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# The Effect of Generalized Wind Characteristics on Annual Power Estimates from Wind Turbine Generators

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by  
William C. Cliff

October 1977

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THE EFFECT OF GENERALIZED WIND CHARACTERISTICS  
ON ANNUAL POWER ESTIMATES FROM WIND  
TURBINE GENERATORS

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October 1977

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SUMMARY

This report presents a technique for estimating the average power output of a wind turbine using, as the wind characteristic input, only the mean annual wind magnitude.

Hourly wind speeds are assumed to have a Rayleigh frequency distribution which requires a single parameter input (e.g., the mean value, variance or higher moment values). Based upon a general shape, for the wind speed versus machine output, a generic set of curves is developed to estimate the average power output of wind turbines. Also, estimates of the percent of time the wind turbine would not produce power (percent down time) and the percent of time the wind turbine would be operating at its rated power are presented.

Two example cases which use typical wind turbine sizes and operating characteristics with a Rayleigh distributed mean wind are given which yield the percent down time, percent running at rated, average power output as well as other statistics of interest (as a function of annual mean wind speed).

The results of the example problems are then compared with results using the same wind turbine operating characteristics and actual wind speed frequency distributions collected at high wind speed sites located around the continental United States, Hawaii and Puerto Rico. These comparisons have been encouraging, especially when the data record length is more than 3 months.

The analysis presented in this document is recommended only for locations where the annual mean wind speed at the expected wind turbine hub (center) height is greater than or equal to 4.5 m/s (10 mph).

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THE EFFECT OF GENERALIZED WIND CHARACTERISTICS ON  
ANNUAL POWER ESTIMATES FROM WIND TURBINE GENERATORS

William C. Cliff

INTRODUCTION

The decision to use wind power at a specific location will depend on a variety of cost and return parameters. The most important return parameter is the expected amount of usable power that a specific wind turbine will produce at a given location. This report presents techniques to estimate a wind turbine's average power, percent down time, and percent time operating at rated power. Only the annual mean wind speed is used as the wind characteristic input.

Historically, selecting a location for a Wind Energy Conversion System relied largely on qualitative measures of the wind.<sup>1</sup> The competitive and cost-conscious world of today demands that quantitative values of expected power be provided and that cost comparisons be made with conventional power systems prior to the purchase and installation of a large wind turbine.

Cost parameters which need to be considered include costs of the installation, transmission, maintenance and operation. However, estimating costing is beyond the scope of this text and will not be performed except as an ancillary spin-off of the average power estimates. An estimation of the average power provides the basic information used in assessing the present worth of a wind turbine.

AVERAGE POWER ESTIMATIONS

The forcing function that drives the wind turbine is obviously the wind. However, the amount of power that is produced is a function of the machine characteristics and the frequency distribution of the wind. Machine operating characteristics (i.e., cut-in, rated, and cut-out speeds) for each specific machine are fixed. Therefore, the power each machine will produce will be a function of the frequency distribution of the wind speed

$p(v)$  (measured at the wind turbine's hub height). If the actual frequency distribution of wind speed at the site is known, an estimate of the wind turbine's expected average power may be computed from:

$$\bar{T}_p = \int_0^{\infty} T_p(v) p(v) dv \quad (1)$$

where  $\bar{T}_p$  = the average power output of the wind turbine  
 $T_p(v)$  = the turbine power output as a function of wind speed  
 $p(v)$  = the frequency distribution of the wind speed (probability density distribution of wind speed usually based upon hourly average wind speeds)  
 $v$  = wind speed at machine hub height

Figure 1 graphically describes the processes involved in Equation 1. Figure 1a depicts a typical wind turbine performance curve. At a cut-in wind speed,  $a$ , the turbine begins to produce power and monotonically produces more power as the wind speed increases until it reaches the rated wind speed,  $b$ . Above this wind speed the wind turbine's power output stays constant until the wind turbine's cut-out wind speed,  $c$ , is reached. At wind speeds greater than the cut-out speed the wind turbine does not produce power. The turbine's blades are usually feathered or are designed to stall above the cut-out speed to prevent damage to the machine.

Figure 1b depicts a typical wind speed frequency distribution. It is noted that only a small percentage of time is spent at either the extremely low or high ends of the distribution. The convolution of the curves in Figure 1a and Figure 1b,  $T_p(v) \times p(v)$ , yields the power frequency distribution of the wind turbine's output as a function of velocity. The integral of this curve (Equation 1) then yields the average expected power output of the wind turbine,  $\bar{T}_p$ . Since the wind turbine only produces power when the wind speed is between the cut-in and the cut-out wind speed, the integration of Equation 1 needs to be performed only over these limits rather than from 0 to  $\infty$ .

The result obtained by the application of Equation 1 should provide a reasonable estimate of the average power produced by a specific wind turbine

