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1997

https://hdl.handle.net/10356/94518

https://doi.org/10.1061/(ASCE)0733-9429(1997)123:8(728)

EFFECT OF CONCENTRATION ON SETTLING VELOCITY OF SEDIMENT PARTICLES

By Nian-Sheng Cheng[1]

ABSTRACT: The drag coefficient for dispersed particles settling in a fluid is related to the particle Reynolds number with the same function as that used for a single particle falling in a clear fluid, after taking into account the effects of concentration both on the effective density of particles and on the viscosity of fluid-sediment mixture. A new method is developed for evaluating the settling velocity of natural sediment particles dispersed in a fluid; it agrees well with the available experimental data. The differences among various empirical expressions are illustrated using the present relationship.

INTRODUCTION

The settling velocity is one of basic properties of sediment particles in a fluid. It depends not only on the size, shape and density of particles, and the density and viscosity of the fluid; but also on the number or concentration of the particles. Much experimental and analytical information on the settling velocity of a single particle is available to guide practical application though the problem is far from being completely solved. However, the case mostly encountered in analyses and predictions concerning sediment transport is that more than a solitary particle falls through a fluid. The presence of other grains will modify the settling velocity of an individual particle in the fluid due to the mutual interference among the the particles. For example, a few closely spaced particles in a fluid will fall faster than a single particle. On the other hand, the settling velocity of the particles uniformly dispersed throughout a fluid will be

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less than that of an isolated identical particle in a clear fluid. The second instance appears most commonly in practice and will be dealt with in this note.

Early investigations regarding the influence of sediment particle concentration on the settling velocity were carried out by Schoklitsch (see Bogardi, 1974) and McNown and Lin (1952). Their results, whilst being quantitatively inconsistent with each other, show that increasing sediment concentration results in reducing in settling velocity of particles. In addition, a survey of the relative motion of fluid and dispersed solid particles in connection with the separation of particles in a suspension or in terms of fluidization has been given by Davidson (1985). Lewis et al (1949) found the applicability to cases of both particle settlement and fluidized-bed of the functional relationship:

$$\frac{w_m}{w} = (1 - c)^n \tag{1}$$

where w = terminal settling velocity of a solitary particle in a fluid; c = sediment concentration by volume; w_m = settling velocity of sediment particles dispersed at the volumetric concentration c; and n = an exponent. Using dimensional analysis, Richardson and Zaki (1951) suggested that n depends on the Reynolds number $Re = wd/v$, where d = diameter of particles and v = kinematic viscosity of fluid; and it approaches different constants for extremely low or high Reynolds numbers, as seen in Fig. 1. Because of its simplicity, (1) has been cited by the subsequent researchers but using various n-values. Fig. 1 shows that the n-values for a given Reynolds number change largely according to different relationships. The difficulty of determining n-values may lead to a low degree of accuracy of prediction associated with (1).

The objective of this note is to theoretically explore the effect of sediment concentration on the settling velocity of uniform cohesionless particles. The changes of n-values in (1) with Reynolds number are then analysed by considering the influences of volumetric concentration and effective density of sediment particles.

THEORETICAL CONSIDERATION

Given a terminal settling velocity w_m of particles dispersed throughout a fluid, the magnitude of relative velocity w' between the falling grains and the fluid is

$$w' = \frac{w_m}{1-c} \tag{2}$$

In a similar manner to the case of solitary settement of particles in the clear fluid, the equilibrium relationship between the drag force and the effective weight force exerting on the particle in a fluid-solid mixture can be written as

$$C_m \frac{\pi}{4} d^2 \frac{\rho' w'^2}{2} = (\rho_s - \rho')g \frac{\pi}{6} d^3 \tag{3}$$

or

$$C_m = \frac{4}{3} \frac{\rho_s - \rho'}{\rho'} \frac{gd}{w'^2} \tag{4}$$

where $\rho' = c\rho_s + (1-c)\rho$ = mean density of the fluid-sediment mixture and C_m = drag coefficient for a grain settling with identical neighbours. It can be found that the drag coefficient for the settlement of particles dispersed in the fluid (C_m) is identical to that for a single particle (C_D) except for the differences of the density between the pure fluid and the fluid-solid suspension. As C_D is well-known to be a function of wd/ν for a particle with a given shape factor, C_m can be expected to be related with the same function to $w'd/\nu'$, where ν' = viscosity of fluid-sediment mixture.

