On the Possible Conflict Between Economic Growth and Social Development

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On the Possible Conflict Between Economic Growth and Social Development*

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Abstract

The present contribution proposes a simple model of private and social capital to study the relationship between economic growth and social development. We investigate whether these two processes move in the same direction or whether they conflict with each other, and show that both outcomes are possible, depending on the initial relative endowment of private and social capital, social technology, and the degree of individual impatience. These dynamics affect and are affected by the choice of time allocation between labour and social participation and by the choice of consumption of both private and relational goods. Taking all these aspects into account allows us to study in an articulated way the interplay between the private and social components of well-being.

JEL-Classification: C73, D62, I31, J22, O41, Z13

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1 Introduction

In the long run, individual and aggregate well-being depend on both material growth and social and cultural development. Although this has perhaps always been true, systematic and sustained material growth has been absent for most of human history, with some positive and negative exceptions (see, e.g., Goodfriend and McDermott 1995). Instead, since the Industrial Revolution, a significant fraction of the world has kept growing at a positive rate, accumulating physical capital, developing better and better technologies, and accumulating human capital. Indeed, these processes have captured most of economists' attention, whereas social and cultural dynamics have remained at the margin of economic analysis. In recent years, however, an increasing number of economists have begun to pay attention to the interplay between material growth and social development.

When material needs have been satisfied to a substantial degree, as is the case in advanced economies, well-being depends to an increasing extent upon social factors, like social environment, individual relative position and social status, and the ability to construct and enjoy meaningful and satisfactory relations with other people[1]. Social status has already received a great deal of attention by economists. Here, we rather focus on the social environment and the enjoyment of social relations, building on the notions of 'social capital' and 'relational goods'.

The present contribution proposes a simple growth model with private and social capital accumulation. We investigate whether these two processes move in the same direction or whether they conflict with each other, and show that both outcomes are possible, depending on the parameters and initial conditions of the economy. Taking into account the effects of these dynamics on the consumption of both private and relational goods, we draw conclusions about well-being that apply to advanced economies. Section 2 clarifies the concepts and motivates our set-up. Sections 3 and 4 introduce the static and dynamic versions of our model. Section 5 concludes.

2 Motivation

Social capital is the collection of those productive assets which are incorporated in the social structure of a group (rather than in physical goods and

[1]Sacco, Vanin and Zamagni (2004) provide an extensive discussion of these issues.

single human beings, like physical and human capital) and which allow cooperation among its members to reach common goals. If we bear in mind that the group considered may range from being very small to including the whole of society, this definition of social capital encompasses most of the definitions to be found in the literature. At one extreme, some scholars even define social capital as an individual asset, but we prefer to consider it as a collective asset, in order to emphasise its interpersonal nature[2]. Examples of social capital range from trust to effective civic norms and to the networks of voluntary associations typical of civil society. A peculiar feature of social capital is that it is not accumulated through a standard mechanism of individual investment, since most of its benefits are not privately appropriable[3]. Rather, or at least to a much greater extent, it is accumulated through social participation in group activities. This participation may only partially be regarded as an investment, since it is, perhaps mainly, an activity that entails the simultaneous production and consumption of a particular kind of goods, namely, relational goods.

Relational goods display two peculiar features: they cannot be enjoyed alone, but exist only inasmuch as they are shared; and their production and consumption very often cannot be separated: relational goods are produced and consumed at the same time through participation in some social activity with other people[4]. Examples range from going out with friends to participating in a choir, a football club, a voluntary organization, and so on.

We focus on two aspects of the relationship between relational goods and social capital. On one hand, a higher social capital increases the return to

[2]Glaeser, Laibson and Sacerdote (2002) call 'social capital' the 'social' component of human capital. Since we distinguish social capital from human capital, we do not follow their approach. DiPasquale and Glaeser (1999) define individual social capital as an individual's connections to others, and argue that it is important for private provision of local amenities and of local public goods. This is in line with our focus, although we emphasize more the role of aggregate social participation.

[3]Glaeser, Laibson and Sacerdote (2002) make the opposite point, namely, that social capital accumulation responds to incentives to investment in exactly the same way as human capital. Indeed, this result is natural if one defines social capital as a component of human capital, but it does not hold if one considers social capital as a group asset rather than as an individual asset.

[4]The concept of relational goods is due to Uhlaner (1989). Corneo and Jeanne (1999) refer to them as to socially provided private goods and study their interplay with social status and growth. Gui (2000) provides a number of interesting contributions on the interpersonal dimension of economic interaction.

the time spent in social participation. For instance, it is easier and more rewarding to participate in an association with people whom we trust and who share our values and norms, and in a social context characterised by a rich network of associative opportunities; similarly, it is more rewarding to go out with friends with whom we share a higher capital of confidence, long-lasting relations and common norms, and in a context that offers many options for socially enjoyed leisure. In other words, social capital may be seen as an input in the production of relational goods[5]. On the other hand, higher social participation brings about social capital accumulation as a by-product. For instance, trust (or empathy) may be reinforced and generalised through social interactions (if individuals do not behave opportunistically). Likewise, high social participation may lead to the formation of new associations, while continuing to feed the existing ones.

Social participation is an activity intrinsically characterised by external effects (generally speaking, there is no market in which other people's participation may be bought, and even less is there a market for social capital). If other people's participation is low, or if the level of social capital is low, the time spent in participating is little productive, and it becomes worthwhile to shift to private activities, that is, to ones which yield private goods. For instance, if my friends do not have time to go out with me, or if they do go out with me but the environment does not offer any interesting social opportunities, I may decide to spend my time better watching television or reading a book. Indeed, Corneo (2001) presents striking empirical evidence that the time devoted to watching television and working are positively correlated across countries, and proposes an explanation based on the substitution between privately enjoyed and socially enjoyed leisure (i.e., between some private goods and relational goods). While our work is quite close in spirit to Corneo's paper, the main difference is that we analyse the dynamics of private and social capital accumulation, whereas he displays a simple static model with multiple equilibria.

Specifically, we propose here a model in which a reduction in social participation implies at the same time an increase in labour supply and a substitution of private for relational goods. On one hand, such a shift stimulates the

[5]Much of the literature on social capital also stresses its positive impact on the productivity of traditional private goods. We do not examine this effect here, thus making our point sharper: as in our framework, a problem of under-accumulation of social capital exists, this problem will become even worse if we also consider the effect of social capital on private production. We discuss this point in more detail in the concluding section.

economy[6]; on the other, it generates a negative externality on the productivity (in terms of relational goods) of social participation. Dynamically, this change has a negative effect on social capital accumulation, whereas the sign of its effect on private capital accumulation depends on whether total savings increase (together with private production) or decrease (due to a more than proportional increase in private consumption)[7]. Theoretically, private and social capital may be both positively or negatively correlated[8].

Both ideas - that private growth brings about social development, and that it generates social disruption - are supported by long-standing traditions of thinking. We do not attempt to reconstruct this fascinating intellectual debate here, but limit ourselves to referring to Hirsch (1976) as a representative of the view that private growth may entail negative social externalities. In particular, Hirsch argues that growth makes individual time constraints increasingly binding, thereby inducing a shift from time-intensive activities (among which there is indeed social participation) to time-saving ones (among which there are many forms of private consumption – e.g., fast food)[9]. We emphasise here that this kind of shift may even reinforce private growth.