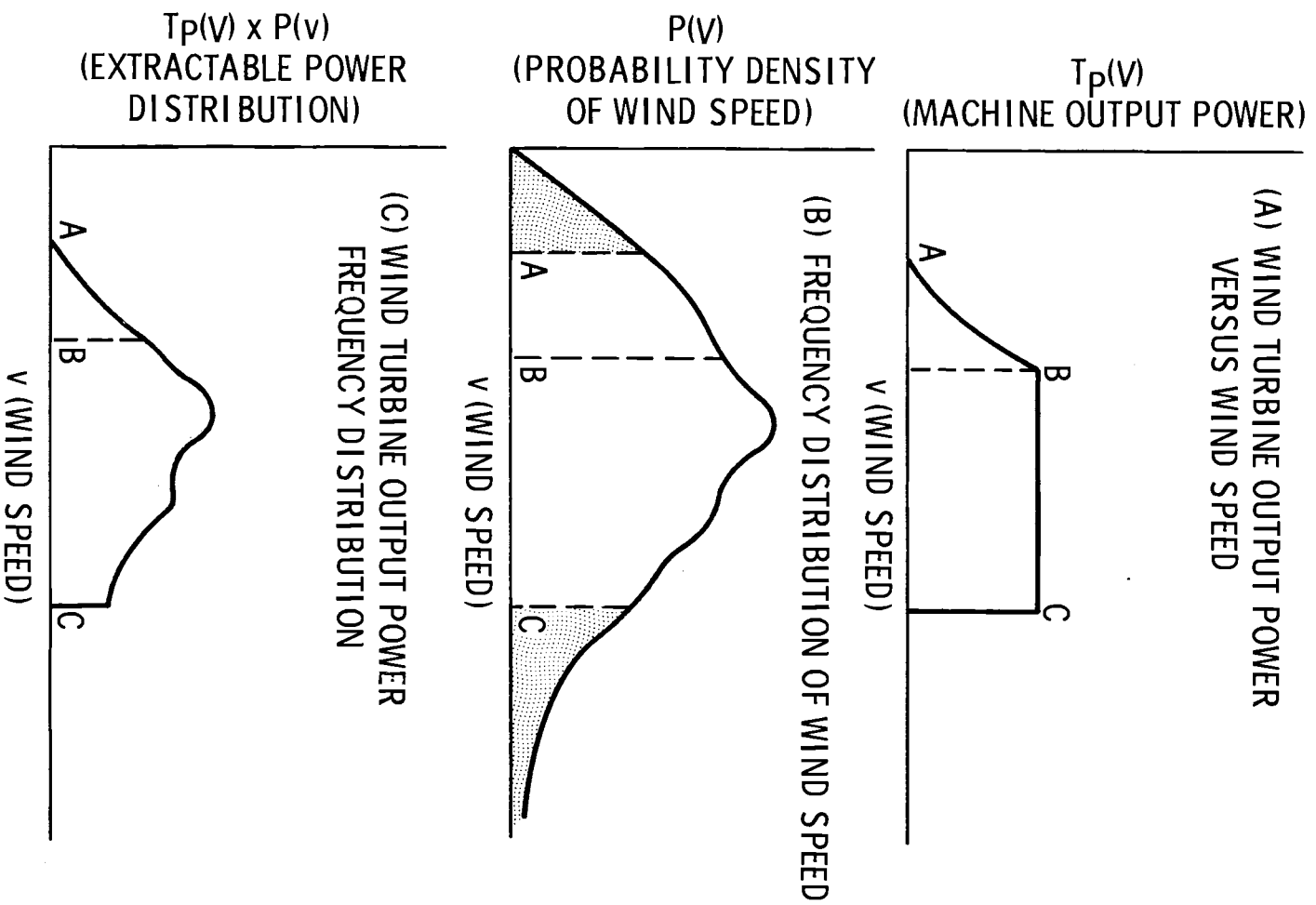


FIGURE 1. Interaction of Wind Turbine Characteristics and Wind Statistics to Produce the Wind Turbine Power Frequency Distribution

operating in an environment described by  $p(v)$ . Nevertheless, the actual power output of the turbine may differ from this estimate somewhat since other factors are not accounted for in Equation 1. Some of these factors are the vertical profile of the horizontal component of wind velocity, the wind direction variability and turbulence. Other parameters of interest associated with a wind turbine's operation are the percent down time and the percent running at rated. The percent down time is the percent of time the wind speed is below cut-in or above cut-out. The percent running at rated is the percent of time the wind speed lies between the rated speed and the cut-out speed, and is therefore the percent of time the turbine is operating at its rated capacity. The percent down time and percent time running at rated may be computed directly from the wind speed frequency distribution.

#### USE OF THE RAYLEIGH DISTRIBUTION

The major aim of this paper is to address the case when the wind speed frequency distribution is not known, but a long term mean wind speed (at hub height) is known or may be estimated. The following text uses the common Rayleigh and a normalized Rayleigh distribution to estimate the wind speed frequency distribution associated with a given annual mean wind speed.

Justus et al.<sup>2</sup> provide an excellent review of the various distributions used to approximate wind speed frequency distributions. In general, Justus et al. recommend using the Weibull distribution, a two-parameter model which may be used if additional wind information is known (such as both the mean and variance of the wind at a site). The Rayleigh distribution is a special case of the Weibull. In the Rayleigh distribution the shape parameter of the Weibull distribution is fixed at a value equal to 2.0, becoming a single parameter distribution depending only on the annual mean wind speed. Justus' report tends to support the hypothesis that the higher mean wind speeds tend to have a Rayleigh frequency distribution. His report also shows that the lower wind speed sites (annual mean less than 4.0 m/s) have distributions which should not be approximated by the

Rayleigh distribution. However, those wind turbine sites with annual mean wind speeds of less than 5 m/s do not presently appear attractive for cost competitive electrical generation by wind turbines.

There has been a concerted effort by the Federal Wind Energy program to examine the wind characteristics at several proposed wind turbine locations throughout the United States, including Hawaii and Puerto Rico. Data have been collected under a variety of topographical and meteorological conditions including sites in the plains, deserts, islands, mountains and on coast lines. Average outputs of typical wind turbines are computed for the actual frequency distributions at 16 of these sites. The results are compared with the expected average output based on the Rayleigh frequency distribution for wind speed using the average wind speed as input. Also, plots of the actual frequency distributions with an overlay of the Rayleigh distribution are presented. As previously noted, the Rayleigh distribution is a single parameter model. That is, once the mean velocity is specified the distribution is completely defined.

#### APPROACH

If a long term frequency distribution of the wind speed for a particular site is known, the investigator should use Equation 1 for expected power output estimates of the machine. When the frequency distribution of the wind speed is not known and the annual mean wind speed is greater than 4.5 m/s (10 mph), it is recommended that the Rayleigh distribution be used to estimate the wind speed frequency distribution.

The Rayleigh distribution may be written as

$$P(v) = \frac{v\pi}{2\bar{v}^2} e^{-\left(\frac{v^2\pi}{4\bar{v}^2}\right)} \quad (2)$$

where

$P(v)$  = frequency distribution of wind speed

$v$  = wind speed

$\bar{v}$  = long term average (mean) wind speed

In some cases the velocity duration curve is found to be useful. The velocity duration curve associated with Equation 2 is equal to the following:

$$VDC = 8760 e^{-\frac{v^2 \pi}{4\bar{v}^2}} \quad (3)$$

The velocity duration curve yields the number of hours that the wind velocity is greater than  $V$ .

The variance of the Rayleigh distribution is given by

$$\text{variance} \approx 0.273 \bar{v}^2 \quad (4)$$

Table 1 lists the values of  $\bar{v} P(v)$  versus  $\frac{v}{\bar{v}}$ . This is called the normalized Rayleigh distribution.

