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Education Enhances the Acuity of the Nonverbal Approximate Number System

Manuela Piazza[1,2,3,4], Pierre Pica[5], Véronique Izard[6,7], Elizabeth S. Spelke[8], and Stanislas Dehaene[1,2,3,9]

[1]Cognitive Neuroimaging Unit, INSERM, Gif sur Yvette, France; [2]NeuroSpin Center, DSV, I2BM, CEA, Gif sur Yvette, France; [3]University Paris 11, Orsay; [4]Center for Mind/Brain Sciences, University of Trento; [5]Unité Mixte de Recherche 7023, Formal Structures of Language, CNRS & Université Paris VIII, Saint-Denis, France; [6]Laboratoire Psychologie de la Perception, Université Paris Descartes; [7]Unité Mixte de Recherche 8158, CNRS, Paris, France; [8]Department of Psychology, Harvard University; and [9]College de France

Abstract

All humans share a universal, evolutionarily ancient approximate number system (ANS) that estimates and combines the numbers of objects in sets with ratio-limited precision. Interindividual variability in the acuity of the ANS correlates with mathematical achievement, but the causes of this correlation have never been established. We acquired psychophysical measures of ANS acuity in child and adult members of an indigene group in the Amazon, the Mundurucú, who have a very restricted numerical lexicon and highly variable access to mathematics education. By comparing Mundurucú subjects with and without access to schooling, we found that education significantly enhances the acuity with which sets of concrete objects are estimated. These results indicate that culture and education have an important effect on basic number perception. We hypothesize that symbolic and nonsymbolic numerical thinking mutually enhance one another over the course of mathematics instruction.

Keywords

perception, sociocultural factors, cross-cultural differences, mathematical ability

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In societies where education is universal, child development, learning, and instruction tend to be inextricably confounded. Their correlation makes it hard to investigate the causes of cognitive change during development. In many domains, competence is present in infancy and continuously improves during childhood, but also correlates with academic scores in school-based tests. Thus, one cannot easily determine how much of this developmental progression relates to brain maturation, to cultural learning, and to schooling.

Here, we approach this question in the number domain. Basic numerical competences, including the ability to estimate and mentally combine approximate numbers of objects in sets, are present very early in life (Izard, Dehaene-Lambertz, & Dehaene, 2008). Such skills (associated with a system in parietal cortex often referred to as the approximate number system, or ANS) follow Weber's law: The extent to which two numerosities can be discriminated is determined by their ratio, which is compatible with the idea that the ANS represents numerosities as random variables, with fixed noise, on a logarithmic scale (Dehaene 2007). The "acuity" of the ANS can therefore be indexed either by the ratio between two quantities that can be discriminated beyond an arbitrary level of accuracy (typically, 75% accuracy; the behavioral Weber fraction) or by the noise of the internal representation of numerosity (the internal Weber fraction, or w), estimated using classical decision-making models (e.g., signal detection models; see Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004, supplementary material, and Dehaene, 2007, for detailed descriptions of the models and equations typically used to retrieve the internal

Corresponding Author:
Manuela Piazza, Center for Mind/Brain Sciences, Functional NeuroImaging Laboratory, 31 Corso Bettini Rovereto, Trento 38068, Italy
E-mail: manuela.piazza@unitn.it

Weber fraction in numerosity-related tasks). The acuity of the ANS undergoes a process of refinement during infancy and through adulthood (for reviews, see Halberda & Feigenson, 2008; Piazza, 2010).

The ANS is thought to be a basic building block for the later cultural construction of abstract, symbolic number concepts (Dehaene, 1997; Gelman & Butterworth, 2005; Gilmore, McCarthy, & Spelke, 2007; Piazza, 2010; Piazza & Dehaene, 2004; but see Butterworth, 2010; Le Corre & Carey, 2007). Indeed, recent studies have revealed significant correlations between ANS acuity and mathematical achievement, in both normally achieving children (Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Mazzocco, Feigenson, & Halberda, 2011b) and dyscalculic children (Mazzocco, Feigenson, & Halberda, 2011a; Mussolin, Mejias, & Noel, 2010; Piazza et al., 2010; Price, Holloway, Rasanen, Vesterinen, & Ansari, 2007). However, because correlations do not prove causation, to date there is no definitive proof of a causal link between the ANS and math proficiency. The ANS may play a causal role in determining later proficiency in mathematics at school, but, conversely, the cultural acquisition of symbolic numbers and arithmetic may also enhance ANS acuity (Mussolin et al., 2010; Piazza et al., 2010). An analogous circular causality pattern is present in the reading domain; phonological abilities determine reading competence but are also improved by literacy (Bradley & Bryant, 1983; Hulme, Bowyer-Crane, Carroll, Duff, & Snowling, 2012; Morais, Cary, Alegria, & Bertelson, 1979).

