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Effect of corrosion on ductility of reinforcing bars

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An experimental investigation into the effect of corrosion on the ductility of steel reinforcement is reported. Both accelerated and simulated corrosion tests were conducted on bare bars and on bars embedded in concrete. The mechanism and degree of the reduction of ductility of reinforcement due to corrosion were examined. The influence of bar type and diameter on ductility of corroded reinforcement is discussed. The experimental results indicate that, since local attack penetration results in a significant variation of residual cross-section along its length, corrosion significantly reduces ductility of reinforcement. Although the strength ratio, elastic modulus and hardening strain only vary with bar type rather than corrosion level, the elongation, ultimate strain and ductile area of corroded reinforcement reduce much more significantly than do those of their yield and ultimate strengths. There is concern regarding bar ductility since about 10% corrosion may possibly decrease the ultimate strain of reinforcement below the minimum requirement specified in CEB Model Code 90 for class S reinforcement. Even though the elongation, ultimate strength and ductile area parameter of corroded small diameter and/or plain bars reduce more than those of large diameter and/or ribbed ones, such differences are not significant and can be neglected. Finally, a set of simple empirical equations is proposed to assess the ductility of corroded reinforcement in practice.

Notation

A^* area parameter proposed by Creazza and Russo[12]
A_d ductile area proposed by Beeby[13]
$A_{\sigma\varepsilon}$ area under stress–strain curve of reinforcing bar
A_{d0} ductile area of non-corroded reinforcements
A_{cor} actual cross-sectional area of corroded reinforcement
A_s average cross-sectional area of corroded reinforcement
A_{s0} initial cross-sectional area of non-corroded reinforcement
d diameter of non-corroded reinforcement
F force of corroded reinforcement
f_0 strength of non-corroded reinforcement
f_y yield strength of corroded reinforcement
f_{y0} yield strength of non-corroded reinforcement
f_u ultimate strength of corroded reinforcement
f_{u0} ultimate strength of non-corroded reinforcement
Q_{cor} corrosion percentage of reinforcement
Q_{cor}^{cr} critical corrosion percentage regarding ductility of reinforcement
L gauge length of reinforcement
r_x ratio of maximum to minimum corrosion penetration
x_{max} maximum local attack penetration of corroded reinforcement
x_{corr} average attack penetration of corroded reinforcement
ρ_{min} minimum value of ratio of ultimate to yield strengths of reinforcement
ε_y yield strain of corroded reinforcement
ε_{yu} hypothetical yield strain of reinforcement at ultimate stress
ε_u ultimate strain of corroded reinforcement
ε_{umin} minimum value of ultimate strain of reinforcement
ε_{sh} strain of reinforcement at the beginning of strain-hardening
σ_i tension stress of reinforcing bars
ε_i tension strain of reinforcing bars
ε_{u0} ultimate strain of non-corroded reinforcement
λ elongation of corroded reinforcement

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λ_0 elongation of non-corroded reinforcement
α_e regression parameters for ultimate strain of corroded reinforcement
α_A regression parameters for ductile area of corroded reinforcement
α_λ regression parameters for elongation of corroded reinforcement
m sectional coefficient of corroded reinforcement
η length coefficient of corroded reinforcement

Introduction

It is well recognised that, for a structure to be capable of undergoing a large inelastic deformation while sustaining a load close to its maximum load-carrying capacity, ductility is of great importance in structural earthquake-resistant design as well as where the redistribution of bending moment from the elastic pattern is taken into account. With significant ductility, a structure can be prevented from failing and collapsing in a brittle fashion without warning, which can save both the lives of people in the area and reduce repair costs. Hence, particular care should be paid to the ductility of a structure in its design.

For commonly used under-reinforced concrete structures, their failure under an applied load is initiated by the yielding of steel reinforcement and, eventually, followed by the crushing of compression concrete. During such a failure process, steel reinforcement must be able to withstand a large post-yielding elongation or compression and not rupture until the crushing of concrete, such that structures can generate significant deformation without substantial reduction of load-bearing capacity. Such a failure mode is therefore ductile with ample warning, as has always been required for practical structures. It is clear that the deformation capacity of under-reinforced concrete structures greatly depends upon the ductility of the reinforcement, in addition to the reinforcement ratio. In other words, ductility of reinforcement substantially affects the deformation capacity of a structure and, in turn, substantially determines whether the structure can survive without collapse if it were to experience a moderate earthquake.

Extensive research has been conducted on the ductility of reinforcement and its influence on the deformation capacity of structures and possible degree of moment redistribution.[1–3] Most of this research, however, focused on the design of new reinforced concrete structures.

Due to the carbonation of concrete cover and/or intrusion of chloride ions, corrosion of reinforcement in concrete may occur in existing structures and corrosion distribution over bar surface and along bar length varies greatly with corrosion initiation mechanism and conditioning regimes. Generally, the carbonation of the concrete cover may produce fairly even corrosion. On the other hand, corrosion in the presence of chloride ions usually results in a much more localised and pitting type of corrosion. The latter type of corrosion not only decreases the actual load-bearing capacity of the reinforcement,[4] but also affects its ductility. For a larger number of earthquake-resistant and moment-redistributed structures, although careful consideration would have been given to their design and detailing to ensure that they would have a significant amount of ductility at the construction stage, once steel starts to corrode substantially, their actual ductile capacity may deteriorate and possibly become much less ductile than had been anticipated. Hence, it is very worthwhile to investigate the ductility of corroded reinforcement.

Some experiments have been performed to examine the effect of corrosion on the ductility of reinforcement.[5–10] In all cases, test specimens were either single bare bars[5,6] or bars embedded in concrete.[7–10] They were exposed to either an outdoor marine environment[5] or indoor accelerated corrosion[6,7] or were removed from actual structures which had suffered from chloride intrusion[8,9] or concrete carbonation.[10] From tension tests conducted on corroded specimens, Maslehuddin *et al*.[5] reported that up to 1·1% corrosion by marine exposure hardly changed the elongation of reinforcement.[5] Andrade *et al*.,[6] Lee *et al*.,[7] Morinaga,[8] Palssom *et al*.,[9] and Zhang *et al*.,[10] however, reported that significant corrosion dramatically decreased the elongation and/or ultimate strain of reinforcement. Regarding the ratio of ultimate to yield strengths, Zhang *et al*. reported that corrosion reduced its value from 1·5 to 1·1. However, the results obtained by Maslehuddin *et al*., Andrade *et al*. and Palssom *et al*. indicate that corrosion hardly affects the strength ratio.[5,6,9]

It is apparent that, although some valuable work has been done, little attention has been paid to the influence of bar type and diameter on the ductility of corroded reinforcement, and the results from different researchers are not consistent.

This paper presents the results of an experimental investigation into the effect of corrosion on ductility of reinforcement of different types and diameters, in addition to the identification of the source and mechanism of such an influence.

Definition of ductility of reinforcement

The conventional definition of reinforcement ductility which is specified in some international codes, such as CEB Model 90, defines the ductility of reinforcement in terms of two independent criteria: the strength ratio and ultimate strain. The first criterion is that the ratio of ultimate strength to yield strength must not be less than a specified minimum value, namely, $f_u/f_y \geqslant \rho_{min}$, and considers the strain-hardening ratio of steel reinforcement which specifies an interval between the ultimate strength and yield strength, as shown in

Fig. 1. The second criterion is that the ultimate strain ε_u must not be less than a specified minimum value, namely, $\varepsilon_u \geqslant \varepsilon_{u\,min}$, and specifies an indicator of maximum deformation capacity. For Class S reinforcement, which is used for highly ductile structures, Model Code 90 specifies that $\rho_{min} = 1{\cdot}15$ and $\varepsilon_{umin} = 6\%$. Because the three parameters, f_u, f_y and ε_u, used in the above two criteria can be obtained directly by simple tensile tests without considering an actual stress–strain curve, these two criteria have been widely adopted to verify bar ductility in engineering practice.