Settling Velocity Formula for a Single Sand Particle

Several formulae are available to evaluate the settling velocity of a solitary particle in a quiescent fluid (Chien and Wan, 1983; Cheng, 1995). Most of them can predict accurately the values of the settling velocity for extremely low and extremely high Reynolds numbers, but nearly all fail to perform well in the transitional region between the viscous-dominated and inertial-dominated flow regimes. Recently, Cheng (1995) derived a simple formula for

determining the settling velocity of natural sediment particles, showing better accuracy than the other published formulae when comparing with the available data bases. The formula is expressed as

$$Re = (\sqrt{25 + 1.2d_*^2} - 5)^{1.5} \tag{5}$$

where $d_* = (\Delta g / \nu^2)^{1/3} d$ and $\Delta = (\rho_s - \rho)/\rho$. Using the drag coefficeint C_D, an equivalent relationship to (5) can be written to be

$$C_D = [(\frac{32}{Re})^{\frac{1}{1.5}} + 1]^{1.5} \tag{6}$$

Settling Velocity of Sediment Particles Dispersed in a Fluid

In view of the above analysis and on the analogy of (6), the drag coefficient C_m is equal to

$$C_m = [(\frac{32}{Re'})^{\frac{1}{1.5}} + 1]^{1.5} \tag{7}$$

where $Re' = w'd/\nu'$. Using (4), one can convert (7) into

$$Re' = (\sqrt{25 + 1.2d_*'^2} - 5)^{1.5} \tag{8}$$

where $d_*' = (\Delta'g / \nu'^2)^{1/3} d$ and $\Delta' = (\rho_s - \rho') / \rho' = (1 - c)\Delta / (1 + c\Delta)$. It can be noted that any application of (7) or (8) in practice depends on information regarding the viscosity ν' of fluid-sediment mixture.

Sha (1965) proposed the following formula to describe the effect of sediment concentration on the viscosity of fluid-sediment mixture.

$$\frac{\nu}{\nu'} = 1 - \frac{c}{c_{max}} \tag{9}$$

where c_{max} = maximum volumetric concentration of sediment. Its value is determined here by using a special model of grain arrangement. Consider a simple packing of identical spheres with a diameter d contained in a cylinder with the same diameter, where each sphere is in contact with

its adjacent spheres. The volume of sphere is $\pi d^3 / 6$. The number of spheres contained in a unit length of cylinder is $1/d$. Thus, the maximum concentration c_{max} is equal to 2/3. Using this value, (9) becomes

$$v' = \frac{2v}{2 - 3c} \qquad (10)$$

The preceding derivation shows that the settling velocity of sediment particles dispersed throughout a fluid can be obtained for the given values of ρ , ρ_s , d, v and c.

Table 1   Summary of Test Conditions (Mints and Shubert, 1957)

No.	No. of tests	Sand diameter (mm)	Viscosity (cm^2/s)	Sand density (g/cm^3)	Average relative error (%)
1	14	0.250	0.0104	2.62	3.3
2	11	0.343	0.0100	2.64	2.4
3	12	0.464	0.0100	2.67	3.2
4	7	0.630	0.0106	2.64	0.5
5	39	0.847	0.0134	2.62	4.9
6	15	0.907	0.0133	2.63	4.8
7	7	1.010	0.0100	2.61	3.2
8	8	1.370	0.0106	2.55	3.5
9	15	2.180	0.0100	2.53	2.4
10	44	3.290	0.0165	2.53	4.3
11	7	3.660	0.0104	2.52	13.7
12	9	5.170	0.0104	2.58	20.3
13	43	5.410	0.0163	2.48	12.7
14	12	8.700	0.0173	2.58	14.7

VERIFICATION

On the basis of the similarity of the fluidized-bed and the settlement of particles dispersed in fluid, a set of data concerning the fluidized-bed experiment conducted by Mints and Shubert (1957) is used in this study to check the validity of the present method for evaluating the settling velocity of sediment particles. The characteristics of all the data are summarised in Table 1. Fig. 2 shows a good agreement in plotting the calculated values of settling velocity against the observed data. The parameter used for describing the accuracy of computation is the average

value of relative error for each run. In Table 1, it can be seen that the relative error is not more than 5 % for sediment particles of diameters less than 3.29 mm but increases for larger sediment particles.

