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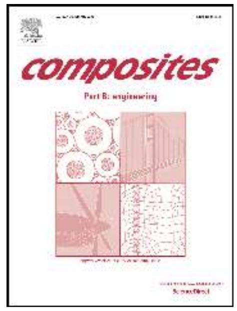

Effect of cutout on stochastic natural frequency of composite curved panels

S. Dey, T. Mukhopadhyay, S.K. Sahu, S. Adhikari

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S. Dey[a], T. Mukhopadhyay[b,*], S. K. Sahu[c], S. Adhikari[b]

[a] Mechanical Engineering Department, National Institute of Technology Silchar, India

[b] College of Engineering, Swansea University, Swansea, United Kingdom

[c] Department of Civil Engineering, National Institute of Technology Rourkela, India

[*]Corresponding author's e-mail: 800712@swansea.ac.uk; URL: www.tmukhopadhyay.com

Abstract

The present computational study investigates on stochastic natural frequency analyses of laminated composite curved panels with cutout based on support vector regression (SVR) model. The SVR based uncertainty quantification (UQ) algorithm in conjunction with Latin hypercube sampling is developed to achieve computational efficiency. The convergence of the present algorithm for laminated composite curved panels with cutout is validated with original finite element (FE) analysis along with traditional Monte Carlo simulation (MCS). The variations of input parameters (both individual and combined cases) are studied to portray their relative effect on the output quantity of interest. The performance of the SVR based uncertainty quantification is found to be satisfactory in the domain of input variables in dealing low and high dimensional spaces. The layer-wise variability of geometric and material properties are included considering the effect of twist angle, cutout sizes and geometries (such as cylindrical, spherical, hyperbolic paraboloid and plate). The sensitivities of input parameters in terms of coefficient of variation are enumerated to project the relative importance of different random inputs on natural frequencies. Subsequently, the noise induced effects on SVR based computational algorithm are presented to map the inevitable variability in practical field of applications.

Keywords: Cutout, composite, support vector regression, random natural frequency, uncertainty quantification, noise

structural components for wide range of aerospace, automotive, nuclear, marine and civil engineering applications. The cutouts of composite panels are inevitable primarily for practical considerations. These are generally utilized not only to access ports for mechanical and electrical systems, but also to serve as doors and windows. Moreover, it is employed for inspection or maintenance purposes of the system. The dynamic behavior composite laminates may fluctuate significantly due to presence of cutout. In other words, the free vibration characteristics of composite curved panels are affected due to variability in shape, size and location of cut outs. Its effects are more difficult to quantify when such composite panels are subjected to random oscillations with uncertain geometric and material properties. The combined effect of cutout along with different stochastic material and geometric parameters may cause wide range of uncertainty in vibration behavior of the structure which may lead to sudden failures due to resonance. It is essential to quantify the uncertain natural frequencies of such composite structural components accurately and thereby follow a design process accounting all the uncertainties appropriately. Composite structures have more uncertainties and variabilities in the structural properties than conventional structures because of large number of structural parameters (inter-dependent in nature) and complex manufacturing and fabrication processes leading to less overall design control. In order to have more exact and realistic analysis, they can be modelled as stochastic structures i.e., structures with uncertain system parameters (both in inputs and outputs). Beside these, the inherent errors involved in finite element modelling lead to inaccuracy in results. A brief review of the literature dealing with the effect of cutout in composite laminates and stochastic analysis of general composite structures is presented in the next paragraph.

primarily confined to buckling and free vibration analysis of composite plates in a deterministic framework. Thornburgh and Hilburger [10] carried out both experimental as well as numerical studies of composite panels with cutouts subjected to compressive load while Dimopoulos and Gantes [11] employed the numerical methods to design the cylindrical steel shells with cutouts. The deterministic free vibration analyses of laminated composite shells with cutout are studied by many researchers [12-18]. The mode shapes and natural frequencies are investigated for cross-ply laminates with square cut-outs by Jenq *et al.* [19]. Eiblmeier and Loughlan [20] studied on buckling analysis of composite panels with circular shaped cut-outs. Sivakumar *et al.* [21] considered large amplitude oscillation on frequency analyses of composite plates with cutout. In past, multi-dimensional deterministic studies are carried out to conduct investigations on behavior of composite and sandwich plate or shells with cutouts such as Anuja and Katukam [22] presented parametric studies on the cutouts in heavily loaded aircraft beams, Mondal *et al.* [23] studied the dynamic performance of sandwich composite plates with circular hole. Recently Venkatachari *et al.* [24] investigated on influence of environment for free vibration of composite laminates with cutouts and Yu et al. [25] studied on buckling and free vibration analyses of laminated composites plates with complicated cutouts employing first order shear deformation theory and level set method. Plenty amount of research is carried out on free vibration analysis of composite plates, shells and sandwich structures [26-40]. A concise review of literature on application of efficient reduced order models in the field of structural analysis and design is presented next. Ample research is conducted on response surface methodologies in conjunction to composite materials in past, such as, Park *et al.* [41/29] conducted a numerical investigation on composite shells employing stochastic finite element response surface method (RSM) to analyze the reliability of turbine blades made of composites. Nik *et al.* [44] conducted an illustrative study of metamodelling methods in conjunction to design optimization of composites subjected to variable stiffness, while Steuben *et al.* [45] carried out the inverse characterization of composite materials by surrogate modeling. Some researcher employed the perturbation-based stochastic multi-scale analyses of composite materials [46, 47]. Considering stochastic nonlinear systems, Gao and Tong [48] employed fuzzy to design composites while Kepple *et al.* [49] incorporated an improved stochastic method for buckling of composite cylindrical shells dealing with modelling errors.

Despite the engineering importance of cutouts involved in composites as pointed out in the preceding paragraphs, the number of rese arch articles and reports in conjunction to the subject topic are found to be limited to deterministic results, possibly due to the computational complexity involved in it. The present study is focused to quantify the uncertain natural frequencies for composite panels with cutouts following an efficient support vector regression based algorithm in conjunction with finite element analysis. In general, the deterministic approach of finite element analysis becomes computationally inefficient and costly when the input parameters considered at each nodal points of each discretized element becomes random with respect to its meshing pattern and boundary condition. The number of elements depends not only on cutout- size but also on its shape and location. Moreover, due to random variability of each input parameters at element level throughout the structure, the application of identical isotropic plate elements for computing the element mass and stiffness matrices will never match with reality. Thus uncertainty quantification for such structures following traditional Monte Carlo simulation based approaches is prohibitively expensive effectively handled by using Support Vector Regression (SVR) which is employed as an efficient surrogate of the expensive finite element model allowing rigorous occurrence of virtual iterations to be exercised with cost-effectiveness. In stochastic structural problems, it can be restricted to consideration of the two-class problems, namely, with linear and non-linear classifier, without loss of generality. In such problems, the aim is to segregate the two classes by means of a function which is induced from known random dataset. In other words, the prime objective is to produce a classifier that will work well on unpredictable random data, i.e. it generalizes well. The conformity of such phenomenon can be efficiently dealt by SVR model. Due to inherent complexity, composite structures have intrusive variability of geometric and material properties in both linear and nonlinear domain while analyzing the structural reliability. Moreover the risk involved in ensuring the reliability of such system can be projected as the accidental loss plus uncertain measure of such loss. In compliance of the same, the present study aimed to predict those uncertain natural frequencies by employing support vector regression model. No literature is found which dealt with uncertainty quantification of natural frequencies in laminated composite panels with cutout using the support vector regression model. The random variation of individual parameters and combined parameters are considered in the present investigation. This article is organized hereafter as, section 2: stochastic finite element formulation for composite curved panels with cutout, section 3: brief description of support vector regression, section 4: support vector regression based uncertainty quantification algorithm for composite laminates with cutout including the effect of noise, section 5: results and discussion, section 6: conclusion.

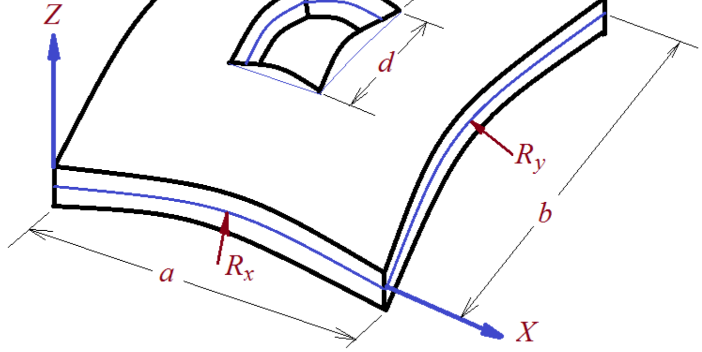


Fig. 1 Laminated composite curved panel with cutout

2. Governing equations

In the present study, the composite panels with central cutout (as shown in Figure 1) are considered. The stress resultants can be expressed in terms of the mid-plane strains and curvatures as

$$\begin{Bmatrix} N_i(\overline{\omega}) \\ M_i(\overline{\omega}) \\ Q_i(\overline{\omega}) \end{Bmatrix} = \begin{bmatrix} A_{ij}(\overline{\omega}) & B_{ij}(\overline{\omega}) & 0 \\ B_{ij}(\overline{\omega}) & D_{ij}(\overline{\omega}) & 0 \\ 0 & 0 & S_{ij}(\overline{\omega}) \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_j \end{Bmatrix} \tag{1}$$

The constitutive equation [50] is given by

$$\{F\} = [\mathrm{D}(\overline{\omega})] \ \{\varepsilon\} \tag{2}$$

Subsequently, extension, bending-stretching coupling and bending terms can be expressed as

$$[A_{ij}(\overline{\omega}), \ \ B_{ij}(\overline{\omega}), \ \ D_{ij}(\overline{\omega})] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\{\overline{Q}_{ij}(\overline{\omega})\}_{on}]_k \ \ [1, \ \ z, \ \ z^2] \, dz \qquad i, j = 1, 2, 6 \tag{3}$$

