EDITORIAL

A New Type of Average for Security Prices

The market averages that are most popular with the American investing public are essentially weighted or unweighted means of security prices at designated intervals. As a rule, they ignore the volume of sales—an element to which experienced traders attribute considerable importance. Such averages endeavor only to reflect the average price level at periodic intervals, and all of those published are entirely satisfactory in this respect.

In this note we shall discuss an *acquisition average* which, instead of being concerned with the price level at a given moment attempts to answer the question, "what is the average price actually paid for the securities by their present owners."

The problem can best be appreciated by presenting two examples of acquisition averages prior to the mathematical theory. The first entry of Table 1 states that for the week ending January 7, 1928 United States Steel common closed at 150 6-8, and that the acquisition average on this date was $137.75. At the time of the market crash in October, 1929, the acquisition average had risen to about $212, and at the present moment this average has receded to about $48. Of course, some of the individuals who bought Steel at about $200 per share are still holding on to it, whereas others among the present holders obtained theirs in 1932 at less than $25 per share. According to our theory, the mean of such acquisition prices is the $48 noted above.

As an illustration of corresponding averages computed on a *daily* basis, table 2 presents the daily closing prices and acquisition averages for Auburn, covering the last half of 1934. This stock was selected because of its relatively small capitalization and frequent activity.

TABLE 1.

WEEKLY CLOSING PRICES AND ACQUISITION AVERAGES FOR U. S. STEEL, 1928-1933.

1928	Close	A.A.	1929	Close	A.A.	1930	Close	A.A.
1- 7	150.6	137.75	1- 5	161.6	152.34	1- 4	169.1	192.63
14	146.0	138.42	12	166.6	153.09	11	169.5	191.32
21	147.4	138.80	19	188.4	156.44	18	169.6	190.31
28	146.3	139.19	26	187.1	160.23	25	179.0	189.43
2- 4	143.4	139.42	2- 2	184.7	162.07	2- 1	184.4	188.86
11	145.7	139.60	9	173.4	163.17	8	182.6	188.59
18	140.0	139.70	16	169.3	163.64	15	186.0	188.37
25	140.0	139.71	23	182.0	164.54	22	183.0	188.15
3- 3	140.2 x	138.00	3- 2	188.4 x	165.62	3- 1	184.3 x	186.21
10	144.7	138.33	9	185.2	166.98	8	181.0	186.07
17	147.7	138.99	16	188.0	168.58	15	179.2	185.88
24	147.5	139.83	23	181.1	169.51	22	187.6	185.68
31	147.3	140.35	30	183.6	170.46	29	193.4	185.97
4- 7	147.1	140.51	4- 6	186.3	172.06	4- 5	196.4	186.50
14	150.0	141.09	13	188.5	173.52	12	193.1	186.98
21	145.5	141.34	20	186.0	174.41	19	195.2	187.30
28	145.3	141.48	27	186.2	174.79	26	188.0	187.51
5- 5	148.0	141.58	5- 4	182.1	175.20	5- 3	170.2	186.73
12	148.6	141.90	11	179.4	175.33	10	172.6	185.65
19	145.6	142.10	18	174.6	175.38	17	172.7	185.30
26	146.7	142.22	25	167.7	175.25	24	172.0	184.80
6- 2	145.2 x	140.70	6- 1	165.0 x	173.28	31	173.5 x	182.87
9	140.6	140.82	8	168.0	173.16	6- 7	164.2	182.48
16	138.5	140.75	15	175.5 r	169.40	14	162.4	181.42
23	133.2	140.52	22	180.6	169.80	21	155.2	179.79
30	137.3	140.39	29	190.6	170.90	28	156.0	178.52
7- 7	140.0	140.35	7- 6	196.3	172.43	7- 5	157.7	177.99
14	136.0	140.26	13	202.3	174.63	12	160.6	177.46
21	138.2	140.16	20	207.7	177.48	19	166.6	177.03
28	144.2	140.21	27	206.0	179.85	26	169.7	176.83
8- 4	140.3	140.28	8- 3	204.4	182.20	8- 2	166.2	176.61
11	142.7	140.32	10	218.0	185.68	9	159.4	176.03
18	149.0	140.78	17	238.5	192.17	16	165.3	175.52
25	151.2	141.45	24	258.2	197.54	23	168.2	175.29
9- 1	154.0 x	140.51	31	256.4 x	198.64	30	171.2 x	173.38
8	155.7	141.54	9- 7	247.4	202.19	9- 6	173.1	173.34
15	159.0	142.71	14	233.2	205.43	13	170.2	173.28
22	157.6	143.62	21	232.1	207.70	20	163.7	173.00
29	159.2	145.12	28	225.0	210.12	27	158.2	172.21
10- 6	158.3	146.31	10- 5	217.6	211.40	10- 4	156.6	171.31
13	164.6	147.56	12	230.6	212.56	11	148.4	169.46
20	162.0	148.26	19	209.0	213.41	18	145.3	168.25
27	161.5	149.16	26	203.4	212.38	25	151.4	167.23
11- 3	160.7	149.68	11- 2	193.2	211.06	11- 1	145.6	166.39
10	164.3	150.34	9	171.0	209.67	8	140.4	165.39
17	171.2	152.26	16	164.2	207.09	15	147.7	163.84
24	167.6	152.95	23	167.0	205.42	22	147.2	163.12
12- 1	165.0 x	151.50	30	162.1 x	201.81	29	145.4	162.59
8	151.2	151.95	12- 7	182.6	200.30	12- 6	142.3 x	160.38
15	151.1	151.92	14	174.0	196.87	13	136.4	159.24
22	156.0	151.97	21	163.0	195.11	20	140.5	158.19
29	159.4	152.11	28	164.4	193.76	27	136.6	157.73