To partially resolve the issue of whether there is a causal link between math proficiency and the ANS, we acquired psychophysical measures of ANS acuity in Mundurucú subjects. The Mundurucú are an indigenous population living in an autonomous territory in Para, Brazil. The Mundurucú language has a very restricted lexicon for number words and no symbolic system for exact numbers and arithmetic. However, the Mundurucú can perform approximate calculations when dealing with concrete quantities; for example, they can mentally compare the numerosity of two arrays or estimate their approximate sum (Pica, Lemer, Izard, & Dehaene, 2004).

In recent years, Brazilian education programs have become available to the Mundurucú, but access to these programs is highly variable across individuals, and mainly determined by the proximity of homes to the few schools. We studied a group of Mundurucú children and adults that included both individuals who had received no education and those who had received some years of schooling. With this unique source of data, we aimed to disentangle the effects of maturation (indexed by chronological age) and of education on ANS acuity.

Our reasoning was straightforward. If the improvement in ANS acuity that comes with age is driven solely by maturation, then analyses controlling for education level should show that older participants have a more refined ANS than younger participants. On the contrary, if ANS acuity is also influenced by educational factors, then analyses controlling for chronological age should show that educated participants have a more refined ANS compared with uneducated participants.

Method

Main experiment (numerosity comparison)

In our main experiment, 38 Mundurucú participants were tested (ages 4–63; 21 males, 17 females). Their degree of instruction varied from no schooling up to several years of attendance at the local schools. According to reports from both the Mundurucú schoolteachers and the education department at the National Indian Foundation, Mundurucú mathematics instruction begins in the 3rd year of formal schooling. During the 1st year of school (Level 1), pupils learn to speak in Portuguese and to read and write single letters. During the 2nd year (Level 2), they learn how to read and write words and basic sentences. In the 3rd year (Level 3), numbers and basic arithmetical operations are introduced. The subsequent levels follow the classical Brazilian primary-school progression.

Participants reported the highest level of schooling that they had attained, which, when possible, was confirmed by schoolteachers, relatives, and village authorities. Of the tested participants, 14 had never attended school or had attended for only a few months (Level 0), 3 had completed Level 1, 9 had completed Level 2, 6 had completed Level 3, and 6 had completed Level 4 or more.

Stimuli, presented via a solar-powered PC, consisted of pairs of arrays of black dots displayed in two white discs on either side of a central white fixation point. On each trial, one of the two arrays contained either 16 or 32 dots (the reference, hereafter referred to as $n1$). The paired array (the target, hereafter referred to as $n2$) contained between 10 and 22 dots (along the following 10-level continuum: 10, 12, 13, 14, 15, 17, 18, 19, 20, or 22 dots) when $n1$ was 16 and double those quantities when $n1$ was 32. Perceptual variables were randomly assigned to each stimulus pair such that, on average, the size of the dots in the $n2$ array was held constant on half the trials and the total occupied area of the dots in the $n2$ array was held constant on the other half; in the $n1$ arrays, these parameters varied simultaneously such that, across trials, they covered all values assigned to the different $n2$ arrays (see Dehaene, Izard, & Piazza, 2005). Stimuli remained on-screen until participants gave their response, which consisted of pressing the button on the computer keyboard that corresponded to the more numerous set or pointing toward that set, without counting. There were 140 experimental trials, preceded by some training.

