EFFECT OF DIFFERENTIAL SETTLEMENT ON FRAME FORCES - A PARAMETRIC STUDY

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Abstract

The dynamic nature of loads, seasonal variation of soil properties, uneven soil strata below footings or inappropriate design of foundation makes differential settlement inevitable. Differential settlement is largely responsible for developing forces or changing the existing forces in the structure and is often considered as the underlying cause of many structural failures. A structural engineer has to take into account these forces while designing the structure. Different structural parameters like stiffness of columns and beams and number of storeys and bays affect the response of the structure to differential settlement. In the present study, a plain portal frame subjected to constant differential settlement of 10 mm is analyzed for different cases using a structural analysis software i.e. STAAD Pro. Different parameters of the frame are varied to understand their influence on frame forces developed due to differential settlement. The parameters taken into account in this study are length of beam, height of column, moment of inertia of beam and column and number of storeys and bays. It is found that the forces in the frame developed due to differential settlement decrease on increasing beam lengths and column heights. Also, decreasing the moment of inertia of beams and columns is effective in reducing these forces. In addition, differential settlement tends to affect forces more in lower storeys as compared to higher storeys. The frame forces are more prominent for bays which are near to the support subjected to settlement. The increase or decrease in frame forces due to differential settlement can be attributed to the change in stiffness of the members. Hence, the study concludes that the stiffer members tend to develop higher forces for constant differential settlement in frames.

Key Words: Differential settlement, Portal frame, Structural response, Frame forces, Storey, Bays

1. INTRODUCTION

The behavior of a building is greatly influenced by the settlement of the soil beneath its foundation. The compression of underlying soil beneath a structure because of increased load during and after the construction causes a vertically downward movement which is defined as settlement. If this settlement is uniform then it does not result in damage to the supported structure. However, when the settlement is non-uniform, it can largely damage the overlying structure. This unequal and non uniform settlement is called as differential settlement and occurs when the soil beneath the structure expands, contracts or shifts away.

Most of the times it is difficult to understand the exact cause of differential settlement but some of the common causes are expulsion of water from the soil mass, flooding, frost heave, inefficient drainage, water leaks from broken water lines, inappropriate design of foundation, the root systems of maturing trees, vibrations from nearby construction or poorly compacted fill soil.

Differential settlement can cause a significant tilt in the structure, making the occupants uncomfortable. Cracks in the foundation and interior walls, non-uniform settling of doors and windows, sinking of chimneys, bulging walls and sunken slabs are often considered as the adverse effects of differential settlement and can be devastating to a building. These effects are a result of increased axial force, shear force and bending moments in the structure. It is common to implement repair and maintenance measures to prevent or reduce the effects of differential settlement.

Due to the unpredictable soil and environmental conditions, differential settlement is often considered as inevitable. Hence, every engineer keeps in mind a certain degree of allowable settlement before designing the structure. The effects of differential settlement can be widely adverse and hence cannot be ignored.

2. LITERATURE REVIEW

Meyerhof [1] analyzed the building frames subjected to unequal settlement by studying the field observations and theoretical investigations. It was concluded that settlement of 1 inch to 2 inch is inevitable and does not usually harm the building. However, a settlement of 3 inch can seriously damage the building's superstructure due to the movement of supports. This is more prominent for structures provided with individual footings or flexible rafts.

DeJong and Morgenstern [2] studied a three-dimensional frame structure to calculate the influence of settlement on the distribution of foundation loads. It was observed that small differential settlement was able to generate large reactions in the structural frame.

Man [3] studied the performance of RCC Frame influenced by differential settlement. The members associated with the settled columns were more seriously damaged. Also, the effect of differential settlement was more prominent for beam members as compared to column members.

D'Orazio et al [4] examined various steel tanks to determine the factors affecting the magnitude of tolerable differential settlement of tanks. A new method for determining the differential settlement was also recommended.

Dutta et al [5] studied the quantitative effect of soil-structure interaction on column axial forces of building frames with isolated footings. The change in column bending moments due to ratio of flexural stiffness of columns and beams, number of storeys and number of bays was analyzed. It was observed that the column moment in settlement condition was significantly higher as compared to no settlement condition. An approximate method of predicting the change in column moments by using the value of change in column axial force is determined.

