetical tabulate deposit, an effective formula is then established. The formula introduced here, together with the procedure adopted, could be used in all similar mining situations by the mining design engineer.

INTRODUCTION

Only a few studies can be traced in the available literature regarding the determination of transition depth from open-pit to underground mining. Majority of these studies have been carried out in the past decade and they all relate to a particular mining situation where a combination of underground and surface methods have had to be used. Some methodologies were established by [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

In this research, in order to establish a fundamental and reliable procedure, the first methodology named "allowable stripping ratio" is considered being the basis of analysis. Allowable stripping ratio is expressed by a relation with emphasis on exploitation cost of 1 ton ore in underground and in open-pit deposits, as well as the removal cost of 1 ton of waste necessary to be removed for the extraction 1 ton of ore. In this way and in choice between open-pit and underground mining methods, it is necessary to compare their operation economic efficiencies, with the exception of when the advantages of one of them are entirely obvious. The main characteristic employed in economic evaluation of open-pit mining is the stripping ratio, by which is on the whole meant the volume of removed waste per unit of mineral (m^3 per m^3, or m^3 per ton).

The parameter known as the stripping ratio is almost universally used and represents the amount of uneconomic material that must be removed to uncover one unit of ore [13]. If a deposit changes abundant in geometry along the dip, above all if the change occurs at the end of the deposit, the stripping ratio will be too large when the whole deposit is mined via open-pit mining [10]. In relation to the practice of surface mining of coal deposits, it is common to describe the stripping ratio in terms of m3 of waste per ton of the mineral, but in operating ore deposits the mentioned ratio is ordinarily given in terms of m3 of waste per m3 of the related mineral. There are various kinds of stripping ratio classified as overall, instantaneous (operating), break-even, and allowable.

Overall stripping ratio is the proportion of the whole volume of overburden in the open-pit to the total reserves of the mineral. In other words, according to Equation 1, the ratio of the total volume of waste to the ore volume is defined as overall stripping ratio [13].

$$OSR = \frac{V_w}{V_o} \qquad (1)$$

Where,

OSR: overall stripping ratio
V_w: Volume of waste removed to a certain depth
V_o: Volume of ore removed to a certain depth

In order to determine maximum depth based on the profitability of the operation, it is essential to know about the overall costs and revenues that will be received by selling the ore and its bye-products, if any [14]. To develop a pit design requires the establishment of the break even stripping ratio. This ratio refers only to the last increment mined along the pit wall. In other words, break even stripping ratio is applied only at the surface of the final pit and must not be confused with the overall stripping ratio, which is always less; otherwise there would be no profit to the operation [1].

The break even stripping ratio is calculated for the point at which break-even occurs and the necessary stripping is paid for by the net value of the ore removed. Generally, the break even stripping ratio can be determined due to Equation 2 [15]:

$$BESR = \frac{I - C_t}{C_{sw}} \qquad (2)$$

Where;

I: revenue per tonne of ore
C_t: production cost per tonne of ore (including all costs to the point of sale, excluding stripping)
C_{sw}: stripping cost per tonne of waste

The allowable stripping ratio characterizes the maximum scope of stripping which is practicable in open-pit operation. The ratio stated in terms of m^3 of waste per ton of the mineral can be determined by Equation 3.

$$ASR = \frac{C_{ug} - C_{op}}{C_w} \qquad (3)$$

Where,

C_{ug}: full prime cost of 1 ton of the mined mineral via underground ($);
C_{op}: prime cost of 1 ton of the mined mineral via open-pit (except stripping costs), ($);
C_w: total costs of ground removal (stripping) in relation to 1 ton of ore extracting using open-pit ($)

The allowable stripping ratio can be engaged during economic evaluation process of open-pit operation and finding out transition depth. It should be also considered that the allowable stripping ratio mainly depends on the nature and extent of mechanization of open-pit mining.

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In most pit designs, the overall stripping ratio is much lower than the allowable stripping ratio. Therefore, the allowable ratio is never apparent in the year to year operating (instantaneous) stripping ratios. Instantaneous stripping ratio is the real relation of the removed waste volumes and the mineral exploited in the pit during a certain and definite period of time.

METHODOLOGY

The authors seek to inference an equation, which can be caused to a formula considering shape of ore and waste located within the pit helping to determine transition depth from open-pit to underground mining. In this case it is assumed that ore deposit be continues (Fig. 1). Figure 1 is considered as the base for mathematical and geometry analysis.