Control experiment (size comparison)

We also probed performance in a control size-comparison task, during a second mission to the Mundurucú territory. Thirty-three Mundurucú subjects were tested (ages 4–67; 20 males, 13 females). Nine had received no instruction, 5 had completed Level 1, 4 had completed Level 2, 6 had completed Level 3, and 9 had completed Level 4 or more. The experimental paradigm was the same as in the numerosity-comparison experiment. Pairs of white discs appeared on either side of a central white fixation point. As in the numerosity-comparison experiment, there were two reference stimuli (diameters = 1.8 and 3.6 cm, respectively), each paired with 10 target stimuli, which differed (in diameter) from the reference stimulus along the same 10 ratios as the ones used for the numerosity experiment. Participants were asked to choose the larger disk, responding with a button press or by pointing. The 140 experimental trials were preceded by some training.

Results

As has been observed for Western subjects, Mundurucú subjects' psychometric functions for the percentage of "larger" responses at each $n2$ value (a separate function for each $n1$ value) were Weberian (sigmoidal with identical slopes once plotted on a log scale of the numerical ratio between $n1$ and $n2$). Subjects responded within an average of 3.1 s (SD = 1.7 s), a speed incompatible with exact counting.

For each participant, we recovered the internal Weber fraction w by fitting the psychometric curves for the 16 and 32 references with a single sigmoid function of the log $n1/n2$ ratio (using Formula 24.9 in Dehaene, 2007). Two young participants (age 5) were excluded after the regressions failed to converge, an indication of random responses. Our w measure was reliable (split-half $r = .68$, $p < .01$).

The average w for the group was 0.25 (range = 0.07–0.45; mean accuracy = 71%, range = 51%–91%). For visualization purposes, we divided the sample into five age groups and plotted the average performance in the different age groups (Fig. 1). The average w across participants decreased from 4 to 13 years of age; however, contrary to results for educated Italian adults tested in another study using the very same task and stimuli (Piazza et al., 2010), w in this sample ceased to decrease after age 13. The mean w of Mundurucú adults (age 18 and up) was 0.23, significantly larger than that observed in educated Italian adults (0.15), $t(38) = 2.95$, $p < .01$ (see Fig. 2a for a graphic comparison of results for individual Mundurucú participants and the average across Italian participants).

We then used an analysis of covariance (ANCOVA) with age as the covariate and education level as a between-groups variable to tease apart the effects of age on w from the effects of education.[1] The effect of education level, $F(7, 27) = 3.12$, $p < .05$, accounted for 45% of the variance of w, controlling for age (see Fig. 2b), which had a marginal effect, $F(1, 27) = 4.04$, $p = .055$. The effect of education became evident for the first educational level at which the school curriculum introduces arithmetic (Level 3). Difference contrasts within the ANCOVA confirmed that only at Level 3 did w become significantly lower than w at preceding levels ($p < .01$).

We next focused on the adult Mundurucús, for whom age and education were best separable: The average w for uneducated adult subjects was 0.31, twice the value of 0.15 observed in educated Italian adults, $t(25) = 5.12$, $p < .01$, but not different from the average w of Italian kindergartners, $p = .29$ (Piazza et al., 2010). The value of w dropped to 0.19 in adult Mundurucús who went to school for at least 1 year. Thus, there was a highly significant difference between w in this group and that in uneducated Mundurucús, $t(18) = 2.69$, $p < .05$ (see Fig. 3), even though the two groups did not differ in age, $t(18) = -1.32$, $p = .20$, or in overall response times, $t(18) = 1.11$, $p = .28$. The ANS acuity in Mundurucús who received some education did not differ from that of Italian adults, $p = .13$.

Finally, we explored the effect of language. In our sample, education and bilingualism were highly correlated ($r = .71$), and their effects inseparable. However, we knew that 20 of the 24 monolingual subjects (who could speak only Mundurucú) could recite some numbers in Portuguese: Six subjects could recite numbers only up to 5, and 14 could count at least up to 10. This difference did not significantly affect w (0.33 for the former group and 0.30 for the latter; $p = .53$).