The idea that negative externalities, either on the natural or the social environment, may foster growth was first studied within an evolutionary framework by Antoci (1996) and Antoci and Bartolini (1997). The environmental economics literature on this subject has subsequently been rapidly expanding. For instance, Bartolini and Bonatti (1997) and several other contributions have further explored the basic idea within a neoclassical framework[10]. While our work is closely related to theirs, the main departure consists in our focus on social capital accumulation, which depends on social participation, whereas the above literature, although it mentions the possibility of a sociological interpretation, is more focused on natural resources, which are typically subject to a spontaneous flow of renewal.

[6] While most private goods enter the GDP, most relational goods do not.

[7] While this is consistent with an interpretation of private capital in terms of physical capital, an extension of the concept to include human capital would not alter the picture significantly.

[8] See also Putnam's (1995, 2000) empirical findings about the rise and decline of US social capital.

[9] See Becker (1965) for a pioneering economic analysis of time allocation.

[10] Among recent contributions, see Antoci and Bartolini (2004) for an evolutionary one and Bartolini and Bonatti (2004) for a neoclassical one.

In two companion papers (Antoci, Sacco and Vanin 2001, 2002), we explore a similar framework, respectively with the tools of evolutionary game theory and of neoclassical economics. In both studies we find that growing economies may fall into social poverty traps, defined as situations in which, although material wealth is high, social poverty forces down overall well-being. For the sake of simplicity, in those models we consider the dynamics of only one asset, social capital. Here, we extend an analysis to include the accumulation of private capital. One might expect that, once the latter is taken into account, possibly together with the positive externalities it causes, material growth may be strong enough, from the point of view of well-being, to more than compensate its negative social externalities. Indeed, we show that this may but need not be the case, and that whether it happens or not depends on the parameters of preferences and technology. An interesting result is that impatience may increase steady-state well-being, since it reduces inefficient over-accumulation of private capital[11].

3 Static model

We now present a simple growth model with private and social capital accumulation. Since some of the basic insights may be appreciated even in a static framework, we first introduce a static version, in which private and social capital are considered as exogenously given in some strictly positive amount, and then introduce their dynamics (in continuous time).

Preferences and technology

We model an economy populated by a continuum of identical, infinitely lived individuals, of size normalised to 1, whose utility depends on three goods: a private consumption good C, used to satisfy basic needs (e.g., food and clothes); a relational good B (e.g., enjoying time with friends); and a private consumption good C_s that serves as a substitute for the relational good (e.g., a luxury good). Instantaneous preferences are described by the utility function $u(C, B, C_s) = \ln C + a \ln(B + bC_s)$, where $a > 0$ is the elasticity of substitution between basic needs satisfied by C on one hand and needs

[11] A similar result is also found in the above-mentioned environmental economics literature, since in that case too growth is the result of a failure of coordination.

satisfied by either B or C_s on the other, and $b > 0$ is the marginal rate of substitution between B and C_s[12].

The key point is how individuals decide to allocate their time (they are endowed with one unit) between social participation, labour and private consumption, besides the allocation of the latter between the two private goods. Since it is out of our focus, we disregard the allocation of time to C and C_s, simply assuming that both require income but not time; on the contrary, B may only be enjoyed if an individual spends time in social participation. Time must therefore be allocated between social participation (fraction s) and labour (fraction $1 - s$). A single individual considers average social participation $\bar{s} = \int_0^1 s(i)\mathrm{d}i$ in the economy as exogenously given.

We assume a backyard technology[13], by which identical individuals produce private goods for their own consumption using labour and private capital K, according to production function $Y = (1-s)^{\epsilon} K^{1-\epsilon} A$, where $\epsilon \in (0, 1)$ is a parameter. Term $A \equiv (1 - \bar{s})^{\sigma} \bar{K}^{\vartheta}$ captures a positive externality in production, which can be due to either the observability of other people's production or the availability of help when needed. Average private capital $\bar{K} = \int_0^1 K(i)\mathrm{d}i$ is considered as exogenously given by each individual and, consequently, the same is true for the whole term A (σ and ϑ are strictly positive parameters).

Besides private capital, our economy is characterised by the presence of social capital K_s. Social capital is not the private property of any individual, but is rather an endowment of the entire economy, that each single individual considers as exogenous.

The quantity of relational good B enjoyed by the representative individual is a function of his own social participation, average social participation and social capital, all of which are indispensable factors: $B = s^{\alpha} \bar{s}^{\beta} K_s^{\gamma}$, where $\alpha,\ \beta,\ \gamma > 0$.

The maximisation problem of the representative individual, and symmetric Nash equilibria

The problem solved by the representative individual is:

[12] The assumption of perfect substitutability between B and C_s greatly simplifies the mathematics. Relaxing this assumption may have non-obvious economic consequences and make closed-form solutions hard to obtain. We simulated a version of this model with the hypothesis of imperfect substitution, but did not gain any interesting insight.

[13] This simplifying assumption allows us to rule out any concern about market structure.

$$\max_{s,C,C_s} u(C,B,C_s) = \ln C + a\ln(s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + bC_s) \quad \text{s.t.} \tag{1}$$

$$C + C_s = Y = (1-s)^{\epsilon}K^{1-\epsilon}(1-\bar{s})^{\sigma}\bar{K}^{\vartheta}, \quad C, C_s \geq 0, \quad s \in [0,1]. \tag{2}$$

A symmetric Nash equilibrium (SNE) is a triplet (s^*, C^*, C_s^*) that solves problem (1) under constraints (2), given that every other individual in the economy chooses s^*, so that, in particular, $\bar{s} = s^*$.

It is easy to show that there is always an SNE with no social participation. To see this, let $\tilde{s} \equiv 0$, $\tilde{C} \equiv \frac{1}{1+a}K^{1+\vartheta-\epsilon}$, and $\tilde{C}_s \equiv \frac{a}{1+a}K^{1+\vartheta-\epsilon}$.

Proposition 1 *The triplet $(\tilde{s}, \tilde{C}, \tilde{C}_s)$ is always an SNE, that is, for any parameter constellation, there exists an SNE with no social participation*[14].

In this equilibrium, no time is devoted to social interaction, since each individual believes that every other one will spend his entire amount of time working, thus rendering social participation not worthwhile.

To be able to investigate analytically the existence of an SNE in which $s > 0$, we make the following simplifying assumption.

Assumption 1 $\alpha + \beta = \epsilon + \sigma = \varphi < 1$*: this implies that, at an SNE, the elasticity of relational goods with respect to social participation equals the elasticity of private production with respect to labour; we call this elasticity φ, and assume that the two functions are concave ($\varphi < 1$)*[15].

Proposition 2 *Under Assumption 1, there exists a unique SNE with strictly positive social participation, namely, the triplet $(\hat{s}, \hat{C}, \hat{C}_s)$, defined as follows:*

$$\text{Case (a)} : K_s < h(K) : \qquad \begin{cases} \hat{s} \equiv \dfrac{1}{1+\left(\frac{b\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^{\gamma}}\right)^{\frac{1}{1-\varphi}}} \\ \hat{C} \equiv \frac{1}{b(1+a)}\hat{s}^{\varphi}K_s^{\gamma} + \frac{1}{(1+a)}(1-\hat{s})^{\varphi}K^{1+\vartheta-\epsilon} \\ \hat{C}_s \equiv \frac{a}{(1+a)}(1-\hat{s})^{\varphi}K^{1+\vartheta-\epsilon} - \frac{1}{b(1+a)}\hat{s}^{\varphi}K_s^{\gamma} \end{cases} \tag{3}$$

$$\text{Case (b)} : K_s \geq h(K) : \qquad \begin{cases} \hat{s} \equiv \frac{a\alpha}{a\alpha+\epsilon} \\ \hat{C} \equiv (1-\hat{s})^{\varphi}K^{1+\vartheta-\epsilon} \\ \hat{C}_s \equiv 0 \end{cases} \tag{4}$$

[14] All proofs are given in the Appendix.