Since one argument was divided by the mean velocity while the other argument was multiplied by the mean velocity, no real change has occurred. The moments of the new function are no longer functions of the mean wind speed but rather have simple numerical values. For instance, the first moment or mean of the Rayleigh distribution is equal to  $\bar{v}$  while the mean of the normalized Rayleigh is equal to 1. The second central moment or variance of the Rayleigh distribution is equal to  $\bar{v}^2 \left( \frac{4}{\pi} - 1 \right)$  while the variance of the normalized Rayleigh is equal to  $\left( \frac{4}{\pi} - 1 \right)$ . Thus, for comparing various frequency distributions with each other as well as with the Rayleigh distribution, the normalized Rayleigh distribution will be used.

#### PERCENT DOWN TIME

The percentage of time which the wind speed is below the wind turbine's cut-in speed plus the percentage of time the wind speed is above the cut-out velocity is called the percent down time. The percent down time may be estimated using the following formula derived from the Rayleigh distribution:

$$\text{Percent down time} = 1 - e^{-a^2/2\sigma^2} + e^{-c^2/2\sigma^2} \quad (5)$$



TABLE 1. Values of Normalized Rayleigh Distribution

RAYLEIGH DISTRIBUTION

$$P(v) = \frac{\pi}{2} \frac{v}{\bar{v}^2} e^{-\left(\frac{v^2 \pi}{4\bar{v}^2}\right)}$$

$\frac{v}{\bar{v}}$	$\bar{v}P(v)$		
0.0	0.000	2.0	0.1358
0.1	0.1559	2.1	0.1033
0.2	0.3044	2.2	0.0772
0.3	0.4391	2.3	0.0567
0.4	0.5541	2.4	0.0409
0.5	0.6454	2.5	0.0290
0.6	0.7104	2.6	0.0202
0.7	0.7483	2.7	0.0138
0.8	0.7602	2.8	0.0093
0.9	0.7483	2.9	0.0062
1.0	0.7162	3.0	0.0040
1.1	0.6680	3.1	0.0026
1.2	0.6083	3.2	0.0016
1.3	0.5415	3.3	0.0010
1.4	0.4717	3.4	0.00061
1.5	0.4025	3.5	0.00036
1.6	0.3365	3.6	0.00021
1.7	0.2759	3.7	0.00012
1.8	0.2219	3.8	0.00007
1.9	0.1752	3.9	0.00004
		4.0	0.00002

where

a = cut-in velocity

c = cut-out velocity

$$\sigma^2 = \frac{2\bar{v}^2}{\pi}$$

Figure 2 presents a graphical representation of Equation 5, giving the percent down time as functions of the ratios of cut-in velocity to mean velocity and cut-out velocity to mean velocity.

Table 2 is a sequential listing of the tabular data used for Figure 2. If the ratio of the cut-out velocity to the annual mean velocity is 5 or greater, the percent down time loses its dependence on this ratio. This effect is seen in Figure 2 and Table 2 and results from very small contributions of the last term of Equation 5 when the ratio of the cut-out velocity to the annual mean velocity exceeds a value of 5.

For a machine with a given cut-in (a) and cut-out speed (c) an estimate of the minimum percent down time would occur with an annual (long term) mean wind speed equal to

$$\bar{v}_{\text{min down time}} = \left[ \frac{\pi}{4} \frac{(c^2 - a^2)}{(\ln c^2 - \ln a^2)} \right]^{1/2} \quad (6)$$

Substituting Equation 6 into Equation 5 yields the following formula for estimating the minimum percent down time for a machine with a cut-in velocity of magnitude a and a cut-out velocity of magnitude c:

$$\text{Minimum percent down time} = 1 - e^{-\frac{a^2(\ln c^2 - \ln a^2)}{c^2 - a^2}} + e^{-\frac{c^2(\ln c^2 - \ln a^2)}{c^2 - a^2}} \quad (7)$$

Figure 3 shows how the estimate of the percent down time would be affected by varying the cut-out velocity for a wind turbine whose cut-in velocity is 6 m/s (13 mph).

