Econophysics: From Game Theory and Information Theory to Quantum Mechanics

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Abstract

Rationality is the universal invariant among human behavior, universe physical laws and ordered and complex biological systems. Econophysics is both the use of physical concepts in Finance and Economics, and the use of Information Economics in Physics. In special, we will show that it is possible to obtain the Quantum Mechanics principles using Information and Economic Theory.

Key words: Quantum Games, Rationality, Physical Optimal Laws.

1 Introduction

At the moment, Information Theory is studied or utilized by multiple perspectives (Economics, Game Theory, Physics, Mathematics, and Computer Science). Economics and Game Theory are interested in the use of information and order state, in order to maximize the utility functions of the rational and intelligent players, which are part of an interest conflict. The players gather and process information. The information can be perfect and complete see ([29], [9], [37] and [36]). On the other hand, Mathematics, Physics and Computer Science are all interested in information representation, entropy (disorder measurement), optimality of physical laws and in the living beings' internal order see ([1], [2], [4], [6], and [7]). Finally, information is stored, transmitted and processed by physical means. Thus, the concept of information and computation can be formulated not only in the context of Economics, Game Theory and Mathematics, but also in the context of physical theory. Therefore, the study of information ultimately requires experimentation and some multidisciplinary approaches such as the introduction of the Optimality Concept ([3], [2], [10], [14], [17], [18] and [22]).

The Optimality Concept is the essence of the economic and natural sciences ([19], [20], [30], [31], and [33]). Economics introduces the optimality concept (maximum utility and minimum risk) as equivalent of rationality and Physics understands action minimum principle, and maximum entropy (maximum information) as the explanation of nature laws ([27] and [28]). If the two sciences have a common backbone, then they should allow certain analogies and to share other elements such us: equilibrium conditions, evolution, uncertainty measurement and the entropy concept. In this paper, the contributions of Physics (Quantum Information Theory) and Mathematics (Classical Information Theory) are used in Game Theory and Economics being able to explain mixed strategy Nash's equilibrium using Shannon's entropy ([16], [17], and [18]).

In Quantum Information Theory, the correlated equilibria in two-player games means that the associated probabilities of each-player strategies are functions of a correlation matrix. Entanglement, according to the Austrian physicist Erwin Shrödinger, which is the essence of Quantum Mechanics, has been known for long time now to be the source of a number of paradoxical and counterintuitive phenomena. Of those, the most remarkable one is the usually called non-locality which is at the heart of the Einstein-Podolsky-Rosen paradox (ERP) see ([3], [4], [5], [22], and [32]), which consider a quantum system consisting of two particles separated long distance.

"ERP suggests that measurement on particle 1 cannot have any actual influence on particle 2 (locality condition); thus the property of particle 2 must be independent of the measurement performed on particle 1."

The experiments verified that two particles in the ERP case are always part of one quantum system and thus measurement on one particle changes the possible predictions that can be made for the whole system and therefore for the other particle ([4]).

This paper is organized as follows. The fist section is a revision of the existent bibliography. Second section is the core of this paper; here we present some Quantum Mechanics principles as a consequence of maximum entropy and minimum action principle (Rationality in Physics). In third section we can see the conclusions of this research. Finally, section fourth is the acknowledment.

2 Model

Elements of Quantum Game Theory

Let $\Gamma = (K, S, v)$ be a game to $n-$players, with K the set of players $k = 1, ..., n$. The finite set S_k of cardinality

$l_k \in N$ is the set of pure strategies of each player where $k \in K$, $s_{kj_k} \in S_k$, $j_k = 1, ..., l_k$ and $S = \Pi_K S_k$ set of pure strategy profiles with $s \in S$ an element of that set, $l = l_1, l_2, ..., l_n$ represent the cardinality of S, ([10], [11], [30], [31], [33] and [36]).

