

Effect of asymmetry of incident wave on the maximum runup height

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ABSTRACT

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Shoaling and runup of long waves on a beach form a classical task for coastal oceanography and engineering. Though many empirical and theoretical formulae have been developed in this field, most of them are targeted to typical waves and situations, while the greatest hazard is caused by extreme events, such as, for example, extreme storms and catastrophic tsunamis. From this point of view it was shown theoretically that one of the most important parameters which influence the wave runup height is the steepness of the incident wave front and the asymmetry of the incident wave. It helped to provide a simple explanation to the extreme runup observed during the catastrophic 2004 Indonesian tsunami event. However, the theoretical results were obtained under many assumptions (ideal fluid, no wave breaking, no bottom friction) and have not been validated. Here we present an experimental study performed in the Large Wave Flume (GWK), Hannover, Germany, which is focused on the influence of the asymmetry caused by the non-linear deformation of incident waves on their runup on a plane beach. The series of experiments are aimed to validate the theoretical formulae for runup height of asymmetric waves. Obtained results are in a good agreement with theoretical predictions and corresponding formulae are recommended to be considered in wave forecasts.

ADDITIONAL INDEX WORDS: *Nonlinear shallow water theory, physical experiment, extreme wave runup, nonlinear waves, long waves, Large Wave Flume (GWK).*

INTRODUCTION

The reliable estimate of the wave runup height on a beach is a key problem of coastal oceanography and engineering. Due to the decreasing water depth in the coastal areas, the nonlinear shallow water equations are often considered as the appropriate theoretical model for description of wave transformation in the coastal zone and their runup on a beach. First rigorous mathematical results for long wave runup were obtained by Carrier and Greenspan (1958) for the beach of a constant slope. After their study, several exact explicit analytical solutions to this problem for various shapes of incident waves have been found. The main difficulty in this problem is the implicit form of the hodograph transformation, which is used for solving nonlinear shallow-water equations. For that reason the detailed analysis of the wave field usually requires numerical methods. Even in the asymptotic limit, when the incident wave is defined far from the shore, runup characteristics are determined by certain integral expressions depending on the wave shape. Various shapes of the periodic incident wave trains such as sine waves (Kaistrenko *et al.*, 1991) and cnoidal waves (Synolakis, 1991) were analyzed in literature. The relevant analysis were also performed for a variety of solitary waves and single pulses such as soliton (Pedersen and Gjevik, 1983;

Synolakis, 1987), sine pulse (Mazova *et al.*, 1991), Lorentz pulse (Pelinovsky and Mazova, 1992), Gaussian pulse (Carrier *et al.*, 2003), and *N*-waves (Tadepalli and Synolakis, 1994). Various, more general “bell-shape” pulses was considered in (Didenkulova *et al.*, 2007a; 2008), where it was proved that the runup of all these symmetric pulses is similar and can be parameterized by means of the basic wave parameters, such as wave length, wave height and the wave travel distance to the shore.

At the same time it was shown in (Didenkulova *et al.*, 2006a; 2007b) that the wave asymmetry plays crucial role for the runup height. Asymmetric waves penetrate over larger distances inland and with greater velocities than the symmetric ones. This effect was also found to exist at some convex beaches (Didenkulova *et al.*, 2009b; Didenkulova and Pelinovsky, 2010).

It is evident that these theoretical results, which are obtained under many assumptions (inviscid fluid, absence of the wave breaking, absence of interaction with air flows and bottom sediments), should be validated in the laboratory experiment. And this work represents an experimental study of the influence of the asymmetry and nonlinearity of incident waves on their runup on a plane beach, performed in the Large Wave Flume (GWK), Hannover, Germany, which is aimed to validate the theoretical results. The flume has a 251 m long section of the constant depth which is attached to the 1:6 slope. Sinusoidal waves of the same period but different amplitudes will transform differently during

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its propagation along the section of constant depth and will gain different asymmetry by the beginning of the slope.

THEORETICAL PREDICTIONS

In this section we will briefly describe the theoretical results for the wave runup on a beach. For more details see (Didenkulova et al., 2007b; Didenkulova, 2009).