Roy and dutta [6] analyzed a building frame with isolated footings to study the effect of differential settlement on frame members. It was found that differential settlement was responsible for redistribution of the column loads. The amount of these loads was dependent on the rigidity of the structure and the load-settlement characteristics of the soil. Also, the variation in amount of force due to soil-structure interaction can be minimized by the use of diagonal braces.

Agarwal and Hora [7] implemented the finite element method to study the non-linear interaction of a two-bay two storey plane building frame-soil system. It is observed that differential settlement causes the forces in the frame members to increase significantly.

Smit [8] analyzed the behavior of a 5-bay by 5-bay, 6 storey flat slab RC frame affected by differential settlement by using the linear-elastic finite element analyses. It was found that the structural behavior was dependent on the structure's overall relative bending stiffness. The axial load in the corner columns increases with the increase in relative bending stiffness. Also, a decrease in soil stiffness or structural stiffness caused the tilt in the structure to increase.

Le et al [9] studied the variation in differential settlement of a rigid strip foundation on an unsaturated soil caused due to rainfall. The magnitude of differential settlement varies with foundation load, variability statistics and rainfall duration. It is inferred that structures resting on unsaturated soils by means of rigid foundation can experience tilting and cracks due to settlement caused by rainfall infiltration. Reddy and Rao [10] performed static vertical load tests on a model building frame supported by pile groups resting on sand to study the effects of rigidity of plinth on displacements, rotation, shear force and bending moments in the building frame. It was inferred that reducing the rigidity of the plinth beam can reduce the shear force and bending moment values considerably.

Sneha et al [11] estimated the vertical settlement and lateral displacement in buildings laid on variety of soils with different sub grade modulus reaction Ks. Buildings with small span of 3m and large span of 6m for both 5 storeys and 10 storeys were taken into consideration which were subjected to dead load, live load, earthquake load and wind loads. It is observed that with the increase in sub grade modulus reaction Ks, there was an increase in the vertical settlement which can be compensated by increasing the size of the footing.

Lin et al [12] studied a 10-storey regular building with three-dimensional nonlinear finite element model. The responses of structure in terms of axial forces and vertical displacements of columns in each floor, bending moments and shear forces in beams were analyzed when the corner, edge and center columns were subjected to a settlement of 25 mm. The deformation in structure is elastic for settlement up to 25 mm and the most critical case was that of the center column. It is also inferred that the adjacent beam of the settling column develops significant bending moment, the settling column develops tensile forces while the adjacent column develops compressive forces. The effect of differential settlement is limited to one span from the settling column.

Paixao et al [13] used the finite element method model to analyze the effect of differential settlement on a train-track system. A numerical model was presented which suggested that differential settlement was largely responsible for track degradation, affecting safety requirements and passenger comfort.

3. PROPOSED WORK

The present study investigates the forces developed in a concrete portal frame subjected to differential settlement of footing. The analysis is performed using STAAD Pro software. A support settlement load is given to the right most support in every case discussed in this paper. This is done so as to provide a settlement of 10 mm to one of the supports of the frame. The other support is kept as fixed support and hence a differential settlement of 10 mm is achieved in the frame. The results of the analyses of axial force, shear force and bending moment for different parameters of the frame are compared to study the effects of these parameters on differential settlement of the portal frame. The different parameters that are taken into consideration are length of beam, height of column, inertia of beam, inertia of column, number of storeys and number of bays.

4. MODELING

Modeling is done using a structural analysis software i.e. STAAD Pro. A plain portal frame as shown in Fig 1 is taken as the reference frame for comparing the results of the analysis. The details of which are specified in table 1.