The transverse shear term can be derived from

$$[S_{ij}(\overline{\omega})] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \alpha_s \ [\overline{Q}_{ij}(\overline{\omega})]_k \ dz \qquad i, j = 4, 5 \tag{4}$$

expressed as

$$
\begin{aligned}
\overline{Q}_{11}(\overline{\omega}) &= Q_{11}(\overline{\omega})\, m^4 + 2[Q_{12}(\overline{\omega}) + 2Q_{66}(\overline{\omega})] m^2 n^2 + Q_{22}(\overline{\omega})\, n^4 \\
\overline{Q}_{12}(\overline{\omega}) &= [Q_{11}(\overline{\omega}) + Q_{22}(\overline{\omega}) - 4Q_{66}(\overline{\omega})] m^2 n^2 + Q_{12}(\overline{\omega})\,(m^4 + n^4) \\
\overline{Q}_{22}(\overline{\omega}) &= Q_{11}(\overline{\omega})\, n^4 + 2[Q_{12}(\overline{\omega}) + 2Q_{66}(\overline{\omega})] m^2 n^2 + Q_{22}(\overline{\omega})\, m^4 \\
\overline{Q}_{16}(\overline{\omega}) &= [Q_{11}(\overline{\omega}) - Q_{12}(\overline{\omega}) - 2Q_{66}(\overline{\omega})]\, n\, m^3 + [Q_{12}(\overline{\omega}) - Q_{22}(\overline{\omega}) + 2Q_{66}(\overline{\omega})]\, m\, n^3 \\
\overline{Q}_{26}(\overline{\omega}) &= [Q_{11}(\overline{\omega}) - Q_{12}(\overline{\omega}) - 2Q_{66}(\overline{\omega})]\, m\, n^3 + [Q_{12}(\overline{\omega}) - Q_{22}(\overline{\omega}) + 2Q_{66}(\overline{\omega})]\, n\, m^3 \\
\overline{Q}_{66}(\overline{\omega}) &= [Q_{11}(\overline{\omega}) + Q_{22}(\overline{\omega}) - 2Q_{12}(\overline{\omega}) - 2Q_{66}(\overline{\omega})] m^2 n^2 + Q_{66}(\overline{\omega})](m^4 + n^4)
\end{aligned}
\tag{5}
$$

The off-axis elastic constant matrix linked with transverse shear deformation can be expressed as

$$
\begin{aligned}
\overline{Q}_{44}(\overline{\omega}) &= G_{13}(\overline{\omega})\, m^2 + G_{23}(\overline{\omega}) n^2 \\
\overline{Q}_{45}(\overline{\omega}) &= [G_{13}(\overline{\omega}) - G_{23}(\overline{\omega})]\, m\, n \\
\overline{Q}_{55}(\overline{\omega}) &= G_{13}(\overline{\omega})\, n^2 + G_{23}(\overline{\omega})\, m^2
\end{aligned}
\tag{6}
$$

where $m = \sin\theta(\overline{\omega})$ and $n = \cos\theta(\overline{\omega})$, wherein $\theta(\overline{\omega})$ is random ply orientation angle. The on-axis terms can be represented as

$$
\begin{aligned}
&[Q_{ij}(\overline{\omega})]_{on} = \begin{bmatrix} Q_{11}(\overline{\omega}) & Q_{12}(\overline{\omega}) & 0 \\ Q_{12}(\overline{\omega}) & Q_{12}(\overline{\omega}) & 0 \\ 0 & 0 & Q_{66}(\overline{\omega}) \end{bmatrix} \quad \text{for } i, j = 1,2,6 \\
&[\overline{Q}_{ij}(\overline{\omega})]_{on} = \begin{bmatrix} Q_{44}(\overline{\omega}) & Q_{45}(\overline{\omega}) \\ Q_{45}(\overline{\omega}) & Q_{55}(\overline{\omega}) \end{bmatrix} \quad \text{for } i, j = 4,5
\end{aligned}
\tag{7}
$$

where

$$
Q_{11} = \frac{E_1(\overline{\omega})}{1 - \nu_{12}(\overline{\omega})\,\nu_{21}(\overline{\omega})} \qquad Q_{22} = \frac{E_2(\overline{\omega})}{1 - \nu_{12}(\overline{\omega})\,\nu_{21}(\overline{\omega})} \qquad Q_{12} = \frac{\nu_{12}(\overline{\omega})\; E_2(\overline{\omega})}{1 - \nu_{12}(\overline{\omega})\,\nu_{21}(\overline{\omega})}
$$

cutout. The differential equations of equilibrium can be expressed as [51]

$$\frac{\partial N_x(\varpi)}{\partial x}+\frac{\partial N_{xy}(\varpi)}{\partial y}-\frac{1}{2}C_2\left(\frac{1}{R_y(\varpi)}-\frac{1}{R_x(\varpi)}\right)\frac{\partial M_{xy}(\varpi)}{\partial y} + C_1\left(\frac{Q_x(\varpi)}{R_x(\varpi)}+\frac{Q_y(\varpi)}{R_{xy}(\varpi)}\right) = P_1(\varpi)\frac{\partial^2 u(\varpi)}{\partial t^2}+P_2(\varpi)\frac{\partial^2\theta_x(\varpi)}{\partial t^2} \tag{8}$$

$$\frac{\partial N_{xy}(\varpi)}{\partial x}+\frac{\partial N_y(\varpi)}{\partial y}+\frac{1}{2}C_2\left(\frac{1}{R_y(\varpi)}-\frac{1}{R_x(\varpi)}\right)\frac{\partial M_{xy}(\varpi)}{\partial x} + C_1\left(\frac{Q_y(\varpi)}{R_y(\varpi)}+\frac{Q_x(\varpi)}{R_{xy}(\varpi)}\right) = P_1(\varpi)\frac{\partial^2 v(\varpi)}{\partial t^2}+P_2(\varpi)\frac{\partial^2\theta_y(\varpi)}{\partial t^2} \tag{9}$$

$$\frac{\partial Q_x(\varpi)}{\partial x}+\frac{\partial Q_y(\varpi)}{\partial y}-\frac{N_x(\varpi)}{R_x(\varpi)}-\frac{N_y(\varpi)}{R_y(\varpi)}-2\frac{N_{xy}(\varpi)}{R_{xy}(\varpi)} + N_x^0(\varpi)\frac{\partial^2 w(\varpi)}{\partial x^2}+N_y^0(\varpi)\frac{\partial^2 w(\varpi)}{\partial y^2} = P_1(\varpi)\frac{\partial^2 w(\varpi)}{\partial t^2} \tag{10}$$

$$\frac{\partial M_x(\varpi)}{\partial x}+\frac{\partial M_{xy}(\varpi)}{\partial y}-Q_x(\varpi) = P_3(\varpi)\frac{\partial^2\theta_x(\varpi)}{\partial t^2}+P_2(\varpi)\frac{\partial^2 u(\varpi)}{\partial t^2} \tag{11}$$

$$\frac{\partial M_{xy}(\varpi)}{\partial x}+\frac{\partial M_y(\varpi)}{\partial y}-Q_y(\varpi) = P_3(\varpi)\frac{\partial^2\theta_y(\varpi)}{\partial t^2}+P_2(\varpi)\frac{\partial^2 u(\varpi)}{\partial t^2} \tag{12}$$

wherein C_1 and C_2 are represented as tracers of shear deformable version of the theories of Sanders ($C_1 = C_2 = 1$), Love ($C_1 = 1$ and $C_2 = 0$), and Donnells ($C_1 = C_2 = 0$). $N_x(\varpi)$, $N_y(\varpi)$ and $N_{xy}(\varpi)$ denote the stochastic in-plane stress resultants, $M_x(\varpi)$, $M_y(\varpi)$ and $M_{xy}(\varpi)$ represents the stochastic moment resultants while $Q_x(\varpi)$ and $Q_y(\varpi)$ depict as the stochastic transverse shear stress resultants. $R_x(\varpi)$, $R_y(\varpi)$ and $R_{xy}(\varpi)$ denote the stochastic radii of curvature along the x and y directions and the radius of twist, respectively.

$$(P_1, P_2, P_3)(\varpi)=\sum_{i=1}^{n}\int_{z_{k-1}}^{z_k}\rho_k(\varpi)\,[1,\ z,\ z^2]\,dz \tag{13}$$

The present study considers eight nodes in isoparametric quadratic element wherein five degrees of freedom (three translations and two rotations) is assumed at each nodal point. Considering Hamilton's principle [52] in conjunction to Lagrange's equation, the dynamic equilibrium equation of motion for free vibration can be expressed as

$$[M(\overline{\omega})][\ddot{\delta}] + [K(\overline{\omega})]\{\delta\} = 0 \tag{14}$$

where $M(\overline{\omega})$, $[K(\overline{\omega})]$, $\{\delta\}$ are represented as mass matrix, elastic stiffness matrix and vector of generalized coordinates. The random natural frequencies $[\omega_m(\overline{\omega})]$ are derived from the standard eigenvalue problem [53] using QR algorithm and are obtained as

$$\omega_m^2(\overline{\omega}) = \frac{1}{\lambda_m(\overline{\omega})} \qquad \text{where } m = 1,2,3............,n_m \tag{15}$$

where n_m denotes the mode number and $\lambda_m(\overline{\omega})$ indicates the m-th eigenvalue of matrix $A = K^{-1}(\overline{\omega})\, M(\overline{\omega})$

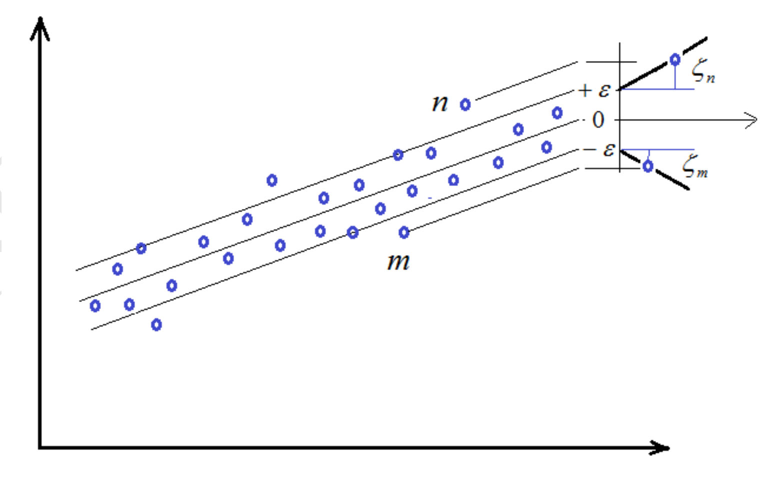


Fig. 2 Soft margin loss setting corresponding to a linear Support Vector machine [54]

