# EFFECT OF CONVECTIVE HEAT TRANSFER ON THAWING OF FROZEN SOIL

Virgil J. Lunardini

*U.S. Army Cold Regions Research and Engineering Laboratory*
*72 Lyme Road*
*Hanover, New Hampshire 03755-1290, USA*
*e-mail: lunard@crrel.usace.army.mil*

**Abstract**

Most analyses of the thawing of frozen soil are based on purely conductive heat transfer, a very good assumption in most cases, but vertical and horizontal water flows occur frequently in permafrost regions. The effect of vertical water movement on the rate of thaw and the thermal regime of the soil is quantified. An exact similarity solution only occurs when the vertical water velocity is proportional to the rate of thaw. This solution indicates that seepage flows (the magnitude of the water velocity is near that of the rate of thaw) have little effect upon the thaw process. Approximate solutions are also given for the case of constant water velocity, using the heat balance integral and quasi-steady methods; they agree with the exact solution if the Stefan number is not too large. Thaw can be greatly accelerated or retarded if the water velocity (Peclet number) is large. The effect upon thawing for the case of horizontal water flow is less than that for the same magnitude of vertical flow.

## Symbols

| | |
|---|---|
| A | constant |
| $b$ | $\exp(v_o X/\alpha)$ |
| $c$ | specific heat |
| C | constant |
| $f$ | similarity parameter |
| $f'$ | $df/dt$ |
| $k$ | thermal conductivity |
| $\ell$ | latent heat |
| $L_c$ | characteristic length |
| $P_e$ | $L_c v_o/\alpha$, Peclet number |
| $S_T$ | $c(T_s - T_f)/\ell$, Stefan number |
| $t$ | time |
| $T$ | temperature |
| $v$ | velocity |
| $x$ | Cartesian depth coordinate |
| $X$ | thaw depth |
| $y$ | $\eta - A\gamma$, dimensionless depth coordinate |
| $\alpha$ | thermal diffusivity |
| $\gamma$ | $X/2\sqrt{\alpha t}$ |
| $\eta$ | similarity independent variable |
| $\rho$ | density |
| $\sigma$ | $X/L_c$, nondimensional thaw depth |
| $\tau$ | $\frac{S_T}{S_T+2}\frac{\alpha}{L_c^2}t$, dimensionless time |

## Subscripts

| | |
|---|---|
| $f$ | freeze |
| $s$ | surface |
| $o$ | constant value |

## Introduction

The most widely used method of estimating the thaw rate in permafrost or frozen soils relies upon heat transfer by pure conduction. This is the well-known Neumann solution (Carslaw and Jaeger, 1959; Lunardini, 1991). However, the movement of water in soil systems must involve convective heat transfer along with conduction. The relative importance of the convection with regard to thaw rates is of considerable interest. Martynov (1959) and Porkhaev (1959) noted that pure conduction is the dominant mode of energy transfer in permafrost or frozen soil systems, based on qualitative reasoning about the possible heat transfer mechanisms. Johansen (1975) presented a qualitative sketch of the heat transfer modes in soil systems based on the pore size and water saturation, inferring that conduction predominated for most practical cases. The effects of water movement are mainly manifested in changes in the thermal properties of the medium. These observations have proved to be useful, at least for natural systems such as permafrost, but they do not indicate the extent of the deviation from pure conduction theory.