For the objective, at the final pit depth, the overall stripping ratio becomes equal to the allowable stripping ratio. Therefore, it is necessary to equate the overall stripping ratio and allowable stripping ratio as Equation 4.

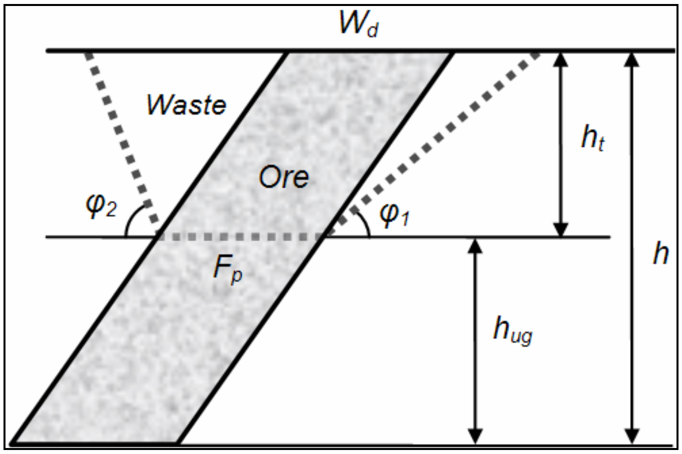

Figure 1. A general schematic of combined mining and transition problem

$$OSR = ASR \Rightarrow \left[\frac{V_w}{V_o}\right] = \left[\frac{C_{ug} - C_{op}}{C_w}\right] \tag{4}$$

Consequently, Equation 5 can be concluded from Equation 4 as below:

$$C_w \cdot V_w + C_{op} \cdot V_o - C_{ug} \cdot V_o = 0 \tag{5}$$

In this case, initially it is necessary to measure the volumes of ore and waste within the pit limit to a certain depth, which here are considered being a function of transition depth (ht) as the Equations 6 and 7, respectively [7, 8].

$$V_w = \int g(h_t) \cdot dh_t \tag{6}$$

$$V_o = \int f(h_t) \cdot dh_t \tag{7}$$

Then, by replacing Equations 6 and 7 into Equation 5, the following Equation (8) can be deduced:

$$\begin{aligned} & C_w \cdot \int_0^{h_t} g(h_t) \cdot dh_t + C_{op} \cdot \int_0^{h_t} f(h_t) \cdot dh_t \\ & - C_{ug} \cdot \int_{h_t}^{h} f(h_t) \cdot dh_t = 0 \end{aligned} \tag{8}$$

In this regard, Equation 8 can be written as Equations 9 and 10, respectively.

Finally, in order to increase the accuracy of Equation 10 the authors take into account both ore recovery coefficient achieved through open-pit and underground mining methods. In this regard, Equation 11 is obtained.

$$\begin{aligned} & C_w \cdot \int_0^{h_t} g(h_t) \cdot dh_t + C_{op} \cdot \int_0^{h_t} f(h_t) \cdot dh_t \\ & - C_{ug} \cdot [\int_0^{h} f(h_t) \cdot dh_t - \int_0^{h_t} f(h_t) \cdot dh_t] = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} & (C_{op} + C_{ug}) \cdot \int_0^{h_t} f(h_t) \cdot dh_t \\ & + C_w \cdot \int_0^{h_t} g(h_t) \cdot dh_t - C_{ug} \cdot \int_0^{h} f(h_t) \cdot dh_t = 0 \end{aligned} \tag{10}$$

$$\begin{aligned} & (R_{op} \cdot C_{op} + R_{ug} \cdot C_{ug}) \cdot \int_0^{h_t} f(h_t) \cdot dh_t \\ & + C_w \cdot \int_0^{h_t} g(h_t) \cdot dh_t \\ & - R_{ug} \cdot C_{ug} \cdot \int_0^{h} f(h_t) \cdot dh_t = 0 \end{aligned} \tag{11}$$

Where,

R_{ug}: ore recovery coefficient due to underground mining method;
R_{op}: ore recovery coefficient due to open-pit mining method;

It is remarkable that an effective formula can be concluded by employing Equation 11 and on the basis of ore deposit shape, open-pit limit, final mining depth, the suitable underground method and its related recovery coefficient, open-pit mining cost per unit of ore volume or tonnage, underground mining cost per unit of ore volume or tonnage, and stripping cost in relation to the unit of ore extracting using open-pit.