x = ex-dividend. r = "rights."

TABLE 1—(*Continued*)

1931	Close	A.A.	1932	Close	A.A.	1933	Close	A.A.
1- 3	143.4	157.38	1- 2	37.1	83.57	1- 7	29.7	46.40
10	143.7	157.06	9	42.7	80.59	14	29.6	46.10
17	139.1	156.65	16	44.1	78.50	21	28.5	45.90
24	142.3	156.30	23	41.4	77.00	28	27.7	45.74
31	139.2	155.99	30	37.3	75.28	2- 4	26.3	45.54
2- 7	140.3	155.67	2- 6	38.5	73.93	11	28.3	45.36
14	145.2	155.30	13	49.0	72.23	18	26.7	45.20
21	148.6	155.06	20	48.4	70.84	25	24.5	44.97
28	147.4	154.87	27	47.0	70.20	3- 4	26.2	44.68
3- 7	146.6 x	152.95	3- 5	50.6 x	69.00	11		
14	144.4	152.82	12	46.7	68.44	18	30.6	44.33
21	147.4	152.65	19	41.5	67.33	25	28.5	44.13
28	141.4	152.50	26	40.2	66.50	4- 1	27.5	44.00
4- 4	140.0	152.23	4- 2	39.0	65.47	8	30.3	43.82
11	137.3	151.80	9	34.6	64.16	15	32.4	43.57
18	132.6	151.22	16	33.2	63.15	22	42.3	43.23
25	125.5	150.14	23	29.1	62.33	29	46.5	43.30
5- 2	115.2	148.10	30	28.2	61.60	5- 6	46.7	43.48
9	111.5	146.24	5- 7	30.0	60.80	13	47.4	43.66
16	101.5	143.97	14	27.0	60.29	20	47.2	43.78
23	98.4	141.33	21	29.0	59.81	27	53.0	44.05
30	91.0 x	136.40	28	27.2	59.34	6- 3	52.1	44.29
6- 6	89.3	132.63	6- 4	30.2	58.58	10	55.4	44.58
13	90.7	130.23	11	26.5	57.57	17	53.1	44.97
20	92.7	128.77	18	25.4	57.04	24	57.4	45.34
27	104.3	125.91	25	23.4	56.62	7- 1	59.7	45.84
7- 4	105.0	124.96	7- 2	23.6	56.13	8	65.2	46.32
11	96.4	123.23	9	21.6	55.83	15	64.2	46.88
18	94.4	121.56	16	23.3	55.43	22	52.2	47.39
25	90.2	120.50	23	24.7	55.18	29	54.3	47.49
8- 1	85.7	118.79	30	28.7	54.51	8- 5	51.4	47.56
8	86.0	118.04	8- 6	41.4	53.70	12	53.4	47.60
15	93.0	116.89	13	37.4	52.86	19	52.7	47.66
22	87.4	116.23	20	40.7	52.38	26	58.4	47.79
29	90.5	115.66	27	48.3	51.91	9- 2	55.3	47.91
9- 5	83.0 x	113.91	9- 3	51.4	51.81	9	51.5	47.96
12	80.5	112.74	10	48.6	51.73	16	55.0	48.05
19	75.2	110.78	17	38.7	51.24	23	49.4	48.17
26	77.1	109.00	24	45.4	50.82	30	45.5	48.16
10- 3	68.4	107.16	10- 1	43.7	50.63	10- 7	47.3	48.14
10	70.7	104.91	8	35.4	50.18	14	43.2	48.11
17	68.6	103.87	15	37.7	49.76	21	35.2	47.82
24	71.5	102.97	22	35.1	49.39	28	39.2	47.57
31	67.4	101.79	29	35.3	49.12	11- 4	40.5	47.43
11- 7	72.3	100.88	11- 5	35.0	48.87	11	42.2	47.35
14	67.7	99.85	12	39.2	48.53	18	43.3	47.26
21	60.6	98.59	19	36.1	48.32	25	45.0	47.21
28	53.5	96.92	26	32.7	48.16	12- 2	44.6	47.18
12- 5	54.0 x	93.36	12- 3	30.4	47.90	9	47.4	47.17
12	44.0	90.78	10	32.2	47.61	16	45.6	47.16
19	41.4	87.09	17	30.1	47.33	23	47.7	47.15
26	37.6	85.55	24	26.5	46.95	30	47.6	47.16
			31	27.4	46.63			