COMPARISON WITH EQUATION (1)

Using (2), (5), (8) and (10), one obtains

$$\frac{w_m}{w} = \frac{2-2c}{2-3c}\left(\frac{\sqrt{25+1.2d_*'^2}-5}{\sqrt{25+1.2d_*^2}-5}\right)^{1.5} \tag{11}$$

In the case of extremely high Reynolds number, (11) reduces to

$$\frac{w_m}{w} = \frac{(1-c)^{1.5}}{\sqrt{1+c\Delta}} \tag{12}$$

For extremely low Reynolds number, (11) can be simplified as

$$\frac{w_m}{w} = \frac{(1-c)^2(2-3c)}{2+2c\Delta} \tag{13}$$

Comparing (11) and (1), the exponent n in (1) can be given in the form of

$$n = \frac{ln(\frac{2-2c}{2-3c})+1.5\,ln\{\dfrac{\sqrt{25+\left(\dfrac{(1-c)(2-3c)^2}{4+4\Delta c}\right)^{2/3}(Re^{4/3}+10\,Re^{2/3})}-5}{\sqrt{25+Re^{4/3}+10\,Re^{2/3}}-5}\}}{ln(1-c)} \tag{14}$$

Eq. (14) indicates that the empirical exponent n in (1) depends not only on the Reynolds number Re, as Richardson and Zaki (1954) suggested, but also on the density coefficient of particles Δ and the volumetric sediment concentration c. Fig. 3 shows the calculated relationship of n versus Re corresponding to (14) in the case of the natural sand particles falling in water. It can be found that the effect of sediment concentration is not negligible. The effective density of sediment particles can result in clear changes in plotting n against Re, as seen in Fig 4. From both Fig. 3 and Fig. 4 in comparison with the empirical curves shown in Fig. 1, the differences among the previous empirical relationships are expected to be caused by the effective density of

particles and sediment concentration.

CONCLUSIONS

A new approach is proposed to evaluate the settling velocity of sediment particles dispersed in a fluid. The calculated results are in good agreement with the published experimental data especially for small sand particles. The detailed analysis based on the present formula shows that the exponent n involved in the empirical relationship (1) is not only a function of the particle Reynolds number, as found by the previous researchers, but also depends on the effective density coefficient of sediment particles and volumetric sand concentration.

APPENDIX I: REFERENCES

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APPENDIX II: NOTATIONS

The following symbols are used in this paper:

c = sediment concentration in volume;

c_{max} = maximum volumetric concentration of sediment;

C_D = drag coefficient for a solitary particle settling in a clear fluid;

C_m = drag coefficient for a grain settling with identical neighbours;

d = diameter of particles;

n = exponent;

Re = wd/ν;

Re' = $w'd/\nu'$;

w = terminal settling velocity of a solitary particle in a clear fluid;

w_m = settling velocity of sediment particles falling with identical neighbours;

w' = relative velocity between grains and fluid;

Δ = $(\rho_s - \rho)/\rho$ = density coefficient of particles in a clear fluid;

Δ' = $(\rho_s - \rho')/\rho'$ = density coefficient of particles in fluid-sediment mixture;

v = kinematic viscosity of pure fluid;

v' = kinematic viscosity of fluid-sediment mixture;

ρ = density of fluid;

ρ_s = density of particles; and

ρ' = mean density of fluid-sediment mixture.

CAPTION FOR FIGURES

Fig. 1 Empirical Relationships of *n* versus *Re*

Fig. 2 Comparison of the Calculated Settling Velocity of Sediment Particles

Subjected to Concentration with the Data of Mints and Shubert (1957)