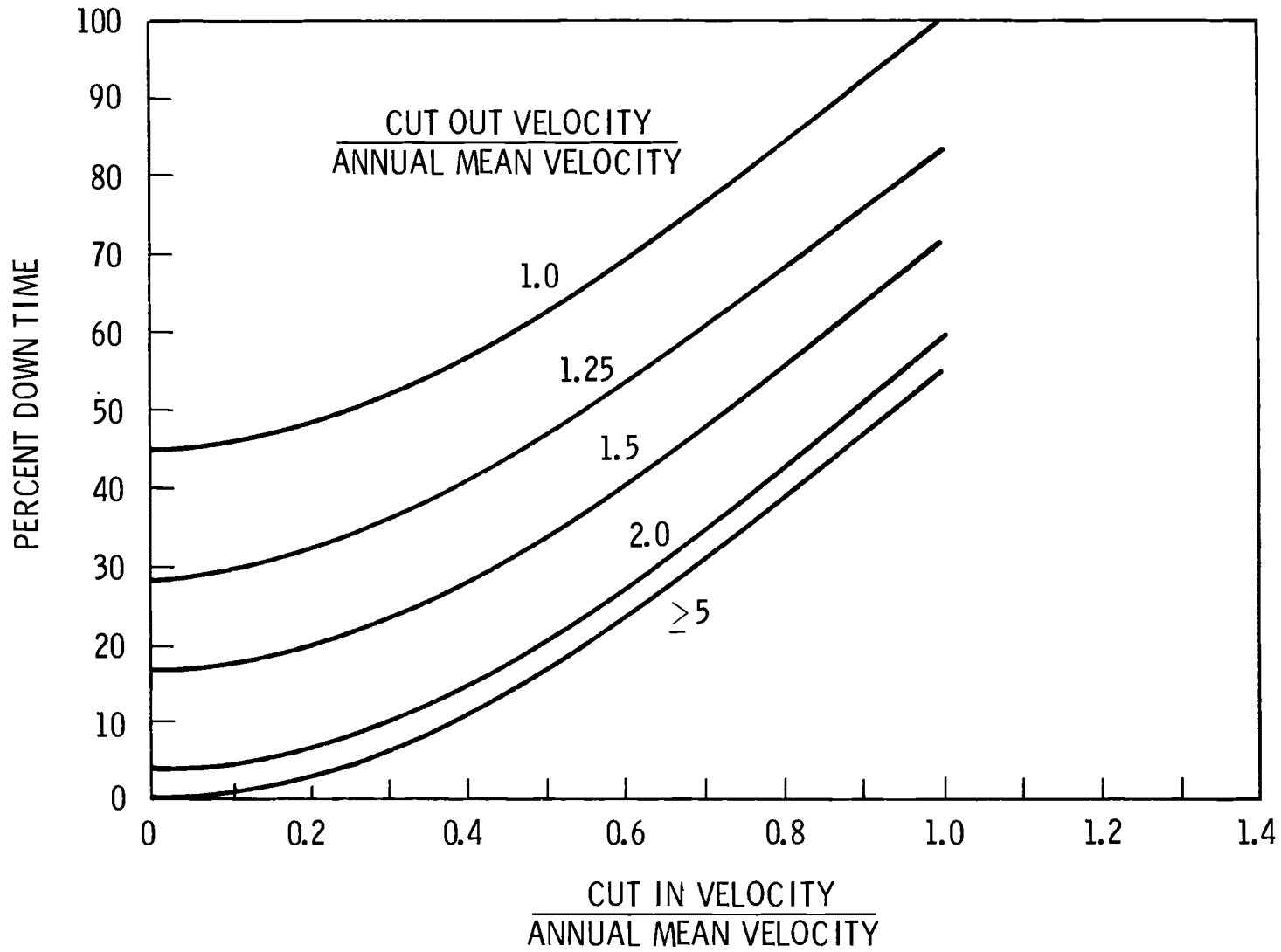


FIGURE 2. Percent Down Time

TABLE 2. Percent Down Time

<u>Percent Down Time</u>	<u>Cut-in Velocity Annual Mean Velocity</u>	<u>Cut-out Velocity Annual Mean Velocity</u>
0.45	0.0	1.0
0.46	0.1	1.0
0.48	0.2	1.0
0.52	0.3	1.0
0.57	0.4	1.0
0.63	0.5	1.0
0.70	0.6	1.0
0.77	0.7	1.0
0.85	0.8	1.0
0.92	0.9	1.0
1.00	1.0	1.0
0.29	0.0	1.25
0.30	0.1	1.25
0.32	0.2	1.25
0.36	0.3	1.25
0.41	0.4	1.25
0.47	0.5	1.25
0.54	0.6	1.25
0.61	0.7	1.25
0.69	0.8	1.25
0.76	0.9	1.25
0.84	1.0	1.25
0.17	0.0	1.5
0.18	0.1	1.5
0.20	0.2	1.5
0.24	0.3	1.5
0.29	0.4	1.5
0.35	0.5	1.5
0.42	0.6	1.5
0.49	0.7	1.5
0.57	0.8	1.5
0.64	0.9	1.5
0.72	1.0	1.5
0.04	0.0	2.0
0.05	0.1	2.0
0.07	0.2	2.0
0.11	0.3	2.0
0.16	0.4	2.0
0.22	0.5	2.0
0.29	0.6	2.0
0.36	0.7	2.0
0.44	0.8	2.0
0.51	0.9	2.0
0.59	1.0	2.0
0.00	0.0	≥5.0
0.01	0.1	≥5.0
0.03	0.2	≥5.0
0.07	0.3	≥5.0
0.12	0.4	≥5.0
0.18	0.5	≥5.0
0.25	0.6	≥5.0
0.32	0.7	≥5.0
0.40	0.8	≥5.0
0.47	0.9	≥5.0
0.55	1.0	≥5.0

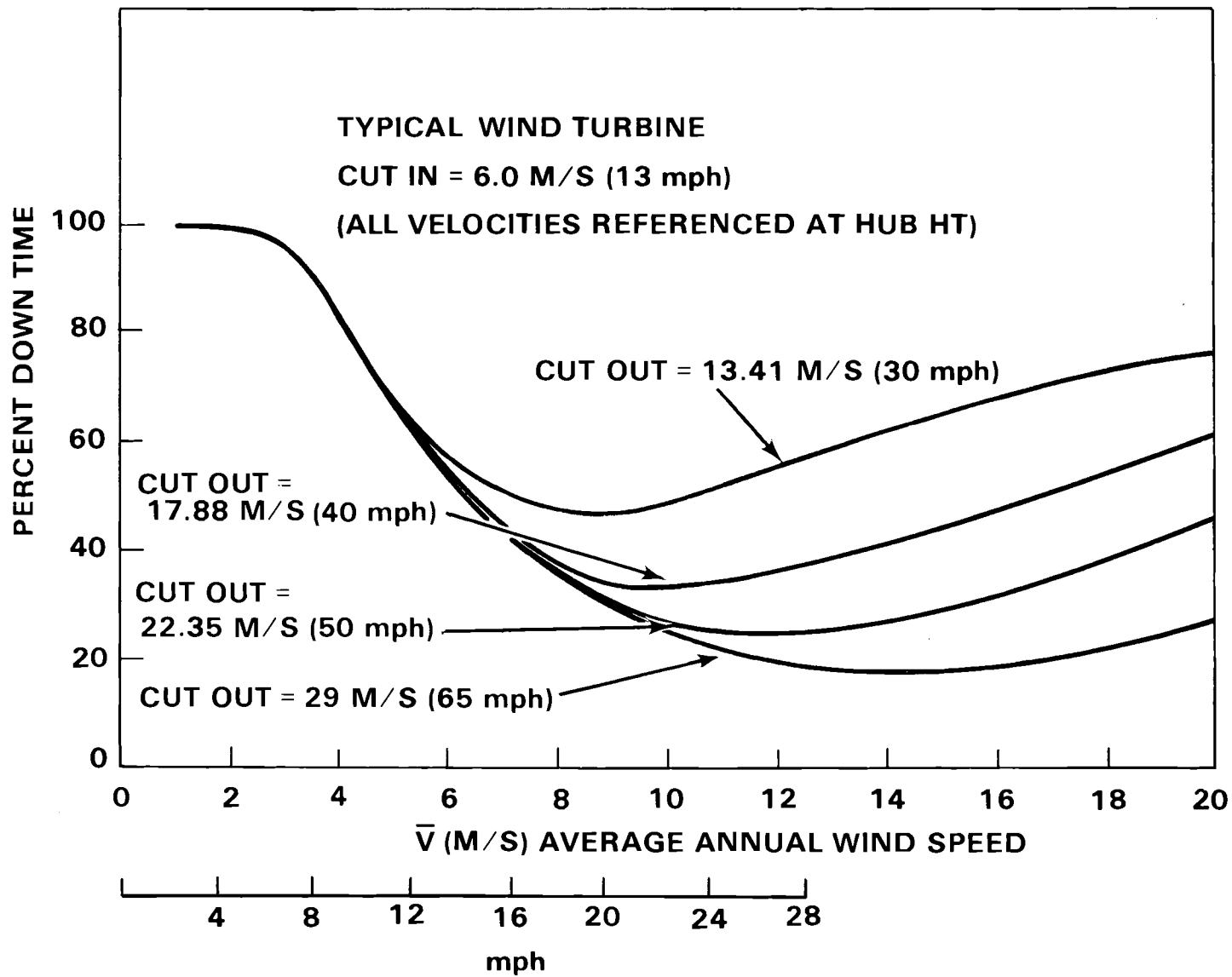


FIGURE 3. Example of Percent Down Time as a Function of Cut-out Speed

### PERCENT TIME RUNNING AT RATED

The percent time running at rated is the percentage of time the wind speed lies between the wind turbine's rated wind speed and the turbine's cut-out wind speed. An estimate of the percent time running at rated may be obtained from the following formula:

$$\text{Percent time running at rated} = e^{-b^2/2\sigma^2} - e^{-c^2/2\sigma^2} \quad (8)$$

where

b = rated velocity

c = cut-out velocity

$$\sigma^2 = \frac{2\bar{v}^2}{\pi}$$

Equation 8 is graphically represented in Figure 4 with Table 3 sequentially listing the tabular data used to produce Figure 4. As with the percent down time, the percent running at rated loses strong dependence on the cut-out velocity when the ratio of the cut-out velocity to the mean velocity is greater than 5. This effect is due to the decreasing contribution from the last term in Equation 8.

### EXPECTED AVERAGE POWER OUTPUT

Equation 1 provides the basic formulation to estimate the expected average power output from a wind turbine. The expected average power output is defined as the mean power output averaged over several months or longer. The two elements that must be known or estimated to evaluate Equation 1 are the machine function  $T_p(v)$  and the wind speed frequency distribution  $P(v)$ . In most cases the machine function  $T_p(v)$  will be estimated by the manufacturer from either theoretical computations or field testing or both. If a long term frequency distribution for  $P(v)$  is known, it should be used in all computations. If, however, only a mean velocity at the wind turbine's hub height is known or can be estimated and this mean wind speed is equal to or greater than 4.5 m/s (10 mph), Equation 2 (the Rayleigh distribution) may be used as an estimate of the wind speed frequency distribution  $P(v)$ . Based on a characteristic machine

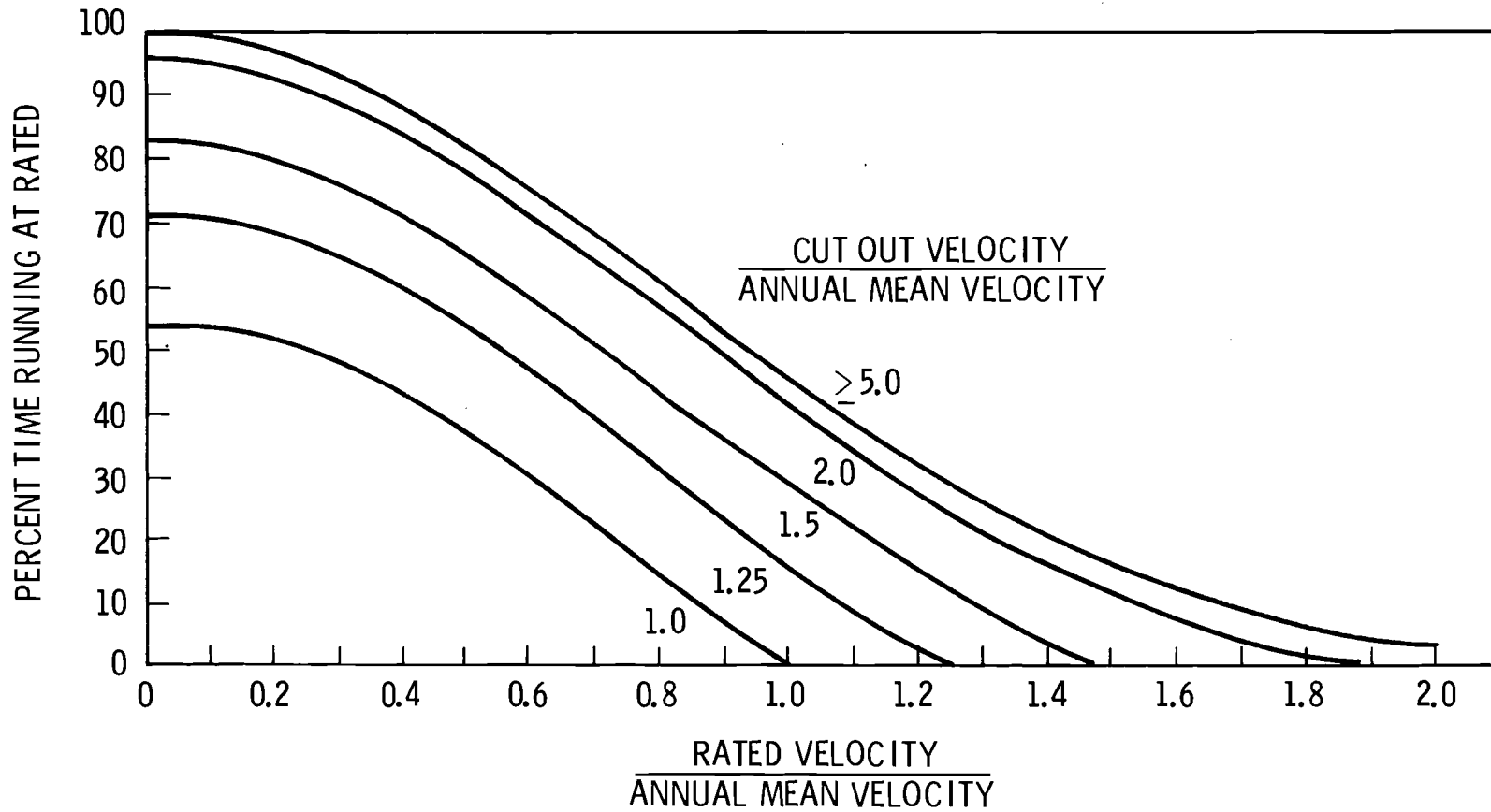


FIGURE 4. Percent Time Running at Rated

TABLE 3. Percent Running at Rated

<u>% Time Running at Rated</u>	<u>Rated Velocity Annual Mean Velocity</u>	<u>Cut-out Velocity Annual Mean Velocity</u>
54	0.0	1.0
54	0.1	1.0
51	0.2	1.0
48	0.3	1.0
43	0.4	1.0
37	0.5	1.0
30	0.6	1.0
22	0.7	1.0
15	0.8	1.0
7	0.9	1.0
0	1.0	1.0
71	0.0	1.25
70	0.1	1.25
68	0.2	1.25
64	0.3	1.25
59	0.4	1.25
53	0.5	1.25
46	0.6	1.25
39	0.7	1.25
31	0.8	1.25
24	0.9	1.25
16	1.0	1.25
9	1.1	1.25
3	1.2	1.25
0	1.25	1.25
83	0.0	1.5
82	0.1	1.5
80	0.2	1.5
76	0.3	1.5
71	0.4	1.5
65	0.5	1.5
58	0.6	1.5
51	0.7	1.5
43	0.8	1.5
36	0.9	1.5
29	1.0	1.5
22	1.1	1.5
15	1.2	1.5
9	1.3	1.5
4	1.4	1.5
0	1.5	1.5