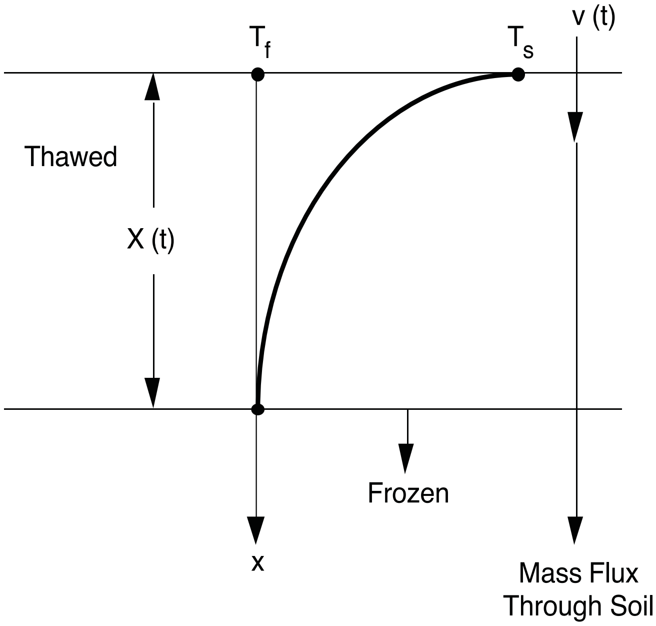


*Figure 1. Thaw of a semi-infinite medium with convection.*

Recently, quantitative results have been presented for free convection in porous media (Bejan, 1989; Chellaiah and Viskanta, 1989; Jany and Bejan, 1988; Sugarawa et al., 1988). These studies revealed that free convection could be significant during melting of porous media under some conditions, but the effects are still unlikely to greatly influence the thawing of frozen soils.

Permafrost thawing will be affected more significantly by forced convection wherein the flow of water is due to potentials other than temperature differences, such as pressure gradients. Numerical work has shown that convection can be significant if the mass flux is fairly large. Sluzalec (1989), for example, showed that water velocities greater than one meter per day could retard freezing around buried circular pipes. The quantitative effects of forced convection upon the thaw rate of frozen soils will be predicated for some simple cases. These results confirm that calculations based on pure conduction have wide validity for permafrost heat transfer.

## Theory

### Exact Similarity Solution

A semi-infinite soil, initially frozen at $T_f$, undergoes a surface temperature change to $T_s$, as shown on Figure 1. A mass flux with velocity *v(t)* occurs in the vertical direction. It is likely that the mass flux will be coupled to the thaw process, but we shall assume that v is a known function of time. The equations governing the thaw process are (Lunardini, 1991)

*Figure 2. Ratio of convective to conductive (Neumann) thaw parameter versus Stefan number.*

