Effect of Channel Estimation Error in OFDM-Based WLAN

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***Abstract*—In this letter, we examine the performance degradation due to the channel estimation error in orthogonal frequency division multiplexing (OFDM)-based wireless LAN (WLAN).The average effective SNR and average bit error probabilities (BEPs) are derived in a Rayleigh fading channel.**

***Index Terms*—Channel estimation, OFDM, wireless LAN.**

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is an efficient technique for mitigating the effects of delay spread in multipath wireless channels. The frequency-selective distortion of the channel is compensated by the zero-forcing (ZF) equalization (one-tap equalization) using estimated channel responses in frequency domain.

In OFDM-based wireless LAN (WLAN) [1], [2], the timing/frequency synchronization and the channel estimation are performed using the short and long preamble which are located in the header of each transmitted packet. The packet error performance is affected by the channel estimation error, which is introduced by the additive noise in the long preamble. Moreover, the received long preamble is probably corrupted by the intercarrier interference (ICI) due to the residual frequency offset.

In this letter, we analyze the effect of channel estimation error in OFDM-based WLAN PHY. To evaluate the performance degradation, we derive the average effective SNR and the bit error probability (BEP) in a Rayleigh fading channel.

II. SIGNAL MODEL AND THE EFFECT OF FREQUENCY OFFSET

The output symbol of the OFDM transmitter is given by the N point complex modulation sequence

$$x_n = \frac{1}{\sqrt{N}} \sum_{k \in \mathcal{K}} X_k e^{j2\pi nk/N} \tag{1}$$

where X_k is a data symbol and $\mathcal{K}$ is a set of the used subcarrier index, i.e., $[-26, 26]$ in OFDM-based WLAN systems.

Given a frequency offset ϵ and a phase offset θ_0, the received OFDM symbol can be expressed by

$$r_n = \frac{e^{j\theta_0}}{\sqrt{N}} \sum_{k \in \mathcal{K}} H_k X_k e^{j2\pi n(k+\epsilon)/N} + w_n \tag{2}$$

where H_k is a channel response of kth subcarrier, and w_n is an additive noise. With perfect symbol timing, the output of the FFT demodulator for ith subcarrier is

$$R_i = \frac{1}{N} \sum_{n=0}^{N-1} r_n e^{-j2\pi ni/N} + W_i = H_i^\epsilon X_i + I_i + W_i \tag{3}$$

where W_i is a frequency domain additive noise, and H_i^ϵ denotes the distorted channel response, which is written as

$$H_i^\epsilon = \frac{H_i \sin \pi\epsilon}{N \sin(\pi\epsilon/N)} e^{j(\pi\epsilon(N-1)/N+\theta_0)}. \tag{4}$$

The ICI due to frequency offset is

$$I_i = \sum_{k \in \mathcal{K}, k \neq i} H_k X_k \frac{\sin \pi\epsilon \cdot e^{j(\pi\epsilon(N-1)/N+\theta_0)}}{N \sin(\pi(k-i+\epsilon)/N)} \cdot e^{-j\pi(k-i)/N}. \tag{5}$$

For a sufficiently large set $\mathcal{K}$, the ICI can be approximated as a zero-mean Gaussian r.v. by central limit theorem [3]. Without loss of generality, we can assume that the channel is normalized and the average channel gain is assumed to be constant, i.e., $E\{|H_i|^2\} = E\{|H|^2\} = 1$. The upper bound of the ICI variance of ith subcarrier can be obtained as follows:

$$\begin{aligned} E\{|I_i|^2\} &= \sum_{k \in \mathcal{K},\, k \neq i} \frac{\sigma_X^2 \sin^2(\pi\epsilon)}{\{N \sin(\pi(k-i+\epsilon)/N)\}^2} \\ &< \frac{2\sigma_X^2 \sin^2(\pi\epsilon)}{N^2} \sum_{m=1}^{M/2} \csc^2\left(\frac{m\pi}{N}\right) \end{aligned} \tag{6}$$

where M is the number of used subcarriers and σ_X^2 is the variance of the transmitted symbol X_i.