[15] The equality plays no other role than to enable us to derive an analytic solution, whereas the assumption that $\varphi < 1$ allows a strictly positive equilibrium social participation, even for a low ratio of social over private capital.

where $h(K) \equiv \left[\left(\frac{\epsilon b}{\alpha}\right)^{\varphi}(ab)^{1-\varphi}K^{1+\vartheta-\epsilon}\right]^{\frac{1}{\gamma}}$.

Note that $\hat{s}$ is an increasing function of K_s and α and a decreasing function of K. We will come back to the interpretation of these findings in the context of the dynamic specification of the model. Observe also that cases (a) and (b) of Proposition 2, defined as $K_s < h(K)$ and $K_s \geq h(K)$, respectively, identify an economy in which social capital is scarce and, respectively, abundant relative to private capital. Whatever the economy's (exogenous) endowment of the two forms of capital, Proposition 2 gives us the SNE with participation $(\hat{s}, \hat{C}, \hat{C}_s)$ as a function of them and of the parameters.

Proposition 3 *Let Assumption 1 hold.*

In case (a), there are both parameter constellations for which the SNE with positive social participation $(\hat{s}, \hat{C}, \hat{C}_s)$ *Pareto-dominates the SNE with no participation* $(\tilde{s}, \tilde{C}, \tilde{C}_s)$*, and other ones for which the reverse is true.*

In case (b), let $g(K) \equiv \left\{\left[\frac{(a\alpha+\epsilon)^{\frac{1+a}{a}}}{(1+a)^{\frac{1+a}{a\varphi}}}\right]^{\varphi}(ab)^{1-\varphi}K^{1+\vartheta-\epsilon}\right\}^{\frac{1}{\gamma}}$. *For any parameter constellation, the SNE with participation* $(\hat{s}, \hat{C}, \hat{C}_s)$ *Pareto-dominates the one with no participation* $(\tilde{s}, \tilde{C}, \tilde{C}_s)$ *if and only if* $K_s > g(K)$*, the reverse being true when* $K_s < g(K)$.

Proposition 3 tells us that in economies relatively poor in social capital and rich in private capital (meaning $K_s < h(K)$), which of the two equilibria Pareto-dominates the other depends on the parameters of preferences and technology; but when social capital is abundant enough relative to private capital ($K_s \geq h(K)$), it eventually (that is, for $K_s > g(K)$) becomes more efficient to devote a positive fraction of time to social participation, thereby foregoing some (or all) luxury consumption but enjoying relational goods. Since, for any parameter constellation and for any strictly positive endowment of both forms of capital, both equilibria are present, it is possible that, due to coordination failure, an economy becomes stuck in the Pareto-inferior equilibrium. The limitation of Proposition 3 is that it does not tell us anything about the sources of the relative abundance of private versus social capital. To investigate this aspect, we have to turn to the dynamic specification of our model.

However, before doing this, a further comment may be made on the externalities which drive the story of this static model. Since both average social participation and average labour time are supposed to exert positive external effects (on the production of the relational good and of the private goods, respectively), it is not *a priori* clear whether, overall, social participation displays positive or negative spillovers[16]. In general, in this game there tend to be positive spillovers from social participation when social capital is high relative to private capital, whereas they are, overall, negative when the reverse is true[17].

Remark 1 *Under Assumption 1, since, generically, in the SNE with positive participation* $(\hat{s}, \hat{C}, \hat{C}_s)$ *spillovers are present, this equilibrium is inefficient even when it Pareto-dominates the SNE with no participation* $(\tilde{s}, \tilde{C}, \tilde{C}_s)$[18].

Remark 1 tells us that the common result that, in the presence of non-internalised externalities, even the best SNE is generally inefficient, also applies to our case.

4 Dynamic model

In the dynamic specification of the model, preferences and technology are the same as above, with the only difference that now private and social capital are endogenously determined. The dynamics of the representative individual's private capital is given by $\dot{K} = Y - C - C_s - \eta K$, where $\eta \geq 0$ is the private capital depreciation rate[19].

Social capital is not accumulated through a process of investment; rather, its stock increases when a high average social participation brings about a high average enjoyment of the relational good (denoted $\bar{B} = \int_0^1 B(i)\mathrm{d}i$). Since relations deteriorate over time if individuals do not actively take care of them, we also assume that K_s depreciates at a rate $\delta > 0$. We can thus summarise the dynamics of social capital as $\dot{K}_s = f(\bar{B}) - \delta K_s$, where f is a strictly

[16]According to Cooper and John's (1988) terminology, social participation has positive (negative) spillovers if an increase in average social participation raises (decreases) individual utility, i.e., if $\frac{\partial u(C,B,Y-C)}{\partial \bar{s}}$ is positive (negative).

[17]Formally, under the reasonable assumption that $\beta, \sigma < 1$, which is even weaker than Assumption 1, $\frac{\partial u(C,B,Y-C)}{\partial \bar{s}} > 0 \Leftrightarrow \beta s^{\alpha} \bar{s}^{\beta-1} K_s^{\gamma} > b\sigma(1-s)^{\epsilon}(1-\bar{s})^{\sigma-1} K^{1+\vartheta-\epsilon}$, i.e., when K_s is high relative to K, s is high and $\bar{s}$ is low.

[18]Precisely, in the SNE $(\hat{s}, \hat{C}, \hat{C}_s)$ there are positive spillovers when $\alpha < \frac{\beta\epsilon}{\sigma}$ and negative ones when the reverse is true. There are no spillovers only in the non-generic case in which $\alpha = \frac{\beta\epsilon}{\sigma}$. Remark 1 then follows from Proposition 2 of Cooper and John (1988).

[19]For notational simplicity, we omit the time index $t \in \Re_+$.

increasing function[20]. The more rewarding social participation is in terms of relational goods, the more it contributes to social capital accumulation[21].

For the sake of simplicity, we make the following assumptions.

Assumption 2 $\eta = 0$*: we ignore private capital depreciation.*

Assumption 3 $f(x) \equiv x$*: this means that* $\dot{K}_s = \bar{B} - \delta K_s$*.*

Assumption 4 $\epsilon > \vartheta$ *and* $\gamma < 1$*: this means that we do not allow either* K *or* K_s *to grow steadily at a strictly positive rate.*

Assumption 2 is an innocent one. Assumption 3 is only made for the sake of analytical simplicity[22]. Assumption 4 means that, in our model, there is no engine for endogenous growth.