(SVM) pertaining to regression analysis. Suppose the training data is given as $\{(x_1, y_1), (x_2, y_2)........(x_l, y_l)\} \subset \chi \times \Re$ where χ and $\Re$ denote the space of the input patterns and Euclidean space vector. In support vector regression, the primary objective is to find a function $\hat{f}(x)$ that has at most ε deviation from the actually obtained targets y_i for all these training data and at the same time, is as flat as possible. The formulation In Figure 2, only the points distributed outside the shaded zone are contributed to the cost linked with insensitive loss function (ζ) at points (*m* and *n*) wherein the deviations are penalized in a linear fashion. Thus the optimization problem is solved more easily in its dual formulation. Moreover, the dual formulation provides the key for extending SV machine to nonlinear functions. Hence it can be used as a standard dualization method utilizing Lagrange multipliers [54]. The errors are neglected as long as they are less than the region of tolerance (say $\pm\varepsilon$) (refer to Figure 2), but it will not accept any deviation larger than this limiting value. The SVR model is constructed by employing the subset sample data and support vectors wherein maximum deviation of ε from the function value of each training data exist. For a linear case, SVR model can be expressed as [55]

$$\hat{f}(x) = \hat{Y}(x) = \langle W \cdot x \rangle + b \tag{16}$$

where $\hat{Y}(x)$, *W* and *b* indicate the predicted value of objective function, weight-vectors and bias, respectively while $\langle \cdot \rangle$ denotes the inner product. The sample data points within the $\pm\varepsilon$ band (known as the ε-tube) are neglected, while the predictors are considered wherein the data points are found on or outside this region. The SVR prediction can be expressed as,

$$\hat{f}(x) = b + \sum_{i=1}^{k} W^{(i)} \psi(x, x^{(i)}) \tag{17}$$

least complex. Despite reducing the risk of using training data for fitting, SVR reduces the upper bound on the calculated risk by employing ε-insensitive loss function, as constrained convex quadratic optimization problem proposed by [51]

$$G(x) = \begin{cases} 0 & \left|Y(x) - \hat{Y}(x)\right| \le \varepsilon \\ \left|Y(x) - \hat{Y}(x)\right| - \varepsilon & Otherwise \end{cases} \tag{18}$$

where ε and G parameters are selected based on the recommendation proposed by Cherkassky and Ma [56]. SVR model performs both linear as well as non-linear regression in conjunction to ε-insensitive loss function, simultaneously. It attempts to decrease the complexity by reducing the weighting vector as the objective function,

$$\text{Minimize} \quad \frac{1}{2}\left|W\right|^2$$

$$\text{Subjected to} \quad \begin{cases} Y_i - \langle W \cdot x^{(i)} \rangle - b \le \varepsilon \\ \langle W \cdot x^{(i)} \rangle + b - Y_i \le \varepsilon \end{cases} \tag{19}$$

A non-linear regression can be formed by replacing the $\langle \cdot \rangle$ in Eq. (16) with a kernel function, K as [57]

$$\hat{f}(x) = \sum_{i=1}^{k} (\alpha_i - \alpha_i^*)\ K(x_i, x) + b \tag{20}$$

In the present study, Gaussian kernel function is used throughout the entire investigation.

4. Stochastic approach using SVR model

The dimension of cutout of laminated composite panel with respect to each layer can be defined as $C_o = c/a$ where c/a denotes the percentage of cutout with respect to overall panel-dimension. The effects of both single variable as well as multi-dimensional random

(a) Variation of only ply-orientation angle: $\theta(\overline{\omega}) = \{\theta_1\ \theta_2\ \theta_3 ....\theta_i ......\theta_l\}$

(b) Variation of only twist angle: $\psi(\overline{\omega}) = \{\psi_1\ \psi_2\ \psi_3 ....\psi_i ......\psi_l\}$

(c) Variation of only thickness: $t(\overline{\omega}) = \{t_1\ t_2\ t_3 ....t_i ......t_l\}$

(d) Variation of only material properties: $p_m(\overline{\omega}) = \{p_{m(1)}\ p_{m(2)}\ p_{m(3)} ....p_{m(i)} ......p_{m(l)}\}$

(e) Combined variation of ply orientation angle, twist angle, thickness and materials properties:

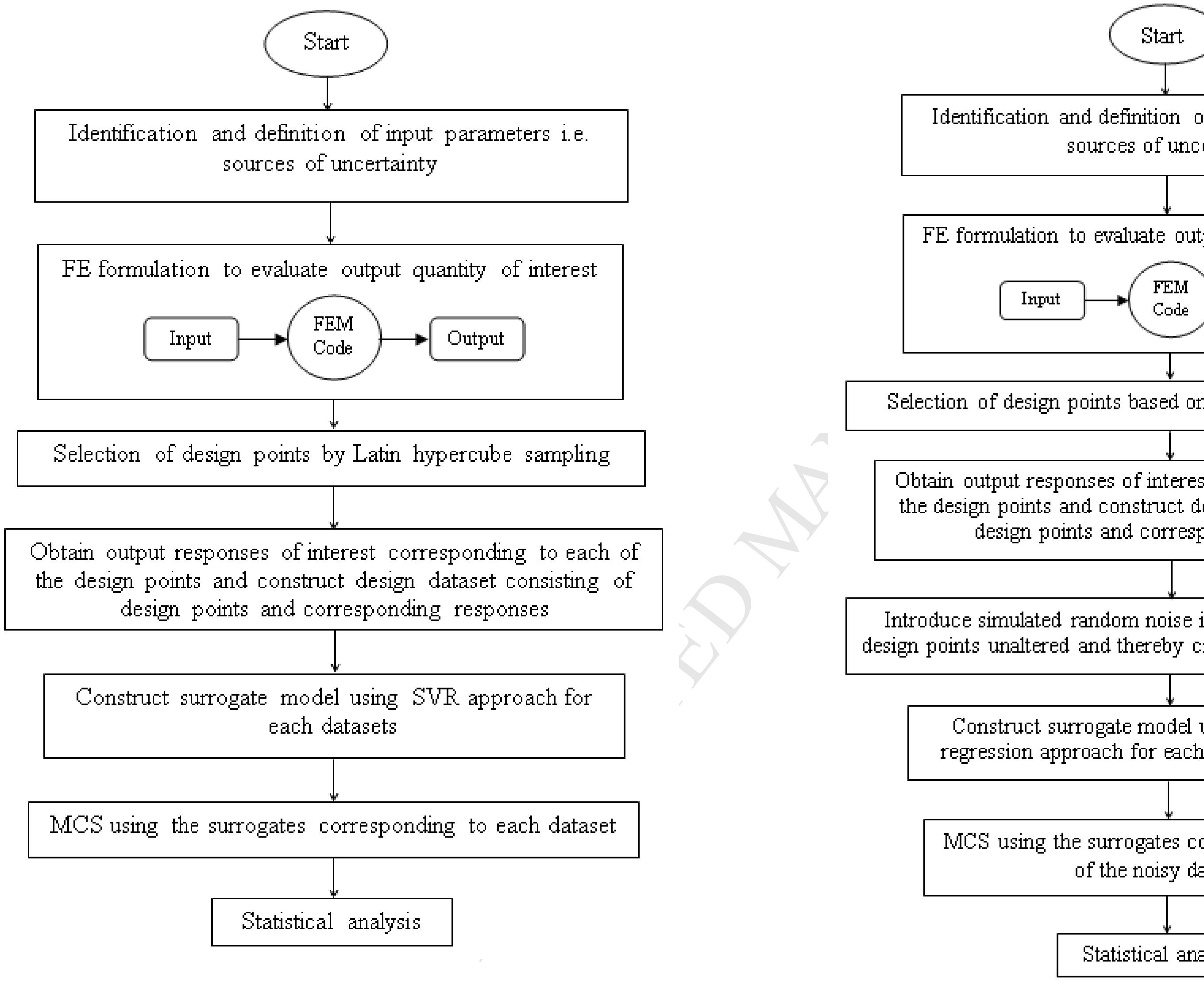
$$g(\overline{\omega}) = \{\Phi_1(\theta_1 ...\theta_l), \Phi_2(\psi_1 ...\psi_l), \Phi_3(t_1 ...t_l), \Phi_4(p_{m(1)} ...p_{m(l)})\}$$

where for i^{th} layer, θ_i, ψ_i, t_i, $p_{m(i)}$ are the ply orientation angle, twist angle, thickness and material properties wherein material properties include $E_{1(i)}$, $E_{2(i)}$, $G_{12(i)}$, $G_{23(i)}$, $\mu_{12(i)}$ and ρ_i denoting the elastic modulus (longitudinal direction), elastic modulus (transverse direction), shear modulus (longitudinal direction), shear modulus (transverse direction), Poisson ratio and mass density, respectively (l is the total layer number, where i = 1, 2, 3…l). In conformity of the same, $\pm 5°$ variability is assumed for ply orientation and twist angle while $\pm 10\%$ variability from respective deterministic mean values of thickness and material properties are considered. The flowchart of uncertainty quantification algorithm based on SVR with and without noise effect using SVR model is shown in Figure 3. Latin hypercube sampling is used for forming the sample dataset of the input space.

The pronounced noise-effect on the proposed SVR based UQ algorithm is also accounted by introducing different levels of noise as depicted in Figure 3(b). In present investigation, Gaussian white noise is employed for SVR model formation

$$f_{ijN} = f_{ij} + p \times \xi_{ij} \qquad (21)$$

where, p and f are the multiplication factor and natural frequency with the subscript i and j frequency number and sample number, respectively. A function generating random numbers



(a) SVR based uncertainty quantification algorithm without noise (b) SVR based uncertainty quan

Fig. 3 Flowchart on uncertainty quantification algorithm based on SVR including the effe noise level. Subscript *N* is used here to indicate the noisy frequency. Thus simulated noisy dataset (i.e. the sampling matrix for SVR model formation) is formed by introducing pseudo random noise in the responses, while the input design points are kept unaltered. Subsequently for each dataset, SVR based MCS is carried out to quantify uncertainty of composite laminates. The noise-effect is found to be investigated previously [58-62] for other problems and related to other surrogates. The assessment of SVR based uncertainty propagation with noise-effect is the first attempt of its kind to the best of authors' knowledge. The root-causes of such inevitable noise-effect can be attributed to the fact of other unknown sources of uncertainty such as measurement-errors, modelling-errors and computer simulation-errors and other system-specific epistemic uncertainties. Thus the present investigation is portrayed with a comprehensive idea about the robustness of SVR based UQ algorithm including noise-effect.