TABLE 3 (continued)

<u>% Time Running at Rated</u>	<u>Rated Velocity Annual Mean Velocity</u>	<u>Cut-out Velocity Annual Mean Velocity</u>
96	0.0	2.0
95	0.1	2.0
93	0.2	2.0
89	0.3	2.0
84	0.4	2.0
78	0.5	2.0
71	0.6	2.0
64	0.7	2.0
56	0.8	2.0
49	0.9	2.0
41	1.0	2.0
34	1.1	2.0
28	1.2	2.0
22	1.3	2.0
17	1.4	2.0
13	1.5	2.0
9	1.6	2.0
6	1.7	2.0
4	1.8	2.0
2	1.9	2.0
0	2.0	2.0
100	0.0	≥5.0
99	0.1	≥5.0
97	0.2	≥5.0
93	0.3	≥5.0
88	0.4	≥5.0
82	0.5	≥5.0
75	0.6	≥5.0
68	0.7	≥5.0
60	0.8	≥5.0
53	0.9	≥5.0
46	1.0	≥5.0
39	1.1	≥5.0
32	1.2	≥5.0
27	1.3	≥5.0
21	1.4	≥5.0
17	1.5	≥5.0
13	1.6	≥5.0
10	1.7	≥5.0
8	1.8	≥5.0
6	1.9	≥5.0
4	2.0	≥5.0

function shape (as shown in Figure 1a), and Equation 2 for  $P(v)$ , Figure 5 was developed to provide rough generic estimates of average power output for wind turbines.

A measure of the overall efficiency of a wind turbine is expressed as the ratio of the average power output to the average energy flux available. The energy flux (potential power available) is defined as  $\frac{1}{2} \rho V^3$  where  $\rho$  is the air density. The average of this value is then equal to  $\frac{1}{2} \overline{\rho V^3}$ , the overbar signifying a time average value.

$\overline{V^3}$  may be related to the annual mean wind speed,  $\bar{V}$ , using the Rayleigh distribution. This relationship is as follows:

$$\overline{V^3} = \bar{V}^3 \left(\frac{4}{\pi}\right)^{3/2} \frac{3}{2} \Gamma\left(\frac{3}{2}\right) \text{ or } \approx 1.91 \bar{V}^3 . \quad (9)$$

Thus based on the Rayleigh distribution, the average total amount of "potential" power available,  $P_A$ , is:

$$P_A \approx 0.955 \rho \bar{V}^3 , \quad (10)$$

where  $\rho$  is in  $\text{Kg/m}^3$  and  $V$  is in  $\text{m/s}$  and the average total potential power is in  $\text{watts/m}^2$ .

#### PRESENT WORTH OF WIND TURBINES

Once an estimate of the average expected power output of a wind turbine has been made, it is a simple matter to multiply this estimate by the number of hours in a year to find the expected yearly energy production for the wind turbine. By estimating the value of one unit of energy from the wind turbine, the present worth of the wind turbine can be computed from the following standard actuarial equation. (The formula given below is for a constant value for a unit of energy. To have an escalating value for a unit cost of energy, the formula below must be modified accordingly.)

$$\text{Present worth} = 8769 \bar{T}_p \left( \frac{1 - (1 + i)^{-n}}{i} \right) C_w \quad (11)$$