The representative individual's maximisation problem

Let $r > 0$ be the inter-temporal discount rate. The representative individual chooses at time $t = 0$ how to allocate, at present and at any point in the future, his own time to participation and labour, and his private production to subsistence and luxury consumption on one hand, and investment in new private capital on the other, in order to maximise lifetime utility. At any given point in time, his control variables are therefore s, C and C_s (which must respect $C, C_s \geq 0$ and $s \in [0, 1]$). A strategy is a time path of controls. Initial stocks of social capital (K_s^0) and private capital (K^0, assumed to be the same for every individual: $K^0 = \bar{K}^0$) are exogenously given. When choosing

[20] The idea that non-material forms of capital may be accumulated through a 'consumption' activity rather than through investment, although unconventional in economics, is neither new (it goes back to Aristotle's analysis of ethical virtues, the influence of which is to be found in Nussbaum's (1986) discussion of relational goods) nor surprising (e.g., knowledge, which is accumulated through the use of knowledge itself).

[21] This specification seems a good first approximation for both main forms of social capital, namely trust and social norms on one hand, and association networks on the other, since the ability of both of them to prosper and expand crucially depends on the reward they yield to the people involved, and this reward consists of a high degree of relational goods. The reason it is a first approximation is that material rewards may also play a role: we discuss this point in the concluding section.

[22] In principle, there is no reason for the 'gross investment' in social capital to be exactly equal to the average benefit from social participation, although it is an increasing function of the latter; however, this specification is by far the easiest one. For instance, all our results would still hold if we assumed $f(x) \equiv \psi x,\ \psi \in (0, 1)$.

his strategy, the representative individual regards as exogenously given the strategies of the rest of the population. Since the time path of social capital and population averages are independent of any single individual's strategy, this amounts to taking the entire future path of $K_s, \bar{K}$ and $\bar{s}$ as given. The set of variables that the representative individual considers as predetermined at any point in time therefore includes these three variables and the state variable that is under his own direct control, namely, his own private capital K[23]. In short, taking for granted the constraints imposed by technology and by the set of admissible controls, the representative individual's problem may be written as follows[24]:

$$\max_{s,C,C_s} \int_0^\infty u(C,B,C_s)\mathrm{e}^{-rt}\mathrm{d}t = \int_0^\infty [\ln C + a\ln(s^\alpha \bar{s}^\beta K_s^\gamma + bC_s)]\mathrm{e}^{-rt}\mathrm{d}t \quad \text{s.t.} \quad (5)$$

$$\begin{aligned} \dot{K}_s &= \bar{s}^{\alpha+\beta} K_s^\gamma - \delta K_s, & (6) \\ \dot{K} &= (1-s)^\epsilon K^{1-\epsilon} A - C - C_s, \qquad A \equiv (1-\bar{s})^\sigma \bar{K}^\vartheta. & (7) \end{aligned}$$

Symmetric Nash equilibrium

An SNE of this economy is a strategy (that is, a time path of the controls s^*, C^* and C_s^*) that solves problem (5) under constraints (6)-(7), given that every other individual in the economy chooses the same strategy.

We now study the SNE of our economy and its dynamic properties[25]. In order to maintain the analytical tractability of the static version also in the dynamic version of the model, we modify Assumption 1 into the following one.

[23]Notice that, in the absence of uncertainty and in the impossibility for the representative individual to affect population averages (which eliminates any incentive to behave strategically to influence other people's future choices), in the present model considering open loop vs. closed loop strategies makes no difference.

[24]Recall that all the variables here (both the controls s, C, C_s and the predetermined variables $K_s, K, \bar{K}, \bar{s}$, as well as the rates of change $\dot{K}_s$ and $\dot{K}$) should appear with a time index t, omitted for notational simplicity. The maximisation is taken over the time path of the three controls, with a slight abuse of notation.

[25]Notice that, although the representative individual *ex ante* (i.e., when deciding) considers the future time path of $\bar{s}$ and $\bar{K}$ as exogenous, *ex post* (i.e., at an SNE) it turns out to be equal to that of his own values s^* and K.

Assumption 5 *$\alpha + \beta = \epsilon + \sigma = \varphi = 1$: this implies that, at any SNE, the relational good is obtained as a linear function of social participation, and private production as a linear function of labour.*

Proposition 4 *Let Assumptions 2 to 5 hold, and consider an SNE of the economy. At any point in time the curve:*

$$K_s = \left(\frac{\epsilon b}{\alpha} K^{1+\vartheta-\epsilon} \right)^{\frac{1}{\gamma}}, \tag{8}$$

separates in the (K, K_s) plane the region in which $s > 0$ and $C_s = 0$ that in which $s = 0$ and $C_s > 0$ (see figure 1)[26].

Precisely, in the two regions, s and C_s are chosen as follows[27]*:*

$$\text{Case (a)} : K_s < \left(\frac{\epsilon b}{\alpha} K^{1+\vartheta-\epsilon} \right)^{\frac{1}{\gamma}} : \qquad \begin{cases} s = 0 \\ C_s = \frac{a}{\lambda} \end{cases}, \tag{9}$$

$$\text{Case (b)} : K_s > \left(\frac{\epsilon b}{\alpha} K^{1+\vartheta-\epsilon} \right)^{\frac{1}{\gamma}} : \qquad \begin{cases} s = \min \left\{ 1, \frac{a\alpha}{\epsilon \lambda K^{1+\vartheta-\epsilon}} \right\} \\ C_s = 0 \end{cases}. \tag{10}$$

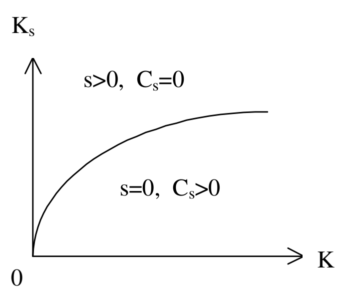

Figure 1

[26] The difference between $\varphi < 1$ and $\varphi = 1$ is that the latter assumption rules out the possibility of a strictly positive equilibrium choice of social participation for low values of social capital relative to private capital (i.e., in the lower region of figure 1). For high values, the choice of $s = 0$ is still an SNE, but not an interesting one, since the resulting dynamics are trivial. Therefore, we only examine the case in which individuals coordinate on the equilibrium with $s > 0$ in the upper region of figure 1.

[27] λ is the shadow price of K. All variables are considered at time $t \in \Re_+$. Observe that the conditions spelled in this proposition do not just hold at steady states, but rather at any point in time.

Case (a) identifies a situation in which, at a given point in time, social capital is scarce relative to private capital, so that, rather than spending time in social participation, the returns of which are low, in equilibrium it is better to choose a high labour supply, which has a high return, and to substitute a high consumption of private goods for the relational good.

On the contrary, case (b) captures a situation of relative scarcity of private capital as compared with social capital. In equilibrium, social interaction (besides subsistence consumption) is the basic source of individual well-being. On one hand, labour productivity is too low to make it worthwhile to work more in order to substitute some private consumption for the relational good; on the other hand, the social environment is rich in opportunities and makes returns in social participation high. The difference between cases (a) and (b) shows why we observe large differences in the patterns of time allocation across countries of comparable size and private capital stock: indeed, these differences may be due to the presence of different relative stocks of private and social capital.