5. Results and Discussion

The present study is dealt with three layered graphite-epoxy angle-ply composite cantilever spherical shallow panel with a central square shaped cutout. Four different types of panels are considered for detail analyses: plate ($R_x = R_y = \infty$), cylindrical ($a/R_x = \infty$, $b/R_y = 0.25$), spherical ($a/R_x = b/R_y = 0.25$) and hyperpolic paraboloid ($a/R_x = -0.25$, $b/R_y = 0.25$). The length, width and thickness of the composite laminate assumed in the present analyses are 1 m, 1 m and 5 mm, respectively and the dimension of the square shaped cutout size is considered as a percentage of its overall length and width [$C_l = c/a$ and $C_b = d/b$ wherein $c = d$ for square cutout] from 0.1 to 0.5 with a step of 0.1. Material properties of graphite–epoxy composite [63] are considered with deterministic mean values

A convegence study is carried out to validate the present formulation and to ascertain the optimal finite element mesh size as shown in Table 1 and Table 2. Table 1 presents the convergence study for non-dimensional fundamental natural frequencies of three layered graphite-epoxy untwisted angle-ply composite plates with finite element sizes (4 × 4), (6 × 6), (8 × 8) and (10 × 10), respectively in addition to comparision with the results obtained by Qatu and Leissa [64]. In contrast, Table 2 presents the convergence of fundamental natural frequencies for a simply supported square plate with specific size of the cutout with finite element sizes (4 × 4), (8 × 8), (12 × 12), (16 × 16) and (20 × 20), respectively in addition to comparision with the results obtained by Reddy [65]. Thus, Table 1 and Table 2 provide validation of the deterministic finite element model. A discretization of (8 × 8) mesh on plan area with 64 elements 225 nodes with natural coordinates of an isoparametric quadratic plate bending element are considered for the present FEM analysis.

In general, the number of expensive finite element analysis required for original Monte Carlo simulation based UQ approach is same as the sampling size. The present approach of SVR based uncertainty quantification develops a predictive and representative surrogate model relating each natural frequency to a number of stochastic input parameters.

Table 1 Convergence study for non-dimensional fundamental natural frequencies [$\omega=\omega_n L^2 \sqrt{(\rho/E_1 t^2)}$] of three layered ($\theta°$/-$\theta°$/$\theta°$) graphite-epoxy untwisted composite plates (a/b=1 and b/t=100)

θ	Present FEM							Ref. [64]
	(4 × 4)	(6 × 6)	(8 × 8)	(10 × 10)	(12 × 12)	(16 × 16)	(20 × 20)	
0°	1.0112	1.0133	1.0107	1.0040	1.0031	1.0028	1.0022	1.0175
90°	0.2553	0.2567	0.2547	0.2542	0.2540	0.2533	0.2530	0.2590

Shell Type	Present FEM					[65]
	(4 × 4)	**(8 × 8)**	**(12 × 12)**	**(16 × 16)**	**(20 × 20)**	
Isotropic	23.8432	23.570	23.4703	23.4364	23.4218	23.489
Orthotropic	51.8546	51.0597	50.7899	50.6944	50.6505	51.232
Composite	48.9546	48.2535	48.0650	48.0222	48.0064	48.414

Thus SVR model for a particular mode represents the result encompassing each possible combination of all stochastic input parameters. A convergence study of sample size for SVR model formation with respect to original MCS is tabulated in Table 3 for the first three modes corresponding to individual (ply-orientation angle) and combined variation (ply-orientation angle, twist angle, thickness, elastic moduli, shear moduli, poission ratio and mass density). By analysing the statistical parameters presented in the Table 3 it is evident that sample size of 256 and 512 are adequate for the SVR model formation corresponding to individual and combined cases, respectively. Figure 4 and Figure 5 present the scatter plot and probability density function plot, respectively considering the converged sample sizes for stochastic natural frequencies using SVR model and traditional MCS approach for angle-ply (45°/-45°/45°) composite curved panels with cutout corresponding to individual and for combined variation, respectively. It is evident from these figures that the results of the proposed SVR based approach are in good agreement with that of direct MCS simulations corroborating accuracy and validity of the proposed approach.

The probability density function plots for stochastic first three modes using SVR approach for individual variation of ply orientation angle considering angle-ply (θ°/- θ °/ θ °) composite curved panels with cutout are presented in Figure 6. The figure reveals that as the ply orientation angle (θ) of the angle-ply composite curved panels with a particular size of cutout increases, the stochastic first three natural frequencies are found to reduce. In figure 6-

Type	Mode	Method	Samples	Maximum	Minimum	Mean	SD
Individual Variation (Only ply-orientation angle)	First	MCS	10,000	17.8750	13.9750	15.8501	0.7935
		Present method	64	17.8810	13.9782	15.8561	0.7948
			128	17.8604	13.9835	15.8464	0.7946
			256	17.8725	13.9905	15.8556	0.7929
			512	17.8770	13.9887	15.8570	0.7941
			1024	17.8758	13.9762	15.8512	0.7945
	Second	MCS	10,000	100.8807	79.8358	90.0001	4.2127
		Present method	64	100.7124	80.99413	90.1040	4.1351
			128	100.6888	80.95801	89.9539	4.1981
			256	100.8305	80.01091	90.0567	4.2017
			512	100.8747	79.91743	90.0316	4.2215
			1024	100.8813	79.84141	90.0016	4.2301
	Third	MCS	10,000	136.1571	122.0387	128.9192	2.7264
		Present method	64	136.1873	122.1453	129.9542	2.7171
			128	136.2222	121.9553	128.9793	2.7513
			256	136.1728	122.1545	129.9441	2.6950
			512	136.1651	122.0598	128.9398	2.7144
			1024	136.1578	122.0436	128.9291	2.7345
Combined Variation	First	MCS	10,000	21.3550	13.1850	17.0607	1.3754
		Present method	128	21.2290	13.2270	17.08304	1.3595
			256	21.4815	13.1062	17.07911	1.3963
			512	21.4277	13.1694	17.0867	1.3848
			1024	21.3673	13.1416	17.07897	1.3822
			2048	21.3778	13.1873	17.07107	1.3787
	Second	MCS	10,000	110.3002	73.0035	90.6014	6.4184
		Present method	128	110.3709	72.9484	90.7125	6.3907
			256	110.9686	72.7500	90.6700	6.5456
			512	110.3437	73.1406	90.6806	6.4109
			1024	110.3767	73.0716	90.6662	6.4375
			2048	110.3034	73.0315	90.6418	6.4297
	Third	MCS	10,000	192.8565	123.9751	158.4033	14.0007
		Present method	128	192.6459	124.6592	158.5274	13.9243
			256	193.2740	124.7608	158.6299	13.6715
			512	192.5198	124.5556	158.4206	13.8215
			1024	193.0577	124.1068	158.3772	13.9229
			2048	192.8565	123.9751	158.4033	14.0007

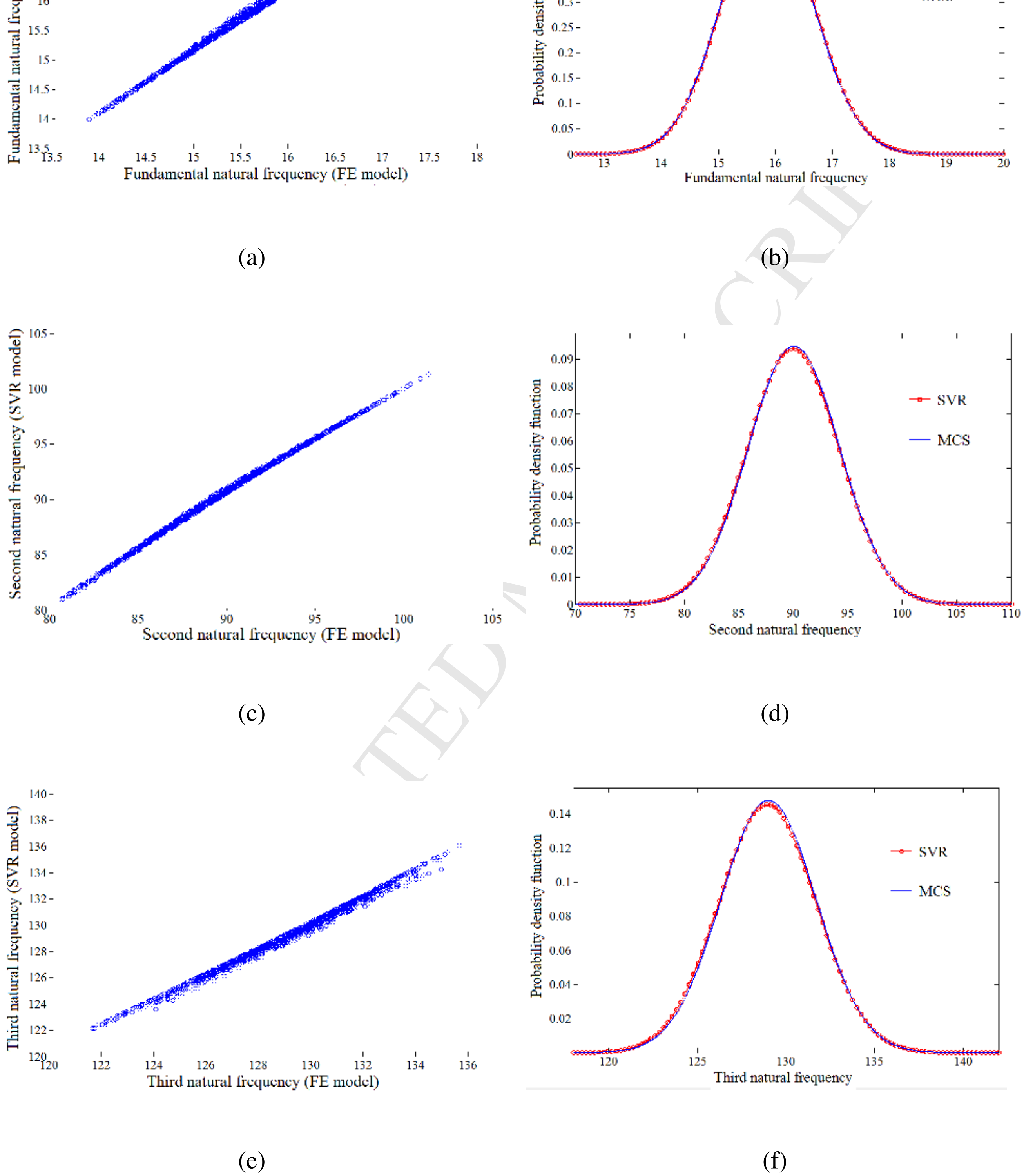


Fig. 4 (a, c, e) Scatter plot and (b, d, f) Probability density function plot for stochastic first three natural frequencies using SVR approach for individual variation of ply orientation angle $[\theta(\overline{\omega})]$ of angle-ply (45°/-45°/45°) composite curved panel with cutout ($C_o = 0.1$)

