Edge Sensitive Variational Image Thresholding

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ABSTRACT

In this paper we propose a locally adaptive image threshold technique via variational energy minimization. The novelty of the proposed method is that from an image it automatically computes the weights on the data fidelity and the regularization terms in the energy functional, unlike many other previously proposed variational formulations that require manual input of these weights by laborious trial and error. To achieve the automatic setting of the weighting parameters we propose a non-linear convex combination of the data fidelity and the regularization terms in the energy functional and seek the optimum threshold surface via minimax principle. Our choice of the novel energy functional allows fast computation of the unique minimax solution. As a specific segmentation application, the proposed technique shows promising results when applied to find lung boundary from MR imagery. Illustrative examples are also provided where the proposed method is observed to retain texture information better than other competing methods.

Index Terms— minimax solution, variational technique.

1. INTRODUCTION

Thresholding is the operation of converting a grayscale image into a binary image. Thresholding is a widely applied preprocessing step for image segmentation. Often the burden of segmentation is on the threshold operation, so that a properly thresholded image leads to better segmentation. There are mainly two types of thresholding techniques available: global and local. In the global thresholding technique a grayscale image is converted into a binary image based on an image intensity value called global threshold. All pixels having values greater than the global threshold values are marked as 1 and the remaining pixels are marked as 0. In local thresholding technique, typically a threshold surface is constructed that is a function on the image domain.

Otsu proposed a global image thresholding technique where the optimal global threshold value is determined by maximizing the between–class variance with an exhaustive search [6]. Although Otsu's method remains one of the most popular choices for global thresholding techniques, it does not work well for many real world images where a significant overlap exists between the pixel intensity values of the objects and the background for un-even and poor illumination.

On the other hand, local thresholding method where the thresholding operation depends on local image characteristics is superior to the global ones for poorly illuminated images. Niblack proposed a local thresholding technique based on the local mean and local standard deviation [5]. The drawback of this algorithm is the determination of the size of the neighborhood that is set by the user and it depends on the content of the images. The window size should be small enough to preserve the local details and at the same time, it should be large enough to suppress noise.

Yan *et al.* proposed a multistage adaptive thresholding method where they introduce two global thresholds [9]. Pixels having gray values lower than the low threshold value are classified as the background. Pixels with intensity greater than the high threshold value are classified as objects. Next, the pixels having gray values in between the two threshold values are classified based on local image statistics of mean and variance within a variable neighborhood. The two global thresholds can be derived using Otsu's multilevel threshold with exhaustive search technique or percentile statistics. The determination of the window size of the neighborhood is once again an ad-hoc procedure here.

Liu *et al.* introduced an active surface model based local thresholding algorithm [4]. They proposed a repulsive external force by which the threshold surface is pulled away from the image surface everywhere except at high gradient locations or edges. The drawback of this model is the presence of two tuning parameters, which are typically determined by an ad-hoc trial and error method. A recent survey on image thresholding accounts for most of the methods available to date [8].

Incidentally, it is noted that these local thresholding techniques have hand tuning parameters that need to be adjusted for differently illuminated images and the values of these parameters vary significantly for different images. However, we propose here an automated adaptive local image thresholding method where no manually-adjusted weighting parameter is present for the data and the regularization terms in the energy functional. We have proposed a novel variational energy functional consisting of a non-linear combination of a data and a regularization term. The energy functional is a function of the threshold surface, the image, as well as the weighting parameter, which makes a balance between the data and the regularization terms. A minimax solution of the proposed energy functional is obtained iteratively by alternating minimization and maximization of the energy functional respectively with regard to the threshold surface and the weighting parameter. Our proposed minimax scheme finds the weighting parameter value by maximizing the energy functional while keeping threshold function fixed and finds the threshold surface function by minimizing energy functional while keeping the weighting parameter fixed at each iterative step. The solution converges to a unique state where the optimal values of the threshold surface and the weighting parameter are achieved.