As it has been stated in Introduction, due to the decreasing water depth in the coastal zone, the dynamics of a wave, climbing the beach can be described in the framework of the shallow water equations. Indeed, all kind of waves become long near the coast. The classical nonlinear shallow water equations for water waves in the ideal fluid with linearly sloping bottom (Figure 1) are:

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} [(-\alpha x + \eta)u] = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \eta}{\partial x} = 0, \tag{2}$$

where x is horizontal coordinate, t is time, $\eta(x,t)$ is the surface displacement, $u(x,t)$ is the depth-averaged velocity, g is the gravity acceleration, and α is the bottom slope.

It is convenient to rewrite Eqs. (1, 2) through their Riemann invariants and to apply the hodograph transformation to the resulting equations (Carrier and Greenspan, 1958). Doing so leads to the linear wave equation with respect to the wave function Φ

$$\frac{\partial^2 \Phi}{\partial \lambda^2} - \frac{\partial^2 \Phi}{\partial \sigma^2} - \frac{1}{\sigma} \frac{\partial \Phi}{\partial \sigma} = 0, \tag{3}$$

where the new (generalized) coordinates λ and σ have been introduced and all variables can be expressed through the wave function $\Phi(\sigma,\lambda)$ as follows:

$$\eta = \frac{1}{2g} \left(\frac{\partial \Phi}{\partial \lambda} - u^2 \right), \tag{4}$$

$$u = \frac{1}{\sigma} \frac{\partial \Phi}{\partial \sigma}, \tag{5}$$

$$t = \frac{1}{\alpha g} \left(\lambda - \frac{1}{\sigma} \frac{\partial \Phi}{\partial \sigma} \right), \tag{6}$$

$$x = \frac{1}{2\alpha g} \left(\frac{\partial \Phi}{\partial \lambda} - u^2 - \frac{\sigma^2}{2} \right), \tag{7}$$

Since σ is proportional to the square root of the depth of the water column

$$\sigma = 2\sqrt{g(-\alpha x + \eta)}, \tag{8}$$

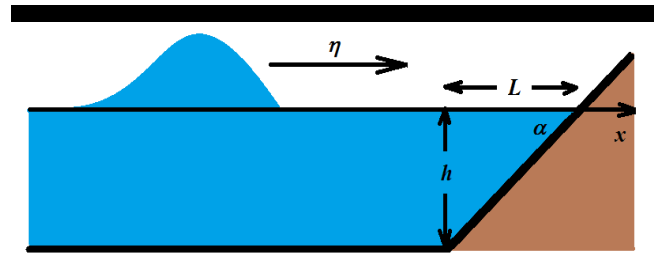


Figure 1. Sketch of geometry for the considered runup problem.

and the point $\sigma=0$ corresponds to the moving shoreline, it is sufficient to solve wave equation (3) on the semi-axis ($0 \leq \sigma < \infty$) with some initial or boundary conditions offshore.

The above-cited literature contains various examples of studies of long wave runup on the plane beach using the hodograph transformation. The well-known bounded solution of the linear wave equation (3) for the runup of a sine wave with frequency ω is expressed in the Bessel functions

$$\Phi(\sigma, \lambda) = \frac{2g^2 \alpha}{\omega} R_0 J_0 \left(\frac{\omega \sigma}{g \alpha} \right) \cos \left(\frac{\omega \lambda}{g \alpha} \right), \tag{9}$$

where R_0 is the maximal runup height, J_0 is the Bessel function of the first kind.

Far from the shoreline the wave field can be presented asymptotically as the superposition of two sine waves of equal amplitude propagating in the opposite directions

$$\eta(x,t) = A(x) \left\{ \sin \left[\omega(t - \tau) + \frac{\pi}{4} \right] + \sin \left[\omega(t + \tau) - \frac{\pi}{4} \right] \right\}, \tag{10}$$

where the instantaneous wave amplitude $A(x)$ is

$$A(x) = R_0 \left(\frac{\alpha g}{16\pi^2 \omega^2 |x|} \right)^{1/4}, \tag{11}$$

and the propagation time of this wave over some distance in a fluid of variable depth is

$$\tau(x) = \int \frac{dx}{\sqrt{gh(x)}}. \tag{12}$$

The maximum amplitude R_0 of the approaching wave with the wavelength λ_0 , determined from the shallow-water dispersion relation

$$\omega = 2\pi \sqrt{gh(L)} / \lambda_0, \tag{13}$$

and the initial amplitude A_0 at the fixed point $|x| = L$ can be found from Eq. (11):