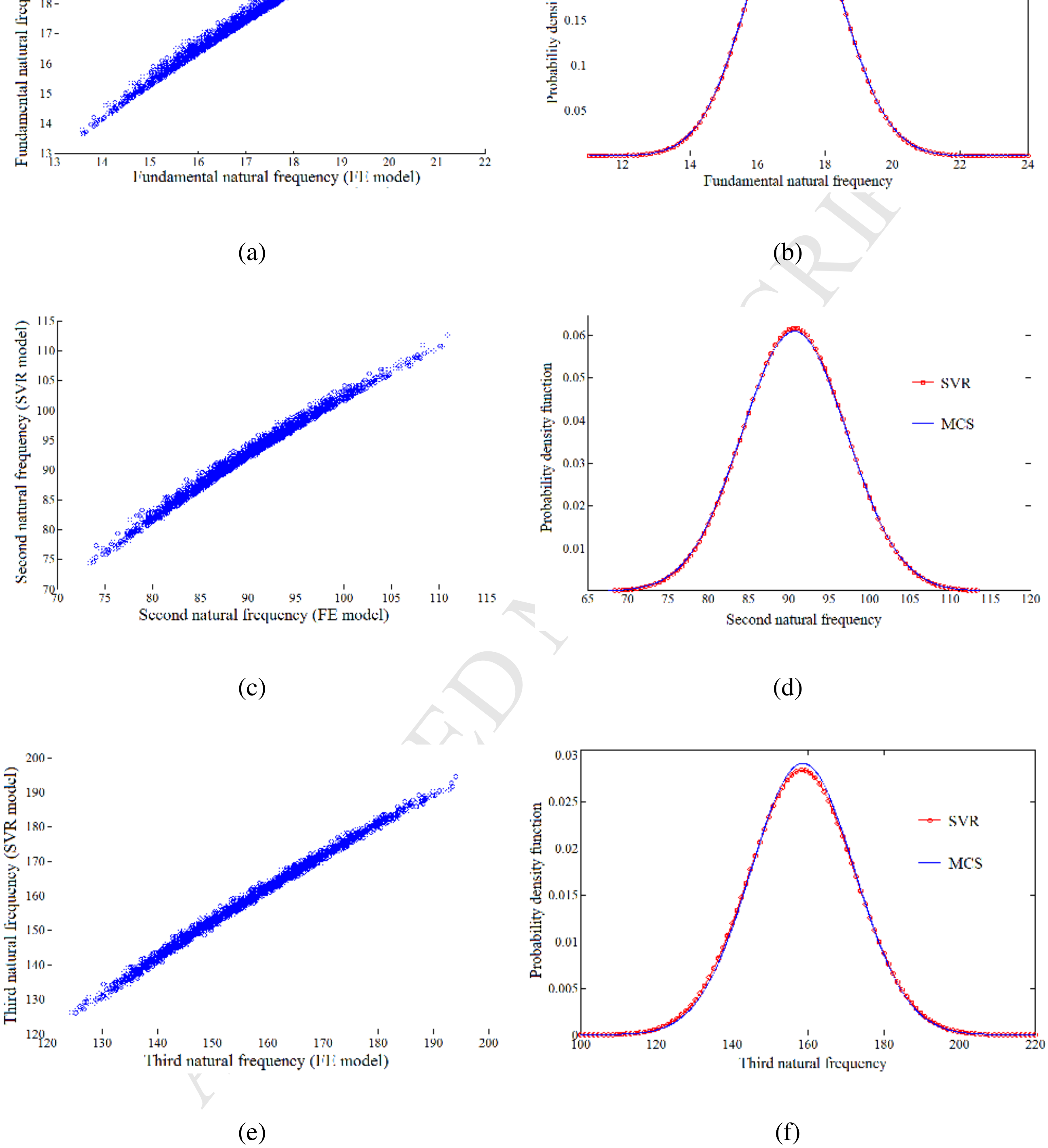


Fig. 5 (a, c, e) Scatter plot and (b, d, f) Probability density function plot for stochastic first three natural frequencies using SVR approach for combined variation of ply angle, elastic modulus, shear modulus, Poisson ratio and mass density $[g(\overline{\omega})]$ of angle-ply (45°/-45°/45°) composite curved panel with cutout ($C_o = 0.1$)

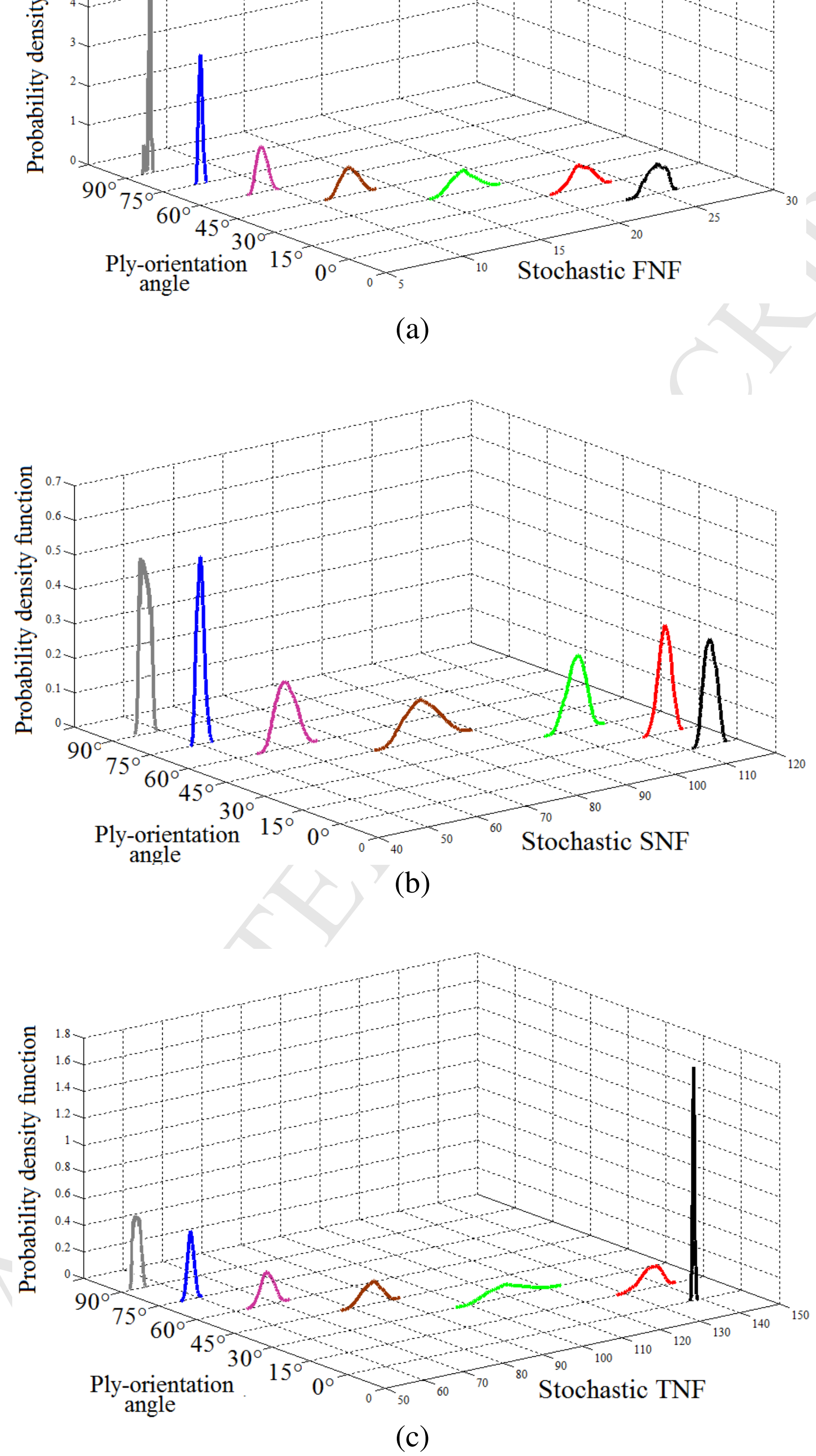
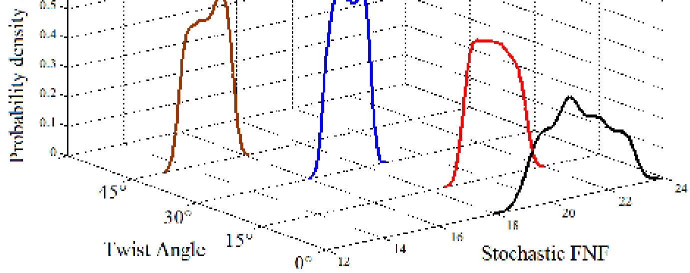