However, the above conventional criteria fail to reflect fully the ductile behaviour of a reinforcing bar under monotonic force. As shown in Figs 1 (a) and (b), even if two specimens of steel reinforcement have the same ultimate strength, the same yield strength and the same ultimate strain, they can still have different ductility and produce different influences on concrete structures.

By considering the strain at the beginning of the strain-hardening phase, ε_{sh}, Creazza and Russo[12] defined bar ductility as an area parameter A^*, as shown in Fig. 1(b). Geometrically, the parameter A^* is the area between the post-hardened stress–strain curve and a horizontal line at the yield strength. By assuming that the strain-hardening curve is a parabolic curve, the area A^* can be determined using equation (1)

$$A^* = \frac{2}{3}(f_u - f_y)(\varepsilon_u - \varepsilon_{sh}) \tag{1}$$

It is clear that Creazza and Russo's criterion for bar ductility not only unequivocally defines the deformation work of a steel bar during its strain-hardening phase, but also simultaneously considers the influence of ultimate strength f_u, yield strength fy_u and ultimate strain ε_u on bar ductility.

Although Creazza and Russo's proposals do indeed improve the conventional definition of bar ductility, some new problems arise. First, as shown in equation (1), one more new parameter ε_{sh} is introduced to represent bar ductility. This parameter is not very objective if there is no distinct point between the yield plateau and the strain-hardening phase on the stress–strain curve (see Fig. 1(a)). Second, some difference exists between the assumed parabolic curve and the shape of the real strain-hardening curve

In studying the influence of bar ductility on ultimate plastic rotation of reinforced concrete beams, Beeby[13] employed another area parameter A_d to describe the bar ductility, as shown in Fig. 1 and defined by equation (2)

$$A_d = f_u(\varepsilon_u - 0{\cdot}5\varepsilon_{yu}) - A_{\sigma\varepsilon} \tag{2}$$

A_d is a ductile area parameter, ε_y is the yield strain, $\varepsilon_{yu} = (f_u/f_y)\varepsilon_y$ is a hypothetical yield strain at ultimate stress, and $A_{\sigma\varepsilon} = \sum_1^N 0{\cdot}5(\sigma_{i+1} + \sigma_i)(\varepsilon_{i+1} - \varepsilon_i)$ is the area under the stress (σ_i)–strain (ε_i) curve.

It is clear that Beeby's area parameter A_d is, geometrically, the area between the post-yield stress–strain curve of a reinforcing bar and a horizontal line at its ultimate strength. It integrates the yield strength, ultimate strength, ultimate strain and the stress–strain curve of a steel bar, and can be easily calculated and used to evaluate bar ductility, on basis of the stress–strain curve of a reinforcing bar under monotonic load.

To date, the area parameters of neither Creazza and Russo nor Beeby, A^* and A_d, respectively, have been widely adopted in practice. Most researchers and practising engineers still adopt the conventional criteria to evaluate the ductility of bars. Hence, in the current research, the two conventional criteria of strength ratio and ultimate strain are used to investigate the ductility of corroded reinforcement, and to compare the authors' results with those of other researchers. In addition, the effect of corrosion on elongation, hardening strain, Beeby's ductile area A_d and elastic modulus are also discussed.

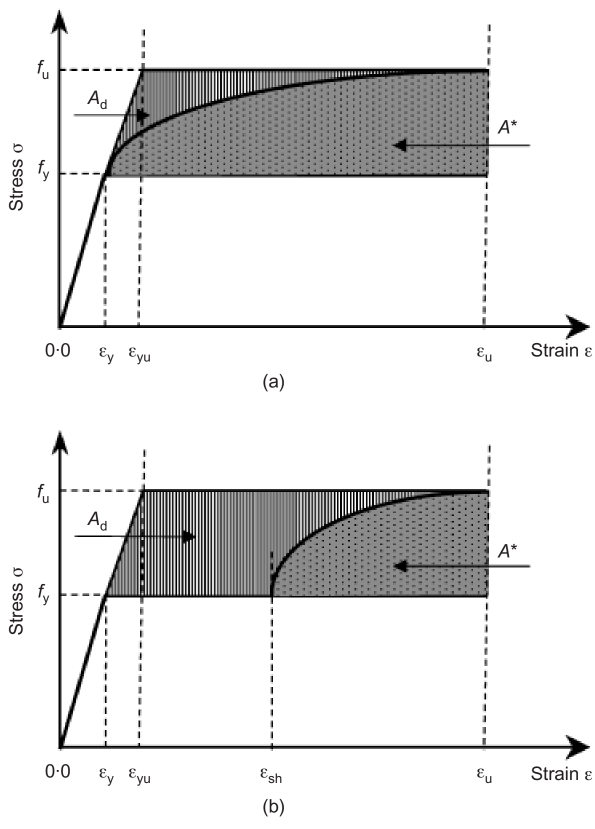

Fig. 1. Ductile areas A_d and A^* of reinforcements (after Creazza and Russo,[12] and Beeby[13]). (a) bar without yield plateau, as R08 used in tests; (b) bar with yield plateau, as R16, T08, T16 and T32 used in tests

Experimental programme

The details of the experimental programme and comments on the practical implications of the techniques used to obtain corrosion information have been reported in a previous paper.[4] Hence, only a brief description is given here.

A total of 108 test specimens, including 30 bars embedded in concrete cylinders of 50 mm diameter (referred to as the CEB specimens) and 78 single bare bars (referred to as the SBB specimens), were manufactured. In addition to 21 non-corroded control specimens, 84 specimens were subjected to accelerated corrosion with a direct current impressed and the three remaining specimens were subjected to simulated corrosion with bare bars mechanically machined in the workshop, before they were tensioned until rupture to measure their residual strength[4] as well as their ductility.

The concrete used for the cylindrical specimens had a water/cement ratio of 0·65 and contained 4% calcium chloride by mass of cement. The average cube compressive strength at the time when the impressed current was first applied on to bar specimens, at an age of 198 and 240 days for specimens RC16 and TC16, respectively, was 46·4 N/mm^2. All reinforcement specimens had the same length of 450 mm with at least three specimens from the same casting for each combination of the following variables.

Specimen type:	single bare bars and bars embedded in concrete
Reinforcement type:	plain bars and ribbed bars
Reinforcement diameter:	8, 16 and 32 mm
Corrosion times:	7, 14, 21 and 28 days

The corrosion apparatus is shown schematically in Fig. 2. Corrosion currents of 0·5, 1·0 and 2·0 mA/cm^2 were impressed onto the 8, 16 and 32 mm diameter bars, respectively, to attain similar corrosion percentages for the same corrosion durations. The corrosion percentage was calculated from the ratio of the weight loss to the initial weight. The corrosion products were removed from the bar surface with acid solution prior to weighing and allowance was made for the influence of such acid cleaning on bar weight loss.

A Denision Testing Machine, equipped with a data-processing computer, an electrical extensometer and a linear variable displacement transducer (LVDT), was used for uni-axial tension tests performed on the reinforcement specimens under a 2 mm/min displacement control rate. Prior to reaching the yield load, extension was measured by means of an electrical extensometer with a maximum stroke of 0·3 mm over its 50 mm gauge. Following initial yield, extension was recorded until bar fracture by the LVDT with a maximum stroke of 35 mm over its 43 mm gauge length. Prior to a tension test, two marks at a spacing of 200 mm were made on a bar's surface for the measurement of bar elongation.