The vector function $\mathbf{v} : S \to R^n$ associates every profile $s \in S$ the vector of utilities $\mathbf{v}(s) = (v^1(s), ..., v^n(s))^T$, where $v^k(s)$ designates the utility of the player k facing the profile s. In order to get facility of calculus we write the function $v^k(s)$ in one explicit way $v^k(s) = v^k(j_1, j_2, ..., j_n)$. The matrix $\mathbf{v}_{n,l}$ represents all points of the Cartesian product $\Pi_{k\in K} S_k$. The vector $\mathbf{v}^k(\mathbf{s})$ is the $k-$ column of $\mathbf{v}$.

If the mixed strategies are allowed, then we have:

$$\Delta(S_k) = \left\{ \mathbf{p}^k \in R^{l_k} : \sum_{j_k=1}^{l_k} p^k_{j_k} = 1 \right\}$$

the unit simplex of the mixed strategies of player $k \in K$, and $\mathbf{p}^k = (p^k_{j_k})$ is the probability vector. The set of profiles in mixed strategies is the polyhedron Δ with $\Delta = \Pi_{k\in K}\ \Delta(S_k)$, where $\mathbf{p} = (p^1_{j_1}, p^2_{j_2} ..., p^n_{j_n})$, and $\mathbf{p}^k = (p^k_1, p^k_2, ..., p^k_{l_n})^T$. Using the Kronecker product $\otimes$ it is possible to write:

$$\mathbf{p} = \mathbf{p}^1 \otimes \mathbf{p}^2 \otimes ... \otimes \mathbf{p}^{k-1} \otimes \mathbf{p}^k \otimes \mathbf{p}^{k+1} \otimes ... \otimes \mathbf{p}^n$$

$$\mathbf{p}^{(-k)} = \mathbf{p}^1 \otimes \mathbf{p}^2 \otimes ... \otimes \mathbf{p}^{k-1} \otimes \mathbf{l}^k \otimes \mathbf{p}^{k+1} \otimes ... \otimes \mathbf{p}^n$$

$$\begin{aligned} \mathbf{l}^k &= (1, 1, ..., 1)^T, \quad \left[\mathbf{l}^k\right]_{l_k,1} \\ \mathbf{o}^k &= (0, 0, ..., 0)^T, \quad \left[\mathbf{o}^k\right]_{l_k,1} \end{aligned}$$

The $n-$ dimensional function $\overline{\mathbf{u}} : \Delta \to R^n$ associates with every profile in mixed strategies the vector of expected utilities $\overline{\mathbf{u}}(\mathbf{p}) = \left(\overline{u^1}\left(\mathbf{p}, \mathbf{v}(s)\right), ..., \overline{u^n}\left(\mathbf{p}, \mathbf{v}(s)\right)\right)^T$, where $\overline{u^k}(\mathbf{p}, \mathbf{v}(s))$ is the expected utility for each player k. Every $\overline{u^k_{j_k}} = \overline{u^k_{j_k}}(\mathbf{p}^{(-k)}, \mathbf{v}(s))$ represents the expected utility for each player's strategy and the vector $\mathbf{u}^k$ is noted $\mathbf{u}^k = (\overline{u^k_1}, \overline{u^k_2}, ..., \overline{u^k_n})^T$.

$$\overline{u_k} = \sum_{j_k=1}^{l_k} \overline{u^k_{j_k}}(\mathbf{p}^{(-k)}, v(s)) p^k_{j_k}$$

$$\overline{\mathbf{u}} = \mathbf{v}'\mathbf{p}$$

$$\mathbf{u}^k = \left(\mathbf{l}^k \otimes \mathbf{v}^k\right)\mathbf{p}^{(-k)}$$

The triplet $(K, \Delta, \overline{\mathbf{u}}(\mathbf{p}))$ designates the extension of the game Γ with the mixed strategies. We get Nash's equilibrium (the maximization of utility) if and only if, $\forall k$, $\mathbf{p}$, the inequality $\overline{u^k}(\mathbf{p}^*) \geq \overline{u^k}((\mathbf{p}^k)^*, \mathbf{p}^{(-k)})$ is respected .