Fig. 6 Probability density function plot for first three stochastic natural frequencies using SVR approach for individual variation of ply orientation angle $[\theta(\bar{\omega})]$ of angle-ply ($\theta°$/- $\theta°$/ $\theta°$) composite curved panel with cutout



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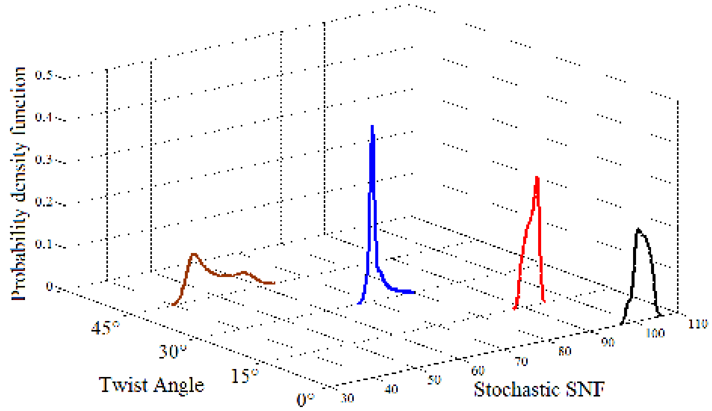

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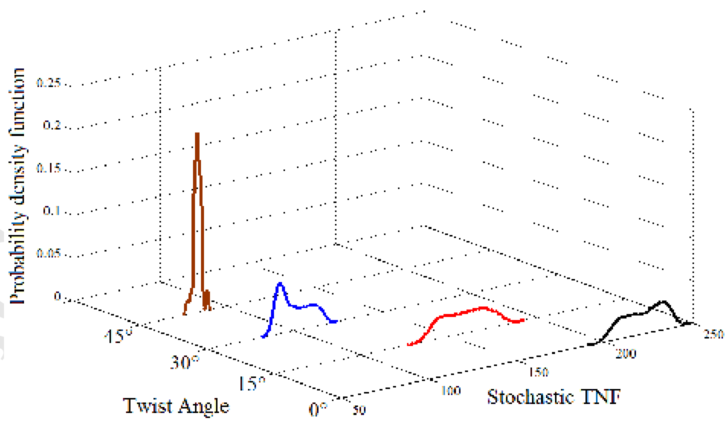

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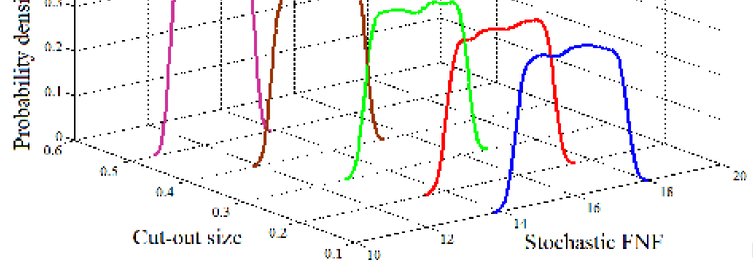
Fig. 7 Probability density function plot for first three stochastic natural frequencies using SVR approach for individual variation of twist angle [$\psi(\overline{\omega})$] of angle-ply (45°/-45°/45°) composite curved panel with cutout



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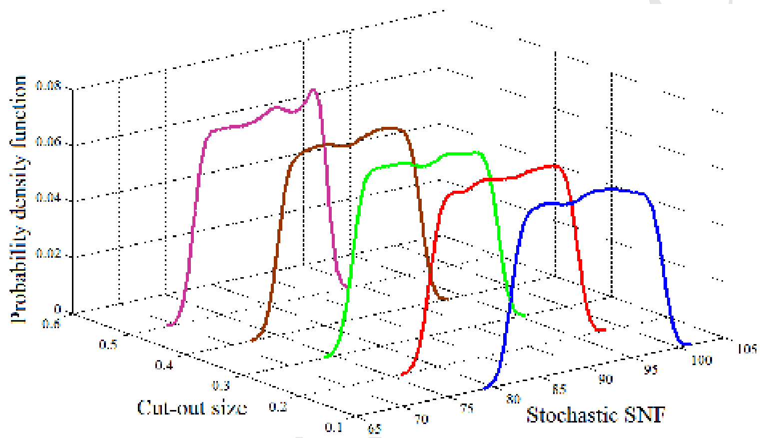

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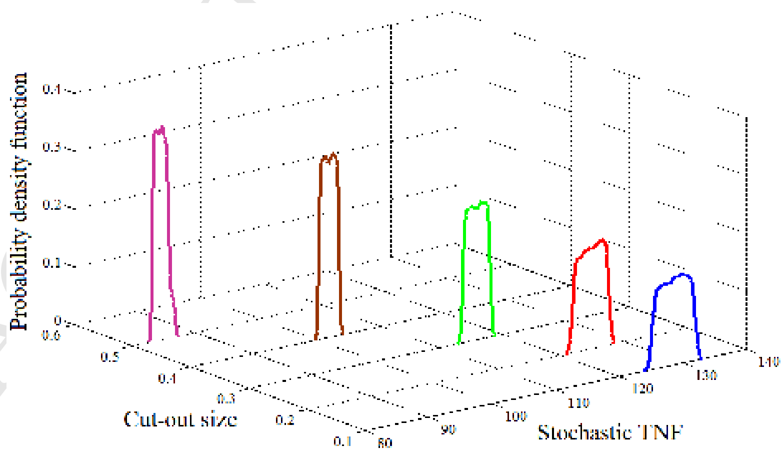

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Fig. 8 Probability density function plots for variation of only thickness $[t(\bar{\varpi})]$ corresponding to different cut out sizes for composite curved shells with cutout

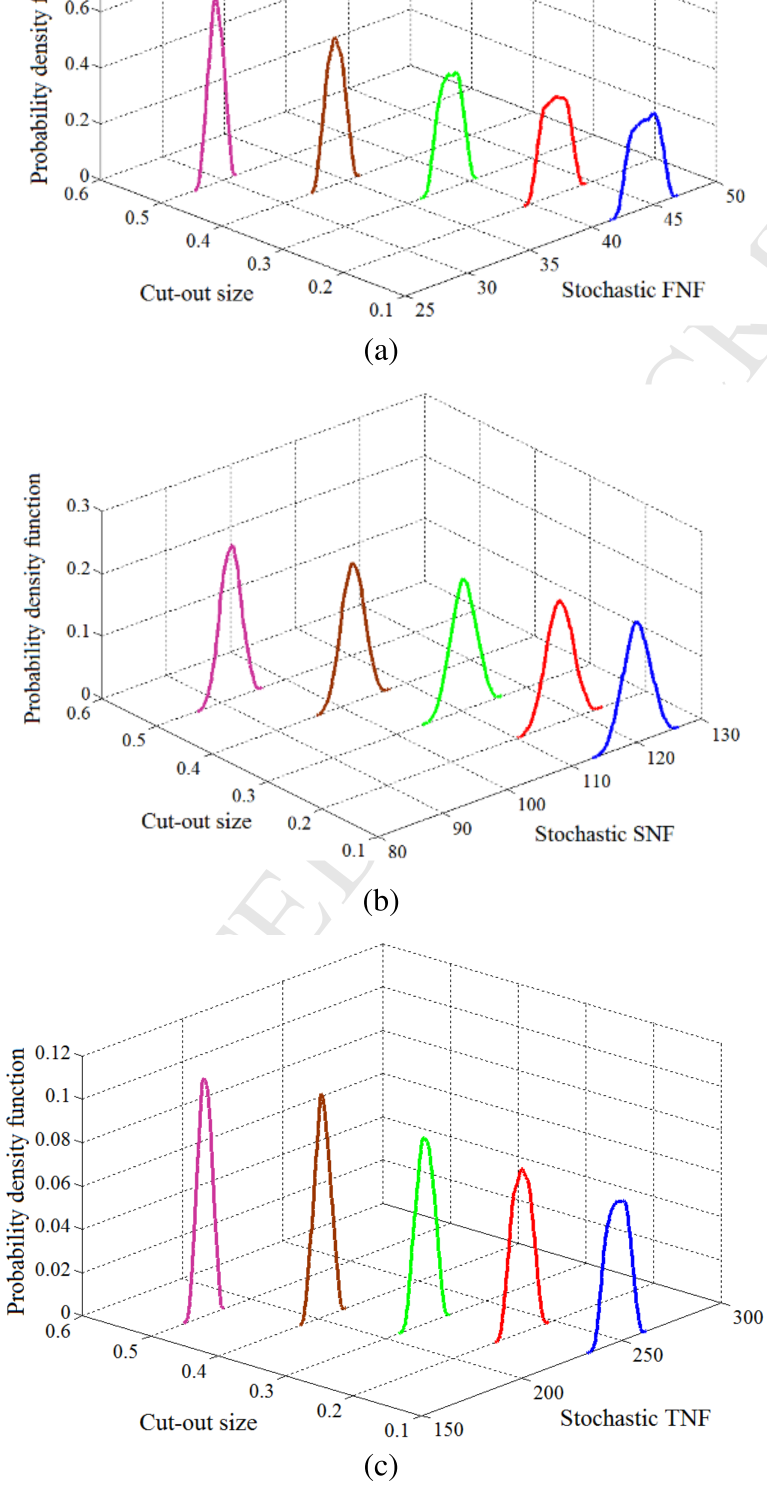
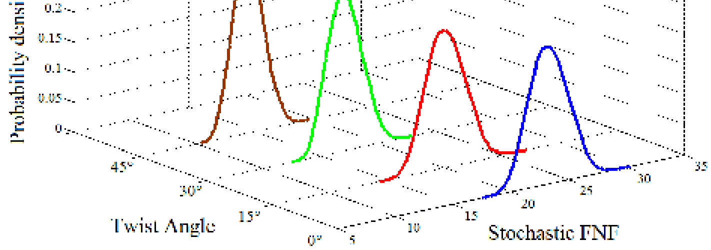


Fig. 9 Probability density function plots for variation of only material properties $[p_m(\bar{\omega})]$ corresponding to different cut out sizes for composite curved shells with cutout