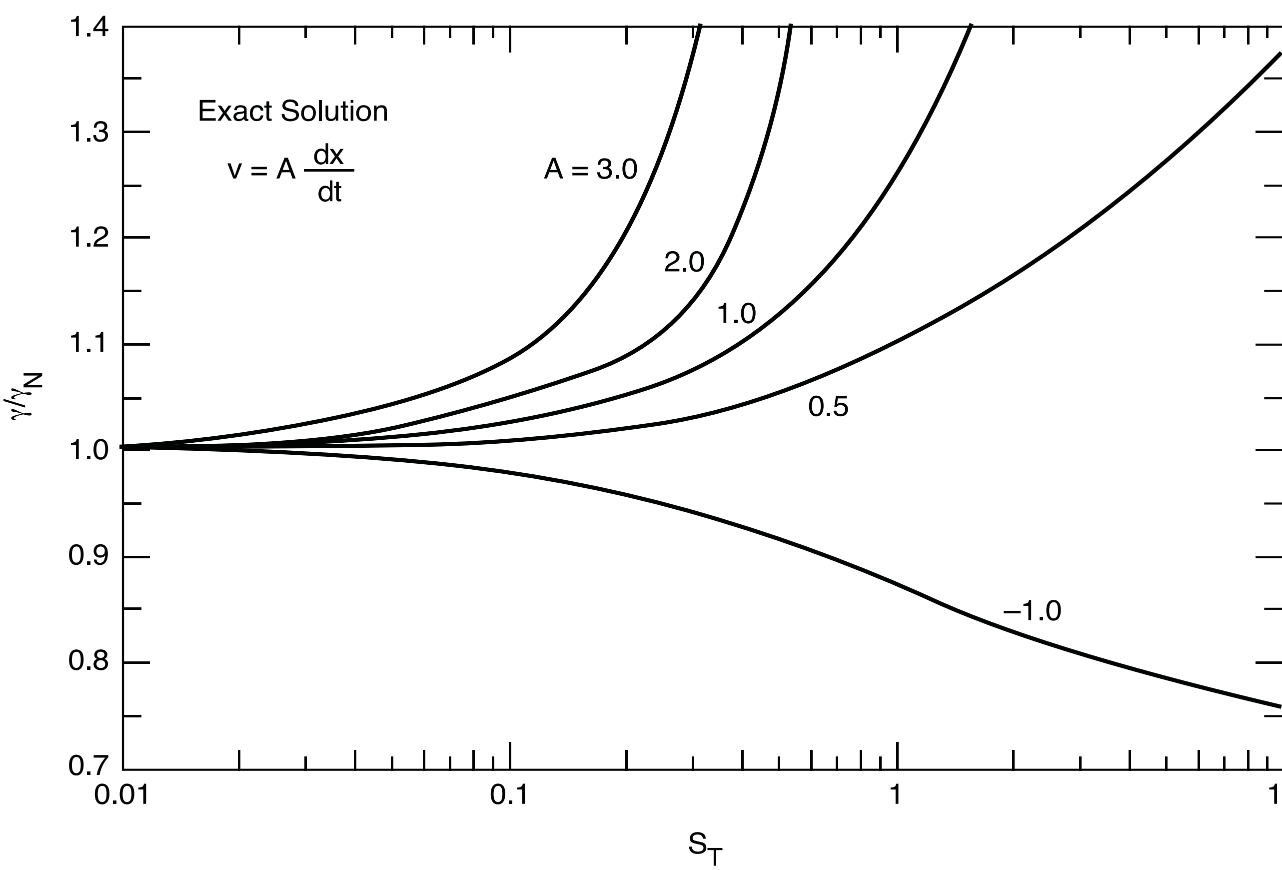

**Table 1. Neumann solution (*v*=0) and convection, *v*= *A dX/dt***

| Value of γ | | | | |
|---|---|---|---|---|
| | Neumann | Convection | | |
| $S_T$ | | A = 1 | A = –1 | A = 10 |
| 0.01 | 0.0706 | 0.0707 | 0.0704 | —□ |
| 0.1 | 0.2209 | 0.226 | 0.2148 | 0.4256 |
| 0.5 | 0.4650 | 0.523 | 0.4269 | —□ |
| 1.0 | 0.6201 | 0.777 | 0.5438 | —□ |
| 10.0 | 1.257 | 5.642 | 0.9393 | |

$$\alpha \frac{\partial^2 T}{\partial x^2} - v(t)\frac{\partial T}{\partial x} = \frac{\partial T}{\partial t} \quad [1]$$

$$T(0,t) = T_s \quad [1a]$$

$$T(x,0) = T_f \quad [1b]$$

$$T(X,t) = T_f \quad [1c]$$

$$k\frac{\partial T(X)}{\partial x} = -\rho\ell\frac{dX}{dt} \quad [2]$$

Under what conditions will an exact similarity solution exist? Let a new independent variable be defined as

$$\eta = \eta(x,t) \quad [3]$$

Then eq (1) is transformed to

$$\alpha\frac{\partial n}{\partial x}\frac{\partial}{\partial \eta}\left(\frac{\partial T}{\partial \eta}\ \frac{\partial \eta}{\partial x}\right) - v\frac{\partial T}{\partial \eta}\ \frac{\partial \eta}{\partial x} = \frac{\partial T}{\partial \eta}\ \frac{\partial \eta}{\partial t} \quad [4]$$

A similarity solution requires that the coefficients of eq (4) be functions of η (or constants); they cannot be explicit function of *x* or *t*. Let

$$\eta = x/f(t) \quad [5]$$

Then eq (4) is

$$\frac{d^2T}{d\eta^2} - \frac{f}{\alpha}\left[v(t) - \eta f'\right]\frac{dT}{d\eta} = 0 \quad [6]$$

where $f' = df/dt$. We know that if $v = 0$, a similarity solution exists only if

$$f = 2\sqrt{\alpha t} \quad [7]$$

If *f* is given by eq (7), then

$$\frac{f}{\alpha}\left[v(t) - \eta f'\right] = \frac{fv}{\alpha} - 2\eta \quad [8]$$

Thus, eq (6) will reduce to an ordinary differential equation in η only if $fv/\alpha$ is a constant or a function of η. But, $fv/\alpha$ cannot be a function of η, so it must be a constant. The only transient velocity function that will work is

$$v = \alpha C\big/\left(2\sqrt{\alpha t}\right) \quad [9]$$

where *C* is an arbitrary constant. From the Neumann solution ($v = 0$), we know that this implies that *v(t)* is proportional to the rate of change of the thaw interface. Let

$$X = 2\gamma\sqrt{\alpha t} \quad [10]$$

$$v = \mathrm{A}\frac{dX}{dt} = \mathrm{A}\gamma\sqrt{\alpha/t} \quad [11]$$

The transformed equations to solve are

$$\frac{d^2T}{dy^2} + 2y\frac{dT}{dy} = 0 \quad [12]$$

$$T(-\mathrm{A}\gamma) = T_s \quad [12a]$$

$$T\left[\gamma(1-\mathrm{A})\right] = T_f \quad [12b]$$

$$\frac{dT\left[\gamma(1-\mathrm{A})\right]}{dy} = -2\gamma\ \ell/c \quad [12c]$$

where $y = \eta - \mathrm{A}\gamma$. The solution to eq (12) is

$$\frac{T - T_s}{T_f - T_s} = \frac{erf y + erf(\mathrm{A}\gamma)}{erf(\mathrm{A}\gamma) + erf\left(\gamma[1-\mathrm{A}]\right)} \quad [13]$$