Another way to calculate the Nash's equilibrium, ([33], [30]), is equaling the values of the expected utilities of each strategy when it is possible.

$$\overline{u^k_1}\left(\mathbf{p}^{(-k)}, v(s)\right) = ... = \overline{u^k_{j_k}}\left(\mathbf{p}^{(-k)}, v(s)\right)$$

$$\sum_{j_k=1}^{l_k} p^k_{j_k} = 1 \qquad \forall k = 1, ..., n$$

$$\sigma^2_k = \sum_{j_k=1}^{l_k} \left(\overline{u^k_{j_k}}\left(\mathbf{p}^{(-k)}, v(s)\right) - \overline{u^k}\right)^2 p^k_{j_k} = 0$$

If the resulting system of equations doesn't have solution $\left(\mathbf{p}^{(-k)}\right)^*$ then we propose the Minimum Entropy Theorem. This method is expressed as $Min_{\mathbf{p}}\left(\sum_k H_k(\mathbf{p})\right)$, where $\sigma^2_k(\mathbf{p}^*)$ standard deviation and $H_k(\mathbf{p}^*)$ entropy of each player k.

$$\sigma^2_k(\mathbf{p}^*) \leq \sigma^2_k\left(\left(\mathbf{p}^k\right)^*, \mathbf{p}^{(-k)}\right) \text{ or}$$

$$H_k(\mathbf{p}^*) \leq H_k\left(\left(\mathbf{p}^k\right)^*, \mathbf{p}^{(-k)}\right)$$

Minimum Entropy Theorem. The game entropy is minimum only in mixed strategy Nash's equilibrium. The entropy minimization program $Min_{\mathbf{p}}\left(\sum_k H_k(\mathbf{p})\right)$, is equal to standard deviation minimization program $Min_{\mathbf{p}}\left(\Pi_k \sigma_k(\mathbf{p})\right)$, when $\left(\overline{u^k_{j_k}}\right)$ has Gaussian density function or multinomial logit see proof in [18].

Theorem 1. Gaussian Density Function permits both maximize I_m and minimize $\overline{\Delta u^2_i}$.

Proof Let it be the next maximization program:

$$\max_{P_i} I_m = \max_{P_i}\left(-\sum_i P_i \ln P_i\right), \text{ subject to}$$

$$1 = \sum_i P_i, \text{ and}$$

$$\overline{\Delta u^2_i} = \sum_i \Delta u^2_i P_i$$

evaluating first derivatives

$$\begin{aligned} \delta I_m &= -\sum_i (\ln P_i + 1)\delta P_i = 0 \\ \alpha &\sum_i \delta P_i = 0 \\ \beta &\sum_i \Delta u^2_i \delta P_i = 0 \end{aligned} \tag{1}$$

which brings us to

$$\delta I_m = -\sum_i (\ln P_i + 1 + \alpha + \beta \Delta u^2_i)\delta P_i = 0 \tag{2}$$

and

$$P_i = A e^{-\beta \Delta u^2_i} \tag{3}$$

In order to determine the integration constants A and β, let's consider the following continuos distribution:

$$\int_{-\infty}^{\infty} Ae^{-\beta\Delta u^2} d\Delta u = 1$$
$$\int_{-\infty}^{\infty} \Delta u^2 Ae^{-\beta\Delta u^2} d\Delta u = \sigma_u^2 \tag{4}$$

Integrating we found:

$$P = \frac{1}{\sqrt{2\pi}\sigma_u} e^{-\frac{\Delta u^2}{2\sigma_u^2}} \tag{5}$$

Equation (5) is a Gaussian density function which also follows Minimum Dispersion Theorem see . ■

From equation (5), considering all given arguments and the fact that action is not a directly measurable quantity, we postulate:

$$P(S) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\Delta S^2}{2\sigma^2}} \tag{6}$$

which contains all the information of the system.

Theorem 2. Plank's constant has a statistical nature. Furthermore, $\sigma = \hbar$, $\Delta p \Delta x \geq \frac{\hbar}{2}$.