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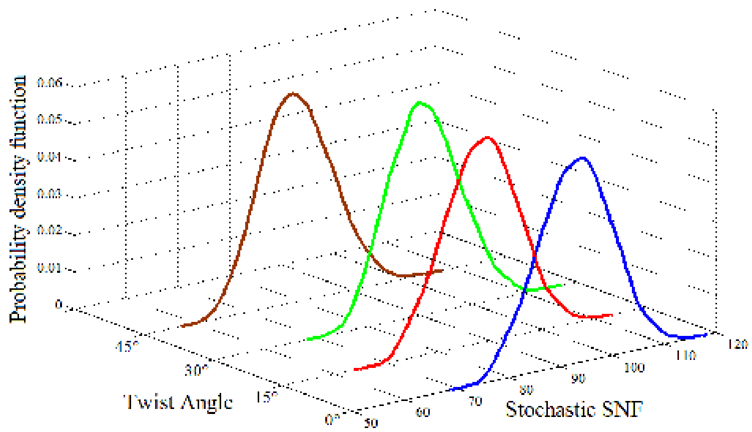

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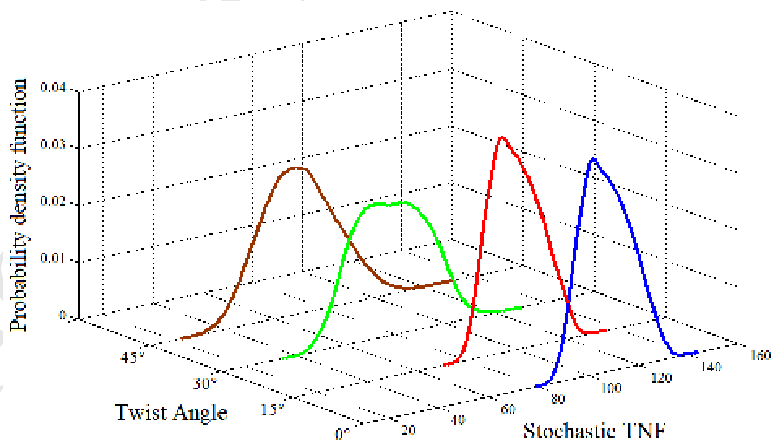

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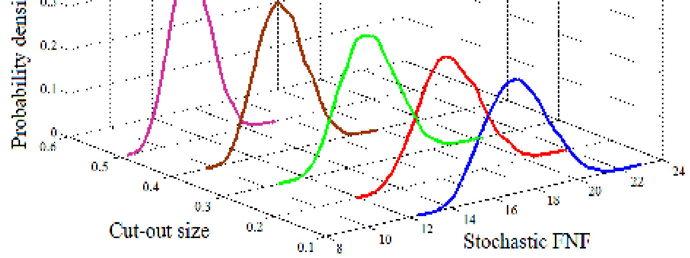
Fig. 10 Probability density function plots for combined variation $[g(\overline{\varpi})]$ corresponding to different twist angle for composite curved shells with cutout



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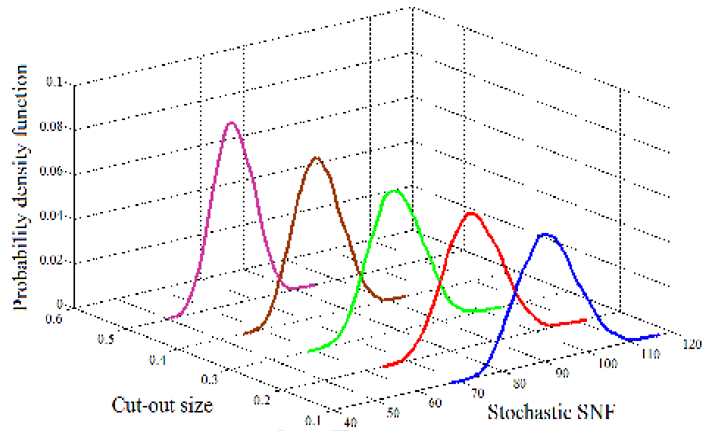

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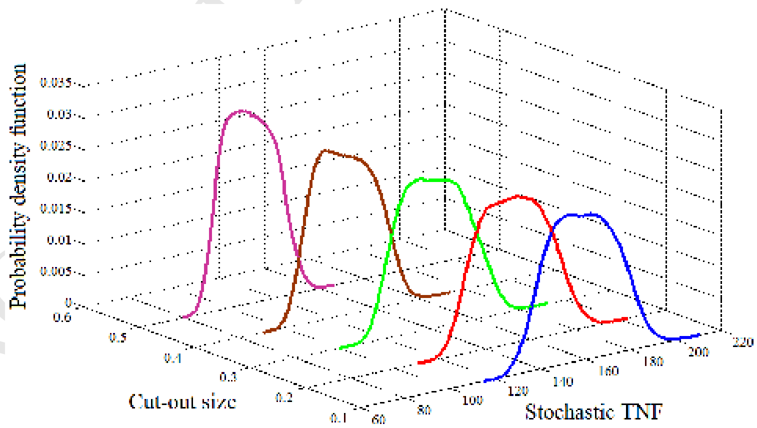
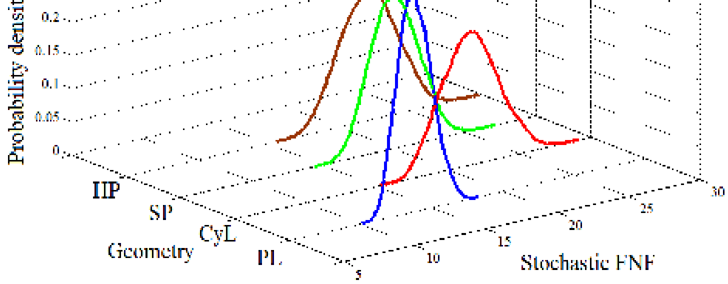


(c)

Fig. 11 Probability density function plots for combined variation of ply orientation angle, thickness, twist angle and material properties $[g(\overline{\varpi})]$ corresponding to different cut out sizes for composite curved shells with cutout



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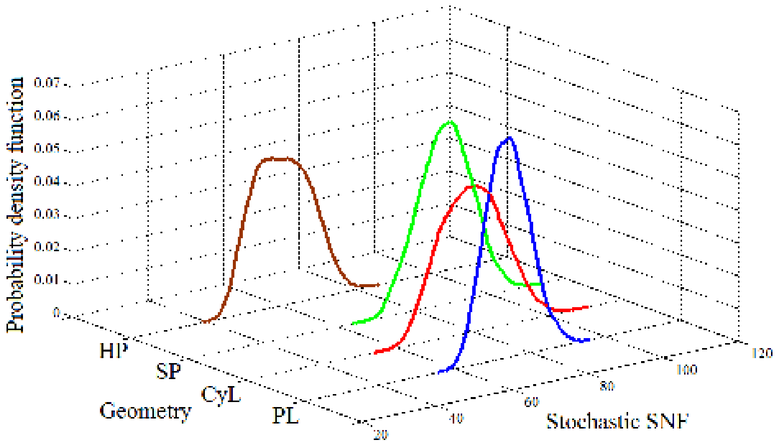

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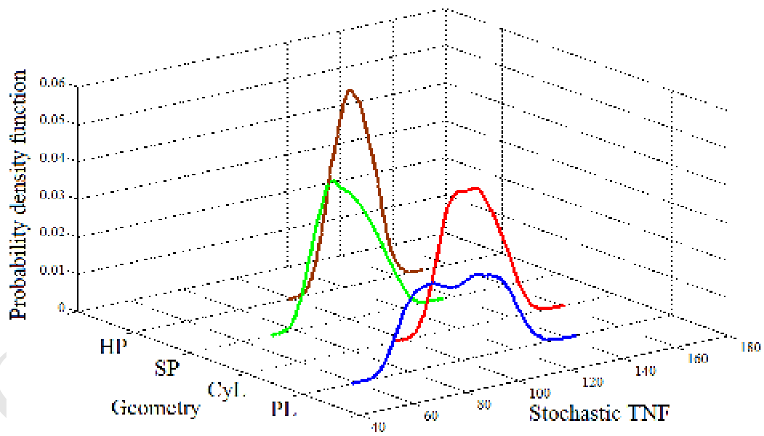

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Fig. 12 Probability density function plots for combined variation $[g(\bar{\varpi})]$ corresponding to different geometry for composite curved shells with cutout (PL-Plate, CyL-Cylindrical, SP-Spherical, HP-Hyperbolic Paraboloid)

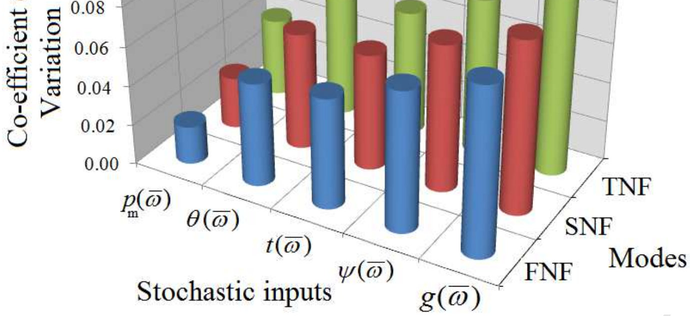


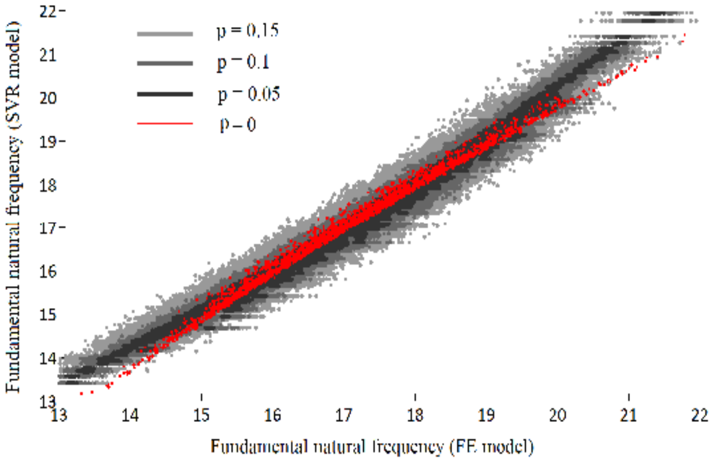
Fig. 13 Coefficient of variation for different of variations of input parameters

12, the first three natural frequencies are referred as fundamental natural frequency (FNF), second natural frequency (SNF) and third natural frequency (TNF) which are stochastic in nature. Figure 7 presents the probability density function plot for stochastic fundamental natural frequency for individual variation of twist angle only of angle-ply (45°/-45°/45°) composite curved panels with cutout. The twist angle in shell-panel structures causes reduction of stiffness which in turn decreases the values of the first three natural frequencies corresponding to a constant cutout size. The probability distributions of the first three natural frequencies for only variation of thickness corresponding to different sizes of cutout for composite shells are furnished in Figure 8. The stochastic mean values of first three natural frequencies are found to reduce with increasing cutout size. Figure 9 presents probability distributions of first three natural frequencies for variation in all the material properties. It is observed that the combined effects of variation of all the material properties follow Gaussian distribution and the mean for all the three natural frequencies reduce with the increase in cutout size.