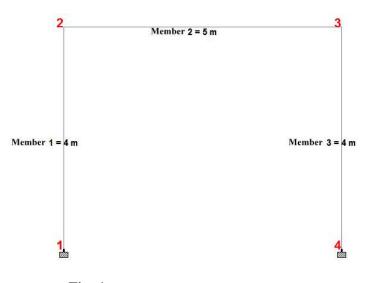


Fig -1: Reference frame showing joint and member numbering

S.	Description	Value/Type
No.	Description	, and , i g pe
1.	Structure type	Dlain portal frame
1.	Structure type	Plain portal frame
2.	Material	Concrete
3.	Modulus of elasticity of	21.72 kN/mm^2
	concrete	
4.	Density of concrete	2402.62 kg/m ³
5.	No. of storeys	One
6.	No. of bays	One
7.	Length of beam	5 m
8.	Height of column	4 m
9.	Size of beam	30 cm x 50 cm
10.	Size of column	40 cm x 40 cm
11.	Moment of inertia of beam	312500 cm ⁴
12.	Moment of inertia of column	213333.3 cm ⁴
13.	Type of supports	Fixed
14.	Type of load	Settlement load
15.	Differential settlement	10 mm

Table -1: Details of reference frame

The different models used in this study apart from the reference frame model are as follows -

- To study the effect of changing beam length on frame forces, models with beam length 6 m and 4 m are used. All other parameters are same as that of the reference frame.
- To study the effect of changing column height, models with column height 5 m and 3 m are used. All other parameters are same as that of the reference frame.
- To study the effect of changing beam inertia, models with beam size 30 cm x 60 cm and 30 cm x 40 cm are used. The moment of inertia of beam in the two models is 540000 cm⁴ and 160000 cm⁴ respectively. All other parameters are same as that of the reference frame.
- To study the effect of changing column inertia, models with column size 50 cm x 50 cm and 40 cm x 40 cm are used. The moment of inertia of columns in the two models is 520833.3 cm⁴ and 67500 cm⁴ respectively. All other parameters are same as that of the reference frame.
- To study the effect of number of storeys on frame forces, two models of two and three storeys are used. All other parameters are same as that of the reference frame.
- To study the effect of number of bays on frame forces, two models of two and three bays are used. All other parameters are same as that of the reference frame.

5. RESULTS AND DISCUSSIONS

In the following section, the effect of various parameters on frame forces is discussed for a portal frame subjected to differential settlement of 10 mm. The increase or decrease in axial force, shear force and bending moment is observed for changing the beam length, column height, beam inertia, column inertia, number of storeys and number of bays.

5.1 Effect of Beam Length on frame forces

From table 2, it is observed that when the beam length is increased to 1.20 times of the reference beam length, the axial force in the column and the shear force in the beam is reduced to 0.68 times whereas the bending moment in the frame is reduced to 0.81 times. Similarly when the beam length is decreased to 0.80 times of the reference beam length, the axial force in the column and the shear force in the beam is increased to 1.59 times where as the bending moment in the frame is increased to 1.28 times. Hence, the smaller the span of beam, larger will be the forces developed due to differential settlement. The axial force, shear force and bending moment diagrams for different lengths of beam are shown in Fig 2.