Proof. The action distribution function, which is local, due to equation (6), is

$$e^{-\gamma\Delta S^2} = e^{-\alpha\Delta x^2} e^{-\beta\Delta p^2} \tag{7}$$

Using the conditions, α, β, and γ constants.

$$\sigma^2 = \frac{\int_{-\infty}^{\infty} \Delta S^2 e^{-\gamma\Delta S^2} d\Delta S}{\int_{-\infty}^{\infty} e^{-\gamma\Delta S^2} d\Delta S} = \frac{\int_{-\infty}^{\infty} 4\Delta p^2 \Delta x^2 e^{-\alpha\Delta x^2} d\Delta x e^{-\beta\Delta p^2} d\Delta p}{\int_{-\infty}^{\infty} e^{-\alpha\Delta x^2} d\Delta x e^{-\beta\Delta p^2} d\Delta p} \tag{8}$$

$$\sigma^2 = 4 \frac{\int_{-\infty}^{\infty} \Delta x^2 e^{-\alpha\Delta x^2} d\Delta x}{\int_{-\infty}^{\infty} e^{-\alpha\Delta x^2} d\Delta x} \frac{\int_{-\infty}^{\infty} \Delta p^2 e^{-\beta\Delta p^2} d\Delta p}{\int_{-\infty}^{\infty} e^{-\beta\Delta p^2} d\Delta p} \tag{9}$$

$$\sigma^2 = 4\sigma_x^2 \sigma_p^2$$

$$\Rightarrow \sigma_x \sigma_p = \frac{\sigma}{2} \tag{10}$$

we obtain the minimum Heisenberg's uncertainty relation.

$$\Rightarrow \sigma = \hbar \tag{11}$$

$\sigma_x \sigma_p$ represents the minimum area in phase space, more generally speaking, any other area will satisfies

$$\Delta p \Delta x \geq \frac{\hbar}{2} \tag{12}$$

We can conclude this because $\delta\overline{\Delta x^2} = 0$ and $\overline{\Delta x^2} \geq 0$ are true only for a minimum. The same arguments are valid for $\overline{\Delta p^2}$. ■

Theorem 3 Optimal wave superposition (equation ( )), which satisfies Gaussian density function (equation (5)), permit us to obtain Plank´s and De Broglie´s equations.

Let d_c be the derivative on space coordinates.

$$\left(-d_c \frac{\Delta S^2}{2\sigma^2}\right)\left(e^{-\frac{\Delta S^2}{2\sigma^2}}\right) = \int A(p, E, k, \omega) d_c f dk d\omega$$

$$-d_c \frac{\Delta S^2}{2\sigma^2} \int A(p, E, k, \omega) f dk d\omega = \int A(p, E, k, \omega) d_c f dk d\omega$$

From here

$$\int A(p, E, k, \omega) \left[f d_c \frac{\Delta S^2}{2\sigma^2} + d_c f \right] dk d\omega = 0$$

this implies the differential equation

$$f d_c \frac{\Delta S^2}{2\sigma^2} + d_c f = 0 \tag{13}$$

$$\frac{\Delta S^2}{2\sigma^2} + lnf = lnA$$

$$\Rightarrow f = Ae^{-\frac{\Delta S^2}{2\sigma^2}} \tag{14}$$

where f is an exponential function on x and t

$$f = Ae^{-\Phi(kx - \omega t)} \tag{15}$$

From where

$$\Phi(kx - \omega t) = \frac{\Delta S^2}{2\sigma^2} - lnA \tag{16}$$

Expanding $\Phi(kx - \omega t)$ in Taylor's series we have

$$\Phi(kx - \omega t) = \Phi(0) + \left.\frac{d\Phi}{du}\right|_0 (kx - \omega t) + \frac{1}{2} \left.\frac{d^2\Phi}{du^2}\right|_0 (kx - \omega t)^2 + \cdots = \frac{\Delta S^2}{2\sigma^2} - lnA \tag{17}$$