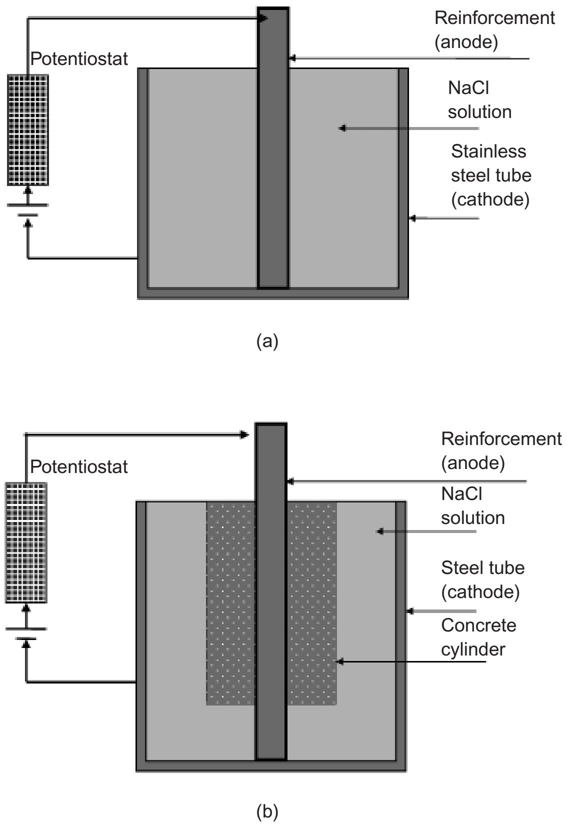

Fig. 2. Corrosion arrangement of reinforced specimen: (a) bare reinforced specimen; (b) reinforcement in concrete

Results and discussion

The main experimental results were a force–extension curve and the elongation of each bar measured after its rupture. On the basis of these results, the effect of corrosion percentage on ductility of reinforcement is analysed and discussed.

In the following analysis, bar stress and strain were calculated, respectively, by dividing force by average cross-sectional area,[4] and extension by the gauge length of either the extensometer or the LVDT, as appropriate. The elongation of a reinforcing bar was taken as the ratio, expressed as a percentage of the increase in the distance between the two marks on the bar surface after its rupture to their initial distance of 200 mm. The ductile area parameter was based on the stress–strain curve of each reinforcing bar and was calculated from equation (2). The elastic modulus was determined by dividing the increment of stress within the middle third of the linear elastic range by the corresponding strain increment. This approach eliminated the influences of initial loading and yielding on the elastic modulus.

Ductility of non-corroded reinforcement

The average of the three measured values of the various ductility parameters of the non-corroded reinforcing bars are summarised in Table 1. The individual values were generally within ± 6·8% of their mean

Table 1. Ductility of non-corroded reinforcements

Reinforcement Type		Plain Bars		Ribbed Bars		
Reinforcement Number		R08	R16	T08	T16	T32
Unit weight	W_g (g/mm)	0·332	1·589	0·386	1·563	6·162
Actual diameter	d_0 (mm)	7·34	16·05	7·91	15·92	31·61
Yield strength	f_{y0} (N/mm^2)	284	273	526	529	498
Ultimate strength	f_{u0} (N/mm^2)	413	362	619	627	604
Ratio of strength	f_{u0}/f_{y0}	1·45	1·33	1·18	1·19	1·21
Elasticity	E_{s0} (kN/mm^2)	196	199	203	201	211
Yield strain	$\varepsilon_{y0} = f_{y0}/E_{s0}$	0·0014	0·0014	0·0026	0·0026	0·0025
Hardening strain	ε_{sh0}	NA	0·025	0·022	0·019	0·017
Ultimate strain	ε_{u0}	0·182	0·203	0·082	0·116	0·123
Bar elongation	λ_0 (%)	23	23	14	16	20
Hardening strain ratio	$\varepsilon_{sh0}/\varepsilon_{y0}$	NA	18	8	7	7
Ultimate strain ratio	$\varepsilon_{u0}/\varepsilon_{y0}$	128	149	31	44	49
Rupture strain ratio	$\varepsilon_r/\varepsilon_{y0}$	162	168	53	60	80
Application in test specimen		SBB	SBB	SBB	SBB	SBB
		—	CEB	—	CEB	—

NA not applicable.

values. It is clear that all reinforcing bars used in the experimental investigation met the minimum requirements for Class S reinforcements for high-ductility structures. The strength ratio, elongation, ultimate strain and ductile area of plain bars are greater than those of ribbed bars and, therefore, they are more ductile than the ribbed bars. The elastic moduli of all of the non-corroded bars are very close to the typical code value of 200 kN/mm^2, with a maximum difference of 5·5%.

The unit weights were obtained by weighing 300 mm lengths of bar, and the actual diameters were determined from the weights by using a density of 7·85 g/cm^2.

It should be pointed out that, except for the 8 mm diameter plain bar R08, the stress–strain curves of all bars exhibited an initial linear elastic portion, a yield plateau and a strain-hardening phase, as shown schematically in Fig. 1(b). The plain bar R08 lacked a well-defined yield plateau. Hence, its maximum linear elastic stress was taken as its nominal yield strength, as shown in Fig. 1(a). A proof stress was not used because corrosion was found not to change the shape of the stress–strain curve.[4]

External surface and residual section of corroded reinforcement

A visual examination of the corroded reinforcement showed that the conditioning corrosion did indeed change the external surface of a steel bar. Compared with the non-corroded reinforcement, as corrosion time increased from 7 days to 28 days under an impressed direct current, the corrosion pits on the reinforcement surface developed in different ways: They increased in number and expanded in size, while some of them joined up with each other to give an appearance between that of the reasonably uniform corrosion due to concrete carbonation and the localised corrosion due to chloride penetration. Corrosion of the reinforcement not only altered the approximately round cross sections into very irregular ones, but it also caused the residual sections to vary significantly along its length.

In addition, for the reinforcement specimens embedded in concrete, some corrosion also occurred on the 75 mm length of bar projecting out of the concrete. There was more corrosion at the location of air voids at the steel interface, although the bars were well protected with a primer coating and an epoxy coating to avoid crevice corrosion. During the tension tests, however, the measurement location was always positioned at the middle of the 300 mm length of bar which had been embedded in concrete to minimise the influence of crevice corrosion on bar properties.

Effect of corrosion on strength ratio, hardening strain and elastic modulus

The effects of corrosion on strength ratio, hardening strain and elastic modulus of reinforcing bars are shown in Figs 3 to 5 for bare bars, and in Figs 6 to 8 for bars RC16 and TC16 embedded in concrete. In the latter figures the embedded bars are compared with bare bars R16 and T16. Linear regression lines are plotted on each figure.

It is clear that, for all situations, the regression lines for strength ratio, hardening strain and elastic modulus are almost horizontal and, hence, these parameters varied little with corrosion percentage. Although a small rise or fall trend could be observed for some bars, no systematic and logical relationship could be found between the strength ratio, hardening strain or elastic modulus of reinforcing bars and the corrosion percentage. Regression lines of strength ratio, hardening strain and elastic modulus of bars embedded in concrete are parallel to those of bare bars. Furthermore, for up to 25% corrosion, the strength ratios of almost all corroded bare bars are greater than the minimum value of