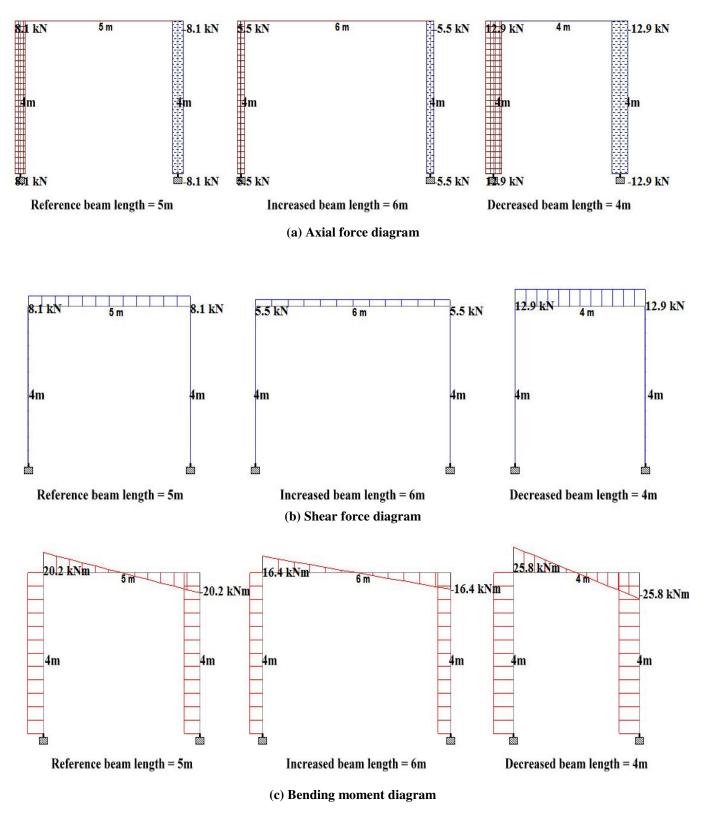


Fig -2: Variation of forces in frame for different beam lengths

5.2 Effect Of Column Height On Frame Forces

From table 3, it is observed that when the column height is increased to 1.25 times of the reference column height, the axial force in the column and the shear force in the beam is reduced to 0.81 times and the bending moment is reduced to 0.82 times when compared to the reference frame. Also, when the column height is reduced to 0.75 times of the

reference column height, the axial force in the column and the shear force in the beam is increased to 1.27 times and the bending moment is increased to 1.28. Hence, the smaller the height of column, larger will be the forces developed due to constant differential settlement. The axial force, shear force and bending moment diagrams for different heights of column are shown in Fig 3.