All terms are zero, except $\Phi(0)$

$$\Phi(0) = -lnA$$

$$\frac{a^2}{2}(kx - \omega t)^2 = \frac{(px - Et)^2}{2\sigma^2} \tag{18}$$

and we obtain

$$p = a\sigma k = \sigma k'$$
$$E = a\sigma\omega = \sigma\omega' \tag{19}$$

The integral equation is

$$e^{-\frac{\Delta S^2}{2\sigma^2}} = \frac{1}{a^2} \int A(p, E, k', \omega') e^{-\frac{(k'x - \omega' t)^2}{2}} dk' d\omega' \tag{20}$$

k' and ω' are dummy variables, then we can do $k' \to k$ and $\omega' \to \omega$

$$e^{-\frac{\Delta S^2}{2\sigma^2}} = \frac{1}{a^2} \int A(p, E, k, \omega) e^{-\frac{(kx - \omega t)^2}{2}} dk d\omega$$

therefore $a = 1$ and

$$p = \sigma k$$
$$E = \sigma\omega \tag{21}$$

which are De Broglie's and Plank's equations, respectively. Again we obtain that

$$\sigma = \hbar \tag{22}$$

■

The new integral equation is

$$e^{-\frac{(px-Et)^2}{2\sigma^2}} = \int A(p,E,k,\omega)e^{-\frac{(kx-\omega t)^2}{2}}dkd\omega$$

this implies that

$$A(p,E,k,\omega) = \delta\left(\frac{E}{\sigma}-\omega\right)\delta\left(\frac{p}{\sigma}-k\right) \tag{23}$$

and obviously

$$e^{-\frac{(px-Et)^2}{2\sigma^2}} = \int \delta\left(\frac{E}{\sigma}-\omega\right)\delta\left(\frac{p}{\sigma}-k\right)e^{-\frac{(kx-\omega t)^2}{2}}dkd\omega \tag{24}$$

Let's analyze every term in equation 24

$$\begin{aligned}\delta\left(\frac{E}{\sigma}-\omega\right) &= \frac{1}{2\pi\sigma}\int_{-\infty}^{\infty} e^{-i\frac{(E-\sigma\omega)}{\sigma}t}dt\\ \delta\left(\frac{p}{\sigma}-k\right) &= \frac{1}{2\pi\sigma}\int_{-\infty}^{\infty} e^{i\frac{(p-\sigma k)}{\sigma}x}dx\end{aligned} \tag{25}$$

from there

$$\frac{1}{(2\pi\sigma)^2}\int e^{i\frac{(px-Et)}{\sigma}}e^{-i\frac{(\sigma kx-\sigma\omega t)}{\sigma}}dxdt = A(p,E,k,\omega)) \tag{26}$$

Now we are able to define the wave functions

$$\begin{aligned}\Psi_{p,E} &= \frac{1}{2\pi\sigma}e^{i\frac{(px-Et)}{\sigma}}\\ \Psi_{p',E'} &= \frac{1}{2\pi\sigma}e^{-i\frac{(p'x-E't)}{\sigma}}\end{aligned} \tag{27}$$

Or considering only p (and the fact that $\sigma = \hbar$)

$$\Psi_p = \frac{1}{\sqrt{2\pi\hbar}}e^{i\frac{px}{\hbar}} \tag{28}$$

$$\int \Psi_p\Psi^*_{p'}dx = \delta(p-p') \tag{29}$$

which is the Dirac's condition of normalization. If we consider only E, we have

$$\Psi_E = \frac{1}{\sqrt{2\pi\hbar}}e^{-i\frac{Et}{\hbar}} \tag{30}$$

$$\int \Psi_E\Psi^*_{E'}dt = \delta(E-E') \tag{31}$$

which is the law of conservation of energy. From equation (28) we see that

$$-i\hbar\frac{\partial}{\partial x}\Psi_p = p\Psi_p \tag{32}$$

and from equation (30) we obtain

$$i\hbar\frac{\partial}{\partial t}\Psi_E = E\Psi_E \tag{33}$$

We see the way in which all concepts of Quantum Mechanics appear naturally.