x = ex-dividend.

TABLE 2.

DAILY CLOSING PRICES AND ACQUISITION AVERAGES FOR AUBURN, JULY 1ST—DECEMBER 30TH, 1933.

1933 July Aug.	Close	A.A.	1933 Sept. Oct.	Close	A.A.	1933 Nov. Dec.	Close	A.A.
1	66.0	61.59	1	61.4	62.13	1	36.4	54.57
3	69.7	62.00	5	58.4	62.12	2	37.0	54.55
5	68.4	62.43	6	59.4	62.06	3	38.6	54.48
6	68.6	62.59	7	59.0	62.04	4	39.0	54.39
7	67.2	62.87	8	58.2	61.99	6	39.0	54.34
8	67.2	62.91	9	58.3	61.99	8	43.0	53.74
10	67.4	63.06	11	62.2	61.94	9	43.0	53.46
11	68.0	63.38	12	61.0	61.93	10	41.0	53.40
12	78.6	66.47	13	61.4	61.93	11	42.4	53.32
13	77.3	68.70	14	61.6	61.92	13	43.0	53.24
14	75.0	69.23	15	59.7	61.90	14	38.0	53.06
15	76.5	69.40	16	62.0	61.90	15	36.0	52.96
17	80.0	70.36	18	59.5	61.88	16	43.6	52.78
18	78.0	70.73	19	60.0	61.83	17	43.0	52.66
19	70.6	71.06	20	56.6 x	61.25	18	43.0	52.63
20	58.1	70.13	21	50.6	60.93	20	46.6	52.45
21	50.0	68.48	22	52.4	60.71	21	45.1	52.31
22	46.4	67.38	23	51.4	60.61	22	45.0	52.26
24	54.4	66.61	25	49.4	60.44	23	43.2	52.13
25	52.5	65.99	26	47.6	60.21	24	45.0	52.07
26	54.2	65.72	27	46.5	59.90	25	44.4	52.07
27	58.0	65.33	28	47.2	59.81	27	42.2	51.98
28	55.4	65.22	29	46.5	59.68	28	43.0	51.91
31	52.1	64.57	30	46.0	59.55	29	44.6	51.89
1	54.6	64.37	2	46.0	59.46	1	45.2	51.81
2	57.2	64.09	3	45.0	59.35	2	44.7	51.80
3	54.6	63.95	4	50.4	59.32	4	45.1	51.77
4	53.4	63.85	5	48.0	59.22	5	48.0	51.68
7	53.4	63.81	6	48.0	59.19	6	46.4	51.64
8	56.0	63.57	7	48.4	59.14	7	48.6	51.46
9	61.4	63.32	9	49.4	59.08	8	49.6	51.35
10	58.4	63.16	10	49.0	59.03	9	56.4	51.51
11	57.0	63.10	11	48.4	58.94	11	57.0	51.95
14	57.6	63.01	13	46.2	58.89	12	55.4	52.10
15	59.0	62.98	14	45.6	58.81	13	54.4	52.17
16	55.0	62.84	16	43.0	58.67	14	57.0	52.41
17	60.6	62.72	17	41.6	58.33	15	57.2	52.76
18	57.4	62.65	18	38.0	57.96	16	55.7	52.89
21	59.0	62.60	19	35.0	56.87	18	55.0	53.04
22	61.0	62.52	20	37.2	56.38	19	53.5	53.08
23	59.0	62.44	21	34.0	56.05	20	49.4 x	52.48
24	58.6	62.38	23	36.0	55.45	21	49.4	52.42
25	61.4	62.25	24	38.0	55.25	22	54.4	52.40
28	62.0	62.23	25	38.6	54.96	23	53.3	52.42
29	61.0	62.19	26	37.0	54.91	26	52.0	52.42
30	60.2	62.16	27	37.4	54.86	27	52.4	52.42
31	59.4	62.14	28	36.7	54.82	28	54.0	52.43
			30	35.4	54.73	29	54.0	52.45
			31	35.0	54.63	30	54.6	52.50