Fixed points

Exploiting Proposition 4, we are now able to characterise the dynamic properties of our economy. In particular, we focus attention on the fixed points by stating the next proposition[28], where we define:

$$K^* \equiv \left(\frac{1-\epsilon}{r}\right)^{\frac{1}{\epsilon-\vartheta}}, \tag{11}$$

$$K_s^* \equiv 0, \tag{12}$$

$$K^{**} \equiv \left[\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)}\right]^{\frac{1}{\epsilon-\vartheta}}, \tag{13}$$

$$K_s^{**} \equiv \left[\frac{a\alpha}{\delta(\epsilon+a\alpha)}\right]^{\frac{1}{1-\gamma}}. \tag{14}$$

Proposition 5 *In the plane (K, K_s), point (K^*, K_s^*) is always a fixed point of the economy. Such point is locally saddle-path stable.*

[28] For expositional purposes, we do not mention here the steady-state values of λ, that are in any case uniquely determined.

There exists at most one more fixed point, namely (K^{**}, K_s^{**}). It is a fixed point if and only if:

$$\frac{a\alpha}{\delta(\epsilon + a\alpha)} > \left(\frac{\epsilon b}{\alpha}\right)^{\frac{1-\gamma}{\gamma}} \left[\frac{\epsilon(1-\epsilon)}{r(\epsilon + a\alpha)}\right]^{\frac{(1-\gamma)(1+\vartheta-\epsilon)}{\gamma(\epsilon-\vartheta)}}. \tag{15}$$

If this condition is met, (K^{**}, K_s^{**}) is locally saddle-path stable.

Remark 2 *Straightforward calculations show that $K^{**} < K^{*}$.*

Remark 2 emphasises the fact that, when both fixed points are present, private capital is lower in the fixed point in which social capital is higher.

Remark 3 *For given values of the other parameters, condition (15) holds if δ and b are low enough and r, α and a are high enough.*

Remark 3 tells us that the fixed point at which social capital is positive exists when:

δ **is low:** social capital does not depreciate too fast (an intuitive condition);

r **is high:** individuals are not too patient: while impatience clearly reduces private accumulation, it may foster social capital accumulation, to the extent that this reflects the external effects of relational consumption;

α **is high:** the relational good is sufficiently a private and not too much of a public good, i.e., its enjoyment is sufficiently sensitive to one's own contribution;

a **is high:** enough weight is attributed to the needs satisfied by either the relational good or its private substitute (again, an intuitive condition);

b **is low:** the balance between the relational good and its private substitute as a means of satisfying preferences for non-subsistence goods is not excessively in favour of the private substitute[29].

[29] To have a numerical intuition, let us parameterise the model in a simple way, so that $a = b = 1$, $\alpha = \epsilon = 0.5$, $\vartheta = 0.1$, $\gamma = 0.8$. In this case, if social capital depreciation rate δ is, for instance, 10%, then condition (15) is met even at a discount rate r of 1%. If we lower γ to 0.5, then, with the same $\delta = 10\%$, condition (15) fails to be met up to a discount rate r of 8%, whereas it is met for $r \geq 9\%$.

It is interesting to speculate on the meaning of such parameters in terms of real world examples. One might argue, for instance, that a high degree of individual mobility is associated with a high δ, the social capital depreciation rate[30]. It is true that mobility gives rise to many 'weak' ties, which are indeed a form of social capital[31]; but they are also a form which depreciates quickly. More generally, individual mobility may make many forms of previously accumulated social capital unproductive (in relational terms). From this point of view, we might speculate that a steady state with high social capital is more likely to exist in Europe than in the US, precisely because individual mobility is lower in the former than in the latter.

Another interesting discussion concerns parameter α, which captures the relative degree to which the relational good is a private rather than a public good[32]. We may associate a high and a low α, respectively, to more active forms of social participation, in which my return crucially depends on my contribution (say, organizing an event and enjoying its success), and to more passive ones, in which my benefit mostly depends on other people's contribution (say, attending the same event as a member of the audience). Another interpretation is that a high α reflects an open context, where integration is easy and my benefits from participation (suppose I am a newcomer or an immigrant) depend to a high degree upon my own choice, whereas a low α reflects closed contexts, where integration is difficult and I may be excluded anyway, despite my efforts to participate.

Therefore, while individual mobility (both geographical and social) may increase the depreciation rate of social capital, it may also render relational goods more private and less public. This second effect would then probably favour the US over Europe as regards the existence of a steady-state with high social capital. However, as we show in the next section, the crucial point is not just whether this steady state exists (e.g., to follow our illustrative speculation, that it exists both in the US and in Europe), but rather whether

[30]Schiff (1999, 2002) analyses the clear-cut difference between the two traditional forms of factor mobility, namely migration and trade, which becomes apparent once we consider their different impact on social capital.

[31]Granovetter (1973) makes the point that weak ties may be economically very important, since they are often the vehicle of new information.

[32]Note that B is a pure public good if $\alpha = 0$, in which case any private incentive to social participation is absent. On the other hand, B is a pure private good if $\beta = 0$, that is, under Assumption 5, if $\alpha = 1$. In general terms, relational goods are an intermediate case between private and public goods.

it is more or less desirable than the other one.

Analysis of well-being

Let us now consider, when both fixed points exist, i.e., in condition (15), which one is Pareto-superior. Let u^* and u^{**} be the representative individual's utility in fixed points (K^*, K_s^*) and (K^{**}, K_s^{**}), respectively.

Proposition 6 *Assume that condition (15) is satisfied. Then fixed point (K^{**}, K_s^{**}) Pareto-dominates (K^*, K_s^*), i.e., $u^{**} > u^*$, if, ceteris paribus, δ is low enough and r and γ are high enough. The reverse is true if δ is high enough and r and γ are low enough.*

Proposition 6 tells us that the same two forces, impatience and low social capital depreciation rate, that let (K^{**}, K_s^{**}) be a fixed point, also make it Pareto-superior. Moreover, as it is natural to expect, high elasticity γ of the relational good to social capital contributes to the comparative efficiency of the fixed point with positive social capital[33].

When fixed point (K^{**}, K_s^{**}) Pareto-dominates (K^*, K_s^*) and the economy becomes stuck in the latter, it may be described as a social poverty trap[34]. The convergence to such a trap may have two basic causes: it may be due to a low initial endowment of social relative to private capital (for instance, Russia)[35]; or to the general problem posed by externalities, the presence of which may lead to inefficient private choices. If the outcome is an over-accumulation of private capital, at the expense of social capital and individual and social well-being, we may say that private growth and social development come into conflict with each other, and that it would

[33]Let us consider again the simple parameterisation $a = b = 1$, $\alpha = \epsilon = 0.5$, $\vartheta = 0.1$, $\gamma = 0.8$. In this case, $u^{**} - u^* = \frac{3}{2}\ln r - 4\ln\delta - 4\ln 2$, which, for instance, is positive for $\delta = 10\%$ and $r = 3\%$, as well as for any lower social capital depreciation rate and higher discount rate. If $\delta = 5\%$, then $u^{**} > u^*$, even at a discount rate of 1%. If we lower γ to 0.5, then $u^* > u^{**}$ for any reasonable value of δ and r.

[34]The use of an infinitely lived agent model may lead to underestimating the consequences of social impoverishment. If, as stressed by Coleman (1988, 1990), social capital is relevant for children's identity formation and for the creation of human capital, the consequences are likely to be more serious than pointed out here.

