EDUCATING CHILDREN IN STOCHASTIC MODELING: GAMES WITH STOCHASTIC URNS AND COLORED TINKER-CUBES

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It is an established opinion that young children have good probabilistic intuitions although their mental framework for reasoning probabilistically may be fragmentary and even contain inconsistencies. We join the camp of mathematics educators who advocate an early training in stochastic thinking. We recognize, however, that such early training can only be based on a heuristic approach to stochastics. The heuristic approach that we propose here is implemented by guiding children to actively construct stochastic situations by means of plastic tinker-cubes and stochastic urns. The aim of this enactive training is to foster the development of a dynamic mental imagery for representing stochastic situations.

INTRODUCTION

Due mostly to historical factors, the pace at which stochastics has been integrated as a component of mathematics school curricula has been comparatively slow: The debate on how early and to what extent children should be taught statistics and probability is far from being concluded. Probability is one of the core concepts in stochastics and aspects of it are being grasped by a child of seven (Piaget, 1952; Wollring, 1994; Falk, 1982, Martignon and Wassner, 2005). Yet its philosophical and mathematical complexity, involving either notions of infinity and convergence in the classical approach, or notions of sets and measures in the modern formalization, has led mathematics educators to think that the inception of probabilistic concepts should be confined to the last years of secondary school. In countries like Germany this late inception had, until 2003, seldom been implemented with real enthusiasm by teachers, although most did acknowledge that stochastics is more useful than other branches of mathematics for everyday life. The influence of the NCTM Standards and the effects PISA Studies 2000 and 2003 operated a change in the attitudes both of teachers and of ministerial organizations towards statistics and probability in schools: After all, 25% of all questions in the PISA 2003 tests dealt with probabilistic or statistical situations. In most of the Länder special recommendations have also been formulated in the programs for primary school to encourage teachers to introduce first notions of descriptive statistics and probabilities.

STOCHASTIC MODELING AND COMMON SENSE IN SCHOOL

One of the mathematical test items of PISA 2003 was the following:

Consider two boxes A and B. Box A contains three marbles, of which one is white and two are black. Box B contains 7 marbles, of which two are white and five are black. You have to draw a marble form one of the boxes with your eyes covered. From which box should you draw if you want a white marble?

The PISA 2003 Report (PISA Consortium 2004) commented that only 27% of the German school students obtained the "correct answer." The answer "I should choose from Box A" without explanation for one's choice would have been insufficient.. Would bare common sense lead a school student to choose Box A? In other words, does a student guided by common sense conclude that the more convenient box is the one with a larger proportion of white marbles? Once this is established the rest is trivial. The next question is whether the proportion of white marbles in "1 out of 3" is larger than in "2 out of 7": Without recurring to fractions a school student could see that "1 out of 3" is like "2 out of 6." The next step would be to compare "2 out of 6" with "2 out of 7," which is again straightforward, without involving fractions. Establishing that the more convenient box is the one with the larger proportion of white marbles was seen by Laplace as based on *unaided* common sense. Modern cognitive psychology has provided a good amount of empirical evidence for this assertion: the discoveries of Gigerenzer and his school corroborated Laplace's views by proving empirically that humans make sound judgments based on "natural proportions of frequencies," or simply "natural frequencies. Hasher, Zacks and Sanft

(1982) proved empirically that humans sample frequencies and make inferences based on sample sizes (1982). When one proportion is markedly larger than another, they know instinctively which one should be chosen even when the choice is between one high proportion and the combination of two low ones (Martignon and Krauss, 2003). Mathematically correct statements on why and when larger proportions in samples correspond to larger chances in the populations do require serious amounts of conceptual work. Therefore, if what has to be conveyed to school students in early grades is the competence to "reduce common sense to a calculus" (as Laplace put it), the recipe seems to boil down to (1) Basic stochastic modeling with natural representation formats and (2) Simple heuristics for operating with these formats.