x = ex-dividend.

We shall now develop the theory on which the preceding tables were constructed. As a simple illustration let us suppose that 100 individuals start an enterprise, that a total of 100 shares of stock are issued, and that each of the individuals purchases one share for $100. The total book value of the issue at the date of issue is therefore,

$$V_0 = \$100 \times 100 = \$10\,000,$$

and the acquisition average then is

$$A_0 = \frac{V_0}{100} = \$100.00.$$

If the first transfer of stock resulted from the sale of a single share at 150, the total amount paid by the group now owning all the issue is obviously

$$V_1 = 99(100) + 150 = 10\,050,$$

and the new acquisition average is

$$A_1 = \frac{V_1}{100} = \left(1 - \frac{1}{100}\right) A_0 + \frac{p_1}{100} = 100.50$$

If somewhat later the next sale of stock is a single share at 50, we may assume that

$$V_2 = 99(100.50) + 50 = 9999.50$$

and consequently

$$A_2 = \frac{V_2}{100} = \left(1 - \frac{1}{100}\right) A_1 + \frac{p_2}{100} = 99.995 .$$

Our first assumption is, therefore, that *whenever the sale of a share of stock is recorded, it is equally likely that any one of the previous holders sold the share.* More will be said of this assumption later.

In generalizing, let us adopt the following notation:

L designates the number of share units listed for an issue

A_0 is the acquisition average at a given initial date.

p_x denotes the price at which the x-th unit of stock is sold following the initial date.

A_x is the acquisition average immediately after the sale of the x-th unit.

We have then that

$$A_1 = (1-\tfrac{1}{L})A_0 + \tfrac{p_1}{L}$$
$$A_2 = (1-\tfrac{1}{L})A_1 + \tfrac{p_2}{L} = (1-\tfrac{1}{L})^2 A_0 + \tfrac{p_1}{L}(1-\tfrac{1}{L}) + \tfrac{p_2}{L}$$
$$A_3 = (1-\tfrac{1}{L})A_2 + \tfrac{p_3}{L} = (1-\tfrac{1}{L})^3 A_0 + \tfrac{p_1}{L}(1-\tfrac{1}{L})^2 + \tfrac{p_2}{L}(1-\tfrac{1}{L}) + \tfrac{p_3}{L}$$
$$\cdots\cdots$$
$$(1)\quad A_x = (1-\tfrac{1}{L})^x A_0 + \tfrac{p_1}{L}(1-\tfrac{1}{L})^{x-1} + \tfrac{p_2}{L}(1-\tfrac{1}{L})^{x-2} + \cdots + \tfrac{p_{x-1}}{L}(1-\tfrac{1}{L}) + \tfrac{p_x}{L}$$