Fig. 3 Effect of Concentration on the Exponent *n* as a Function *Re* for Quartz

Sand Particles Settling in Water

Fig. 4 The Exponent *n* as a Function *Re* at Volumetric Concentration of 0.1

CAPTION FOR FIGURES
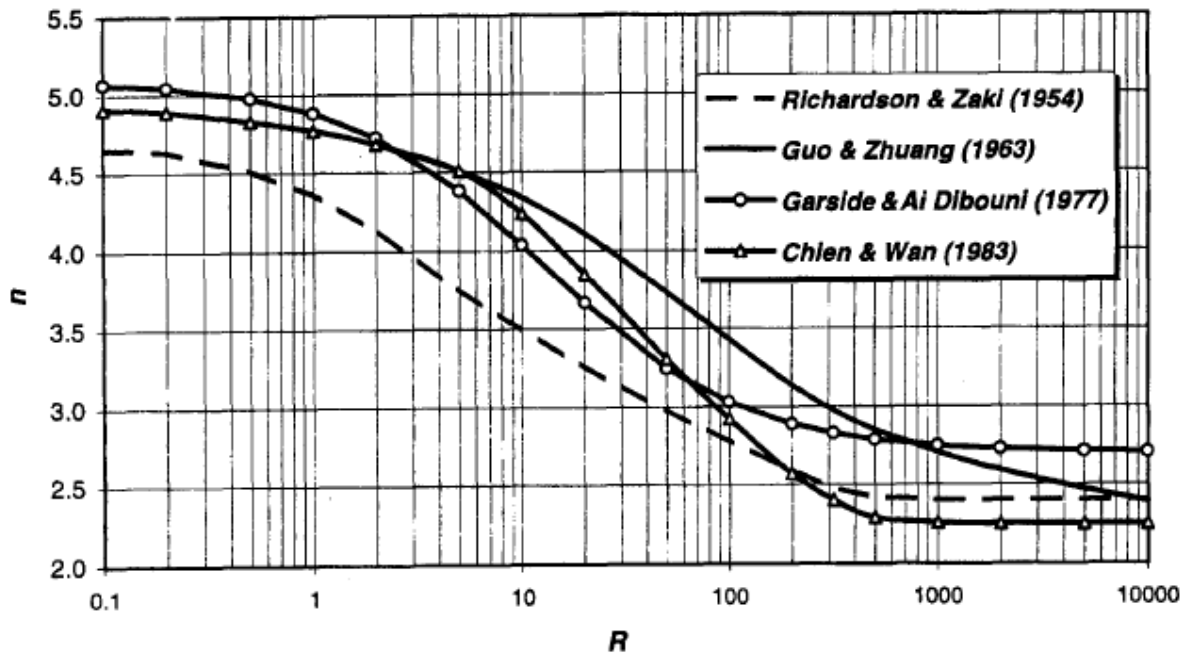
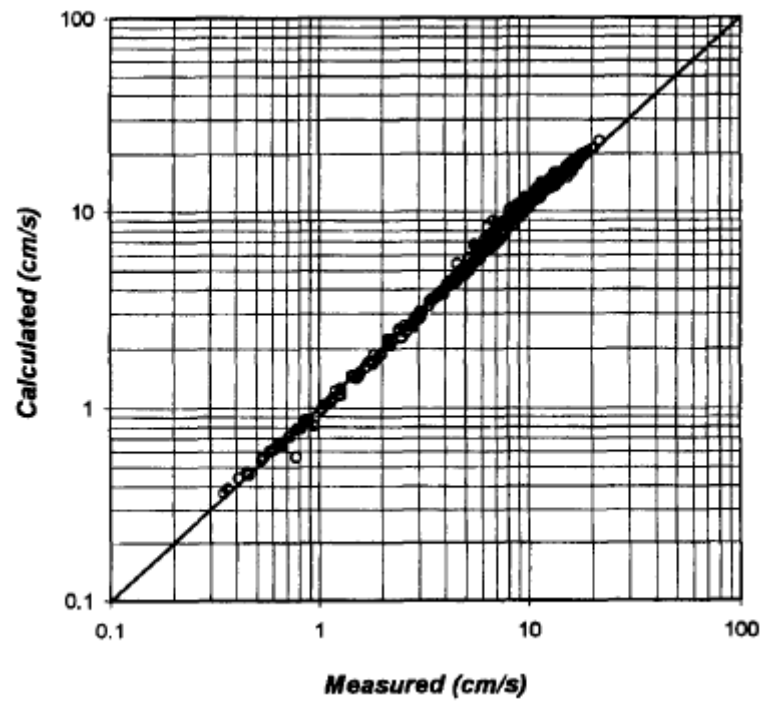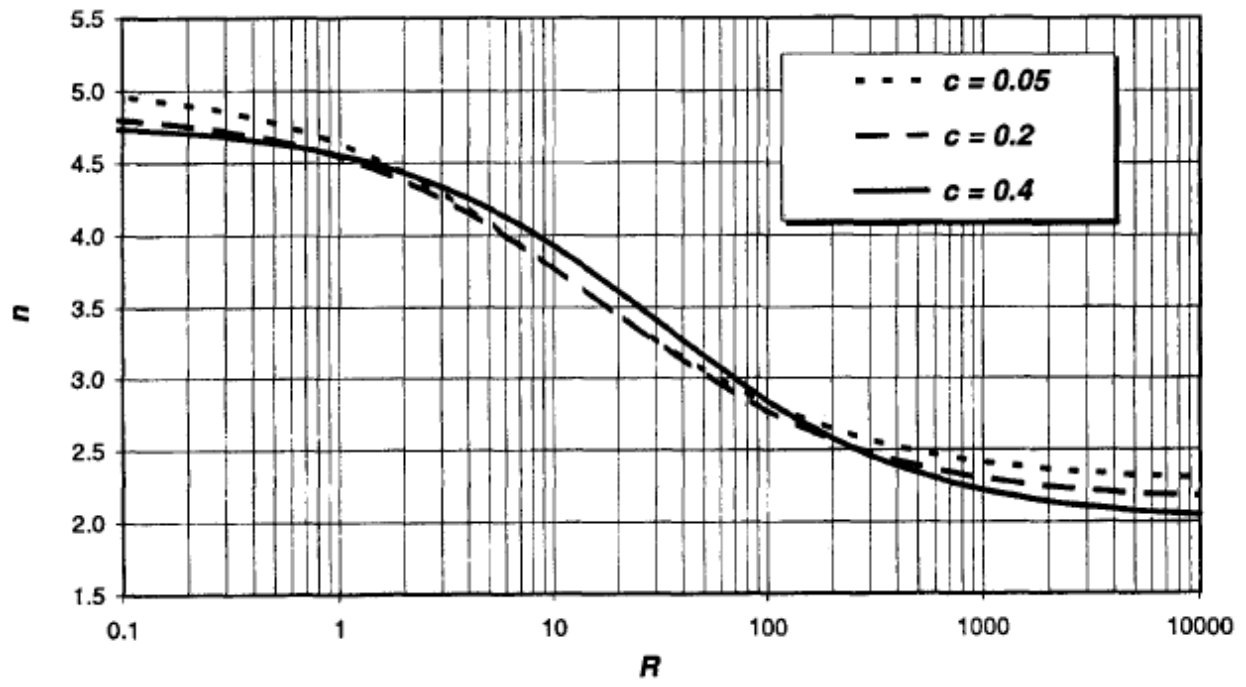**TABLE 1. Summary of test conditions (Mints and Shubert 1957)**