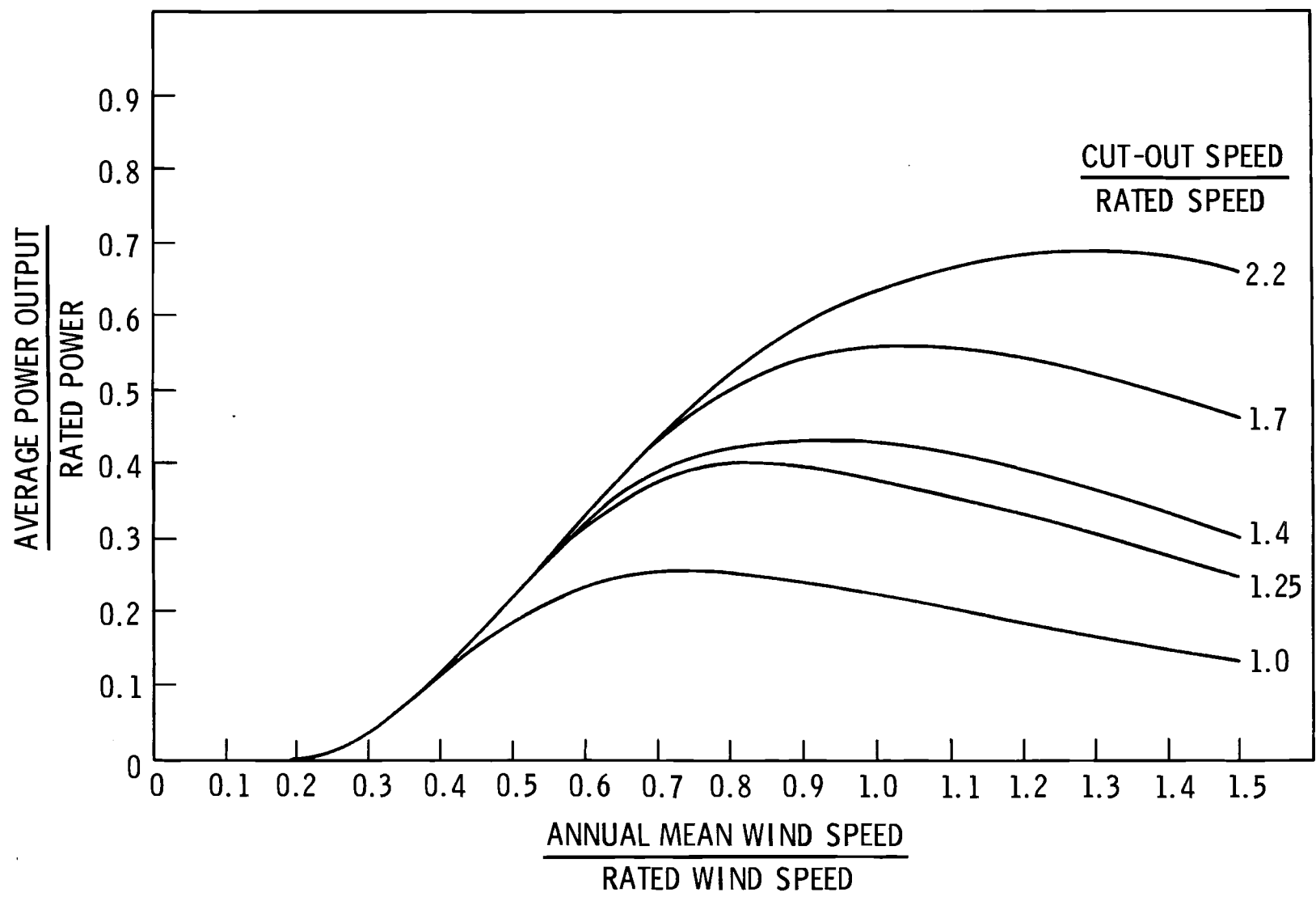


FIGURE 5. Estimate of Expected Average Power Output for Wind Turbines

where

$\bar{T}_p$  = average expected power output

$i$  = interest rate per payment interval (i.e., interest per month, year, etc.)

$n$  = total number of payment intervals over which the wind turbine will be amortized

$C_w$  = estimate of the worth of a unit of energy from the wind turbine (i.e., 2 cents/kW-h, etc.)

Using the Rayleigh distribution for  $P(v)$ ,  $\bar{T}_p$  becomes a function only of the mean wind speed. Thus, differences in the present worth of a specific wind turbine for two different mean wind speeds may be computed. This could be valuable information to consider if one is trying to assess whether it is better to place a wind turbine in a higher wind location with greater costs (installation, distribution, etc.) or place the wind turbine where these costs are lower but also the mean wind is lower. Examples of the use of all the preceding material will be presented in the next section along with a comparison of the computations using actual field data versus the estimates using the Rayleigh distribution.

### ANALYSIS

To illustrate the process of estimating the behavior of a wind turbine, two typical examples (with different cut-in, cut-out and rated wind speeds) will be presented here. Hypothetical machine functions and sizes have been used to add a note of realism. Of course, the rated power could be scaled up or down to fit a particular case.

#### EXAMPLE 1

In this example assume a wind turbine with the following characteristics:

Cut-in velocity (at hub height) = 6 m/s (13 mph)

Rated velocity (at hub height) = 13 m/s (29 mph)

Cut-out velocity (at hub height) = 29 m/s (65 mph)

Blade diameter = 61 m (200 ft)

Rated power = 1500 kW

Using a characteristic shape for the machine function as in Figure 1a and Equation 2 for  $P(v)$ ,  $\bar{T}_p$  may be computed by numerically integrating Equation 1. Equations 5 and 8 are used to compute percent down time and percent time rated. These results, presented in Table 4, are estimates of the example wind turbine's performance as a function of the annual mean velocity. Table 4 also presents a few terms which have not previously been used, but may be of interest to the reader and are defined on page 20.

From Equation 6 we find that the minimum percent down time would occur at a mean wind speed of 14.16 m/s, which is confirmed in Table 4. The values of Table 4 are presented in graphical form in Figures 6 and 7. Wind frequencies from 16 sites (with high mean winds) from around the United States and Puerto Rico were analyzed using the same typical machine characteristics and compared with the results obtained using the Rayleigh distribution for the wind speeds frequency distribution. These results are also given in Figures 6 and 7. The actual distributions used along with their location and length of record analyzed are included in Appendix A. As an overlay to the actual distributions the normalized Rayleigh distribution is included on each graph to give an indication of the ability of the Rayleigh distribution to represent the actual distribution.

As indicated in Figure 7, the long term mean value used for the test sites ranges in length from 1 to 6 months. Even with these short data collection periods, the estimate curves generated using the Rayleigh distribution compare remarkably well. The integration process involved in computing  $\bar{T}_p$  has a tendency to smooth out small discrepancies and the flat portion of the machine function makes this region relatively insensitive to minor shape variations of  $P(v)$  in this range. As shown in Appendix A, some of the distributions appear to vary significantly from the Rayleigh distribution but still lead to values quite close to the estimates predicted using the Rayleigh distribution. The West Coast sites experienced an anomalous wind condition the months of January, February and March of 1977. The greatest deviation occurred at San Gorgonio where

#### DEFINITIONS OF TERMS IN TABLE 4

- Mean velocity - The long term mean wind speed (long term denotes several months, preferably 6 months to 1 year or longer).
- Average extractable power - The long term average power the turbine would produce.
- Total power - The total energy flux available to the wind turbine, equal to
- $$\frac{1}{2} \overline{\rho v^3} A = \frac{1}{2} \rho A \int_0^{\infty} v^3 P(v) dv \approx 0.955 \rho \overline{v^3} A$$
- for Rayleigh  $P(v)$ . (A = swept area of the blades.)
- CP - The overall coefficient of performance and is equal to the ratio of the extractable power to the total power.
- $T_1$  - The average amount of extractable power generated from wind speeds between cut-in and rated.
- $T_2$  - The average amount of extractable power generated from wind speeds between rated and cut-out (i.e., when the turbine is operating at its rated capacity).