If we multiply both sides of this last equation by $(1-\frac{1}{L})$ and then subtract the resulting equation from (1), we obtain

$$(2)\quad A_x = A_0(1-\tfrac{1}{L})^x - p_1(1-\tfrac{1}{L})^x + (1-\tfrac{1}{L})^{x-1}(p_1-p_2) + (1-\tfrac{1}{L})^{x-2}(p_2-p_3) + \cdots\cdots + (1-\tfrac{1}{L})(p_{x-1}-p_x) + p_x .$$

We shall now make a second assumption, namely, that the prices vary linearly from 0 to x. To illustrate, if Steel closes one week at 54 and during the next week 100,000 shares are sold after which the close is 59, our assumption means that after 20,000 shares were sold the quotation is 55, at 40,000 shares the price is 56, etc. Actually the price trend between two dates is not a straight line but rather a scattering of points. However, the linear assumption introduces compensating errors which have been found to result in only negligible variations in the resulting acquisition averages. We may write, therefore,

$$(3)\quad \begin{cases} p_1 = p_0 + \dfrac{p_x - p_0}{x} \\ p_{i+1} - p_i = \dfrac{p_x - p_0}{x} \end{cases}$$

and equation (2) then reduces to

$$(4)\quad A_x = A_0(1-\tfrac{1}{L})^x + p_0\left[\tfrac{L-1}{x} - (1-\tfrac{1}{L})^x(\tfrac{L-1}{x}+1)\right] + p_x\left[1 - \tfrac{L-1}{x} + (1-\tfrac{1}{L})^x\tfrac{L-1}{x}\right].$$

But since in practice both L and x are large integers we may write

$$(5) \qquad \left(1-\frac{1}{L}\right)^{x} = e^{-\lambda}, \qquad \text{where} \quad \lambda = \frac{x}{L},$$

and (4) then becomes

$$(6) \qquad A_x = \alpha A_0 + \beta p_0 + \gamma p_x$$

where

$$(7) \qquad \alpha = e^{-\lambda}, \quad \beta = \frac{1-e^{-\lambda}}{\lambda} - e^{-\lambda}, \quad \gamma = 1 - \frac{1-e^{-\lambda}}{\lambda}.$$

Tables of α, β and γ have been computed for the interval — rate-of-turnover, λ. With the aid of these, tables 1 and 2 are readily extended. A slight difficulty is encountered in determining the acquisition average at an initial point. At the outset it is necessary to assume two initial acquisition averages, one equal to the "high" at some point in the past, the other equal to the corresponding "low." The true acquisition average certainly lies between these two limits. It is necessary to start computations sufficiently far before the date of the first desired acquisition average so that the two series derived respectively from the highs and lows will converge to a single average. The length of the past experience period required will depend upon the rate of turnover of the stock. The activity in grains is frequently so great that the two series will converge over a period of two weeks.

I wish to point out emphatically that this acquisition average is an average and nothing more. Like any other average its value depends largely upon the ability of the individual using it. Although the use of this average might prove of value to an investor, it can not rightly be said that this is a forecasting formula. I doubt the existence of any valid method of forecasting—mathematical or otherwise. The acquisition average merely measures *secondary* phenomena, and provides a tool for recogniz-

ing an unfavorable condition that might very easily be changed into a favorable situation by any one of numerous causes. Thus, if the market quotation is greater than the acquisition average, it follows that the average owner of the stock in question has a "paper profit." Moreover, since a sale is made when the owner of the stock and the prospective purchaser can agree on a price, and because of the peculiar psychology usually affecting one possessing a paper profit, the excess of the market price over the acquisition average tends through bidding to increase both prices and acquisition averages. This vicious circle carries prices too far in either direction until some "impressed force" changes the trend abruptly.

Since the price of a security at a given time depends upon the status of the entire market as well as the intrinsic value of that security, it follows that a general average for the acquisition figures for a number of the "market leaders" would probably be of value to certain investors. In fact, any of the popular market averages can be accompanied by corresponding acquisition averages.

Since in many cases fifty percent of the stock is kept to protect control, it is evident that one might be justified in using one-half the share units listed for the value of L in formula (5) for λ. Again, if one desires to investigate the status of the group operating on margins, the amount of the "floating stock" and the brokers' loans must be taken into consideration in determining λ.

In conclusion let me point out that under the most favorable conditions our method of determining the acquisition average can do no more than a 100% successful questionaire inquiring of stockholders the price at which each share was purchased. Of course all stockholders would not give such information if they could, and couldn't if they would.