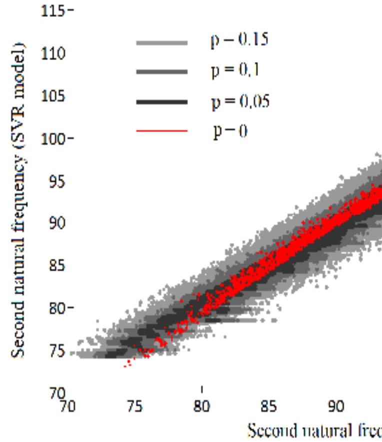
The probability density function plots of first three modes with combined variation (ply-orientation angle, twist angle, thickness, elastic moduli, shear moduli, poission ratio and mass density) corresponding to different twist angles are shown in Figure 10. The mean of

Gaussian distribution, in contrast to the probabilistic characters depicted for individual variation of twist angle only (Figure 7). Probability distributions for first three modes with combined variation corresponding to different sizes of cutout are shown in Figure 11, wherein it is evident that the distributions are of Gaussian nature and mean of stochastic natural frequencies decrease with the increase in cutout size. The probability density function plots of the stochastic first three natural frequencies for combined variation corresponding to different geometry such as composite plate, cylindrical, spherical, hyperbolic paraboloid curved shells as shown in Figure 12. Even though the probability distributions are found to follow Gaussian distributions, the mean and standard deviation of natural frequencies for different modes are highly dependent on the type of shell geometry. Figure 13 presents the cofficient of variation (ratio of standard deviation and mean for a distribution) of the stochastic first three natural frequencies corresponding to individual and combined variation of stochastic input paramenters. Cofficient of variation is highest for combined variation of all input parameters, as expected. The analysis presented in Figure 13 provides a measure of relative sensitivity of different input parameters towards the natural frequencies. Among the stochastic geometric features, twist angle is found to be the most sensitive parameter followed by thickness, ply orientation angle and material properties, respectively.

It is worthy to note here that all the probabilistic results furnished in this paper are obtained on the basis of 10,000 simulations. Application of support vector regression based approach allows us to obtain these results by means of efficient virtual simulations instead of actual expensive finite element simulation. 512 and 256 samples are required to construct the SVR model for layer-wise combined variation and individual variation of the stochastic input parameters, respectively. Thus for the purpose of stochastic analysis, same number of actual



(a)



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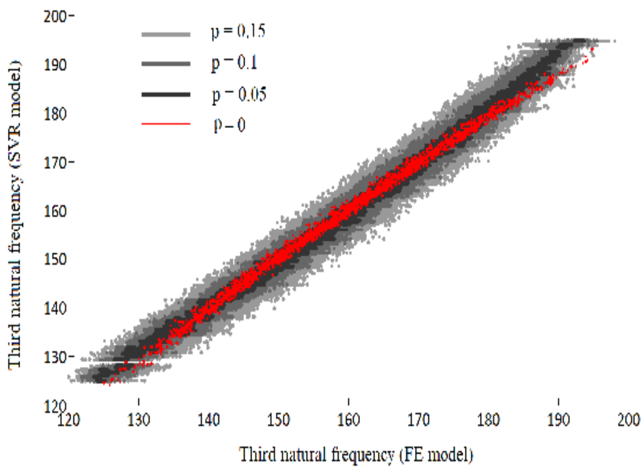

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Fig. 14 Effect of noise on prediction capability of SVR model for first three natural frequencies consideri

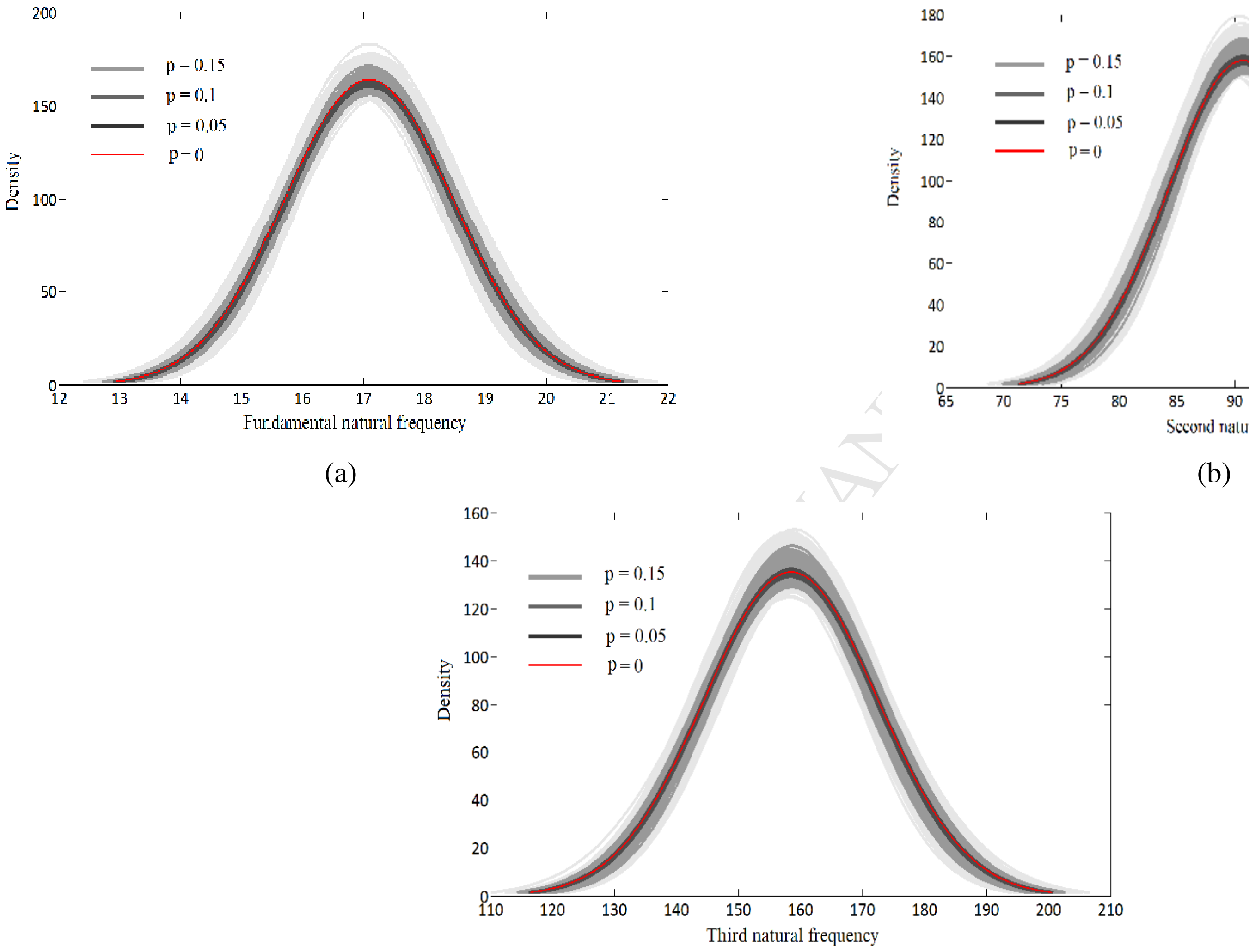

Fig. 15 Effect of noise on SVR based uncertainty quantification algorithm for first three natural frequencies cons

uncertainty quantification is more computationally efficient than traditional MCS approach in terms of FE simulations.

Figure 14 shows the effect of noise on prediction using SVR considering combined variation of all input parameters. Representative results are presented in figure 15 showing the effect of noise on first three natural frequencies considering different levels of noise (p) ranging from 0 to 0.15. The results presented in this paper are obtained on the basis of 1000 such noisy datasets, which involves construction of SVR model and thereby performing MCS for each dataset using corresponding SVR models as explained in figure 3(b). A comparative assessment of the effect of different levels of noise with noise free data (p = 0) provides a comprehensive idea about the influence of such noise in the probability distributions of first three natural frequencies.

6. Conclusion

This paper presents an efficient support vector regression based stochastic natural frequency analysis for laminated composite curved panels with cutout including the effect of twist angle and variation in shell-panel geometry (such as cylindrical, spherical, hyperpolic paraboloid and plate). First three stochastic natural frequencies are analyzed considering layer-wise variation of individual (low dimensional input parameter space) as well as combined effect (relatively higher dimensional input parameter space) of random input parameters (such as ply orientation, twist angle, thickness and material properties). The computational time and cost are reduced significantly by using the present SVR based approach compared to traditional Monte Carlo simulation method. A sensitivity analysis among the stochastic material and geometric features is carried out within the analysis domain to ascertain their relative importance. The effect of noise on SVR based uncertainty structures.

The novelty of the present study includes the stochastic natural frequencies analysis of composite curved panels with cutout and development of the efficient SVR based uncertainty quantification algorithm for laminated composites. Moreover, the effect of noise on SVR based uncertainty quantification algorithm is first analyzed in this article. Even though it is concentrated on stochastic natural frequency analysis of laminated composite curved panels in this paper, the SVR based approach for uncertainty quantification including the effect of noise can be extended to deal with other computationally intensive problems in different fields of science and engineering.

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