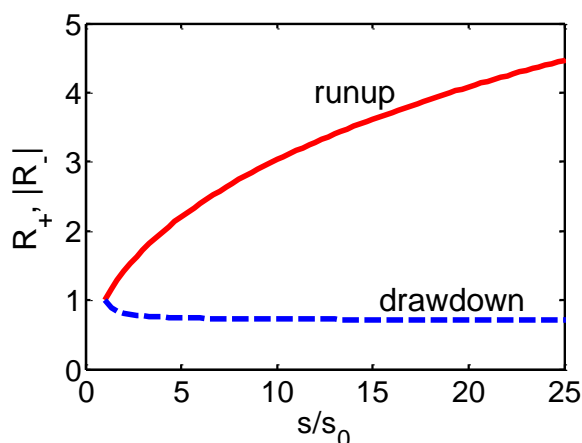


Figure 2. Dependence of extreme runup (R_+) and drawdown (R_-) heights (dimensionless variables) on the wave steepness of deformed wave s with respect to the steepness of the corresponding sine wave s_0 of the same amplitude and period.

$$\frac{R_0}{A_0} = \left(\frac{16\pi^2 \omega^2 L}{g\alpha} \right)^{1/4} = 2\pi \sqrt{\frac{2L}{\lambda_0}}. \quad (14)$$

Using Fourier superposition of solutions (9) of the linear Eq. (3), we can find solutions of the initial problem for nonlinear waves of arbitrary shape (see Didenkulova *et al.*, 2007b; 2008 for details).

An important class of incident waves of specific interest form asymmetric waves, with the steepness of front slope exceeding that of the rear slope. Such waves are naturally formed within the framework of the classical solution of the nonlinear shallow water theory in a basin of a constant depth (Didenkulova *et al.*, 2007b). The shape of waves in this framework is gradually deformed due to the difference in speed of the crest and trough so that a wave gains and increases asymmetry as it propagates. As a result, the wave approaching the beach frequently has a very steep front.

The runup properties of such asymmetric waves on a plane beach are substantially different from those of symmetric waves. They are studied within the above-described approach using a spectral presentation of the nonlinear deformed wave (Didenkulova *et al.*, 2006b; 2007b). Dimensionless runup and drawdown amplitudes (normalized by R_0) behave differently and turn out to be functions of wave steepness (normalized by s_0 – the steepness of the sine wave) only. The drawdown amplitude (Figure 2) weakly depends on the wave steepness. The difference from the drawdown caused by a sine wave is no more than 30%, and Eq. (14) can still be used for express evaluation of its magnitude.

The runup height, R_+ , however, strongly depends on the wave steepness and can be approximated by (Didenkulova *et al.*, 2007b).

$$R_+ = \frac{R_{\max}}{A_0} = 2\pi \sqrt{\frac{2Ls}{\lambda s_0}}. \quad (15)$$



Figure 3. Large Wave Flume (GWK) in Hannover, Germany (a view from the slope).

It tends to infinity for shock wave within given model of the non-breaking wave. The realistic runup is still limited by wave breaking.

Eq. (15) shows that the wave steepness is the most significant parameter of the runup process. Further, Eq. (15) confirms that from all the waves of a fixed height and length from the class of waves in question, the wave with steepest front penetrates inland to the largest distance, and that all asymmetrical waves with the front steeper than the back create larger flooding than a wave with a symmetrical shape. Many examples of extremely strong penetration of tsunami waves to inland (including observations during the 2004 Indonesian and 2011 Japanese tsunamis) can be interpreted as the confirmation of the important role of the wave steepness.

EXPERIMENT

In order to check the described theoretical results the corresponding experiment was set up in a 307 m long, 7 m deep and 5 m wide Large Wave Flume (GWK) in Hannover, Germany (Figure 3) on 15-26 October 2012.

Table 1. Positions of the wave gauges.