The ICI variance is independent of i, so that we denote the normalized ICI variance by $\sigma_{\text{ICI}}^2 = E\{|I_i|^2\}/\sigma_X^2$. Taking the first-order term of the taylor expansion of $\csc^2(m\pi/N)/N^2$, the normalized ICI for small ϵ can be approximated as

$$\sigma_{\text{ICI}}^2 \approx 2(\pi\epsilon)^2 \sum_{m=1}^{\infty} \left(\frac{1}{m\pi}\right)^2 = \frac{(\pi\epsilon)^2}{3}. \tag{7}$$

III. EFFECT OF CHANNEL ESTIMATION ERROR

The channel information is estimated by dividing the demodulated pilot pattern with the known symbol P_i. In this case, the estimated channel response can be

$$\hat{H}_i^\epsilon = H_i^\epsilon + \underbrace{(I_i^P + W_i^P)/P_i}_{\eta_i} \tag{8}$$

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where η_i denotes the channel estimation error of ith subcarrier, and the I_i^P and W_i^P are the ICI and the additive noise in demodulating the received preamble. For sufficiently large M, η_i is assumed to be a Gaussian r.v.

Using the estimated channel response, the ZF equalized symbol can be written as

$$\hat{X}_i = \frac{R_i}{\hat{H}_i^\epsilon} = X_i - \frac{\eta_i X_i}{\hat{H}_i^\epsilon} + \frac{I_i + W_i}{\hat{H}_i^\epsilon}. \tag{9}$$

Assuming that the channel response is stationary during one packet duration and the transmitted symbols are mutually uncorrelated, the instantaneous effective SNR of ith subcarrier is obtained as

$$\gamma_{\text{eff}}(i) = \frac{\left|\hat{H}_i^\epsilon\right|^2 \sigma_X^2}{|\eta_i|^2\sigma_X^2 + \sigma_I^2 + \sigma_W^2} = \frac{|H_i^\epsilon + \eta_i|^2 \overline{\gamma}}{|\eta_i|^2\overline{\gamma} + \sigma_{\text{ICI}}^2\overline{\gamma} + 1} \tag{10}$$

where $\overline{\gamma}$ is the average SNR when there is no ICI, i.e., $\overline{\gamma} = \sigma_X^2/\sigma_W^2$.

The instantaneous SNR can be rewritten as

$$\gamma_{\text{eff}}(i) = \frac{|H_i^\epsilon|^2 + |\eta_i|^2 + 2\Re\{H_i^\epsilon \eta_i^*\}}{|\eta_i|^2 + \sigma_{\text{ICI}}^2 + 1/\overline{\gamma}} \tag{11}$$

where $\Re(x)$ is a real part of a complex value x. Since H_i^ϵ and η_i are uncorrelated, the average effective SNR can be obtained as

$$\overline{\gamma}_{\text{eff}} = \left[\frac{\sin^2 \pi\epsilon}{N^2 \sin^2(\pi\epsilon/N)} - \sigma_{\text{ICI}}^2 - \frac{1}{\overline{\gamma}}\right] \mathcal{I}\left(\sigma_{\text{ICI}}^2 + \frac{1}{\overline{\gamma}}, \sigma_\eta^2\right) + 1 \tag{12}$$

where σ_η^2 is the variance of η_i, and the definite exponential integral $\mathcal{I}(x,y)$ is

$$\mathcal{I}(x,y) = \begin{cases} \dfrac{1}{x}, & \text{for } y = 0 \\ \displaystyle\int_0^\infty \frac{1}{y(t+x)} e^{-t/y} dt = e^{x/y}\Gamma\left(0, \frac{x}{y}\right), & \text{for } y > 0 \end{cases} \tag{13}$$

where $\Gamma(a,b)$ is the incomplete gamma function.

Fig. 1 shows the average effective SNR versus the pilot SNR when the average SNR $\overline{\gamma}$ is 30 dB. In this case, when the pilot SNR is equal to the average received SNR, the expected SNR degradation is about 3 dB.

IV. Bit Error Probability in a Rayleigh Fading Channel

In this section, the performance degradation due to the channel estimation error is analyzed. Note that the effect of the phase rotation due to the residual frequency offset is compensated using the pilot subcarriers in OFDM-based WLAN.

The exact BEP can be obtained as the expectation of Gaussian Q function using the instantaneous SNR in (11). However, it is hard to evaluate, so that we provide an approximation of the average BEP when the channel estimation error exists.

For small σ_η^2, the estimated channel response $\hat{H}_i^\epsilon$ and the channel estimation error η_i are almost uncorrelated. Since the estimated channel response and the channel estimation error are complex Gaussian r.v.s, we can assume that the two r.v.s are independent.