In general, the wave function is

$$\Psi = Ae^{\frac{i\Delta S}{\hbar}} \tag{34}$$

Given that

$$\Delta S = \sum_i p_i q_i - Et \tag{35}$$

where p_i and q_i are physical conjugate observables.

$$-i\hbar\frac{\partial\Psi}{\partial q_i} = p_i\Psi \tag{36}$$

in this way, every physical (p_i) observable has associated a Hermitian operator ($\hat{p_i} = -i\hbar\frac{\partial}{\partial q_i}$), such that its mean values are the eigenvalues of that operator ([20], [21], [27], [34] and [28]).

Let's consider a photon

$$E = pc$$

$$\frac{E^2}{c^2} - p^2 = 0 \tag{37}$$

given that $E = -\frac{\partial S}{\partial t}$ and $p = \frac{\partial S}{\partial x}$ then

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left(\frac{\partial S}{\partial x}\right)^2 = 0 \tag{38}$$

This bring us to define the Lagrangian density

$$\mathcal{L} = \left(\frac{\partial S}{\partial x}\right)^2 - \frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 \tag{39}$$

From Euler - Lagrange equation

$$\frac{\partial}{\partial x}\left[\frac{\partial\mathcal{L}}{\left(\frac{\partial S}{\partial x}\right)}\right] + \frac{\partial}{\partial t}\left[\frac{\partial\mathcal{L}}{\left(\frac{\partial S}{\partial t}\right)}\right] = 0 \tag{40}$$

$$\frac{\partial^2 S}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 S}{\partial t^2} = 0 \tag{41}$$

where we obtain

$$S = S_0 e^{i(kx-\omega t)} \tag{42}$$

the energy

$$E = \frac{\partial S}{\partial t} = -i\omega S_0 e^{i(kx-\omega t)} \tag{43}$$

The square modulus

$$|E|^2 = \omega^2|S_0|^2 \tag{44}$$

The average is

$$\langle|E|^2\rangle = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty}\omega^2|S_0|^2 e^{-\frac{\Delta S_0^2}{2\sigma^2}}d\Delta S_0 = \omega^2\hbar^2 \tag{45}$$

i. e.

$$E_{fot\acute{o}n} = \sqrt{\langle|E|^2\rangle} = \omega\hbar \tag{46}$$

What we measure, in fact, is the mean quadratic value of the random fluctuations of energy.

Given that

$$\frac{\Delta S_0^2}{2\sigma^2} = \frac{\Delta E^2}{2\sigma_E^2} + \frac{\Delta t^2}{2\sigma_t^2} \tag{47}$$

we find

$$\sigma_E\sigma_t = \frac{\sigma}{2} = \frac{\hbar}{2} \tag{48}$$

or in general

$$\Delta E\Delta t \geq \frac{\hbar}{2} \tag{49}$$

which is the other Heisenberg's uncertainty equation. Equation (47) explains the reason there is a Gaussian dispersion of energy in a monochromatic LASER and not a Dirac's delta.

3 Conclusions

1. The Universe is structured in optimal laws. Random processes are those that maximize the mean information and are strongly related to symmetries, therefore, to conservational laws. Random processes have to do with optimal processes to manage information, but they do not have anything to do with the contents of it, this is the anthropic principle: Laws and physical constants are designed to produce life and conscience see [12].

2. Every physical process satisfies that the action is a locally minimum. It is the most important physical magnitude, after information, because through it is possible to obtain the energy, angular momentum, momentum, charge, etc. Every physical theory must satisfy the necessary (but not sufficient) condition $\delta S = 0$, because it is an objective principle.

3. We have demonstrated that the concepts of information and the principle of minimum action $\delta S = 0$ leads us to develop the concepts of Quantum Mechanics and to explain the spontaneous decay transitions. Also, we have understood that all physical magnitudes are quadratic mean values of random fluctuations.

4. Information connects every thing in nature, each phenomenon is an expression of totality.

5. Mass appears were certain symmetries are broken, or equivalently, when the mean information decreases. A very small decrease in information produces enormous amounts of energy. Also, we have shown that there exist a relation between the amount of information and the energy of a system (equation ( )). Information, mass and energy are conservative quantities which are able to transform one into another.

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