Number of the wave gauge	Distance from the wave maker (m)
TG1	50.0
TG2	51.9
TG3	55.2
TG4	60.0
TG5	140.0
TG6	150.0
TG7	160.0
TG8	161.9
TG9	165.2
TG10	170.0
TG11	180.0
TG12	190.0
TG13	200.0
TG14	210.0
TG15	220.0
TG16	230.0
TG17	240.0
TG18	250.0

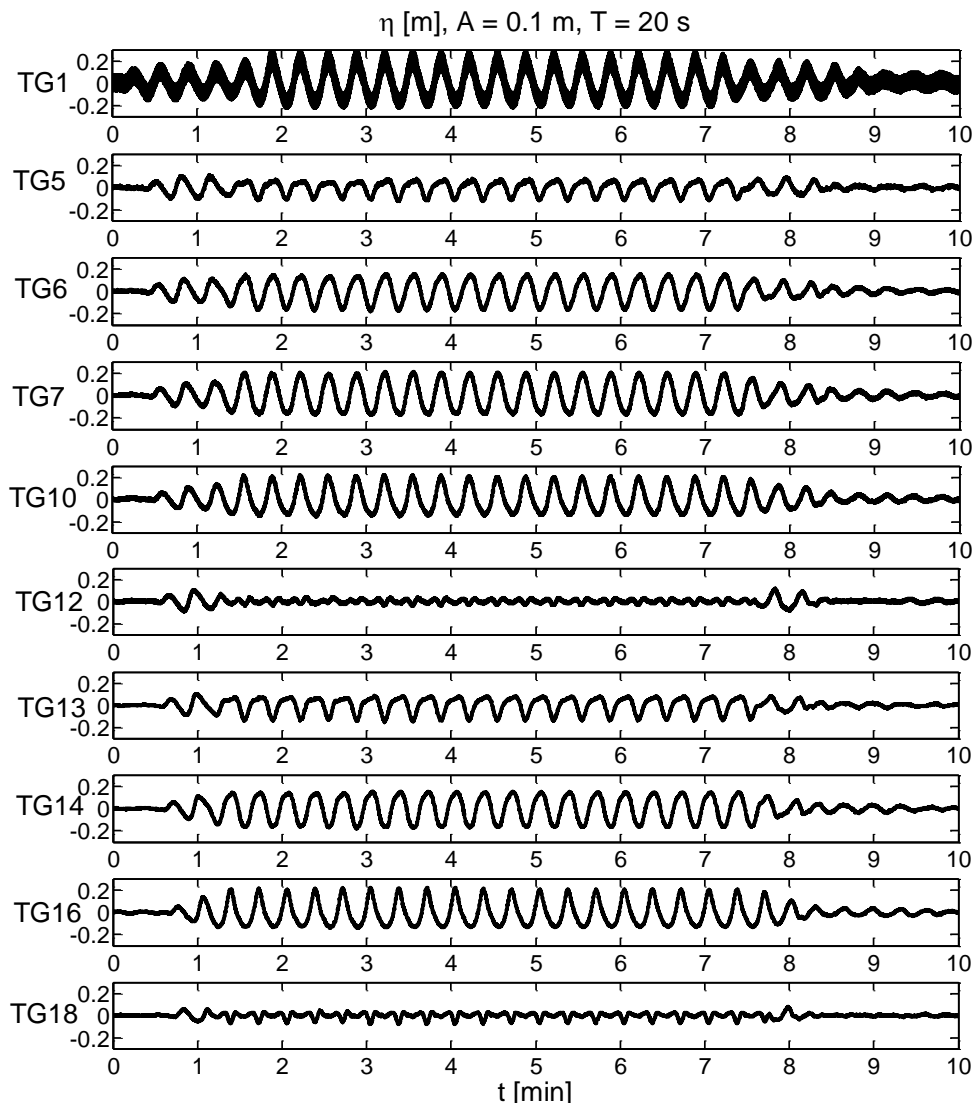


Figure 4. Wave gauge records for the sinusoidal wave of amplitude 0.1 m and of a period 20 s.

The experimental set-up was in agreement with the schematic geometry shown in Figure 1. The water depth was kept constant at $h = 3.5$ m during all tests. The distance between the wave maker and the beginning of the 1:6 slope was 251 m, the horizontal slope length was $L = 21$ m.