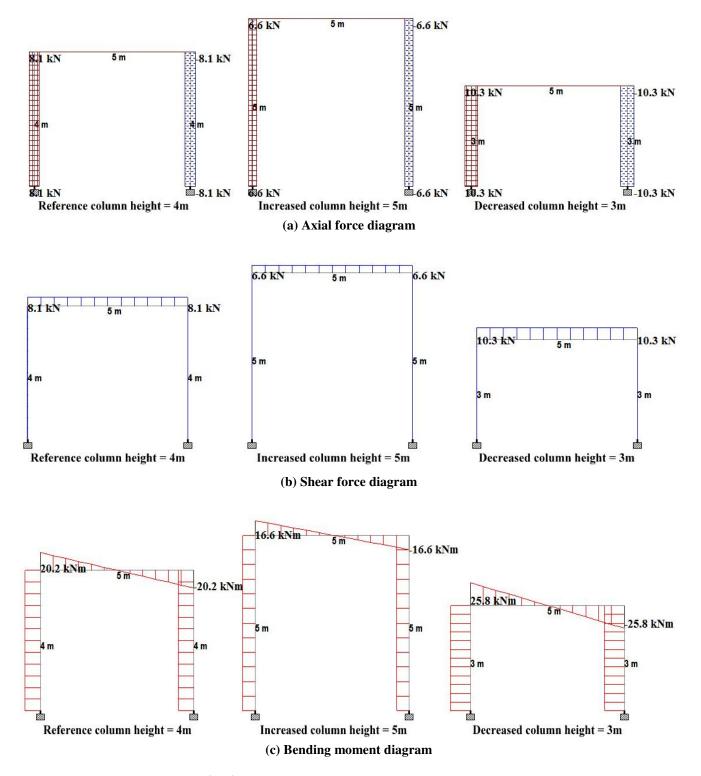


Fig -3: Variation of forces in frame for different column heights

				Frame	forces for	different	Comparison of Analyses									
		Referer	ice beam l	ength 5m	Increa	Increased beam length 6m			ased beam 4m	length		f increase m/5m = 1.2		Effect of decrease in length (4m/5m = 0.80)		
Member No.	Node	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force	Shear Force	Bending Moment	Axial Force	Shear Force	Bending Moment
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	Ratio (4)/(1)	Ratio (5)/(2)	Ratio (6)/(3)	Ratio (7)/(1)	Ratio (8)/(2)	Ratio (9)/(3)
1	1	8.10	0.00	20.20	5.50	0.00	16.40	12.90	0.00	25.80	0.68	-	0.81	1.59	-	1.28
1	2	-8.10	0.00	-20.20	-5.50	0.00	-16.40	-12.90	0.00	-25.80	0.68	-	0.81	1.59	-	1.28
2	2	0.00	8.10	20.20	0.00	5.50	16.40	0.00	12.90	25.80	-	0.68	0.81	-	1.59	1.28
2	3	0.00	-8.10	20.20	0.00	-5.50	16.40	0.00	-12.90	25.80	-	0.68	0.81	-	1.59	1.28
3	3	-8.10	0.00	-20.20	-5.50	0.00	-16.40	-12.90	0.00	-25.80	0.68	-	0.81	1.59	-	1.28
3	4	8.10	0.00	20.20	5.50	0.00	16.40	12.90	0.00	25.80	0.68	-	0.81	1.59	-	1.28

Table -2: Effect of variation of beam length on frame forces developed due to differential settlement

Table -3: Effect of variation of column height on frame forces developed due to differential settlement

				Frame	forces for	r differen	t height of	Comparison of Analyses								
		Referen	nce colum 4m	nn height	Increased column height 5m			Decreas	sed column 3m	n height		f increase i m/4m = 1.2		Effect of decrease in height $(3m/4m = 0.75)$		
Member No.	Node	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force	Shear Force	Bending Moment	Axial Force	Shear Force	Bending Moment
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	Ratio (4)/(1)	Ratio (5)/(2)	Ratio (6)/(3)	Ratio (7)/(1)	Ratio (8)/(2)	Ratio (9)/(3)
1	1	8.10	0.00	20.20	6.60	0.00	16.60	10.30	0.00	25.80	0.81	-	0.82	1.27	-	1.28
1	2	-8.10	0.00	-20.20	-6.60	0.00	-16.60	-10.30	0.00	-25.80	0.81	-	0.82	1.27	-	1.28
2	2	0.00	8.10	20.20	0.00	6.60	16.60	0.00	10.30	25.80	-	0.81	0.82	-	1.27	1.28
2	3	0.00	-8.10	20.20	0.00	-6.60	16.60	0.00	-10.30	25.80	-	0.81	0.82	-	1.27	1.28
3	3	-8.10	0.00	-20.20	-6.60	0.00	-16.60	-10.30	0.00	-25.80	0.81	-	0.82	1.27	-	1.28
3	4	8.10	0.00	20.20	6.60	0.00	16.60	10.30	0.00	25.80	0.81	-	0.82	1.27	-	1.28

5.3 Effect Of Moment Of Inertia Of Beam On

Frame Forces

From table 4, it is observed that when the beam inertia is increased to 1.73 times of the reference beam inertia, the axial force in the column, the shear force in the beam and the bending moment in the frame is increased to 1.05 times. Similarly when the beam inertia is decreased to 0.51 times

of the reference beam inertia, the axial force in the column, the shear force in the beam and the bending moment in the frame is reduced to 0.89 times. Therefore, on increasing the moment of inertia of beam, the forces developed due to constant differential settlement will also increase. The axial force, shear force and bending moment diagrams for different inertias of beam are shown in Fig 4.