The automated weighting parameter makes an appropriate balance between the effect of data term and regularization term on energy functional in the minimax sense and prevents each term to dominate over the other. The data term is edge sensitive and it leads to proper segmentation even when clear modes for objects and background in the gray level histogram are absent *i.e.*, there is no clear sign of different peaks and valleys in the gray level histogram representing the existence of a gray level threshold to threshold the image based on its pixel intensity values.

The energy functional is chosen in such a way that it is concave with respect to the weighting parameter and convex with respect to the threshold surface so that iterative minimax method can be applied to the energy functional to find the optimal threshold function automatically without any manually tuned weighting parameter. Gennert and Yuille proposed a minimax method for multi-component energy functional [3]. However, they proposed a Fibonacci search technique to find the optimal value of the weighting parameters that is computationally expensive, because of the minima computation of the energy functional a number of times– each time with different weighting parameter values. We deliberately avoid multiple minima computation by making the energy functional convex with respect to the threshold surface. Our proposed iterative scheme reaches to the global minima after a few iterations.

2. PROPOSED VARIATIONAL MINIMAX METHOD

Let the image and threshold surface function be denoted by $I(x, y)$ and $T(x, y)$ respectively. Then the proposed energy functional is as follows:

$$E(T;\alpha) = \sqrt{1-\alpha^2}\,E_1(T) + \alpha E_2(T), \tag{1}$$

where,

$$E_1(T) = \frac{1}{2}\iint g(x,y)\big(I(x,y) - T(x,y)\big)^2\,dxdy, \tag{2}$$

$$E_2(T) = \frac{1}{2}\iint |\nabla T(x,y)|^2\,dxdy, \tag{3}$$

$$g(x,y) = \frac{|\nabla I(x,y)|^q}{\max\big(|\nabla I(x,y)|^q\big)}$$

and $\alpha \in [0,1]$ is the weighting parameter. The first energy component E_1 is edge sensitive as it encourages the threshold surface T to intersect the image surface I where the value of the edge indicator function $g(x, y)$ is large. E_2 is the regularization term with the L_2 norm that enforces smoothness in the threshold surface. The value of the exponent q can be generally chosen to be 2 as usually done in the image processing literature to indicate edge strength. However it can also be experimentally determined for a specific application. A typical behavior of the proposed energy functional has been illustrated in Fig.1, where it is observed that desired threshold profile follows the image profile in such a way that they meet only at high gradient places or equivalently at the edges of the desired objects to be segmented.

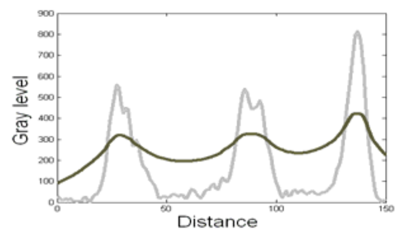


Fig.1. Image intensity profile and optimal threshold surface profile. The dark line represents the threshold surface profile and light one represents the image profile.

In order to compute the optimum threshold surface, we seek the minimax solution $\max_{\alpha}\min_{T} E(T;\alpha)$ for the energy functional (1). Here, E is a concave function in α and a convex functional in T. Because of this convex-concave nature of E, to find the minimax solution we can interchange the order of the max and the min operations to obtain:

$$T^* = \arg\min_{T}\max_{\alpha} E(T;\alpha) \tag{4}$$

Thus, we first differentiate E in (1) with respect to α and find the maximum value α^* by equating the derivative to zero:

$$\alpha^* = \frac{E_2(T)}{\sqrt{(E_1(T))^2 + (E_2(T))^2}} \tag{5}$$

Then, we minimize E via gradient descent equation for T keeping this value α^* for α be fixed:

$$\frac{\partial T}{\partial t}(x,y) = \sqrt{1-(\alpha^*)^2}\big(g(x,y)\big(I(x,y) - T(x,y)\big)\big) + \alpha^*\big(\nabla^2 T(x,y)\big). \tag{6}$$

The algorithm Variational Minimax iteratively solves the minimax problem:

Algorithm Variational Minimax

Initialize: $T \leftarrow I$

While convergence/maximum iterations not reached

    Compute $E_1(T)$ and $E_2(T)$ via equation (2) and (3)