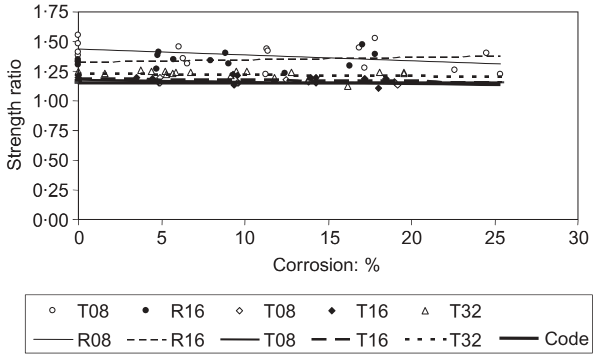

Fig. 3. Effect of corrosion on strength ratio of bare bars

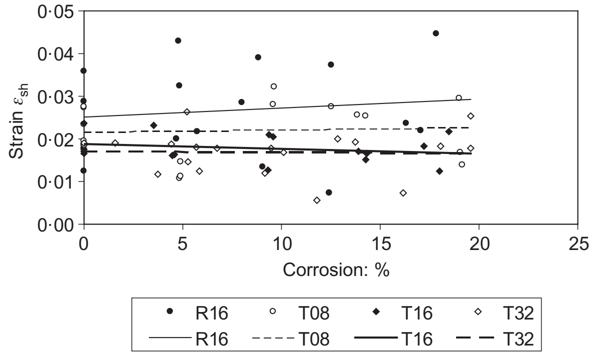

Fig. 4. Effect of corrosion on hardening strain of bare bars

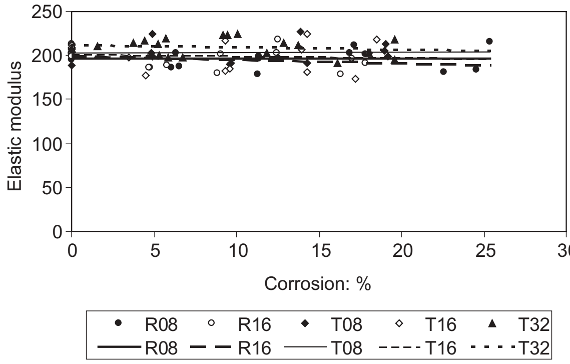

Fig. 5. Effect of corrosion on elastic modulus of bare bars

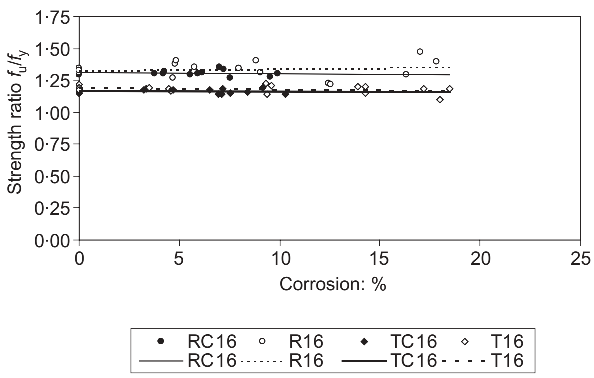

Fig. 6. Effect of corrosion on strength ratio of bars in concrete

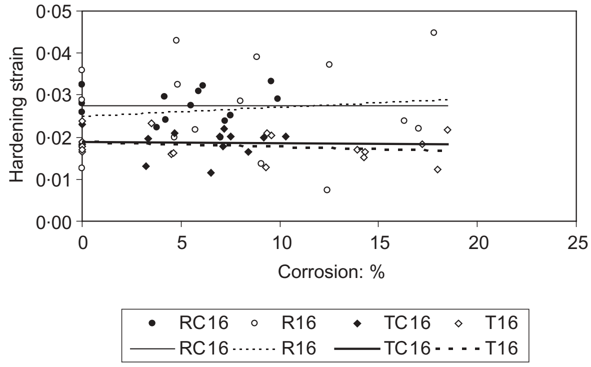

Fig. 7. Effect of corrosion on hardening strain of bars in concrete

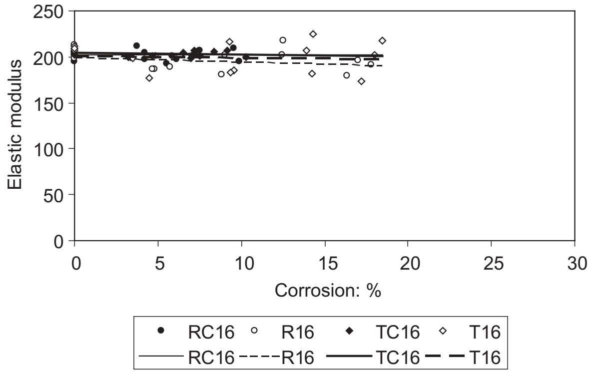

Fig. 8. Effect of corrosion on elastic modulus of bars in concrete

1·15 specified in CEB Model Code 90 for class S reinforcement. Hence, corrosion of reinforcement hardly affects strength ratio, hardening strain and elastic modulus of reinforcing bars.

Significance tests performed on the experimental results of the bare bars at the 5% significance level showed that there were significant differences between the mean values of the strength ratio, hardening strain and elastic modulus of bars of different diameter and/or type.[11] However, at the same significance level, the mean values of strength ratios, hardening strain and elastic modulus of bars in concrete were not significantly different to those of bare bars. Hence, strength ratio, hardening strain and elastic modulus of corroded bars varied mainly with bar type and bar diameter, rather than corrosion condition (i.e. whether a bare bar or a bar embedded in concrete).

The explanation for the above observation is that the strength ratio of a reinforcing bar is determined by its yield and ultimate strengths, which are both functions of the minimum cross-sectional area. Hence, when corrosion reduces bar section, the yield and ultimate strengths are reduced by similar amounts and the strength ratio is largely unaffected.[4] Furthermore, the hardening strain and elastic modulus of a reinforcing bar are dependent on their chemical composition and manufacturing process. Corrosion removes iron ions only from the bar surface and does not change the nature and composition of the remaining steel bar. As a result, corrosion of reinforcement does not change significantly the strength ratio, hardening strain and elastic modulus.

With regard to the effect of corrosion on the strength ratio, different researchers have reported apparently contradictory results. Regression results of the tabulated test data of Andrade *et al*. show that corrosion did not decrease bar strength ratio.[6] However, Zhang *et al*. reported[10] that corrosion reduced bar strength ratio f_u/f_y from 1·5 to 1·1.

In the authors' opinion, the reason for such contradictory results is the use of different types of bar and different testing techniques.[4] An extensometer with the 60 mm gauge length used by Andrade *et al*.[6] may be more sensitive to the yielding of a steel bar than the measurement of displacement from the movement of a testing machine's platens over a distance of 200 to 400 mm as used by Zhang *et al*.[10]

Effect of corrosion on ultimate strain, ductile area and elongation

The effect of corrosion percentage on ultimate strain, ductile area and elongation of bare bars is shown in Fig. 9, and on the ultimate strain of bars embedded in concrete in Fig. 10 where embedded bars are compared with bare bars. Since the effects of corrosion on ductile area and elongation of bars embedded in concrete are very similar to that on ultimate strain, they are not shown in Fig. 10, although they are included in the following analysis. Here, $\varepsilon_u/\varepsilon_{u0}$, A_d/A_{d0} and λ/λ_0 and are ratios of ultimate strength ε_u, ductile area A_d and elongation λ of corroded bars to the non-corroded value, indicated by the subscript 0, as given in Table 1.