[35]Rose (1998) considers in detail how the centralisation of the Soviet Union may have eroded ample forms of social capital, inducing individuals to rely on a narrow circle of family ties, which represents at the same time a response to the state of affairs and a social trap which inhibits the mechanism of social development.

be efficient to increase social participation and decrease labour supply, sacrificing some accumulation of private capital, but gaining in terms of an improved social environment. Of course, this remains true only if fixed point (K^{**}, K_s^{**}) Pareto-dominates (K^*, K_s^*) and the economy becomes stuck in the latter; since the former fixed point is also locally stable, the economy will converge to it if its initial endowment of social capital is high enough[36]. When convergence to fixed point (K^{**}, K_s^{**}) takes place from below along both dimensions, social development and economic growth move together[37].

Instead, we have seen that (K^*, K_s^*) may Pareto-dominate fixed point (K^{**}, K_s^{**}) if the 'social technology' is 'bad' (high δ and low γ), and if individuals are very patient (low r). Moreover, we have shown that, in the same conditions, (K^{**}, K_s^{**}) may even fail to be a fixed point. In the first case, (K^{**}, K_s^{**}) should be regarded as a situation in which individuals devote too much time to socially enjoyed leisure, while working and saving too little to reach a more efficient steady state. In the second case, since there is no alternative, there is no comparative discussion.

5 Conclusions

The present contribution sheds light on the interplay between the private and social components of well-being in a scenario in which both private and social capital are present, relational goods play a role, and their substitutability with some private goods is taken into account.

We first present a static model, in which social and private capital are constant at some exogenously given stock. This model displays two equilibria: a privately oriented one, in which labour time and private production are high and relational goods are substituted by private goods, and a socially oriented one, in which labour supply is low and social participation high, so that, besides private consumption, relational goods become a key determinant of well-being. If social capital is low relative to private capital, the privately oriented equilibrium tends to be Pareto-superior; if the reverse is true, the socially oriented equilibrium definitely becomes more efficient. Since equilibrium selection is a matter of coordination, it is possible for the

[36]More precisely, if initial endowment (K^0, K_s^0) is close enough to (K^{**}, K_s^{**}). Note that even this case, although more favourable, does not solve the problem of externalities.

[37]However, recall that, because of Assumption 4, neither private growth nor social development may be endogenously sustained forever.

economy to become stuck in the Pareto-inferior equilibrium.

The static model does not explain the determinants of the relative endowment of social and private capital. We confront this issue in the dynamic extension of the model, in which we assume that private capital is accumulated, as usual, through savings, while the accumulation of social capital is a by-product of the generation of relational goods. The most interesting case is when parameters are such that the dynamics of the system admit two fixed points: one in which there is only private capital, and another in which both forms of capital are present. We further discuss the conditions in which the latter steady-state Pareto-dominates the former and show that, in this case, both equilibria are saddle-path stable, so the system can converge to the Pareto-inferior one. In this case, we witness a conflict between economic growth and social development, since growth drives the economy into a social poverty trap. If, instead, the economy converges to the Pareto-superior fixed point, we may have economic growth and social development moving in the same direction. The distinction between these two cases once again depends upon the initial relative endowment of private and social capital, but also upon the technology of social interaction and the degree of impatience.

Some of the assumptions under which we derive our results deserve a short discussion. First, we assume, for simplicity, that neither social capital matters for the production of private goods, nor private capital for relational goods. An interesting future extension could include these cross influences[38]. Second, while we consider positive learning-by-doing externalities in private production, we do not allow them to be strong enough to generate endogenous growth. This is another possible extension of the model. Third, we assume that private consumption does not require time, so that all leisure time is devoted to social participation. Although unrealistic, we make this modelling choice because, generally speaking, social participation is a more time-intensive activity than private consumption[39]. Fourth, the assumption that the 'gross investment' in social capital is exactly equal to the average production of the relational good could easily be generalised (for instance, by assuming that only a fraction of the relational good produced accumulates as social capital), without changing any of the results of the model: it has simply been dictated by notational economizing. Lastly, perfect substitutability

[38]See, along similar lines, Bartolini and Bonatti (2004).

[39]Clearly, the consumption of some private substitutes of the relational good (e.g., watching television) is also time-intensive, so that an interesting extension would be to take this into account, along the lines set by Corneo (2001).

and Assumptions 1 and 5 are crucial to obtain simple analytical solutions[40]. Relaxing Assumption 1 to some extent would not alter the results of the static model, although it would preclude the possibility of expressing them in closed form[41]. As regards Assumption 5, a comparison with Antoci, Sacco and Vanin (2002) allows us to conjecture that its main effect is to rule out a repulsive fixed point that separates the two stable ones. Since our mathematical findings are supported by a clear economic intuition, we are quite confident in their general validity.

The model and its results may aid better understanding of a number of concrete situations. For instance, it is widely recognised that, besides more traditional economic fundamentals, social factors played important roles in the crises of both Russia and Argentina. In cases, past political history was responsible for widespread disruption of the essential structures of civil society. In terms of our model, this amounts to a sharp reduction in the stock of social capital. Indeed, both countries faced a very low ratio of social to private capital, a situation to which individuals reacted by shifting to privately oriented strategies, thus worsening the problem. While the present version of our model allows us clearly to understand these dynamics and their socially disruptive consequences, the above extension to include the effects of social capital on private productivity may help in explaining the relatively low success of the private sectors of these economies.

Another quite different situation which may be explained by our model is the case of successful professionals, who devote much of their time to working, earning high incomes, and consuming great quantities of luxury goods, but who have poor social lives and are overall dissatisfied. Casual observation tells us that this is quite a common case in advanced societies. In terms of our model, this is precisely what one would observe along the convergence path towards a social poverty trap. Moreover, when this situation is widespread, individual reactions tend to be to invest even more time in private activities, thus exacerbating the problem. Again, pursuing this application rigorously would probably require an extension of the model towards a non-homogeneous society or an asymmetric equilibrium. Our hope is that

[40] Recall that Assumption 1, made for the static model, implies the equality in equilibrium of the degree of concavity of the relational good and private production as functions, respectively, of social participation and labour; Assumption 5, made for the dynamic model, implies a linear specification for both functions in equilibrium.

[41] More precisely, this would be the case if one just assumed $\alpha + \beta < 1$ and $\epsilon + \sigma < 1$ without requiring them to be equal.

our contribution may serve as a starting point for future research.

Appendix

Proof of Proposition 1

Using the production function and the budget constraint to substitute for C_s, and calling $v(s,C) = u(C, B, Y - C)$, we can re-write problem (1)-(2) as:

$$\max_{s,C} \; v(s,C) = \tag{16}$$
$$= \ln C + a \ln\{s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + b[(1-s)^{\epsilon}(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon} - C]\} \quad \text{s.t.}$$
$$C \geq 0, \qquad (1-s)^{\epsilon}(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon} - C \geq 0, \qquad s \in [0,1]. \tag{17}$$

If $0 < C < Y$, the FOCs of this problem are:

$$\frac{\partial v}{\partial C} = 0, \tag{18}$$
$$\frac{\partial v}{\partial s} \leq 0, \qquad s\frac{\partial v}{\partial s} = 0, \qquad 0 \leq s \leq 1. \tag{19}$$

Equation (18) immediately yields:

$$C = \frac{1}{b(1+a)}[s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + b(1-s)^{\epsilon}(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon}], \tag{20}$$

which, substituted into inequality (19), after rearranging, yields:

$$\frac{(1-s)^{1-\epsilon}}{s^{1-\alpha}} \leq \frac{b\epsilon(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon}}{\alpha\bar{s}^{\beta}K_s^{\gamma}}, \text{ with equality if s} > 0, \quad 0 \leq s \leq 1. \tag{21}$$

When $\bar{s} = 0$, we have $B = 0$ whatever the individual choice of s. Hence, the optimal individual response to $\bar{s} = 0$ is to choose $s = 0$. The rest of the proposition follows from equation (20) and from the production function, which also shows that, for $s = \bar{s} = 0$, constraint $0 \leq C \leq Y$ is not binding.