In this study, for clarity explanation of the considered parameters during the concluded economic-mathematical equation, a two dimensional section of a tabulate-shape ore deposit with transition problem is considered. For the target, an analytic geometry procedure is used.

TWO DIMENSIONAL TABULATE ORE DEPOSIT AND THE TRANSITION DEPTH

For the target of the section and proving the suitable formulas to ascertain transition depth over from open-pit to underground, two states as the following are considered:

State 1- If deposit includes outcrops and maximum width of pit floor
State 2- If deposit includes outcrops and minimum possible width of pit floor

State 1: In first state it is assumed a tabulate ore deposit includes outcrop with an equal width of the deposit and pit floor (Fig. 1). In this case, it is initially necessary to measure the volume of covered waste rocks and the related ore within the pit limits area. Then, utilizing a geometric analytical procedure and the main Equation obtained, Equation 12 can be proved.

$$h_t = \frac{W_d \cdot \left(R_{ug} \cdot C_{ug} - R_{op} \cdot C_{op}\right)}{C_w \cdot \cot\varphi_1 + \cot\varphi_2} \tag{12}$$

Where,

h_t: transition depth (m)
W_d: horizontal thickness of the ore body (m)
φ_1: pit side slope angle along the foot-wall (deg)
φ_2: pit side slope angle along the hanging-wall (deg)

State 2: This state is the same as the first state, with the exception of this case that only minimum possible width of pit floor may be mineable (Fig. 2). It takes into consideration the eventual deepening of the open-pit without extending to sidewalls. Due to the difference and basis of the main obtained Equation, to determine transition depth from open-pit to underground Equation 13 is concluded.

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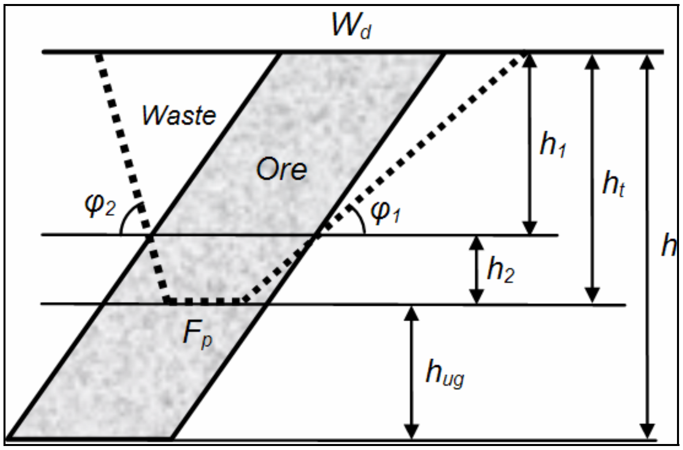

Figure 2. Transition problem including minimum possible width of pit floor

$$h_t = h_1 + h_2 \Rightarrow$$
$$h_t = \frac{W_d \cdot \left(R_{ug} \cdot C_{ug} - R_{op} \cdot C_{op}\right) + (W_d - F_p) \cdot C_w}{C_w \cdot \cot\varphi_1 + \cot\varphi_2} \qquad (13)$$

Where,

h_1: pit depth in ore with extension sideways (m)
h_2: deepening of pit depth without extension sidewalls (m)
F_p: minimum possible width of the pit floor (m)

CONCLUSION

In relation to the deposits which have potential of using the combined mining of open-pit and underground in vertical direction, the most significant problem is the determination of transition depth over from open-pit to underground mining. For this target, in the study using allowable stripping ratio and overall stripping ratio as well as an economic-mathematical analysis an equation was initially proved. Then, to introduce more clarity of the procedure and the effectiveness of the considered parameters in this regard, a two dimensional section of a tabulate-shape ore deposit with transition problem during two states were considered. To find the volumes of extracted ore and waste, an analytic geometry procedure was used. First, in regard to the tabulate deposits including outcrops and considering the maximum width of pit floor for exploitation, a simple formula was proved. Then during the second state, to get the eventual deepening of the open-pit without extending it sideways, minimum possible width of pit floor was contemplated and consequently a formula is devised that can be used to determine the most economical transition depth in similar situations. The procedure explained in this paper can also serve as a useful tool for the mining design engineer when attempting to analyse varying depths or in different mining conditions.

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