We used the size-comparison task to verify that the effect of education was specific to the numerosity-comparison task. Performance on the size-comparison task varied consistently with the ratio of the sizes, which allowed extraction of w. On average, w was much lower for size (group average = 0.04, range = 0.002–0.10; accuracy = 95%, range = 86%–99%) than for numerosity, which indicated a higher sensitivity to differences in size compared with number. However, the size w varied considerably across subjects, and decreased throughout the life span, from 0.059 in children (< 10 years) to 0.029 in older adults (> 40 years). An ANCOVA with age as the covariate and education as a between-groups variable showed that, unlike in the numerosity task, there was no effect of education over and above age, $F(5, 26) = 0.77$,

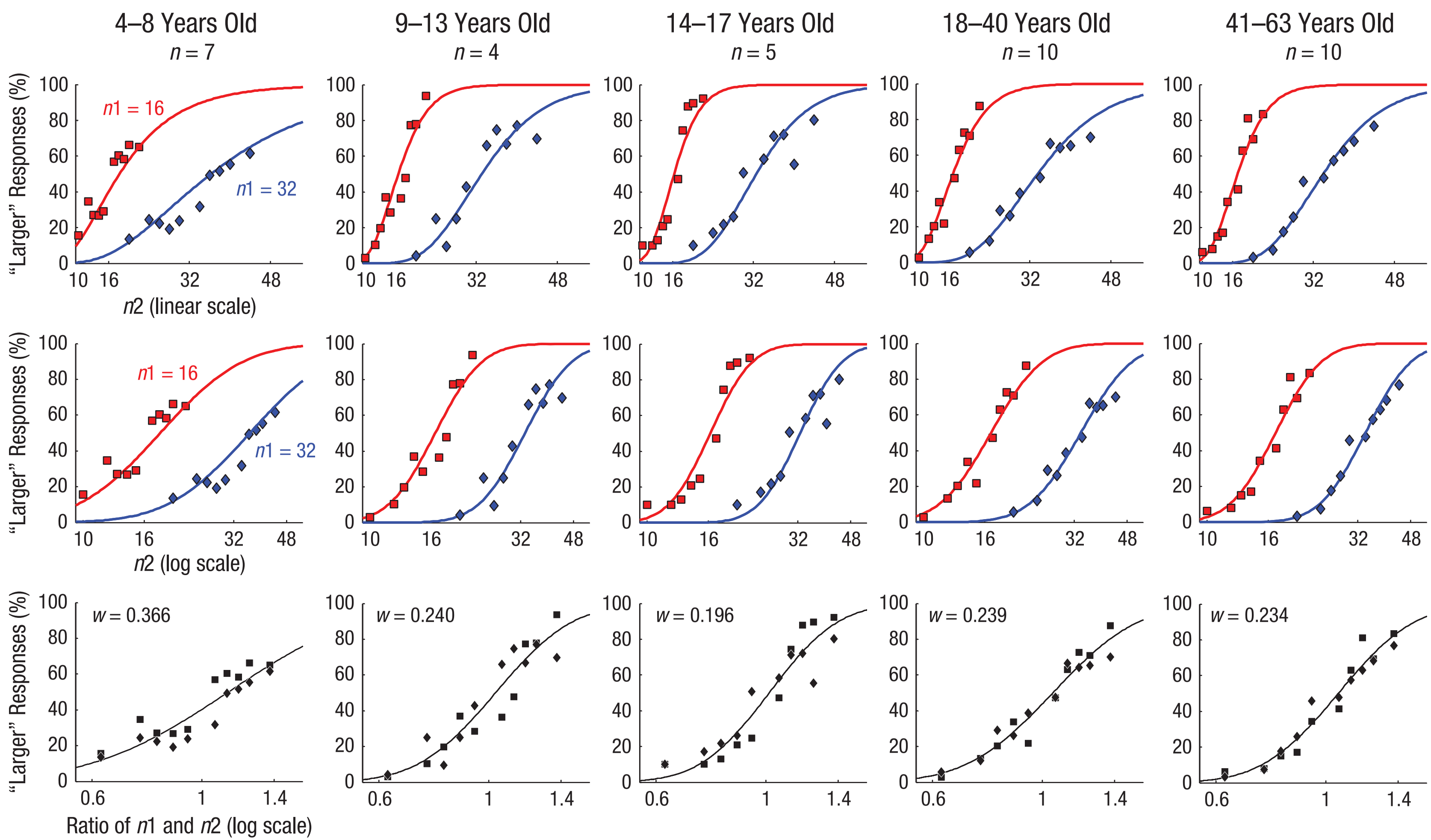

Fig. 1. Results of the numerosity-comparison experiment. In the top row, the percentage of responses in which the comparison array ($n2$) was reported as larger than the reference array ($n1$) is plotted as a function of $n2$ numerosity, separately for each of five age groups (red squares: $n1 = 16$; blue diamonds: $n1 = 32$). The normal cumulative-distribution fitting functions (with a variable mean and standard deviation) for the two values of $n1$ (red and blue lines, respectively) became parallel when responses were plotted on a log scale of $n2$ (middle row), and they