EDUCATING STOCHASTIC MODELLING THROUGH ENACTIVE REPRESENTATIONS: STOCHASTIC URNS AND TINKER-CUBES

Along the lines of the recipe suggested above we have designed a program for canalizing young children's aptitudes in stochastic thinking. The program we propose is being developed as an online guide for primary school instructors (Martignon and Kurz-Milcke, 2006). It consists of instructions for a series of units distributed through the sequence of years in primary school, aiming at educating children in modeling stochastic situations. This is achieved by guiding them to actively construct representations of situations and learn to answer questions based on the features of these representations. In these playful yet structured activities children develop an urn concept that goes far beyond the material perception of an object called *urn*. The material urns with which children work are transparent plastic containers, which children can easily hold with their two hands. Children's modeling of situations by means of small plastic cubes, denominated *tinker-cubes*, which can be assembled in little towers, is performed with great ease and pleasure. After all, children are good at "imagining" scenarios, say by mentally converting sofas into cockpits and tables into airports. They have even less difficulties than an adult in representing and modeling situations by means of available tokens and objects.

As a possible instance of urn construction children can construct the urn for "our class," where each girl is represented by a red tinker-cube and each boy by a blue one. Other features can be encoded easily by adjoining cubes in other colors for say, blond hair, or brown hair. Thus urns end up containing towers of tinker-cubes, which children construct, encouraged by the teacher. In recent explorative studies we have had children of the fourth grade construct urns for describing concrete situations like the following: *In a class there are 12 girls and 11 boys. Of the girls 8 have long hair, and of the boys 2 have long hair. All other children have short hair. Construct an urn representing the class so that you will easily detect the boys with long hair.*

One possibility is, for instance, that of encoding "long hair" with a yellow cube and "short hair" with a "green" cube. As a possible test of children's probabilistic intuitions the teacher picks one tower composed of two cubes and hides it behind her back, making sure that the children have not seen, which tower she has picked. She says "My tower has one yellow cube. What do you think, is it a boy or a girl?" This activity is repeated many times with different settings. Due to the feedback provided by the teacher showing the tinker-cube tower she has hidden, children develop a propensity to bet for the more probable situation. These activities are performed by children in the fourth class. They learn to construct urns and to make bets on whether a tower represents a certain item in a cell of a certain partition. Categorical partitions are characterized by constructing the apposite urns. Children do even count the towers and answer questions concerning quantified categorizations. These tasks contain first elements of Bayesian reasoning at a heuristic level. Kurz-Milcke has systematically guided children in the fourth grade to construct tree diagrams, initially using concrete urns and colored string to produce tree-like layouts on the class room floor (Kurz-Milcke and Martignon, in press). What surprised us was the extraordinary readiness of children to reason and play with representatives (i.e., the towers of tinker-cubes) and always know what was meant. On the other hand it is well known that children games are based on simulation and representation: chairs become rockets and tables become airports! This capacity to simulate and encode is one of the fundamental resources of scientific thought and teachers can help canalizing this resource in useful directions. Urns and tinker-cubes are just an example of enactive simulation. The studies illustrated above concern the fourth grade but our conceptual framework envisages urns and tinker-cubes as enactive representations from a very early stage. Already in the Kindergarten urns are used for categorizations: red and green toys of two types, balls and tinker-cubes of different sizes, are sorted by children and placed in urns. At one instance they place all red toys together and all green toys together (Martignon and Wassner, 2005). Between five and six years of age children begin to play with dice. These games fascinate children because of the combination of luck and chance. They throw two dice and bet on the outcome. It is common to see them blowing on the dice to ensure good luck.

We have tested a game of dice in 4 first grade classes in Stuttgart and surroundings. Children were divided into groups of four, two boys and two girls. Every group had to play at a table after having participated in completing a table drawn on the blackboard displaying the possible outcomes of the dot sum of two dice. Every pair of teams alternated in throwing a pair of dice. Each team made bets on the corresponding outcome. At every instance the winner was the team whose bet was closer to the actual outcome. A total of 30 bets were performed and the sequence of bets of each team was then analyzed. The question we analyzed was to which extent having completed a table of the possible outcomes of the pairs of dice had influenced children's bets. Feedback provided by the sequence of outcomes clearly had an impact on children's bets. We could trace three factors that influenced children's bets: feedback, knowledge of the possible combinations for each outcome, and finally affective relationships towards special numbers, like 3, 7 and 12. After the bets, there were discussions on the bets and their outcomes. The concept discussed was "strategy": the teacher explained the meaning of the word and asked children about theirs. Most children (60% of the teams, N = 18 teams) could talk about their betting strategies and detect principles that had guided them through the game. Finally the urns describing the possible outcomes were constructed, by means of tinker-cubes in 11 different colors.