Then using eq (12c), the parameter γ can be found from the solution to

$$S_T e^{-\gamma^2(1-\mathrm{A})^2} = \sqrt{\pi}\,\gamma\left[erf(\mathrm{A}\gamma) + erf\left(\gamma[1-\mathrm{A}]\right)\right] \quad [14]$$

These relations reduce to the Neumann solution, as expected, if the mass flux velocity is zero. Table 1 compares the effect of convection to the Neumann solution for thaw.

For moderate Stefan numbers and small seepage flows (–1 < A < 1), flow in the direction of thaw accelerates the thaw by modest amounts, while flow in the opposite direction decelerates the thaw process. The ratio of the convective thaw rate to the Neumann thaw rate is not a function of time. As can be seen from Figure 2, the effect of convection is sensitive to the Stefan number if ST > 0.15 and A > 1.0.

This exact solution is only valid for the particular form of v(t). It cannot be used for any other mass velocity function. It is important, however, since the mass velocity very likely will be coupled to the thaw rate and thus will be of similar form. Nixon (1975) and Lunardini (1987) presented exact solutions for this kind of convection, due to soil consolidation and density differences. The convection itself did not have a great effect upon the thaw, while the accumulation of the excess pore water at the free surface was more significant for the thaw rates.

#### Heat Balance Integral Solution

The exact solution severely limits the transient form of the mass velocity. The heat balance integral (HBI) method can be used to obtain approximate solutions for the thaw rate, with v given by more general functions of time, in particular, for a constant velocity. Equation 1 is integrated over the space coordinate x.

$$\alpha\int_0^X \frac{\partial^2 T}{\partial x^2}dx - \int_0^X v(t)\frac{\partial T}{\partial x}dx = \int_0^X \frac{\partial T}{\partial t}dx \quad [15]$$

Carrying out the integration in the usual manner leads to

$$\alpha\left[-\frac{\rho\ell}{k}\frac{dX}{dt} - \frac{\partial T(0,t)}{\partial x}\right] - v(t)\left[T_f - T_s\right] = \frac{d}{dt}\left[\int_0^X T dx - T_f X\right] \quad [16]$$

## Velocity proportional to *dX/dt*

We can compare the HBI solution with the exact solution. The following temperature approximation will be used.

$$T(x,t) = T_f + \frac{T_f - T_s}{X}(x - X) \quad [17]$$

Equation (16) then reduces to

$$X\frac{dX}{dt} = \frac{\alpha}{1/S_T + 1/2 - A} \quad [18]$$

The solution to this equation is

$$X^2 = \frac{2\alpha S_T t}{1 + S_T(1/2 - A)} \quad [19]$$

The thaw parameter γ is then

$$\gamma^2 = \frac{S_T}{2 + S_T(1 - 2A)} \quad [20]$$

The accuracy of eq (20) is compared to the exact result in Table 2 for A = 1. As can be seen, the heat balance integral approximation is quite good for $S_T < 0.5$.

## Constant velocity *vo*

The percolation velocity is often assumed to be constant. For this case, eq (16) reduces to

$$-\alpha\left[\frac{\rho\ell}{k}\frac{dX}{dt} + \frac{\partial T(0,t)}{\partial x}\right] - v_o\left[T_f - T_s\right] = \frac{d}{dt}\left[\int_0^X T dx - T_f X\right] \quad [21]$$

The assumed temperature is again given by eq (17) and

$$\frac{dX}{dt} = \frac{\alpha/X + v_o}{1/S_T + 1/2} \quad [22]$$

The solution to this equation is

$$X - \frac{\alpha}{v_o}\ln\left(v_o X/\alpha + 1\right) = \frac{v_o t}{1/S_T + 1/2} \quad [23]$$

This equation can be nondimensionalized as

$$\sigma - \frac{1}{P_e}\ln\left(P_e\sigma + 1\right) = 2P_e\tau \quad [24]$$

where $\sigma = X/L_c$, $P_e = L_c v_o/\alpha$, $\tau = \frac{S_T}{2 + S_T}\frac{\alpha}{L_c^2}t$,

and $L_c$ is a characteristic length.