became superimposable when responses were plotted as a function of the log $n1/n2$ ratio (bottom row; black squares: $n1 = 16$; black diamonds: $n1 = 32$; black line: single fitting function). Thus, performance obeyed Weber's law.

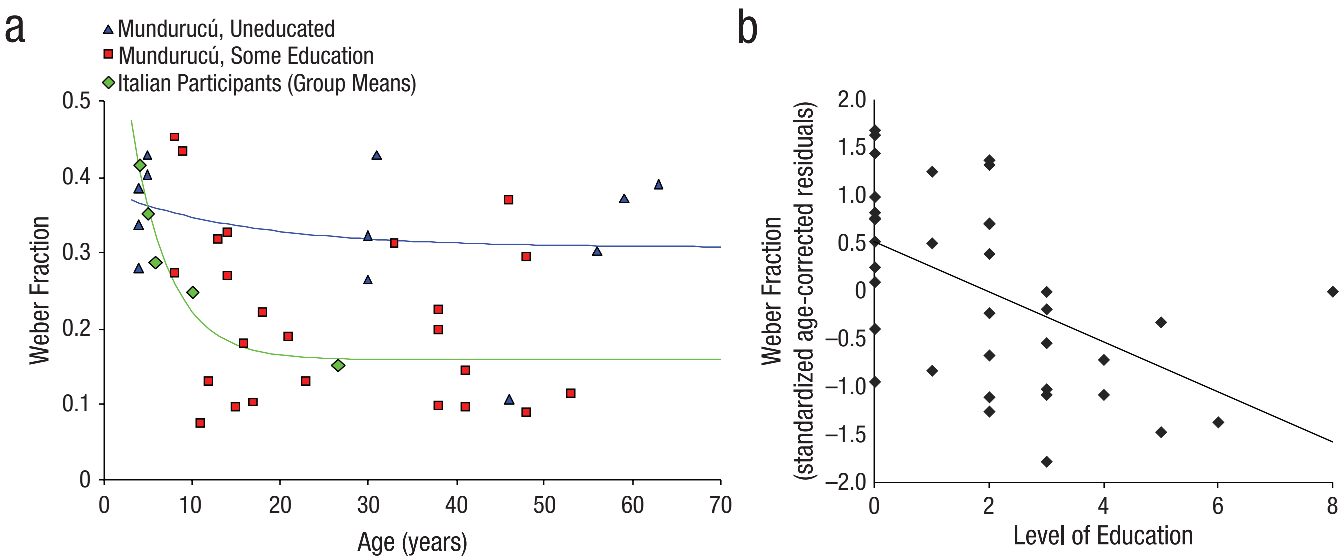

Fig. 2. Changes in the Weber fraction with age and education. In (a), the Weber fractions of individual Mundurucú participants with and without formal education are plotted as a function of age. For reference, the Weber fractions of five age groups of Italian participants (4-, 5-, 6-, and 10-year-olds and adults; Piazza et al., 2010) are also shown. The data points for Mundurucú who received no education and for Italian participants have been fitted with decreasing exponential functions with a variable decay rate and asymptote. In (b), individual Mundurucú participants' Weber fractions (age corrected) are plotted as a function of education level; the best-fitting regression line is also shown.

p = .58; the effect of age was again only marginal, $F(1, 26) = 3.33$, $p = .08$. Indeed, the size w was equal for Mundurucú adults with and without education (0.033 and 0.034, p = .91). The distinct effects of education on size- and numerosity-comparison performance was confirmed by a significant Task × Education interaction, $F(5, 54) = 2.87$, $p < .05$, in an ANCOVA with age as the covariate, and with task and education level as between-subjects variables.