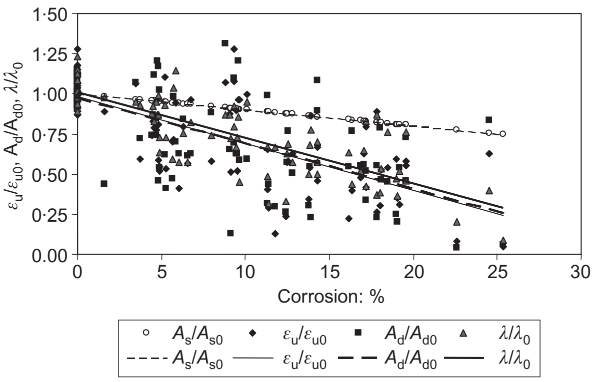

Fig. 9. Effect of corrosion on ductility of bare bars

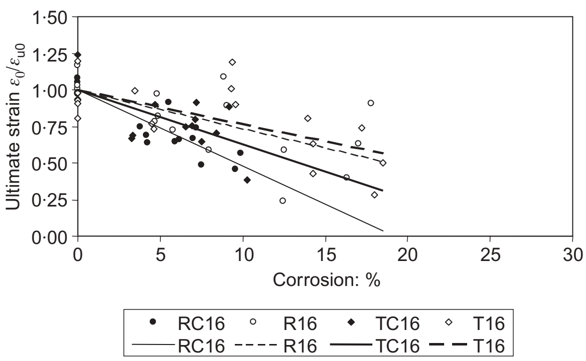

Fig. 10. Effect of corrosion on ultimate strain ratio of bars in concrete

Figures 9 and 10 show that the ultimate strain, ductile area and elongation of reinforcing bars were reduced significantly due to corrosion. For bare bars, 10% corrosion resulted in about a 29% reduction of ultimate strain, ductile area and elongation, which is much greater than the approximately 15 and 5% reductions of their force capacities and strengths, respectively.[4] Furthermore, for bars embedded in concrete, although their reductions of strength are similar to those of bare bars,[4] their ultimate strain, ductile area and elongation decrease more than those of bare bars. As shown in Fig. 10, typically, 10% corrosion reduced the ultimate strain of bars RC16 and TC16 embedded in concrete by 52 and 37%, respectively, but the same corrosion percentage reduced the ultimate strain of bare bars R16 and T16 only by 26 and 23%, respectively. Hence, due to corrosion, the ductility of reinforcement in terms of ultimate strain, ductile area and elongation decreases more than its strength. The reason for this difference is discussed later.

Regression analyses indicated that ultimate strain, ductile area and elongation of corroded bars can be generally described in the form of equations (3) to (5).

$$\varepsilon_u = (1{\cdot}0 - \alpha_e Q_{cor})\varepsilon_{u0} \tag{3}$$

$$A_d = (1{\cdot}0 - \alpha_A Q_{cor})A_{d0} \tag{4}$$

$$\lambda = (1{\cdot}0 - \alpha_\lambda Q_{cor})\lambda_0 \tag{5}$$

where α_e, α_A and α_λ are regression coefficients for ultimate strain, ε_u, ductile area, A_d, and elongation, λ, respectively, of reinforcing bars, and are listed in Table 2 for different bar types and diameters and different corrosion conditions.

Equations (3) to (5) show that, for a positive value of a regression coefficient, an increase of corrosion percentage decreases the ultimate strain, ductile area and elongation of corroded bars. Furthermore, the larger is a regression coefficient, the greater is the rate of reduction of a bar's ultimate strain, ductile area and elongation.

Table 2 indicates that, except for the bar T32 that was corroded under a 2·0 mA/cm^2 current,[4] for the same type of reinforcement specimen, the smaller the bar diameter, the greater are the regression coefficients α_e, α_A and α_λ. For the same reinforcement diameter, the regression coefficients α_e, α_A and α_λ for plain bars are always larger than those for ribbed bars. Furthermore, for the same type of reinforcements with an identical diameter, the regression coefficients α_e, α_A and α_λ of bars embedded in concrete are much greater

Table 2. Regression and correlation coefficients and correlation for bar ductility in terms of ultimate strain, ductile area and elongation

Specimen type	Bar type	Ultimate strain $\varepsilon_u = (1{\cdot}0 - \alpha_e Q_{corr})\varepsilon_{u0}$		Ductile area $A_d = (1{\cdot}0 - \alpha_A Q_{corr})A_{d0}$		Elongation $\lambda = (1{\cdot}0 - \alpha_A Q_{corr})\lambda_0$	
Bare bars	All bars	0·029	0·660	0·029	0·608	0·028	0·807
	R08	0·032	0·781	0·030	0·733	0·034	0·890
	R16	0·026	0·632	0·023	0·564	0·024	0·705
	T08	0·027	0·572	0·026	0·647	0·028	0·870
	T16	0·023	0·660	0·019	0·494	0·022	0·806
	T32	0·031	0·586	0·031	0·524	0·024	0·740
Bars in concrete	All bars	0·044	0·735	0·037	0·622	0·039	0·704
	RC16	0·052	0·868	0·047	0·875	0·048	0·729
	TC16	0·037	0·648	0·029	0·468	0·031	0·771

than those of bare bars. Hence, for an identical corrosion percentage, the ultimate strain, ductile area and elongation of smaller diameter bars and of plain bars decrease more than those of larger diameter bars or ribbed bars. It is again apparent that the ductility of bars embedded in concrete reduces more rapidly than that of bare bars in terms of ultimate strain, ductile area and elongation.

Although bar diameter and bar type do indeed affect the reduction of ultimate strain, ductile area and elongation of corroded bars, their influences are not significant at the 5% significance level. Consequently, the reductions of ultimate strain, ductile area and elongation of different type and diameter bars are determined primarily by the corrosion percentage, rather than bar type or diameter. Hence, by taking the results of all bars as one population, equations (6) to (8) are obtained.

$$\varepsilon_u = (1{\cdot}0 - 0{\cdot}029 Q_{cor})\varepsilon_{u0} \quad (6)$$

$$A_d = (1{\cdot}0 - 0{\cdot}029 Q_{cor})A_{d0} \quad (7)$$

$$\lambda = (1{\cdot}0 - 0{\cdot}028 Q_{cor})\lambda_0 \quad (8)$$

It should be noted that the coefficients 0·029 and 0·028 in equations (6) to (8) have almost the same value. Hence, the reduction coefficient, due to corrosion, of ultimate strain, ductile area and elongation of bare bars are similar to each other and can be taken conservatively as 0·03 for practical use.

Although the influence of both bar type and its diameter on reductions of ultimate strain, ductile area and elongation of both bare bars and bars in concrete is not significant, the effect of corrosion condition, that is, bare bars or bars in concrete, is significant. A regression analysis for bars embedded in concrete, similar to that described above for bare bars, resulted in equations (9) to (11).

$$\varepsilon_u = (1{\cdot}0 - 0{\cdot}044 Q_{cor})\varepsilon_{u0} \quad (9)$$

$$A_d = (1{\cdot}0 - 0{\cdot}037 Q_{cor})A_{d0} \quad (10)$$

$$\lambda = (1{\cdot}0 - 0{\cdot}039 Q_{cor})\lambda_0 \quad (11)$$

It should be pointed out that, in contrast to those in equations (6) to (8), the coefficients of 0·037 to 0·044 in equations (9) to (11) are not very close to each other. The scatter of the test data, as well as the smaller number of specimens, may possibly have been the reason for these results. The number of bars corroded in concrete was 24, which is less than half of the 63 bare bars. It is the authors' view that a coefficient with a conservative value of 0·05 may be adopted. Hence, it is suggested that the ductility of corroded reinforcement can be determined by equations (12) and (13) for bare bars and bars embedded in concrete, respectively

$$\varepsilon_u = (1{\cdot}0 - 0{\cdot}03 Q_{cor})\varepsilon_{u0} \quad (12)$$

$$\varepsilon_u = (1{\cdot}0 - 0{\cdot}05 Q_{cor})\varepsilon_{u0} \quad (13)$$

Critical corrosion and critical aspects of bar ductility

From equations (12) and (13), a critical corrosion percentage can be determined above which the reinforcement would not meet the minimum ductility requirements. For example, the critical percentage to ensure that the ultimate strain criterion is achieved is given by equation (14).

$$Q_{cor}^{cr} = \left(1 - \frac{\varepsilon_{umin}}{\varepsilon_{u0}}\right)\frac{1}{\alpha_e} \quad (14)$$

where ε_{umin} is the minimum requirement for bar ductility specified in the appropriate design standard, ε_{u0} is the ultimate strain of the reinforcement prior to corrosion, α_e is the regression coefficient which can be taken as 0·03 for bare bars and 0·05 for bars in concrete.

