# EFFECT OF CARDIOID AND LIMAÇON DIRECTIONAL SENSORS ON TOWED ARRAY REVERBERATION RESPONSE

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# 1. INTRODUCTION

Reverberation is often the limiting factor affecting the performance of active sonar systems, so receivers with directional beam patterns are used to improve the echo-toreverberation ratio. Towed horizontal line arrays are often used, but the axial symmetry of their beam pattern gives rise to what is often called "left-right" ambiguity. Elements with directional response can be used to reduce the ambiguity; e.g., the DRDC Atlantic DASM array [Theriault *et al.*, 2006] where the omni-directional elements are augmented with crossed dipole pairs allowing cardioid or limaçon beam patterns to be formed. In this paper, we investigate some of the effects on reverberation response.

#### 2. BEAM PATTERNS

When a receiver beam pattern has azimuthal dependence, the effective beam pattern approach is a useful technique for efficiently computing monostatic reverberation in a range-independent environment [Ellis, 1991]. The effective beam pattern response  $B^*$  at vertical angle  $\theta$  is simply the 3-D beam pattern  $B_{\beta_0}(\beta)$  averaged over all azimuth angles  $\phi$ :

$$B^*_{\beta_0}(\theta) = (2\pi)^{-1} \int_0^{2\pi} \left[ B_{\beta_0}(\theta,\phi) \right] d\phi,$$

where  $\beta_0$  is the towed array beam steering angle,  $\cos \beta = \cos \theta \cos \phi$ , and all angles are measured from forward endfire of the array. The beam patterns for linear arrays with point elements and various weightings are well known; when the elements have identical directional responses  $D(\beta)$ 

and the same weightings, the usual line array beam response is simply multiplied by  $D(\beta)$ .

An omnidirectional element and a horizontal dipole, equally weighted, will produce a broadside cardioid response of

$$D(\beta) = \left[ (1 + \sin \beta)/2 \right]^2$$

By weighting the omni by  $\sin \beta_0$ , the general limacon formula can steer a null in the centre of the ambiguous beam [Franklin, 1984]

$$D(\beta) = \left[ (\sin\beta_0 + \sin\beta) / (2\sin\beta_0) \right]^2,$$

where the denominator normalizes to unit response at horizontal in the beam steering direction.

Fig. 1 shows polar plots of the various responses in the horizontal plane, for a ~15-wavelength array. In the left plot the linear array has equal response (blue solid) at  $60^{\circ}$  and  $300^{\circ}$ ; when multiplied by the broadside cardioid (green dash-dot line), the combined response (red dashed line) has a much reduced response at  $300^{\circ}$ , which is only obvious on a dB plot (middle). In the right plot, the linear array has equal response (blue solid) at  $30^{\circ}$  and  $330^{\circ}$ , the normalized limaçon response (green dash-dot line) has a null at  $330^{\circ}$  and the multiplied response (red dashed line) has a single lobe at  $30^{\circ}$ . Even on a dB plot (not illustrated), the limaçon shows no ambiguous beam.

Fig. 2 shows some effective beam patterns for the broadside beams and beams 60° off broadside, for a ~44-wavelength array. Note at broadside that the cardioid (or limaçon) directionality reduces the response at vertical incidence by



Fig. 1. Polar plots: (left) including cardioid response; (middle) normalized response as a dB plot; (right) including limaçon response.

6 dB, and elsewhere by about 3 dB, producing a much "flatter" effective beam pattern as a function of grazing angle. For off-broadside beams the limaçon response produces a slightly lower and flatter response between the "cusps".



Fig. 2. Effective beam patterns for a 44  $\lambda$  array: (upper) broadside; (lower) 60° from broadside.

### 3. REVERBERATION CALCULATIONS

A normal mode reverberation model with beam patterns [Ellis, 1993, 1995] was used for the calculations. A typical "Pekeris" environment was used: water of depth 100 m, and sound speed 1500 m/s, over a homogeneous bottom halfspace of sound speed 1800 m/s, relative density 2.0, and attenuation 0.36 dB/wavelength. The bottom scattering was Lambert's rule, with a -27 dB strength. A 1.4 kHz CW pulse for a duration of 0.1 s and 10 dB source level was used. The receiving array had 96 elements (omni-dipole pairs) spaced at 0.5 m (~44 wavelengths), with uniform weighting along the array.

Figure 3 shows reverberation predictions using the effective beam patterns of Fig. 2. The horizontal array reduces the reverberation by about 20 dB compared to an omni, and the cardioid response reduces it by about another 3 dB. Even at 60° from broadside, the limaçon produces only a marginal additional reduction in reverberation.

## 4. DISCUSSION

Previous calculations have illustrated results for horizontal line arrays with omni-directional elements. In this paper, calculations are presented for towed array beam patterns with cardioid or limaçon directional sensors replacing the omni-directional elements. As one would expect, the cardioid sensors reduce the reverberation by about 3 dB, and suppress the ambiguous beam, particularly near broadside. The limaçon sensors, even with the null steered in the direction of the ambiguous beam, provide only a marginal improvement in the predicted reverberation. They, however, reduce the ambiguous beam much more effectively than the cardioid, and so are very useful for target detection [Theriault *et al.*, 2006].



Fig. 3. Reverberation predictions: (upper) omni and broadside beams; (lower) omni and beams 60° from broadside.

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