    Compute:

$$\alpha^* \leftarrow \frac{E_{21}(T)}{\sqrt{(E_1(T))^2 + (E_2(T))^2}}$$

    Update T via equation (6)

End while

3. NUMERICAL IMPLEMENTATION

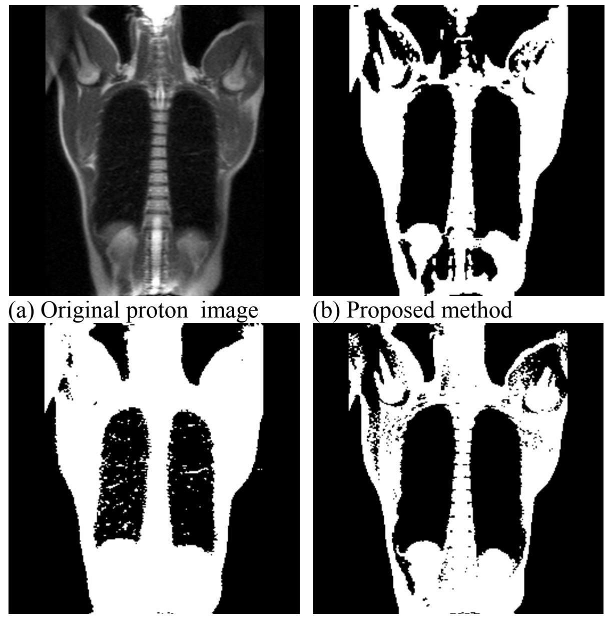
The gradient descent equation (6) is a parabolic equation characterized by the heat equation with a source term $\alpha^*\left(|\nabla I|^2 (I - T)\right)$[2]. We have discretized and implemented (6) using explicit numerical scheme as follows:

$$T_{i,j}^{t+1} = T_{i,j}^t + \tau\sqrt{1-(\alpha^*)^2}\left(|\nabla I|^2\left(I_{i,j} - T_{i,j}^t\right)\right) + \tau\alpha^* \left(T_{i+1,j}^t + T_{i-1,j}^t + T_{i,j+1}^t + T_{i,j-1}^t - 4T_{i,j}^t\right) \quad (7)$$

Where τ is the time step size of the iterative numerical scheme. In the explicit scheme, the finite difference analogs for $\nabla^2 T$ are written in known time level indexed by t. Convergence analysis shows that the step size (τ) should be less than or equal to 0.25 [2].

4. RESULTS AND DISCUSSIONS

We have applied the proposed edge sensitive variational minimax thresholding algoritm to find out the lung boundaries in proton MRI slices shown in Fig.2(a). The value of edge indicator's exponent q has been found experimentally. The experimental result is displayed in Fig. 3. Here the value has been chosen 8 as it shows consistently better segmentation score on a series on proton MRI slices. From Fig.3 it is observed that initially the Pratt's figure of merit (PFOM) [1] (defined shortly after) increases as the value of q increases and it stabilizes at q =6. If the value of q is increased without bound, $g(x, y)$ will tend to zero for most pixel locations (x, y) and consequently the effect of edge sensitive data term on energy functional (1) decreases. Then the regularization term dominates over the data term. With the negligible data fidelity term the solution turns out to be that of an isotropic heat equation and will essentially be a global threshold value. Thus we limit the value of the exponent as 8. To find the two lung cavities/lung boundaries in the thresholded binary image of Fig. 2(b) we first obtain the largest and the second largest black connected components from the middle portion of the image. Next, morphological operations are used to eliminate any remaining small white connected component within the lung cavity. Further, any possible extraneous portions connected to the lung cavities via "necks" are also eliminated by morphological operations. Same parameter setting (such as area threshold and neck width) in the morphological operations has been used for all the data sets. The results of Otsu and Fuzzy c-means algorithm are shown in Fig. 2(c) and 2(d) respectively.



(a) Original proton image (b) Proposed method

(c) Otsu's method (d) Fuzzy c-means

Fig.2. Thresholded Binary lung Image.