Proof of Proposition 2

The value of $\hat{s}$ in case (a) follows from equation (21) after applying the SNE condition $\bar{s} = s$ and Assumption 1. The values of $\hat{C}$ and of $\hat{C}_s$ then follow from equation (20) and from the budget constraint. The definition of function h, and therefore the distinction between case (a) and case (b), follows from equation (21) and the production function, setting $C = Y$. When this constraint is binding, i.e., in case (b), the representative individual sets $\frac{\partial v}{\partial s} = -\frac{\partial v}{\partial C}\frac{\partial Y}{\partial s}$. The values of $\hat{s}$, $\hat{C}$, $\hat{C}_s$ in case (b) follow from this equation, conditions $C = Y$, $s = \hat{s}$, and the budget constraint.

Proof of Proposition 3

Let $\tilde{u}$ and $\hat{u}$ be the representative individual's utility in the two SNE $(\tilde{s}, \tilde{C}, \tilde{C}_s)$ and $(\hat{s}, \hat{C}, \hat{C}_s)$, respectively.

Consider first case (a). Using condition $K_s < h(K)$, it is easy to show that $\hat{u} - \tilde{u} > (1+a)\varphi \ln\left(\frac{\alpha}{\alpha + ab^2\epsilon}\right) + (1-a)\ln b$, and that this term is $\simeq 0.41$ for $a = \frac{1}{9}$, $b = 3$, $\alpha = \epsilon = 0.5$, $\beta = \sigma = 0.3$; therefore, for such parameters $\hat{u} > \tilde{u}$. Analogously, we show that $\hat{u} - \tilde{u} < (1+a)\ln\left[\left(\frac{\epsilon}{\alpha + ab^2\epsilon}\right)^{\varphi} a^{1-\varphi} + 1\right] + (1-a)\ln b$, where this term is $\simeq -0.21$ for $a = 0.5$, $b = 0.1$, $\alpha = \epsilon = 0.5$ and $\beta = \sigma = 0.3$; therefore, for such parameters, $\tilde{u} > \hat{u}$.

Consider now case (b). The definition of function g comes from a straightforward substitution of the equilibrium values in the utility function. Note that g is strictly increasing. A comparison between functions h and g shows that a sufficient condition for $\hat{u} > \tilde{u}$ to hold is $\frac{b\epsilon}{\alpha} > \frac{(a\alpha+\epsilon)^{\frac{1+a}{a}}}{(1+a)^{\frac{1+a}{a\varphi}}}$.

Proof of Proposition 4

The current Hamiltonian value for problem (5) under constraints (6)-(7) is

$$\begin{aligned} H &= \ln C + a\ln(s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + bC_s) + \lambda[(1-s)^{\epsilon}K^{1-\epsilon}A - C - C_s] + \\ &\quad +\mu[\bar{s}^{\alpha+\beta}K_s^{\gamma} - \delta K_s]. \end{aligned} \tag{22}$$

By the maximum principle, we have:

$$\dot{K} = \frac{\partial H}{\partial \lambda} = (1-s)^{\epsilon}K^{1-\epsilon}A - C - C_s, \tag{23}$$

$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial K} = \lambda[r - (1-\epsilon)(1-s)^{\epsilon}K^{-\epsilon}A], \tag{24}$$

$$\dot{K}_s = \frac{\partial H}{\partial \mu} = \bar{s}^{\alpha+\beta}K_s^{\gamma} - \delta K_s. \tag{25}$$

We omit the dynamics of μ, the 'shadow price' of social capital, since equations (23) to (25) are independent of it, due to the fact that the representative individual considers both $\bar{s}$ and K_s (and therefore $\dot{K}_s$) as exogenous.

The first-order conditions are:

$$\begin{aligned}
\frac{\partial H}{\partial C} &= \frac{1}{C} - \lambda = 0, \qquad C > 0, &(26)\\
\frac{\partial H}{\partial C_s} &= \frac{ab}{s^\alpha \bar{s}^\beta K_s^\gamma + bC_s} - \lambda \leq 0, \qquad C_s \frac{\partial H}{\partial C_s} = 0, \qquad C_s \geq 0, &(27)\\
\frac{\partial H}{\partial s} &= \frac{a\alpha s^{\alpha-1} \bar{s}^\beta K_s^\gamma}{s^\alpha \bar{s}^\beta K_s^\gamma + bC_s} - \epsilon\lambda(1-s)^{\epsilon-1} K^{1-\epsilon} A \leq 0, &(28)\\
& s\frac{\partial H}{\partial s} = 0, \qquad s \in [0,1].
\end{aligned}$$

Note that s and C_s cannot both be set at zero. Thus, either condition (27) or condition (28) must hold with equality.

The transversality condition for private capital is:

$$\lim_{t\to\infty} \mathrm{e}^{-rt}\lambda(t)K(t) = 0. \tag{29}$$

Substituting Assumption 5 and equilibrium conditions $\bar{s} = s$ and $\bar{K} = K$ into equations (26) to (28), we obtain:

$$\begin{aligned}
C &= \frac{1}{\lambda}, &(30)\\
\frac{\partial H}{\partial C_s} &= \frac{ab}{sK_s^\gamma + bC_s} - \lambda \leq 0, \qquad C_s \frac{\partial H}{\partial C_s} = 0, \qquad C_s \geq 0, &(31)\\
\frac{\partial H}{\partial s} &= \frac{a\alpha K_s^\gamma}{sK_s^\gamma + bC_s} - \epsilon\lambda K^{1+\vartheta-\epsilon} \leq 0, \qquad s\frac{\partial H}{\partial s} = 0, \qquad s \in [0,1]. &(32)
\end{aligned}$$

Inequality $\frac{\partial H}{\partial C_s} \leq 0$ may be re-written in the form $\frac{a}{sK_s^\gamma + bC_s} - \frac{\lambda}{b} \leq 0$.

For $K_s > 0$, inequality $\frac{\partial H}{\partial s} \leq 0$ may be re-written in the form $\frac{a}{sK_s^\gamma + bC_s} - \frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^\gamma}\lambda \leq 0$.

Hence, if $\frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^\gamma} > \frac{1}{b}$, $\frac{\partial H}{\partial C_s} = 0$ and $\frac{\partial H}{\partial s} < 0$ hold, so that the representative individual's equilibrium choice is such that $C_s > 0$ and $s = 0$. If, on the contrary, $\frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^\gamma} < \frac{1}{b}$, then we have $C_s = 0$ and $s > 0$. If, lastly, $\frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^\gamma} = \frac{1}{b}$, we remain with one equation for two unknowns, and the choice of C_s and s is indeterminate. The remainder of Proposition 4 follows from a straightforward substitution in equations (31) and (32).