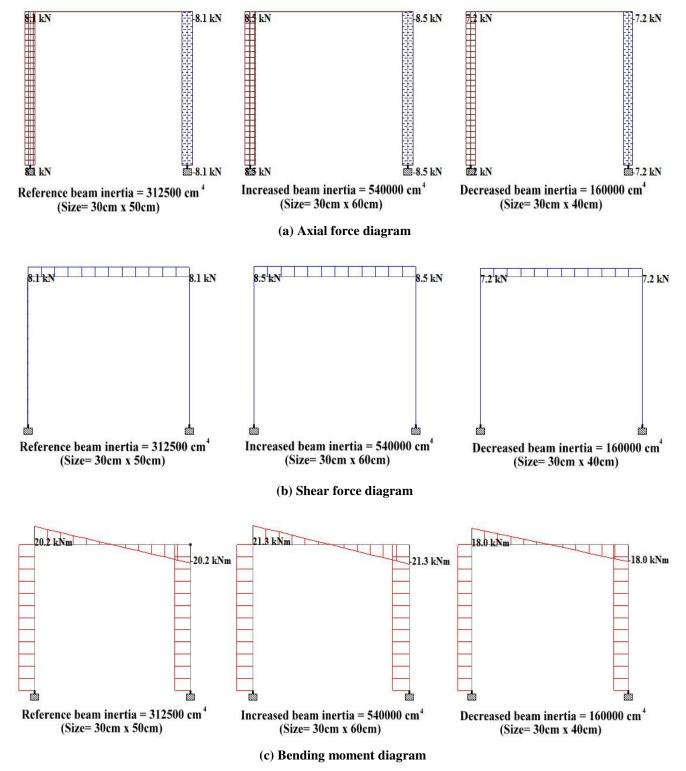


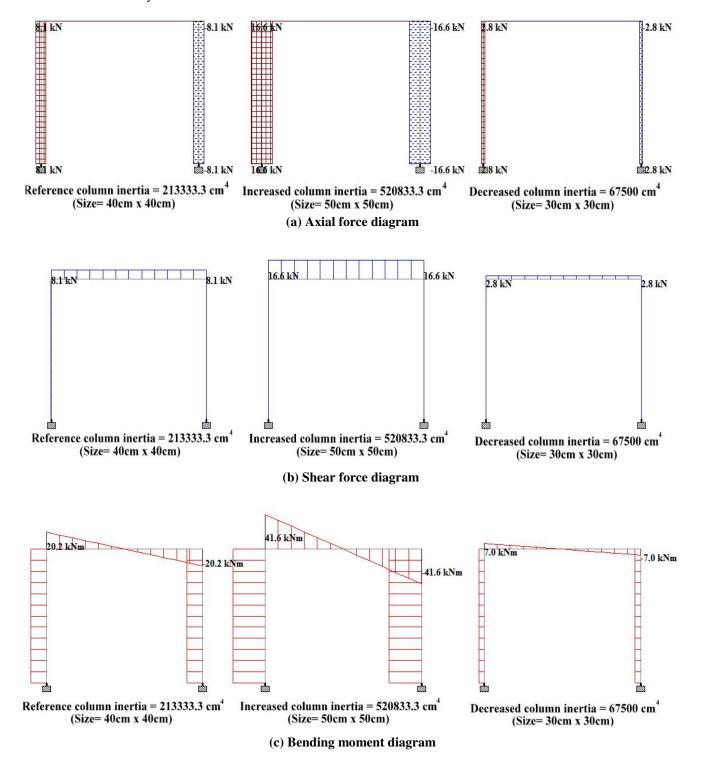
Fig -4: Variation of forces in frame for different beam inertia

5.4 Effect Of Moment Of Inertia Of Columns On

Frame Forces

From table 5, it is observed that when the column inertia is increased to 2.44 times of the reference column inertia, the axial force in the column and the shear force in the beam is increased to 2.05 times and the bending moment in the frame is increased to 2.06 times as compared to that of reference frame. Similarly when the column inertia is

decreased to 0.32 times of the reference column inertia, the axial force in the column, the shear force in the beam and the bending moment in the frame is reduced to 0.35 times. Therefore, on increasing the moment of inertia of column, the forces developed due to constant differential settlement will also increase. The axial force, shear force and bending moment diagrams for different inertias of column are shown in Fig 5.