When the actual corrosion percentage exceeds its critical value, that is, $Q_{cor} \geqslant Q_{cor}^{cr}$, there is a significant probability that the ultimate strain may be reduced to below its minimum requirement. For the reinforcements used in the current tests, their ultimate strains prior to corrosion, ε_{u0}, were in the ranges of 0·086 to 0·203 for bare bars and 0·116 and 0·203 for bars in concrete. These values exceed by a significant margin the minimum requirement of 0·06. However, only 10% corrosion would be sufficient to decrease the ultimate strain to a value less than the minimum requirement for some combinations of variables tested.

The previous discussion of the experimental results indicated that corrosion does not affect ductility in terms of strength ratio, but does decrease it in terms of ultimate strain, ductile area and elongation. However, the effect on structural ductility and robustness depends not only on the reinforcement properties but also on the characteristics of the applied load and of the structural response.

For structures subjected mainly to heavy static loads, such as live loads in a library, that are independent of structural response, the strength ratio of reinforcement plays a more dominant role in structural robustness than ultimate strength or elongation. A high value of strength ratio gives a significant margin between the yield and ultimate strengths and allows a structure to accommodate an overload. Hence, corrosion of reinforcement used in such structures, which mainly resist static loads, hardly influences structural robustness.

For structures located in a seismic zone or subjected mainly to dynamic loads, however, the ultimate strain or elongation of the reinforcement is more critical than its strength ratio. The reason is that a dynamic load depends not only upon the level of action itself, such as earthquake intensity, but also on structural response. An intense earthquake may cause a moderate effect on a ductile structure, but a moderate earthquake can induce an intense effect on a less ductile structure. This is because the dynamic loads due to such an action are dependent on structural stiffness, and reduce as the reinforcement yields and plastic rotations of sections occur. Hence, although an action may theoretically cause the ultimate strength of reinforcement with a low strength ratio to be exceeded, in practice the reinforcement will not rupture if it has sufficient elongation capacity. Hence, corrosion of reinforcement used in structures which are mainly subjected to dynamic loads can decrease their ductility and robustness significantly, because the ultimate strain and elongation of the reinforcement are reduced.

From the above discussion it is apparent that, from the point of view of structural ductility and robustness, the ultimate strain of reinforcement is more critical than its strength ratio. Consequently, henceforth, ultimate strain is taken as the critical ductility parameter.

Analysis of ductility of corroded reinforcement

As discussed previously, corrosion reduces the ductility of reinforcement, in terms of ultimate strain, ductile area and elongation, much more than strength. The reason is that the reduction of bar strength is caused by a decrease of bar section with local penetration and is dominated by the minimum cross-sectional area of the reinforcement. The reduction of bar ductility, however, is a function of the non-uniform distribution of residual sections along the bar length, as discussed below.

Due to irregular and local attack penetration, the residual cross-sectional area of a corroded bar varies significantly along its length,[4] as shown schematically in Fig. 11(a). From non-corroded reinforcement with an initial section A_{s0} over a gauge length of L, the corroded bar can be idealised as a composite with three different portions. Portion I has an average cross-sectional area of A_s over a length of $(1-\eta)L$, whereas portions II and III are assumed to have minimum and maximum residual cross-sectional areas of mA_s and $(2-m)A_s$, respectively, over identical lengths of $0{\cdot}5\eta L$. The coefficient m is the ratio of the minimum residual cross-sectional area of the corroded reinforcement to its average value and, as shown in Reference 4, can be determined by equation (15).

$$m = \frac{A_{\text{smin}}}{A_s} = \frac{1 - 0{\cdot}01Q_{\text{cor}}[2r_x/(1+r_x)]}{1 - 0{\cdot}01Q_{\text{cor}}} \qquad (15)$$

where r_x is the ratio of maximum corrosion penetration $x_{\max}$ to the minimum value $x_{\min}$. Q_{cor} is the corrosion percentage. It is clear that, for either $Q_{\text{cor}} = 0$ (non-corroded bars) or ratio of penetration $r_x = 1{\cdot}0$ (uniform residual section), the coefficient m is always equal to 1·0. Otherwise, it is less than 1·0 and decreases with the corrosion percentage as well as the ratio of penetration.

The coefficient η describes the distribution of different residual sections along the bar length. It is clear that both the sectional coefficient m and the length

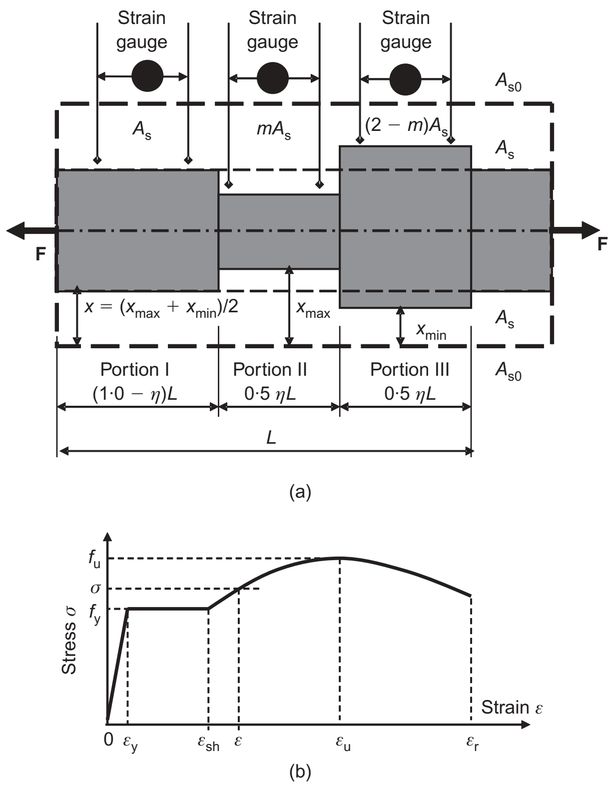

Fig. 11. Analytical model for ductility of corroded concrete. (a) Analytical model; (b) stress–strain curve

coefficient η are constants ($\leqslant 1{\cdot}0$) for a given corrosion condition and degree of corrosion.

Furthermore, since corrosion does not change substantially the shape of the stress–strain curve of reinforcement,[4] it is assumed that corroded reinforcement has a similar curve to that of non-corroded bars and has a definite yield plateau, as shown in Fig. 11(b). On the basis of the experimental results given in Table 1, the hardening, ultimate and rupture strains of plain bars are assumed to be 18, 139 and 166 times their yield strain, respectively; those of ribbed bars are assumed to be 7, 41 and 65 times the yield value. Assuming that the hardening strain curve is a parabolic curve, the strain of reinforcement after hardening can be determined by equation (16)

$$\varepsilon = \varepsilon_h + (\varepsilon_u - \varepsilon_h)\sqrt{\frac{\sigma/f_y - 1}{f_u/f_y - 1}} \quad (16)$$

Figure 11(a) shows that, when portion I with an average section yields, portions II and III with minimum and maximum sections must be in stages of strain-hardening and linear elasticity, respectively. In other words, for a particular tensile force, different residual sections have different tensile stresses and strains. Four possible cases of bar rupture are discussed below with the detailed derivations given in the Appendix Table. In each case the bar ruptures within portion II, which has the minimum section, but the states of portions III and I depend on the critical values $1/(f_u/f_y)$ and $2/(1 - f_u/f_y)$ of sectional coefficient m. By averaging the test data for non-corroded reinforcement l in Table 1, the critical values of $1/(f_u/f_y)$ are 0·72 and 0·83 for tested plain and ribbed bars, respectively. Those of $2/(1 - f_u/f_y)$ are 0·84 and 0·91, respectively.