If the Peclet number is zero (mass flux is zero) the solution reduces to the Neumann form

$$\sigma^2 = 4\tau \quad [25]$$

The effect of convection now increases with time, even if $P_e$ is small. Figure 3 shows the impact of convection with time. For example, if α= 0.0025 $m^2$/hr and $S_T$ = 0.2, then the thaw depth is 20% greater than the Neumann value after 440 hours and 75% greater after 4400 hours. Figure 4 can be used to evaluate the time

**Table 2. Comparison of exact, HBI and quasi-steady solutions, *v=dX/dt***

| | Value of γ | | |
|---|---|---|---|
| $S_T$ | Exact | Heat balance integral | Quasi-steady |
| 0.01 | 0.0707 | 0.0709 | 0.0709 |
| 0.1 | 0.226 | 0.2294 | 0.2295 |
| 0.5 | 0.523 | 0.5774 | 0.5887 |
| 1.0 | 0.777 | 1.000 | —□ |

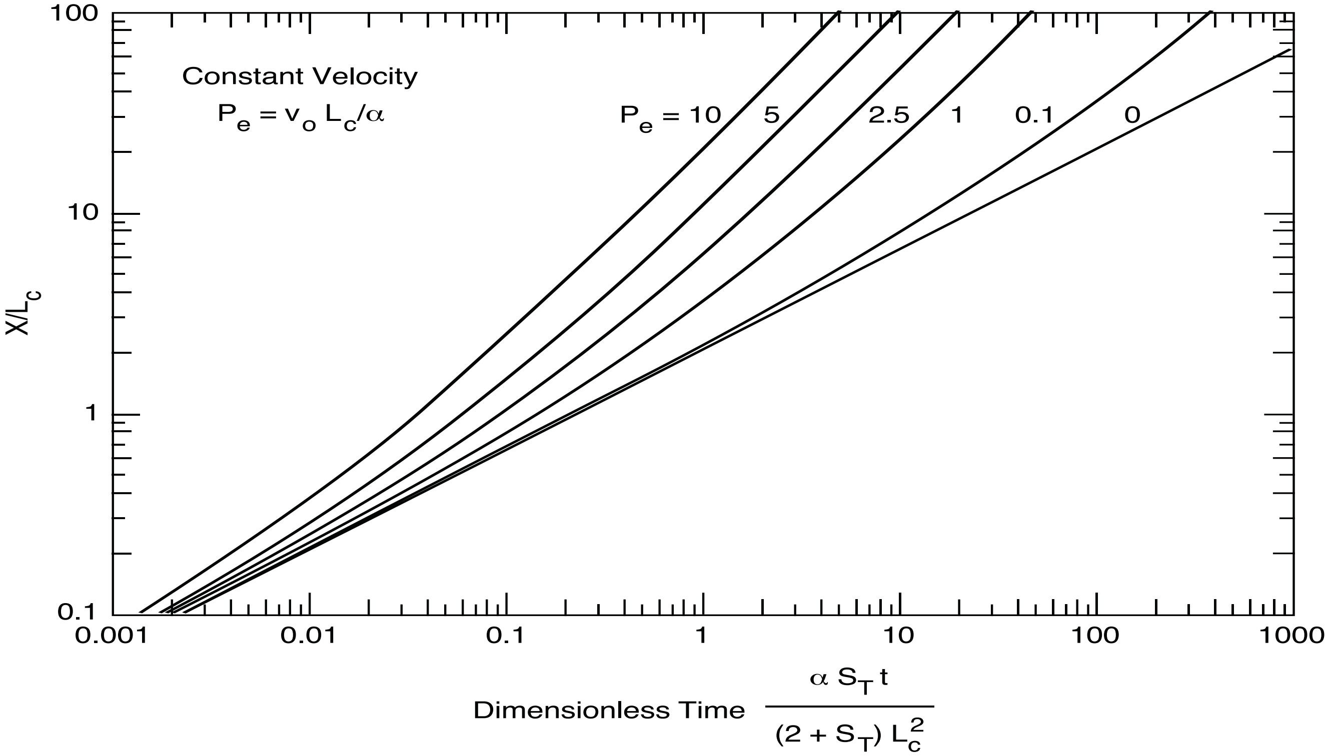


*Figure 3. Thaw depth versus Peclet number, constant velocity.*

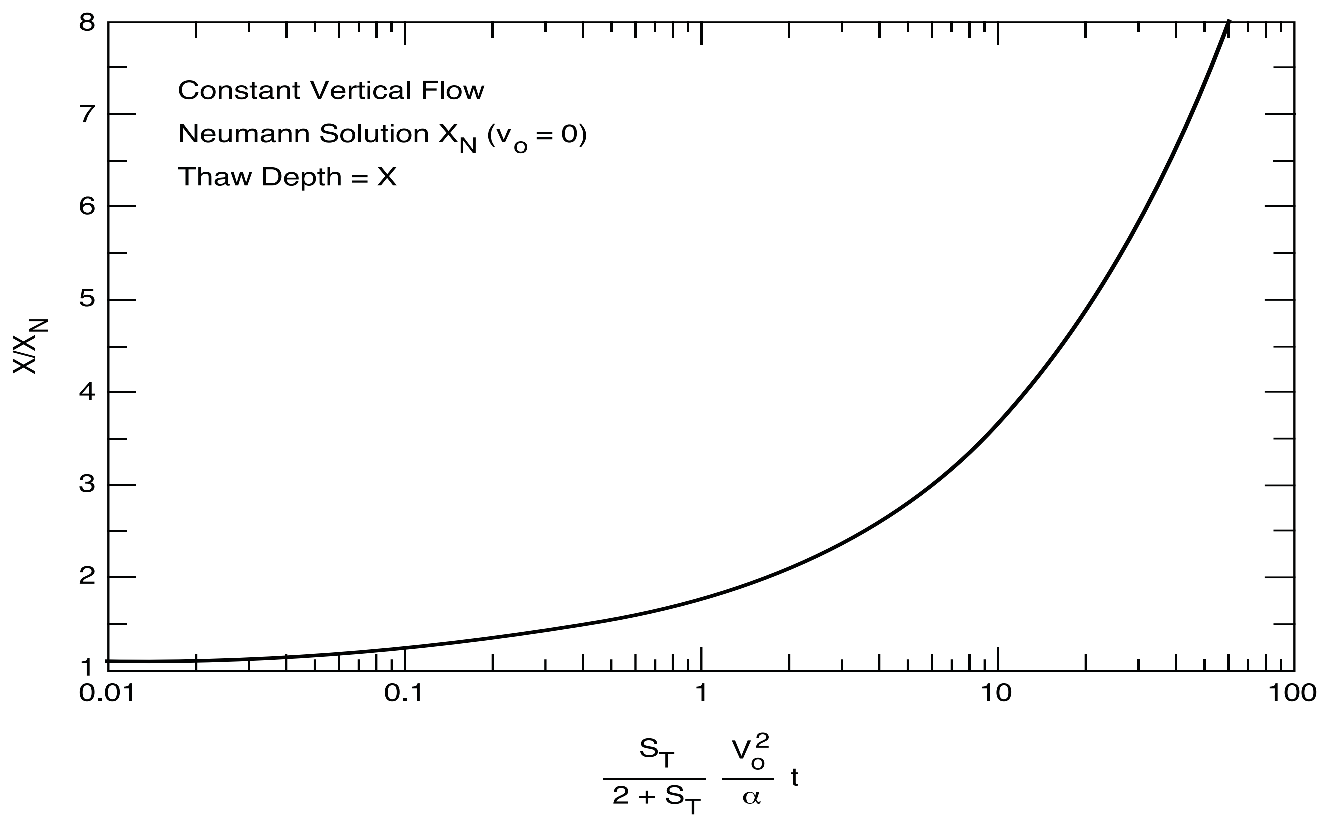


*Figure 4. Ratio of convective to Neumann thaw versus time, constant velocity.*

required for $X/X_N$ to exceed a given amount for any values of percolation velocity, Stefan number, and thermal diffusivity.

## Quasi-steady solution

It is a simple matter to obtain the quasi-steady approximation. The quasi-steady equations are

$$\alpha \frac{\partial^2 T}{\partial x^2} - v(t)\frac{\partial T}{\partial x} = 0 \qquad [26]$$

along with eq (la–c,2).