Member No.				Frame	forces fo	r differen	t inertia of	Comparison of Analyses								
		Reference beam inertia 312500 cm ⁴ (Size= 30cm x 50cm)			Increased beam inertia 540000 cm ⁴ (Size= 30cm x 60cm)				ased bear 160000 cr = 30cm x	m ⁴		f increase in 0/312500 =		Effect of decrease in inertia (160000/312500 = 0.51)		
	Node	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force	Shear Force	Bending Moment	Axial Force	Shear Force	Bending Moment
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	Ratio (4)/(1)	Ratio (5)/(2)	Ratio (6)/(3)	Ratio (7)/(1)	Ratio (8)/(2)	Ratio (9)/(3)
1	1	8.10	0.00	20.20	8.50	0.00	21.30	7.20	0.00	18.00	1.05	-	1.05	0.89	-	0.89
1	2	-8.10	0.00	-20.20	-8.50	0.00	-21.30	-7.20	0.00	-18.00	1.05	-	1.05	0.89	-	0.89
2	2	0.00	8.10	20.20	0.00	8.50	21.30	0.00	7.20	18.00	-	1.05	1.05	-	0.89	0.89
2	3	0.00	-8.10	20.20	0.00	-8.50	21.30	0.00	-7.20	18.00	-	1.05	1.05	-	0.89	0.89
3	3	-8.10	0.00	-20.20	-8.50	0.00	-21.30	-7.20	0.00	-18.00	1.05	-	1.05	0.89	-	0.89
3	4	8.10	0.00	20.20	8.50	0.00	21.30	7.20	0.00	18.00	1.05	-	1.05	0.89	-	0.89

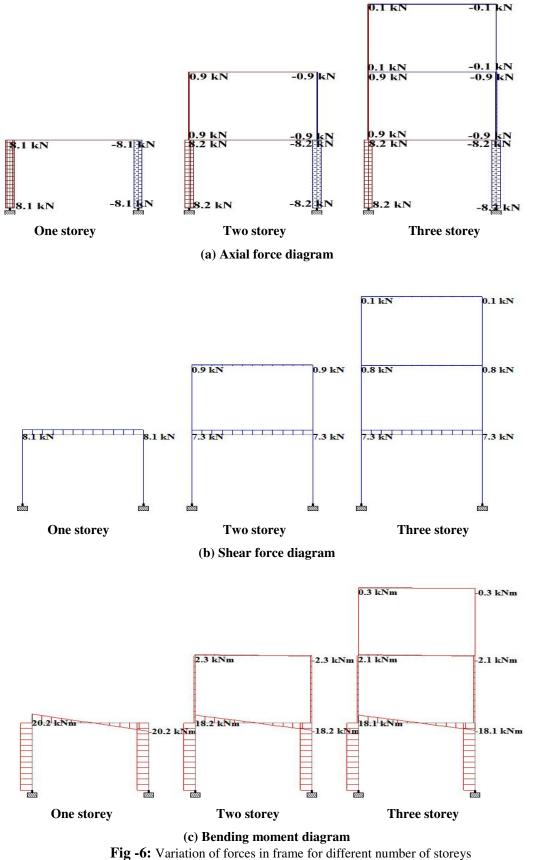
Table -4: Effect of variation of moment of inertia of beam on frame forces developed due to differential settlement

Table -5: Effect of variation of moment of inertia of column on frame forces developed due to differential settlement

				Frame	forces for o	lifferent ir	nertia of co	Comparison of Analyses								
		2	erence co inertia 13333.3 c = 40cm x	cm ⁴	Increased column inertia 520833.3 cm ⁴ (Size= 50cm x 50cm)			Decreased column inertia 67500 cm ⁴ (Size= 30cm x 30cm)				f increase i .3/213333.		Effect of decrease in inertia (67500/213333.3 = 0.32)		
Member No.	Node	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force (kN)	Shear Force (kN)	Bending Moment (kNm)	Axial Force	Shear Force	Bending Moment	Axial Force	Shear Force	Bending Moment
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	Ratio (4)/(1)	Ratio (5)/(2)	Ratio (6)/(3)	Ratio (7)/(1)	Ratio (8)/(2)	Ratio (9)/(3)
1	1	8.10	0.00	20.20	16.60	0.00	41.60	2.80	0.00	7.00	2.05	-	2.06	0.35	-	0.35
1	2	-8.10	0.00	-20.20	-16.60	0.00	-41.60	-2.80	0.00	-7.00	2.05	-	2.06	0.35	-	0.35
2	2	0.00	8.10	20.20	0.00	16.60	41.60	0.00	2.80	7.00	-	2.05	2.06	-	0.35	0.35
2	3	0.00	-8.10	20.20	0.00	-16.60	41.60	0.00	-2.80	7.00	-	2.05	2.06	-	0.35	0.35
3	3	-8.10	0.00	-20.20	-16.60	0.00	-41.60	-2.80	0.00	-7.00	2.05	-	2.06	0.35	-	0.35
5	4	8.10	0.00	20.20	16.60	0.00	41.60	2.80	0.00	7.00	2.05	-	2.06	0.35	-	0.35