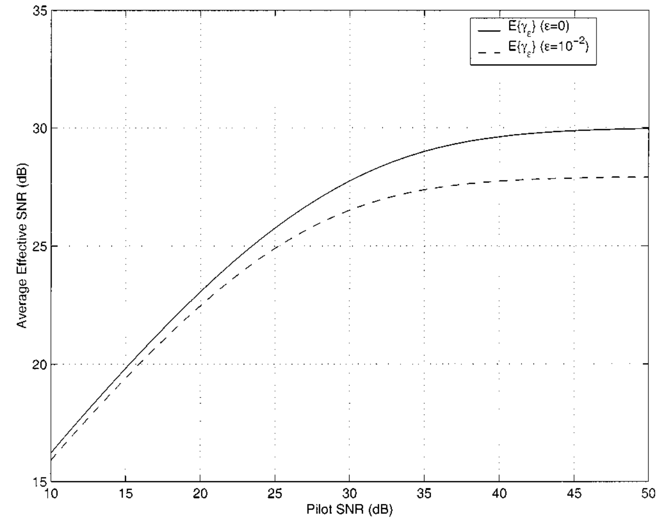

Fig. 1. Degradation due to the effect of channel estimation error ($\overline{\gamma} = 30$ dB).

Denoting $|\hat{H}_i^\epsilon|^2$ and $|\eta_i|^2$ by α and β, α and β are assumed to independent chi-square r.v.s with their mean $\overline{\alpha}$ and $\overline{\beta}$, respectively. Thus, the average BEP in QPSK mode can be approximated as the follows:

$$\begin{aligned} P_b(E) &\approx \int_0^\infty \int_0^\infty Q\left(\sqrt{\frac{\alpha\overline{\gamma}}{\beta\overline{\gamma} + \sigma_{\text{ICI}}^2\overline{\gamma} + 1}}\right) p(\alpha)p(\beta)\, d\alpha\, d\beta \\ &= \int_0^\infty \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\alpha\gamma}}{\overline{\alpha\gamma} + 2(\beta + \sigma_{\text{ICI}}^2)\overline{\gamma} + 2}}\right) p(\beta)\, d\beta \\ &= \frac{1}{2}\left(1 - \sqrt{\frac{\pi\overline{\alpha}}{2\overline{\beta}}} \exp(\xi)\text{erfc}\left(\sqrt{\xi}\right)\right) \end{aligned} \tag{14}$$

where $\text{erfc}(\cdot)$ is the complementary error function and

$$\xi = \frac{\overline{\alpha\gamma} + 2\sigma_{\text{ICI}}^2\overline{\gamma} + 2}{2\overline{\beta\gamma}}. \tag{15}$$

Using the approximation in [4], the average BEP of square M-QAM with Gray bit mapping is

$$\begin{aligned} P_b(E) \approx & \frac{2}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right) \\ & \cdot \left(1 - \sqrt{\frac{3\pi\overline{\alpha}}{2(M-1)\overline{\beta}}} \exp(\xi_M)\text{erfc}\left(\sqrt{\xi_M}\right)\right) \end{aligned} \tag{16}$$

where

$$\xi_M = \frac{3\overline{\alpha}\,\overline{\gamma} + 2(M-1)(\sigma_{\text{ICI}}^2\overline{\gamma} + 1)}{2(M-1)\overline{\beta}\,\overline{\gamma}}. \tag{17}$$

In Fig. 2, we compare the average BEP of QPSK in a Rayleigh fading channel when the channel estimation error exists. The approximated average BEP is very close to the exact BEP, which is obtained Monte Carlo integration. Therefore, using the provided approximation, the BEP when the channel estimation error exists can be evaluated analytically and easily calculated with tightness to the exact BEP.