Discussion

By studying a remote Amazonian population, we separated the effects of education and age on the acuity of the ANS. These effects are nearly impossible to separate in societies where virtually all children receive an early education in counting and arithmetic. Previous research established that an approximation strategy is available to both Mundurucú and Western subjects. We developed a finer-grained experimental design to quantify its precision at the individual level. The results indicate that education is associated with a significant increase in the acuity of the ANS, and that this relationship is independent of maturation. Effects of education were observed especially for Mundurucú participants who had advanced far enough in the educational system to receive instruction in symbolic enumeration and arithmetic.

In members of industrialized societies, the developmental trajectory of the ANS is characterized by an initial sharp improvement followed by progressively smaller

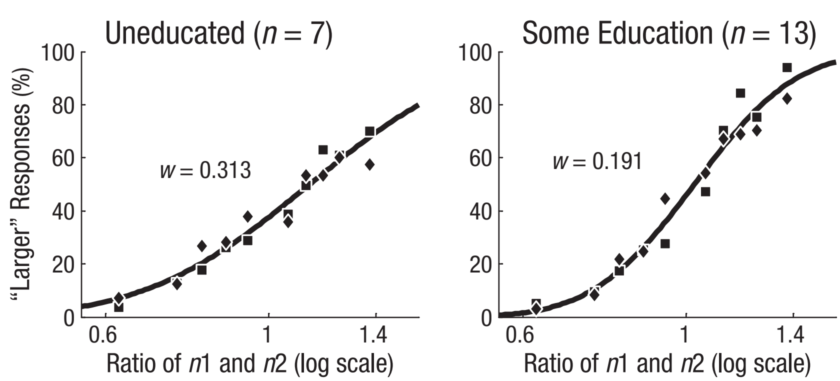

Fig. 3. Performance of educated (left) and uneducated (right) Mundurucú adults on the numerosity-comparison task. The graphs show individual data points along with best-fitting regression lines.

but long-lasting change (Halberda & Feigenson, 2008; Piazza et al., 2010). Although the initial improvement likely reflects intrinsic maturational and sensory factors, the present data suggest that the later ANS improvements are almost entirely imputable to education: In Mundurucú subjects not exposed to formal education, number acuity ceases to increase beyond the level reached by North American and European children at about 6 years of age, around the time when formal schooling starts.

The value of w for the uneducated Mundurucú adults (0.31, twice the value observed in educated Italian adults) is also comparable to the observed w of a group of Italian dyscalculic children (0.35) tested with exactly the same experimental paradigm and procedure (Piazza et al., 2010). This suggests that the impairment in w in dyscalculics (Mazzocco et al., 2011a; Mussolin et al., 2010; Piazza et al., 2010; Price et al., 2007) may be partially a consequence of poor school-based acquisition of numeracy. Such an educational "confound" may well apply to other reports of a correlation between ANS acuity and math achievement (Halberda et al., 2008; Mazzocco et al., 2011b).

Two observations suggest that the effect of education on ANS acuity is not a generic effect of schooling but a specific effect of numeracy instruction. First, the most significant reduction of the Weber fraction was observed at the level of schooling where the current Mundurucú system introduces counting and the arithmetical operations. Second, education had no effect on a nonnumerical perceptual comparison task, which suggests that our results may not be interpreted in terms of improvements of a generic magnitude-representation system (Feigenson, 2007; Walsh, 2003) and, a fortiori, of a generic schooling effect. Taken together, these data suggest that symbolic and nonsymbolic numerical thinking mutually enhance one another over the course of mathematics instruction.

Because the present study did not randomly assign participants to the different groups, however, we cannot completely rule out the possibility that the educated and uneducated subjects differed on variables other than education (even though at the time of testing they had similar occupations and were equally socially integrated). Whether other demographic factors might contribute to the enhancement of the ANS system in educated subjects is a question for future studies. Another important open question is which specific aspects of numeracy influence ANS acuity. Our data suggest that the mere ability to recite the counting list does not suffice to affect the ANS. Longer-term practice with counting, leading to the emergence of a full-blown referential symbolic system (Deacon, 1997), is likely to be necessary to sharpen number sense.

In conclusion, the present study provides evidence that education plays a significant role in sharpening the sense of approximate numerical quantity: Number sense is coarser in a culture without symbols for exact numbers, and it becomes more precise in members of that culture who are introduced to the concepts of exact number and calculation.