5.5 Effect Of Number Of Storeys On Frame Forces

To study the effect of number of storeys on frame forces, three models of one, two and three storey are taken into consideration. The variation of axial force, shear force and bending moment with number of storey is shown in Fig 6. It is found that the effect of differential settlement on frame forces is significant for ground storey only. As the number of storey increases, this effect diminishes drastically.



5.6 Effect of number of bays on frame forces

Three models of one, two and three bays are considered to study the effect of number of bays on frame forces. The variation of axial force, shear force and bending moment with number of bays is shown in Fig 7. It is found that the effect of differential settlement on frame forces is higher for two bays as compared to one bay because the two bay system provides more stiffness due to continuation of beam. However, force pattern in two bay and three bay frame is found to be similar. The effect of differential settlement on frame forces decreases for the bays located far from the support subjected to settlement.

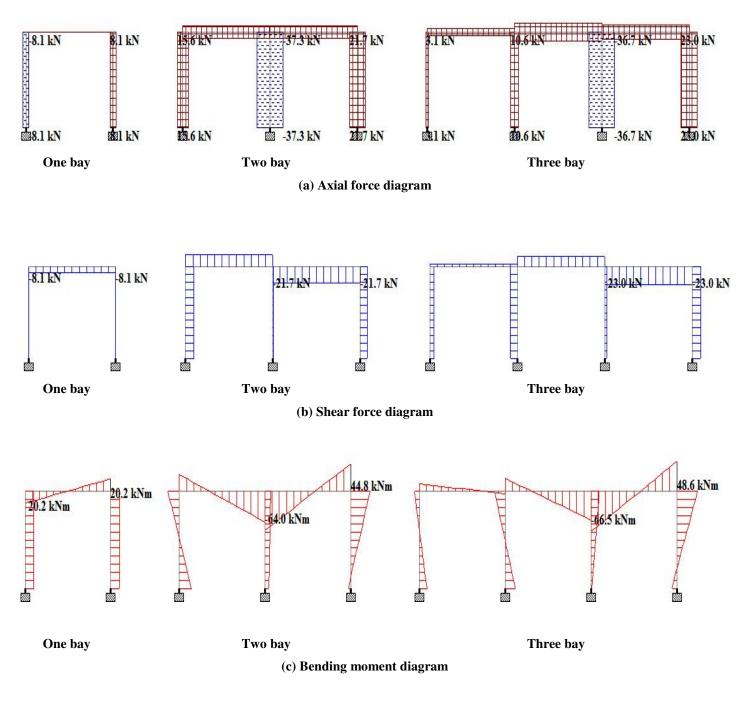


Fig -7: Variation of forces in frame for different number of bays

6. CONCLUSIONS

Effect of different structural parameters like stiffness of columns and beams and number of storeys and bays affect the frame forces developed due to differential settlement.

- a) The effect of differential settlement is more prominent in frames having beams of shorter span as compared to longer span. Similarly, the effect of differential settlement is more prominent in frames having columns of lesser height.
- b) The beams and columns having greater moment of inertia attract more structural forces for a constant differential settlement.
- c) The effect of differential settlement on frame forces is more prominent for lower stories and this effect diminishes rapidly in higher storeys.
- d) The effect of differential settlement is much higher for two bay frame as compared to one bay frame. This is because two bay system provides more rigidity due to the continuation of beam. Moreover, frame forces are more prominent for bays which are near to the support subjected to settlement.

The increase or decrease in frame forces due to differential settlement can be attributed to the change in stiffness of the members. Hence, it can be concluded that stiffer members tend to develop higher forces for constant differential settlement in frame.

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