Case 1. $m \leqslant 1/(f_u/f_y)$

Portion I would have just yielded and portion III would still be linear elastic. The total strain can be determined from equations (17) and (18) for plain and ribbed bars, respectively.

$$\varepsilon_u = (0{\cdot}007 + 0{\cdot}5\eta)\varepsilon_{u0} \quad (17)$$

$$\varepsilon_u = (0{\cdot}024 + 0{\cdot}5\eta)\varepsilon_{u0} \quad (18)$$

Case 2. $m \leqslant 2/(1 - f_u/f_y)$

Portion I would have strain-hardened and portion III would have just yielded. The total strain can be determined from equations (19) and (20) for plain and ribbed bars, respectively.

$$\varepsilon_u = (0{\cdot}69 - 0{\cdot}19\eta)\varepsilon_{u0} \quad (19)$$

$$\varepsilon_u = (0{\cdot}73 - 0{\cdot}21\eta)\varepsilon_{u0} \quad (20)$$

Case 3. $m > 2/(1 - f_u/f_y)$

Portion I would have strain hardened and portion III would have just strain hardened. The total strain can be determined from equations (21) and (22) for plain and ribbed bars, respectively.

$$\varepsilon_u = (0{\cdot}70 - 0{\cdot}13\eta)\varepsilon_{u0} \quad (21)$$

$$\varepsilon_u = (0{\cdot}74 - 0{\cdot}15\eta)\varepsilon_{u0} \quad (22)$$

Case 4. $m \cong 1{\cdot}0$

Rupture would occur within either portion I or II, when portion III had sufficiently strain hardened. The total strain can be determined from equation (23) for both plain and ribbed bars.

$$\varepsilon_u = (1{\cdot}0 - 0{\cdot}12\eta)\varepsilon_{u0} \quad (23)$$

Equations (15) to (23) indicate that, in addition to the negative influence of the length coefficient η, a decrease of sectional coefficient m from about 1·0 in Case 4 to 0·72 or 0·84 in Case 1 causes a significant reduction of ultimate strain. As shown in equation (15), the reduction of sectional coefficient m actually is a combined function of increases both of penetration ratio r_x and corrosion percentage Q_{cor}. Equation (15) indicates that, for an assumed penetration ratio $r_x = 5$, an increase of corrosion from 0 to 10% causes a decrease of sectional coefficient m from 1·0 to 0·93. As a result, for a length coefficient $\eta = 0{\cdot}5$, the ultimate strains of plain and ribbed bars would theoretically reduce from $\varepsilon_u = 0{\cdot}94\varepsilon_{u0}$ by using equation (23) to $\varepsilon_u = 0{\cdot}64\varepsilon_{u0}$ and $\varepsilon_u = 0{\cdot}67\varepsilon_{u0}$ by equations (21) and (22), respectively.

It is apparent that the substantial reduction of ductility of corroded reinforcement, in terms of ultimate strain, ductile area and elongation, could be the result of small extensions occurring over some lengths of a bar when the smallest residual section ruptures.

The above conclusion is supported by the experimental results of the three plain bars that were mechanically machined to 6·40 mm from their initial diameter of 7·34 mm to simulate a 24% removal of metal from the bar surface by corrosion.[4] The experimental results showed that, for 24% reduction of cross-sectional area, although the average yield and ultimate strengths of the three machined bars were almost equal to those of three non-machined bars, their ultimate strain decreased from 0·17 to 0·11. This is because the machining process induced little local penetration over the bar section, but resulted in different residual sections at the machined and non-machined locations.

It should be pointed that equations (15) to (23) have been used only to explore qualitatively the source and mechanism of the reduction of ductility of reinforcement. Due to the lack of test data on the ratio of corrosion penetration and on the distribution of residual sections along a bar's length, neither of the sectional and length coefficients m and η can be determined analytically.

Figure 11(a) also shows that, for the same corroded bar, due to different residual sections along the bar length, different positions of a strain sensor would produce different experimental results. If a sensor were attached to portion II, where the residual section is the smallest, a complete force–extension curve with definite yield plateau would be obtained. However, if the sensor were located at portion III, where the residual section is largest, an incomplete force–extension curve without a yield plateau might be obtained, since the bar's yield and rupture would be concentrated within portion II. Hence, when discussing the characteristics of the stress–strain curve and ultimate strain of corroded bars, it is essential to know the specific testing techniques and details, and care must be taken when interpreting the results from a few test data.

The greater reduction of ductility of bars embedded in concrete, compared with bare bars, arises from different corrosion conditions around the bars. As shown in Fig. 2, compared with uniform sodium chloride solution around a single bare bar, the cover concrete surrounding a bar is much less uniform as an electrolyte, because of its non-uniformly distributed material components and pores. As a result, more irregular residual sections are induced along the length of a bar embedded in concrete. This causes such bars to rupture at their weakest cross section, before they experience a significant extension over a significant length. Hence, ductility of reinforcements embedded in concrete, in terms of ultimate strain, ductile area and elongation, decrease at a much greater rate than those of bare bars.

Comparison with other researchers' results

A comparison of the authors' results with those of other researchers is made in Table 3 in terms of regression coefficients for elongation, strength ratio and elastic modulus. It should be noted that, with the exception of Palssom *et al*.[9] who measured the minimum cross-section with vernier callipers, all of the researchers determined corrosion by means of weight loss

The authors' strength ratio and elastic modulus results are consistent with those of other researchers, except for the strength ratio data of Zhang *et al*.[10] The latter data indicate a 27% decrease in strength ratio for 67% corrosion whereas the data of the authors, Maslehuddin *et al*.,[5] Andrade *et al*.[6] and Palssom *et al*.,[9] for up to 30% corrosion, suggest no significant effect of corrosion on strength ratio. However, the authors' concerns over the experimental technique of Zhang *et al*. have been expressed earlier.

Regarding the elongation of bare plain bars, the authors' data and those of Andrade *et al*.[6] are in reasonable agreement with coefficients of 0·024 and 0·017, respectively. Independent data for bare ribbed bars are not available.

For bars embedded in concrete, the data of Zhang *et al*.[10] indicate the smallest coefficient. This is to be expected, because corrosion of the bars was induced by

Table 3. Comparison of authors' results with those of other researchers

Researcher	Authors		Maslehuddin *et al.*[5]		Andrade *et al.*[6]	Lee *et al.*[7]	Morinaga[8]	Palsson *et al.*[9]	Zhang *et al.*[10]	
Specimen	Bare bars Bars in concrete		Bare bars		Bare bars	Bars in concrete	Bars in concrete	Bars in concrete	Bars in concrete	
Bar type (Diameter mm)	Plain (8, 16)	Ribbed (8, 16, 32)	Plain (8–32)	Ribbed (8–32)	Ribbed (12)	Ribbed (10)	Not stated	Not stated	Plain (8–14)	Ribbed (10–25)
Corrosion condition	Accelerated 0·5–2·0 mA/cm^2		Marine exposure		Accelerated 1·0 mA/cm^2	Accelerated 13·0 mA/cm^2	Service chloride	Service chloride	Service Carbonation	
Corrosion amount (%)	0–25%		0–0·5%		0–11%	0–25%	0–25%	0–>30%	0–67%	
Elongation α_λ	0·029 0·048	0·024 0·031	No significant change		0·0166	Not stated	0·0603	0·018	0·0137	
Strength ratio α_u	No significant change		No significant change		No significant change	Not stated	Not stated	No significant change	Reduced from 1·50 to 1·10	
Elastic modulus α_e	0·0099	0·0108	Not stated		Not stated	0·0123	Not stated	Not stated	Not stated	

carbonation and one would expect more uniform corrosion under such conditions. However, there is considerable variation in the coefficients for the other sets of data. The coefficients for the authors' accelerated data are significantly greater than those of the accelerated data of Andrade *et al*.[6] at a comparable corrosion rate, and are within the range of the data from bars in service chloride environments of Morinaga[8] and Palssom *et al*.[9] Hence, it appears that the effect of corrosion on ductility is highly dependent on the actual corrosion conditions. As a consequence, when using laboratory test data or field data obtained from another structure to assist with the prediction of the response of a corroded structure, it is essential that the assessing engineer should have a clear understanding of the corrosion mechanism and environment of the structure under consideration, and of the conditions pertaining to the laboratory and/or field data being used to inform the assessment. However, in this context it is useful to note that the data obtained in the current investigation lie towards the higher end of the range of available laboratory and field data. Hence, it would be conservative to apply them in practice.