Fig. 1
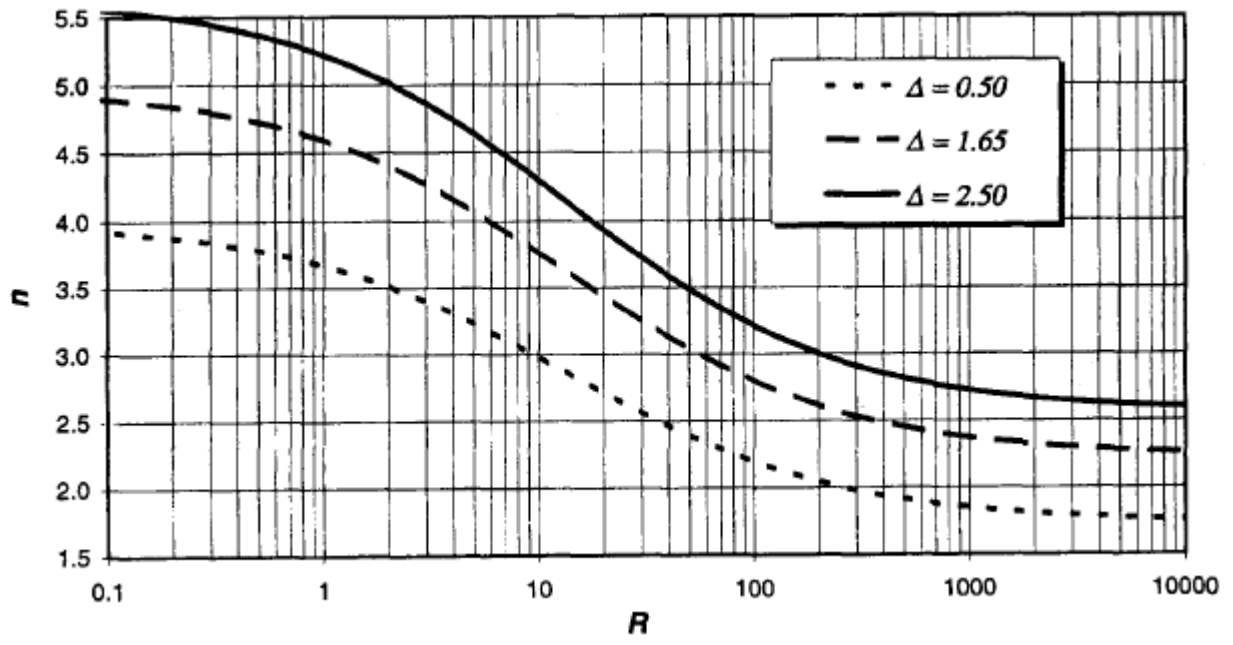
Fig. 2

Fig. 3

Fig. 4

Number (1)	Number of tests (2)	Sand diameter (mm) (3)	Viscosity (cm²/s) (4)	Sand density (g/cm³) (5)	Average relative error (%) (6)
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