A total of 18 wave gauges were mounted along the flume to reconstruct the incident wave field and to study its nonlinear deformation. Their positions along the flume are shown in Table 1. During the tests, two video cameras and a capacitance probe were used to measure wave runup on a sloping beach. The capacitance probe consisted of the two isolated copper wires suspended at 5 mm above the slope. A several volt 100 kHz signal was applied to the one of the wires. The signal from the other wire was measured by a lock-in amplifier and its amplitude was logged with the sampling frequency of 200 Hz (see Denissenko *et al.*, 2011 for details). The signal from wave gauges was also recorded with the sampling frequency of 200 Hz. One video record was used to calibrate runup data measured by the capacitance probe.

Another video record was used for determining the shape of the water surface, which was illuminated by a laser sheet along the direction of wave propagation (see the green line in Figure 3).

We have studied runup of sinusoidal waves of a period of 20 s and of the amplitude varying from 0.025 to 0.3 m, which corresponds to a non-breaking regime. The wave generation was controlled by an online absorption system. This special system works with all kinds of regular and irregular wave trains. Thus, the tests are unaffected by re-reflections at the wave maker and can be carried out over nearly unlimited duration.

Original wave gauge records for the sinusoidal wave of amplitude 0.1 m and of a period 20 s are plotted in Figure 4. After the wave reflects from the slope, the standing wave is formed in the flume as it can be seen from Figure 4 displaying signals from wave gauges located differently with respect to nodes and antinodes (locations of wave gauges along the wave flume are given in Table 1).

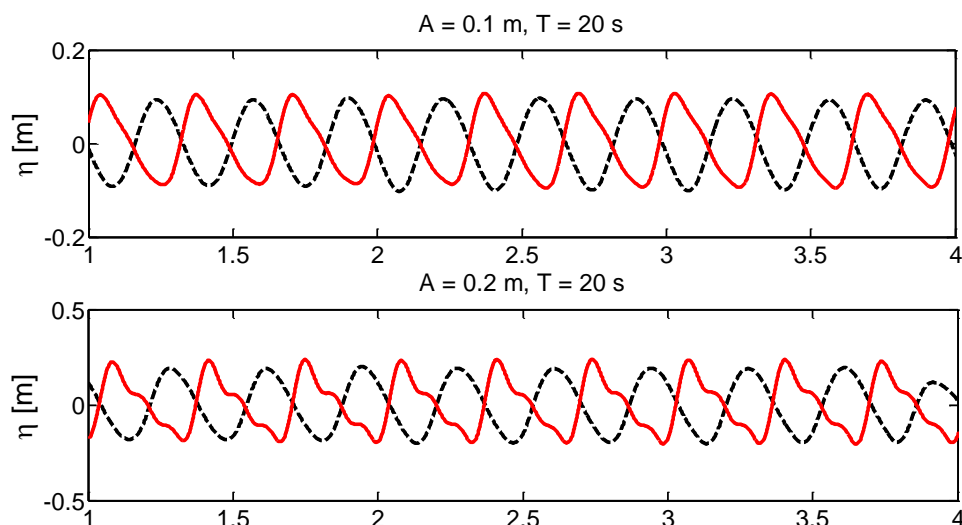


Figure 5. Reconstructed incident wave next to the wave maker (black dashed line) and next to the slope (red solid line) for two different wave amplitudes.

Incident waves near the wave maker and near the slope were reconstructed from the signals recorded by wave gauges using the L~Davis software based on Mansard and Funke method (Mansard and Funke, 1980). Reconstructed incident waves for runs with amplitudes of 0.1 m and 0.2 m are shown in Figure 5. It can be seen that waves become asymmetric by the beginning of the slope and that the wave front is steeper for waves of greater amplitude, which is the manifestation of the nonlinear wave deformation along an even bottom.

The runup (i.e. position of the shoreline) measured by the capacitance probe is plotted in Figure 6. The waveform is similar to the theoretical prediction (see Didenkulova *et al.*, 2007b; Didenkulova, 2009). The mean shoreline position shifts up with respect to the mean sea level and the runup signal becomes more asymmetric with an increase in the amplitude of the incident wave (see Figure 6).