URN ARITHMETIC IN THE FOURTH GRADE

The fourth grade has proven to be the ideal time interval for making children familiar with a simple urn Arithmetic, which leads to simple solutions of problems like the Two Boxes selection task above. We denote by U(a:b) the urn with *a red* and *b green* tinker-cubes. We have two urns, namely U(1:2) and U(2:5): Which urn is more convenient if we want a red tinker-cube? For answering this type of question fractions are not required. We do though have to transform each urn in an equivalent urn, so that the resulting urns are easily compared. The concept of equivalent or similar urn requires at least 6 hours of playful constructions in the fourth grade, before it is consistently primed in the children's minds. Why is U(1:1) equivalent to U(2:2)? Children assemble two red tinker-cubes and two green tinker-cubes. They learn that if you add the same amount of tinker-cubes to each single tinker-cube, the proportion does not change. They construct pyramids composed of 1:1 red and green cubes at the top, 2:2 red and green cubes in the second row, 3:3 in the third row, etc. Children then construct the pyramid for the proportion 1:2. Pyramids for several different proportions are placed on a table where they remain e and remain several days for all children to see. There comes the day when pyramids are dissembled and the corresponding urns are constructed by throwing pyramids of a pyramid row in an urn. For each pyramid several *similar* urns are constructed. The next step is to treat the comparison of two different urns; by different we mean urns, which are not similar. Here the instructor has to lead children to construct two similar urns, which become comparable. Given U(a:b) and U(c:d), one possibility is to construct U(ac:bc) and U(ac:ad) and the compare these urns, which becomes easy. This urn arithmetic requires a certain amount of discipline because progress in grasping the concepts of similarity and comparison demands discipline and time. We consider this first confrontation with comparison of proportions and similarity of proportions a fundamental previous step before fractions are introduced.

PREPARING CHILDREN FOR BAYASIAN REASONING: FOSTERING STOCHASTIC HEURISTICS IN PRIMARY SCHOOL

We have conducted studies and interviews with German school teachers concerning the aim of school education in probabilistic thinking. The core question was: Should the aim be to communicate formalized knowledge of probabilities or foster the use of heuristics based on sound (in the sense of Hogarth, 2001) modeling of elementary probabilistic situations (Wassner and Martignon, 2005)? Our conclusion is that teachers consider the education of elementary probabilistic modeling a task to be placed prior – both with regard to the temporal sequence and in importance – to the theoretical treatment of probabilistic inference. Elements of theoretical treatment have to be provided, of course, when possible, but should be the completion and not the initiation of stochastics education in school. Historically, this attitude has been the same in the case of other fields of Mathematics, although the process has been less conscious. Other fields were incorporated in school curricula before and educational principles had been formalized and tested empirically. Basic arithmetic and geometry are taught first as heuristic competences; only at a later stage, school students are given a glimpse into these fields as more formalized bodies of theoretical development. The PISA studies, the recommendations of the NCTM Standards and the reformed German Curricula since 2005 advocate the education of mathematical competencies prior to, yet not substitutes of, theoretical knowledge. In direct analogy with arithmetic and geometry children's aptitude should be *prepared* during primary school by means of basic heuristics for coping with elementary tasks of probability and statistics. By heuristic we do not mean a rule of thumb; we use the term in the sense made famous by Einstein (1905) and, in recent years, by Gigerenzer and his colleagues, namely, as a correct yet simple, partial approximation of the full approach.

A CONCLUDING REMARK ON TINKER-CUBES

We have chosen tinker-cubes because they are neutral toys: other toys like Legos polarize boys and girls and girls are less prone to use them in a spontaneous way. Our tinker-cubes can be assembled into towers and towers can be disassembled. Towers placed on a table can also serve as "living" histograms. Thus tinker-cubes display two stochastic functions: They can motivate probabilistic questions but also describe statistical situations.

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