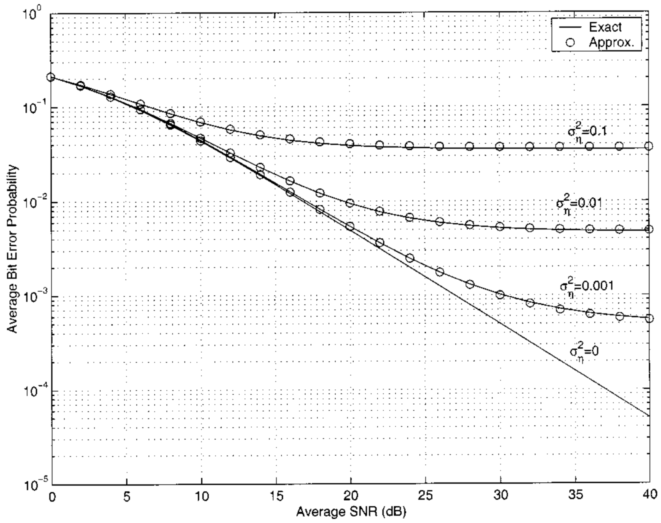

Fig. 2. Average BEP in Rayleigh fading channels (QPSK).

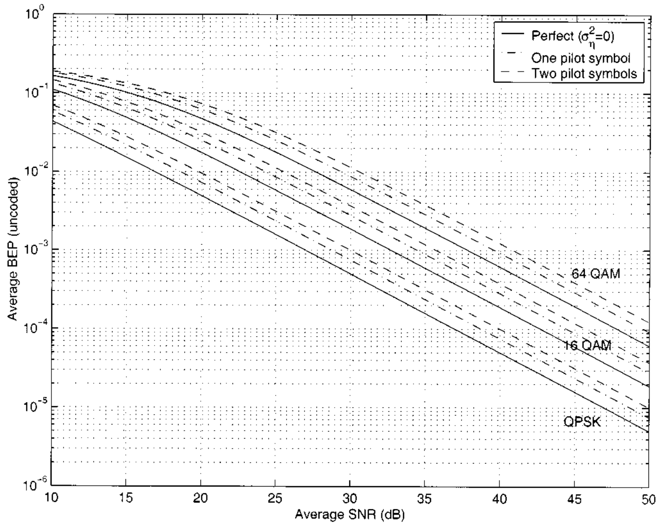

Fig. 3. Average BEP of OFDM-based WLAN in Rayleigh fading channels (uncoded, $\epsilon = 0$).

In OFDM-based WLAN, the preamble and the data symbols are transmitted with the same power, and the channel estimation is performed using one or two pilot symbols in the long preamble, depending on the practical implementation.

In Fig. 3, we investigate the average uncoded BEP of QPSK, 16 QAM, and 64 QAM in OFDM-based WLAN with no residual frequency offset. It can be seen that the performance degradation due to the channel estimation error is about 3 dB

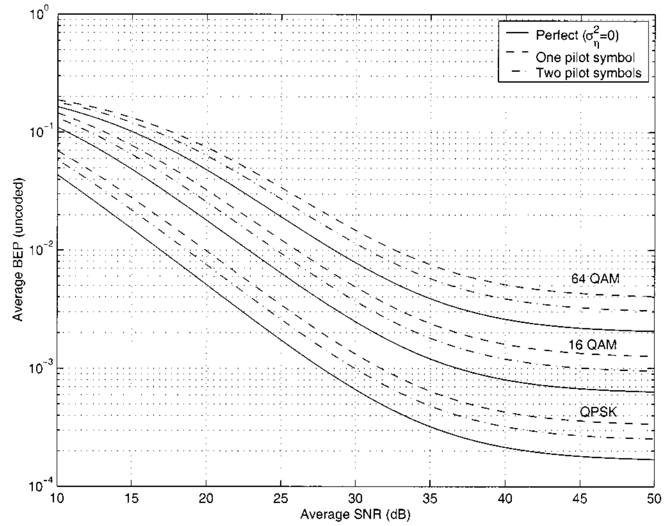

Fig. 4. Average BEP of OFDM-based WLAN in Rayleigh fading channels (uncoded, $\epsilon = 0.01$).

when one pilot symbol is used, and below 2 dB when two pilot symbols are used. Fig. 4 shows that the ICI due to the residual frequency offset ($\epsilon = 0.01$) increases the channel estimation error and causes the error floor.

V. Conclusions

The effect of channel estimation error in OFDM-based WLAN is considered in the average SNR and the average uncoded BEP. The average BEP is approximated analytically under the assumption that the estimated channel response and the channel estimation error are almost uncorrelated when the channel estimation error is sufficiently small. The approximated average BEP shows tightness to the exact average BEP.

The examples provided are for OFDM-based WLAN PHY, and may serve as the basis for practical OFDM PHY design.

References

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