Results for maximum runup and minimum drawdown are shown in Figure 7. Red solid and blue dashed lines correspond to the theoretical calculations of maximum runup and minimum drawdown respectively as it has been demonstrated in the previous section (see also Figure 2). Red circles and blue diamonds indicate corresponding experimental values of extreme runup and drawdown and demonstrate a relatively good agreement with the theoretical curves. The experimental data spread with respect to the theoretical curve

$$\delta = \sqrt{\frac{1}{N} \sum_{n=1}^N (R_n^{exp} / R_n^{th} - 1)^2}, \quad (16)$$

where R^{exp} and R^{th} are experimental and theoretical values

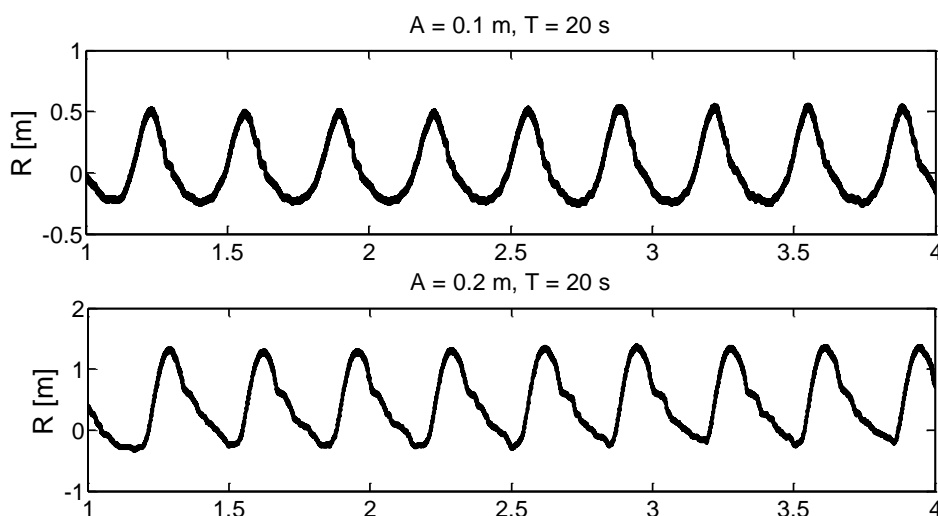


Figure 6. Vertical displacement of the moving shoreline for two different amplitudes of incident waves.

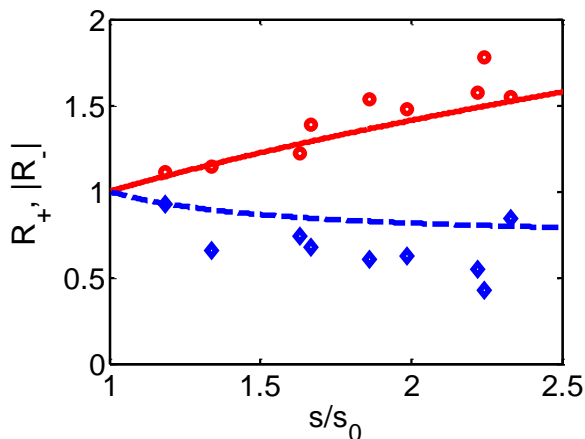


Figure 7. Theoretical dependence of extreme runup, R_+ (red solid line) and drawdown R_- (blue dashed line) dimensionless heights on wave steepness, red circles and blue diamonds correspond to the experimentally measured values.

respectively and N is a number of measurements, is less than 0.03 for runup measurements and less than 0.09 for drawdown that demonstrates a good agreement between the theory and the experiment.

It should also be noted that while the minimum drawdown is overestimated by the theoretical curve, the maximum runup has a tendency to be slightly underestimated, which should also be considered for different wave forecasting systems, for instance, for tsunami early warning systems.

CONCLUSION

The main result of this work is the strong influence of the wave front steepness on the runup characteristics of the long waves, found theoretically and approved experimentally. Among waves of a fixed amplitude and frequency (length), the steepest wave penetrates inland to the largest distance. Consequently, the least dangerous are symmetric sinusoidal waves.

The described influence of the front steepness has been often observed during large tsunamis, for example, during the 2004 Indonesian Ocean and 2011 Japanese tsunamis, when the wave penetrated over an extremely large distance inland and also for waves induced by high-speed ferries, which were able to excite extreme runup (Didenkulova *et al.*, 2009a; Soomere *et al.*, 2009; Torsvik *et al.*, 2009).

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