MRI data sets	PFOM value (proposed method)	PFOM value (Ray *et al.*'s method [7])
1	0.7586	0.7156
2	0.7546	0.7120
3	0.755	0.7085

Table 1. Pratt's figure of merit comparison

To measure the quantitative evaluation of our segmentation results we have calculated PFOM. PFOM is a subjective edge evaluation which is defined as:

$$F = \frac{1}{\max\{I_I, I_A\}} \sum_{i=1}^{I_A} \frac{1}{1 + \beta d^2(i)}$$

where, I_I and I_A are the number of ideal and actual edge pixels, $d(i)$ is the pixel miss distance of the i^{th} edge pixel detected, and β is a scaling constant chosen to be 1/9 to provide a relative penalty between smeared edges and isolated, but offset, edges [1]. PFOM is a dimensionless number between 0 and 1. The maximum value of PFOM is 1 for ideal segmentation. To calculate PFOM we need the information of ideal edges which is computed based on ground truth. We have carried out automated segmentation in three MRI data sets. The PFOM results of all MRI slices over the three data sets are shown in Fig.4. Table 1 summarizes the average value of PFOM of all three data sets and it compares the results found by Ray *et al.*'s active contour method [7]. Results found by our method shows superiority of the quality over the results of [7].

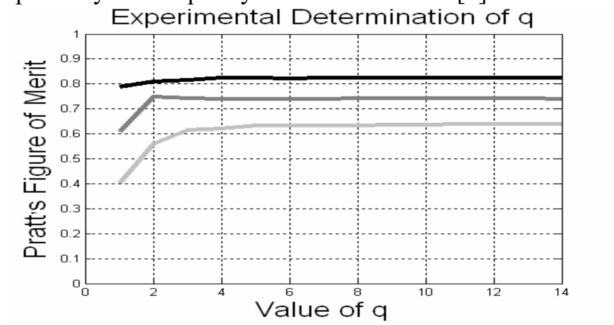


Fig.3. PFOM for different values of q is shown. Different lines represent PFOM for different MRI slices.

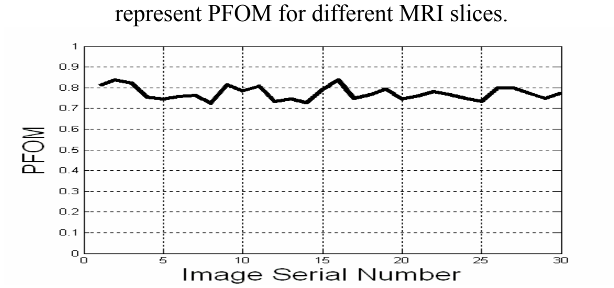


Fig 4. PFOM for our proposed method on lung data sets.

We have applied the proposed method on 512 x 512 grayscale Barbara image, and the result is shown in Fig.5(a). Here, the value of q is taken as 1.5. We have implemented Liu *et al.*'s method on the Barbara image and we have chosen the two tuning parameters of their method $\sigma = 16$, $w = 1$ from their article [4]. We have also implemented Otsu's global thresholding and fuzzy c-means on the same image as shown in Fig.5. The proposed method is observed to better preserve the texture.

5. SUMMARY AND FUTURE WORK

We have proposed a manually tuned weighting parameter free novel variational adaptive image thresholding technique. Our proposed novel energy functional is made of a non-linear convex combination of an edge sensitive data term and a regularization term. The edge indicator function used in our energy functional has an exponent as a parameter. The value of this exponent is found experimentally and we indicated its connection with isotropic heat equation leading to global threshold. The optimal values of the threshold function and the weighting parameter are determined by an iterative minimax method. Our proposed novel technique has been successfully applied on MRI slices for the purpose of finding lung boundaries. The proposed method is observed to preserve texture and details better than other competing techniques.

In the future, we would like to concentrate on how to automatically set the value of the exponent q from the image data itself.

(a) Proposed method (b) Liu *et al.*'s method

(c) Otsu's method (d) Fuzzy c-means.

Fig.5. Thresholded Barbara image.

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