Proof of Proposition 5

For case (a), i.e., in condition (9), the equilibrium dynamics of our economy are described by:

$$\begin{aligned} \dot{K} &= K^{1+\vartheta-\epsilon} - \frac{1+a}{\lambda}, && (33)\\ \dot{\lambda} &= \lambda[r - (1-\epsilon)K^{\vartheta-\epsilon}], && (34)\\ \dot{K}_s &= -\delta K_s. && (35) \end{aligned}$$

For case (b), i.e., in condition (10), if $\frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} \leq 1$,[42] the equilibrium dynamics are:

$$\begin{aligned} \dot{K} &= K^{1+\vartheta-\epsilon} - \left(1+\frac{a\alpha}{\epsilon}\right)\frac{1}{\lambda}, && (36)\\ \dot{\lambda} &= \lambda\left[r - (1-\epsilon)\left(K^{\vartheta-\epsilon} - \frac{a\alpha}{\epsilon\lambda K}\right)\right], && (37)\\ \dot{K}_s &= K_s^{\gamma}\left(\frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} - \delta K_s^{1-\gamma}\right). && (38) \end{aligned}$$

The analytical determination of (K^*, K_s^*) and (K^{**}, K_s^{**}) follows from a straightforward substitution in the systems (33) to (35) and (36) to (38), setting the LHS of each equation at zero. (K^*, K_s^*) satisfies the condition of case (a): $K_s^* < \left(\frac{\epsilon b}{\alpha} K^{*1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}}$, and is thus indeed a fixed point. (K^{**}, K_s^{**}) is a fixed point if and only if it satisfies the condition of case (b): $K_s^{**} > \left(\frac{\epsilon b}{\alpha} K^{**1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}}$. Equation (15) is simply a re-writing of this condition.

The stability properties are determined as follows. The Jacobian matrix of the system (33) to (35), evaluated at (K^*, K_s^*), is:

$$A = \begin{bmatrix} (1+\vartheta-\epsilon)K^{\vartheta-\epsilon} & \frac{1+a}{\lambda^2} & 0 \\ (1-\epsilon)(\epsilon-\vartheta)\lambda K^{\vartheta-\epsilon-1} & 0 & 0 \\ 0 & 0 & -\delta \end{bmatrix}.$$

[42]Since we are interested in the fixed points of these dynamics, we do not consider, in case (b), the possibility that $\frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} > 1$, since in this case $\dot{K} = -\frac{1}{\lambda}$ and there is no fixed point. Note, moreover, that this possibility is not a relevant one, since it means that individuals do not work at all, and derive their private consumption only from 'eating' their existing stock of private capital.

One eigenvalue is therefore $-\delta < 0$, and the other two have opposite signs, since the determinant of the sub-matrix obtained from A by deleting the third row and the third column is negative. Therefore, if (K, K_s) is initially close enough to (K^*, K_s^*), there exists a single initial value of λ that puts the representative agent on the stable arm (which, in turn, has dimension 2).

Observe now that the Jacobian matrix of the system (36) to (38), evaluated at (K^{**}, K_s^{**}), is such that $\frac{\partial \dot{K}}{\partial K_s} = \frac{\partial \dot{\lambda}}{\partial K_s} = 0$ and $\frac{\partial \dot{K}_s}{\partial K_s} = -\delta(1-\gamma) < 0$. Therefore, the latter value is one of the eigenvalues of the Jacobian matrix and the other two have opposite signs, since the determinant of the sub-matrix is negative:

$$B = \begin{bmatrix} \frac{\partial \dot{K}}{\partial K} & \frac{\partial \dot{K}}{\partial \lambda} \\ \frac{\partial \dot{\lambda}}{\partial K} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{bmatrix}.$$

To see this, observe that $\frac{\partial \dot{K}}{\partial K} = (1+\vartheta-\epsilon)K^{\vartheta-\epsilon} > 0$, $\frac{\partial \dot{K}}{\partial \lambda} = \left(1 + \frac{a\alpha}{\epsilon}\right)\frac{1}{\lambda^2} > 0$, $\frac{\partial \dot{\lambda}}{\partial K} = -(1-\epsilon)\lambda\left[-(\epsilon-\vartheta)K^{\vartheta-\epsilon-1} + \frac{a\alpha}{\epsilon\lambda K^2}\right]$, $\frac{\partial \dot{\lambda}}{\partial \lambda} = -\frac{a\alpha(1-\epsilon)}{\epsilon K\lambda} < 0$.

It is then easy to obtain $\operatorname{Det} B = -\frac{(1-\epsilon)(\epsilon-\vartheta)}{\lambda^2 K^2}\left(1 + \frac{a\alpha}{\epsilon}\right) < 0$.

Proof of Proposition 6

In order to calculate u^*, observe first that, since we are in case (a), $s = 0$ and $u^* = \ln C + a \ln bC_s$. From equations (30) and (31), it follows immediately that $C = \frac{1}{\lambda}$ and $C_s = \frac{a}{\lambda}$, so that $C_s = aC$. Equations (33) and (11) then imply $C = \frac{1}{1+a}K^{*1+\vartheta-\epsilon} = \frac{1}{1+a}\left(\frac{1-\epsilon}{r}\right)^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}$ and $C_s = \frac{a}{1+a}\left(\frac{1-\epsilon}{r}\right)^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}$. Therefore, $u^* = \ln\frac{1}{1+a} + a\ln\frac{ab}{1+a} + (1+a)\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}\ln\frac{1-\epsilon}{r}$ is easily yielded.

Let us now calculate u^{**} in an analogous way. Since we are in case (b), $C_s = 0$ and $u^{**} = \ln C + a\ln sK_s^{\gamma}$. Remember that in the fixed point $\lambda K^{1+\vartheta-\epsilon} = 1 + \frac{a\alpha}{\epsilon}$, equations (13) and (30) yield $C = \frac{1}{\lambda} = \frac{K^{**1+\vartheta-\epsilon}}{1+\frac{a\alpha}{\epsilon}} = \frac{\left[\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)}\right]^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}}{1+\frac{a\alpha}{\epsilon}}$ and equation (32) yields $s = \frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} = \frac{a\alpha}{\epsilon+a\alpha}$. Since K_s is given by equation (14), we obtain $u^{**} = \ln\frac{\epsilon}{\epsilon+a\alpha} + \frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}\ln\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)} + a\ln\frac{a\alpha}{\epsilon+a\alpha} + $ $+a\frac{\gamma}{1-\gamma}\ln\frac{a\alpha}{\delta(\epsilon+a\alpha)}$.

Proposition 6 follows from an analysis of the following expression[43]:

[43] Note that the term in the last square brackets is negative if $\delta < \frac{a\alpha}{\epsilon+a\alpha}$, that $\frac{\gamma}{1-\gamma}$ increases rapidly with γ, and that the absolute value of $\ln r$ is a decreasing function of r.

$$
\begin{aligned}
u^* - u^{**} &= \ln\frac{\epsilon + a\alpha}{\epsilon + a\epsilon} + \frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}\left[a\ln(1-\epsilon) - a\ln r + \ln\frac{\epsilon + a\alpha}{\epsilon}\right] + \\
&\quad + a\ln\frac{\epsilon + a\alpha}{\alpha + a\alpha} + a\ln b + a\frac{\gamma}{1-\gamma}\left[\ln\delta + \ln\frac{\epsilon + a\alpha}{a\alpha}\right]. \qquad (39)
\end{aligned}
$$

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