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Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

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Note

1. Assumptions underlying ANCOVA were met: First, w was normally distributed at all education levels (Shapiro-Wilk tests, all $ps > .05$), excluding Levels 6 and 8, at each of which there was only 1 subject. Second, the variance of w did not differ across the education levels (Levene's test), $F(7, 28) = 1.380$, $p = .25$. Third, the slopes of the regression functions relating w to age did not differ across the education levels, $F(5, 22) = 0.82$, $p = .55$. Results were also confirmed by a nonparametric ANCOVA performed on the rank-transformed data; this analysis revealed a significant effect of education, controlling for age, $F(7, 27) = 3.36$, $p = .01$.

References

Bradley, L., & Bryant, P. E. (1983). Categorizing sounds and learning to read—a causal connection. *Nature*, *301*, 419–421.

Butterworth, B. (2010). Foundational numerical capacities and the origins of dyscalculia. *Trends in Cognitive Sciences*, *14*, 534–541.

Deacon, T. W. (1997). *The symbolic species: The co-evolution of language and the brain*. New York, NY: W. W. Norton & Co.

Dehaene, S. (1997). *The number sense*. New York, NY: Oxford University Press.

Dehaene, S. (2007). Symbols and quantities in parietal cortex: Elements of a mathematical theory of number representation and manipulation. In P. Haggard, Y. Rossetti, & M. Kawato (Eds.), *Attention & performance XXII: Sensorimotor foundations of higher cognition* (pp. 527–574). New York, NY: Oxford University Press.

Dehaene, S., Izard, V., & Piazza, M. (2005). *Control over non-numerical parameters in numerosity experiments*. Retrieved from http://www.unicog.org/docs/DocumentationDotsGeneration.doc

Feigenson, L. (2007). The equality of quantity. *Trends in Cognitive Sciences*, *11*, 185–187.

Gelman, R., & Butterworth, B. (2005). Number and language: How are they related? *Trends in Cognitive Sciences*, *9*, 6–10.

Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2007). Symbolic arithmetic knowledge without instruction. *Nature*, *447*, 589–591.

Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2010). Non-symbolic arithmetic abilities and mathematics achievement in the first year of formal schooling. *Cognition*, *115*, 394–406.

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, *44*, 1457–1465.

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*, 665–668.

Hulme, C., Bowyer-Crane, C., Carroll, J. M., Duff, F. J., & Snowling, M. J. (2012). The causal role of phoneme awareness and letter-sound knowledge in learning to read. *Psychological Science*, *23*, 572–577.

Izard, V., Dehaene-Lambertz, G., & Dehaene, S. (2008). Distinct cerebral pathways for object identity and number in human infants. *PLoS Biology*, *6*(2), e11. Retrieved from http://www.plosbiology.org/article/info%3Adoi%2F10.1371%2Fjournal.pbio.0060011

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, *105*, 395–438.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011a). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Development*, *8*, 1224–1237.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011b). Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PLoS ONE*, *6*(9), e23749. Retrieved from http://www.plosone.org/article/info:doi/10.1371/journal.pone.0023749

Morais, J., Cary, L., Alegria, J., & Bertelson, P. (1979). Does awareness of speech as a sequence of phones arise spontaneously? *Cognition*, *7*, 323–331.

Mussolin, C., Mejias, S., & Noel, M. P. (2010). Symbolic and nonsymbolic number comparison in children with and without dyscalculia. *Cognition*, *115*, 10–25.

Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences*, *14*, 542–551.

Piazza, M., & Dehaene, S. (2004). From number neurons to mental arithmetic: The cognitive neuroscience of number sense. In M. Gazzaniga (Ed.), *The cognitive neurosciences III* (pp. 865–875). Cambridge, MA: MIT Press.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., & Lucangeli, D. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, *116*, 33–41.

Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. *Neuron*, *44*, 547–555.

Pica, P., Lemer, C., Izard, W., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, *306*, 499–503.

Price, G. R., Holloway, I., Rasanen, P., Vesterinen, M., & Ansari, D. (2007). Impaired parietal magnitude processing in developmental dyscalculia. *Current Biology*, *17*, R1042–R1043.

Walsh, V. (2003). A theory of magnitude: Common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences*, *7*, 483–488.