Conclusions

(*a*) The strength ratio, hardening strain and elastic modulus of reinforcement are not significantly affected by corrosion and consequently the corresponding values for non-corroded bars can be adopted in practice.

(*b*) Corrosion creates a non-uniform distribution of cross-sections along the length of a bar. As result ultimate strain, ductile area and elongation are reduced significantly by corrosion. The reductions in these various measurements of ductility are greater than those of yield and ultimate strength which have been reported previously by the authors.[4]

(*c*) The ductility of bars corroded whilst embedded in concrete decreases more significantly than that of bare bars.

(*d*) Only an average value of about 10% of non-uniform corrosion is sufficient to reduce the ductility of bars embedded in concrete to below the minimum requirement specified in CEB Mode Code 90 for class S reinforcement for use in high ductility situations.

(*e*) Although the measured reductions in ultimate strain, ductile area and elongation of smaller diameter plain bars were generally greater than those of larger diameter ribbed bars, the observed differences were not significant at the 5% significance level. Hence, the reduction of ductility of corroded reinforcement is primarily a function of the amount of corrosion, rather than the bar type and diameter.

(*f*) When using laboratory test data or field data obtained from another structure to assist with the prediction of the response of a corroded structure, it is essential that the assessing engineer should have a clear understanding of the corrosion mechanism and environment of the structure under consideration, and of the conditions pertaining to the laboratory and/or field data being used to inform the assessment. However, in this context it is useful to note that the data obtained in the current investigation lie towards the higher end of the range of available laboratory and field data. Hence, it would be conservative to apply them in practice.

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Discussion contributions on this paper should reach the editor by 1 March 2006

Appendix Table. Analysis of elongation strain of corroded reinforcements

		Portion I	Portion II	Portion III
Length (mm)		$(1-\eta)L$	$0{\cdot}5\eta L$	$0{\cdot}5\eta L$
Area (mm^2)		A_s	mA_s	$(2-m)A_s$
Force (N)		F	F	F
Elasticity (N/mm^2)		E_s	E_s	E_s
Case 1	Force (N)	$F_u^1 = F_y$	$F_u^1 = F_y$	$F_u^1 = F_y$
Mean yield and local rupture	Stress σ_s (N/mm^2)	$\frac{F_y}{A_s} = f_y$	$\frac{F_y}{mA_s} = f_u$	$\frac{F_y}{(2-m)A_s} < f_y$
	Strain ε (mm/mm)	$\frac{F_y}{E_s A_s} = \varepsilon_y$	$\varepsilon_u = (41/139)\varepsilon_y$	$\frac{F_y}{E_s(2-m)A_s} < \varepsilon_y$
$m \leqslant \frac{1}{\left(\frac{f_u}{f_y}\right)}$ $= 0{\cdot}72$ plain $= 0{\cdot}83$ rib	Elongation ΔL_i(mm)	$\varepsilon_y(1-\eta)L$	$(41/139)\varepsilon_y(0{\cdot}5\eta L)$	$< \varepsilon_y(0{\cdot}5\eta L)$
	Overall strain $\varepsilon_u = \sum \Delta L_i/L$	$\varepsilon_y[1+69\eta]$ for plain bars; $\varepsilon_y[1+20\eta]$ for ribbed bars $(0{\cdot}007 + 0{\cdot}5\eta)\varepsilon_u$ plain bars; $(0{\cdot}024 + 0{\cdot}48\eta)\varepsilon_u$ for ribbed bars		
Case 2	Force (N)	$F_u^2 = (2-m)F_y$	$F_u^2 = (2-m)F_y$	$F_u^2 = (2-m)F_y$
Whole yield and local rupture	Stress σ_s (N/mm^2)	$\frac{(2-m)F_y}{A_s} > f_y$	$\frac{(2-m)F_y}{mA_s} = f_u$	$\frac{(2-m)F_y}{(2-m)A_s} = f_y$
	Strain ε (mm/mm)	$= (29{\cdot}8/96)\varepsilon_y$	ε_u	ε_y
$m \leqslant \frac{2}{\left(1+\frac{f_u}{f_y}\right)}$ $= 0{\cdot}84$ plain $= 0{\cdot}91$ rib	Elongation ΔL_i(mm)	$(29{\cdot}8/96)\varepsilon_y(1-\eta)L$	$(41/139)\varepsilon_y(0{\cdot}5\eta L)$	$\varepsilon_y(0{\cdot}5\eta L)$
	Overall strain $\varepsilon_u = \sum \Delta L_i/L$	$\varepsilon_y[96 - 26\eta]$ for plain bars; $\varepsilon_y[29{\cdot}8 - 8{\cdot}8\eta]$ for ribbed bars $(0{\cdot}69 - 0{\cdot}19\eta)\varepsilon_u$ for plain bars; $(0{\cdot}73 - 0{\cdot}21\eta)\varepsilon_u$ for ribbed bars		
Case 3	Force (N)	$F_u^3 = mF_u$	$F_u^3 = mF_u$	$F_u^3 = mF_u$
Local rupture	Stress σ_s (N/mm^2)	$\frac{mF_u}{A_s} > f_y$ but $< f_u$	$\frac{mF_u}{mA_s} = f_u$	$\frac{mF_u}{(2-m)A_s} > f_y$
$m > \frac{2}{\left(1+\frac{f_u}{f_y}\right)}$	Strain ε (mm/mm)	$= (30{\cdot}2/96{\cdot}8)\varepsilon_y$	ε_u	$= \varepsilon_h = (7/18)\varepsilon_y$
	Elongation ΔL_i(mm)	$(30{\cdot}2/96{\cdot}8)\varepsilon_y(1-\eta)L$	$(41/139)\varepsilon_y(0{\cdot}5\eta L)$	$(7/18)\varepsilon_y(0{\cdot}5\eta L)$
	Overall strain $\varepsilon_u = \sum \Delta L_i/L$	$\varepsilon_y[96{\cdot}8 - 18{\cdot}3\eta]$ for plain bars; $\varepsilon_y[30{\cdot}2 - 6{\cdot}2\eta]$ for ribbed bars $(0{\cdot}70 - 0{\cdot}13\eta)\varepsilon_u$ for plain bars; $(0{\cdot}74 - 0{\cdot}15\eta)\varepsilon_u$ for ribbed bars		
Case 4	Force (N)	$F_u^4 = F_u$	$F_u^4 = F_u$	$F_u^4 = F_u$
Whole rupture	Stress σ_s (N/mm^2)	$\frac{F_u}{A_s} = f_u$	$\frac{F_u}{mA_s} \cong f_u$	$\frac{F_u}{(2-m)A_s} > f_y$
	Strain ε (mm/mm)	ε_u	ε_u	$= (31/104)\varepsilon_y$
m close to $1{\cdot}0$	Elongation ΔL_i(mm)	$(41/139)\varepsilon_u(1-\eta)L$	$(41/139)\varepsilon_y(0{\cdot}5\eta L)$	$(31/104)\varepsilon_y(0{\cdot}5\eta L)$
	Overall strain $\varepsilon_u = \sum \Delta L_i/L$	$\varepsilon_y[139 - 17{\cdot}5\eta]$ for plain bars; $\varepsilon_y[41 - 5\eta]$ for ribbed bars $(1{\cdot}0 - 0{\cdot}12\eta)\varepsilon_u$ for plain bars; $(1{\cdot}0 - 0{\cdot}12\eta)\varepsilon_u$ for ribbed bars		