CASE 1. CONSTANT VELOCITY

When the velocity is constant, there is an exact solution, with the temperature and phase-change position given by

$$\frac{T - T_s}{T_f - T_s} = \frac{e^{vx/\alpha} - 1}{b - 1} \qquad [27]$$

$$X + \frac{\alpha}{v_o}\left(e^{-v_o X/\alpha} - 1\right) = S_T v_o t \qquad [28]$$

where (28i) Fel'dman (1972) noted that if (28ii),

$$b = e^{v_o X/\alpha} \quad [28i]$$

$$\frac{v_o X}{\alpha} > 3 \quad [28ii]$$

then

$$X = 0.95\frac{\alpha}{v_o} + S_T v_o t \qquad [29]$$

The thaw rate for the quasi-steady solution is only about 5% different than that for the heat balance approximation.

CASE 2. $v = A\frac{dX}{dt}$

Using eq (2, 27) yields a solution for the phase-change interface as

$$X^2 = \frac{2\alpha t}{A}\ln\left(\frac{1}{1 - S_T A}\right) \qquad [30]$$

This quasi-steady solution compares very well to the exact solution for $S_T < 0.5$, but it rapidly breaks down as the Stefan number increases (see Table 2).

## Conclusion

Concern about the effect of a mass flux (convective heat transfer) on the thaw rate and temperature field in permafrost arises commonly, since most predictions rely on a purely conductive mode of heat transfer. The effect of vertical water flow is unlikely to be significant if the water flux is proportional to the thaw rate itself. This is the case for water flows induced by the thaw process for which the water velocity is not greater than the thaw velocity. Both the exact and the heat balance integral solutions quantitatively demonstrate this. For water flows greater than seepage flows, such as percolation, the effect of convection can be quite large. The magnitude of the effects are given for the simple case of single-phase flow, with a constant water velocity of any magnitude.

Under natural conditions, it is also possible to have horizontal water flows. A quasi-steady solution showed that, for the same water velocity, the effect of horizontal water flow is less than that of vertical flow (Lunardini, 1991).

## References


**Bejan, A.** (1989). Theory of melting with natural convection in an enclosed porous medium. *Journal of Heat Transfer,* **111**, 407–415.

**Carslaw, H.S. and Jaeger, J.C.** (1959). *Conduction of Heat in Solids***.** Oxford University Press, London.

**Chellaiah, S. and Viskanta, R.** (1989). Freezing of water-saturated porous media in the presence of natural convection: Experiments and analysis. *Journal of Heat Transfer,* **111**, 425–432.

**Fel'dman, G.M.** (1972). *Calculation of Ground Thawing Allowing for Water Seepage***,** USACRREL TL 334, Hanover, N.H.

**Jany, P. and Bejan, A.** (1988). Scales of melting in the presence of natural convection in a rectangular cavity filled with porous medium. *Journal of Heat Transfer,* **110**, 526–529.

**Johansen, O.** (1975). *Thermal Conductivity of Soils.* USACRREL TL 637, (1977), Hanover, N.H.

**Lunardini, V.J.** (1991). *Heat Transfer With Freezing and Thawing*. Elsevier, Amsterdam, Oxford, New York, Tokyo.

**Lunardini, V.J.** (1987). Exact solution for melting of frozen soil with thaw consolidation. In *Proceedings of 6th International Offshore Mechanics and Arctic Engineering Symposium,* Houston, Texas, 97–102.

**Martynov, G.A.** (1959). *Principles of Geocryology, Part 1, Chapter VI.* Academy of Sciences, USSR, NRC TT–1065,(1963), Ottawa, Canada.

**Nixon, J.F.** (1975). The role of convective heat flow in the thawing of frozen soils. *Canadian Geotechnical Journal,* **12**, 425–429.

**Porkhaev, G.V.** (1959). *Principles of Geocryology, Part 2, Chapter IV.* Academy of Sciences, USSR, NRC TT–1249, (1966), Ottawa, Canada.

**Sluzalec, A.** (1989). Groundwater flow effects in processes of soil freezing. *Numerical Heat Transfer,* Part A, **15**, 399–409.

**Sugarawa, M., Inaba, H. and Seki, N.** (1988). Effect of maximum density of water on freezing of a water-saturated horizontal porous layer. *Journal of Heat Transfer,* **110**, 155–159.