TABLE 4. Example 1 Wind Turbines' Performance Characteristics

EXAMPLE 1  
WIND TURBINE DESIGN VALUES

CUT IN WIND SPEED = 6.0 M/S (13.5 mph)  
 RATED WIND SPEED = 13.0 M/S (29 mph)  
 CUT OUT WIND SPEED = 29 M/S (65 mph)  
 RATED POWER = 1500 kW  
 BLADE RADIUS = 30.48 M (100 ft)

MEAN VELOCITY (M/S)	EXTRACTABLE POWER (kW)	TOTAL POWER (kW)	CP	PERCENT DOWN TIME	PERCENT TIME RATED	T1	T2
3.50	23.74	146.39	0.162	90.1	0.0	23.71	0.03
4.00	51.48	218.52	0.236	82.9	0.0	51.10	0.38
4.50	91.03	311.14	0.293	75.2	0.1	88.89	2.14
5.00	140.86	426.80	0.330	67.7	0.5	133.43	7.43
5.50	198.58	568.08	0.350	60.7	1.2	179.93	18.66
6.00	261.62	737.52	0.355	54.4	2.5	224.03	37.59
6.50	327.51	937.69	0.349	48.8	4.3	262.66	64.85
7.00	394.17	1171.15	0.337	43.8	6.7	294.22	99.95
7.50	459.98	1440.47	0.319	39.5	9.4	318.28	141.69
8.00	523.75	1748.19	0.300	35.7	12.6	335.24	188.52
8.50	584.65	2096.89	0.279	32.4	15.9	345.87	238.78
9.00	642.13	2489.13	0.258	29.5	19.4	351.18	290.95
9.50	695.81	2927.45	0.238	27.0	22.9	352.14	343.67
10.00	745.44	3414.44	0.218	24.8	26.4	349.67	395.77
10.50	790.85	3952.64	0.200	22.9	29.8	344.56	446.29
11.00	831.95	4544.62	0.183	21.3	33.0	337.50	494.45
11.50	868.68	5192.93	0.167	19.9	36.0	329.02	539.66
12.00	901.04	5900.15	0.153	18.8	38.8	319.58	581.47
12.50	929.08	6668.82	0.139	18.0	41.3	309.50	619.57
13.00	952.88	7501.52	0.127	17.4	43.6	299.08	653.80
13.50	972.60	8400.80	0.116	17.0	45.6	288.51	684.09
14.00	988.41	9369.22	0.105	16.9	47.4	277.95	710.47
14.50	1000.54	10409.34	0.096	16.9	48.9	267.51	733.02
15.00	1009.20	11523.73	0.088	17.1	50.1	257.30	751.91
15.50	1014.67	12714.94	0.080	17.5	51.2	247.35	767.32
16.00	1017.21	13985.54	0.073	18.0	52.0	237.73	779.48
16.50	1017.07	15338.08	0.066	18.7	52.6	228.44	788.63
17.00	1014.54	16775.14	0.060	19.5	53.0	219.52	795.02
17.50	1009.85	18299.26	0.055	20.4	53.3	210.96	798.89
18.00	1003.25	19913.00	0.50	21.4	53.4	202.76	800.49

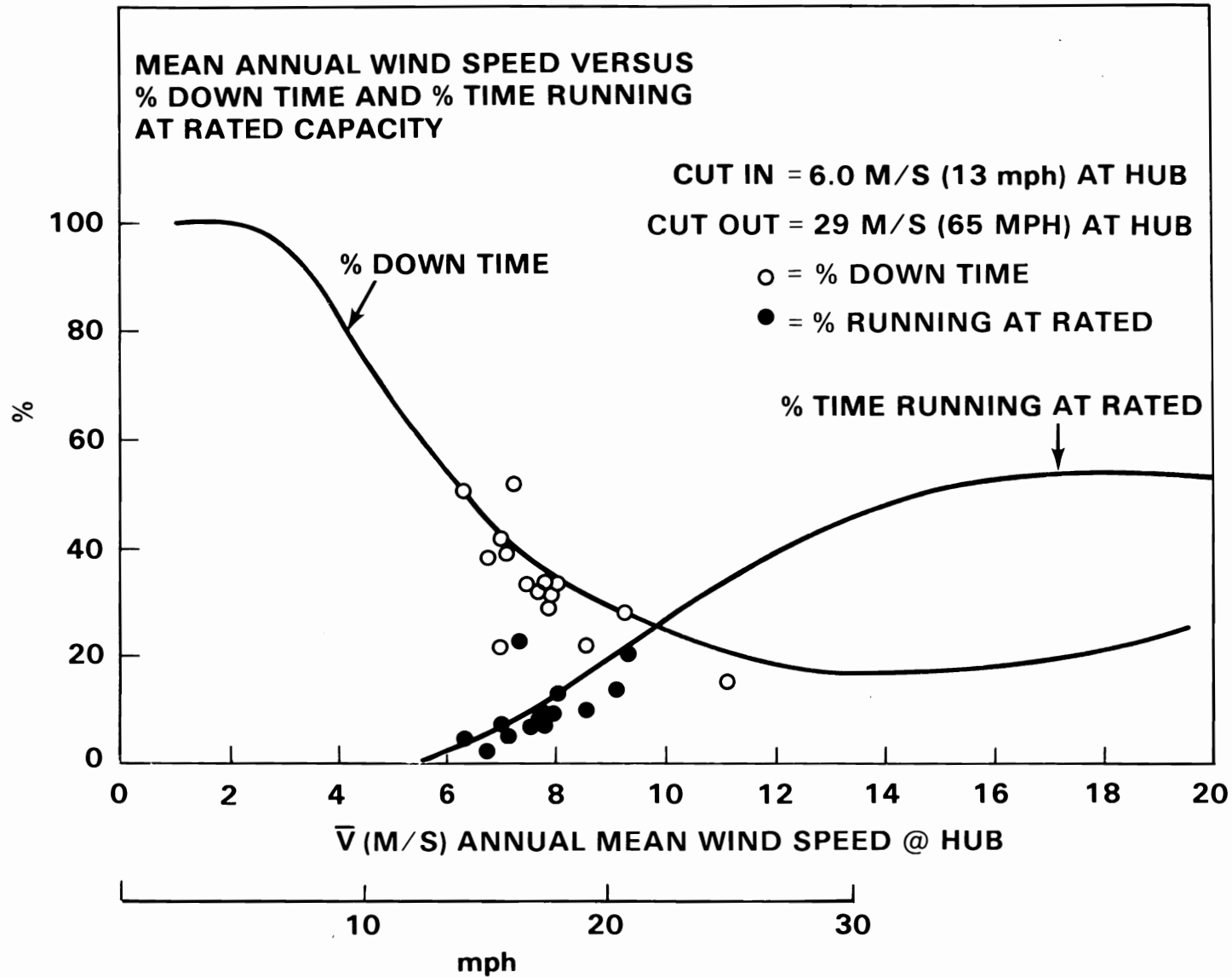


FIGURE 6. Example 1 Wind Turbines' Percent Down Time and Percent Running at Rated (dots represent data from sites listed in Appendix A)



# TYPICAL WIND TURBINE (1500 Kw)

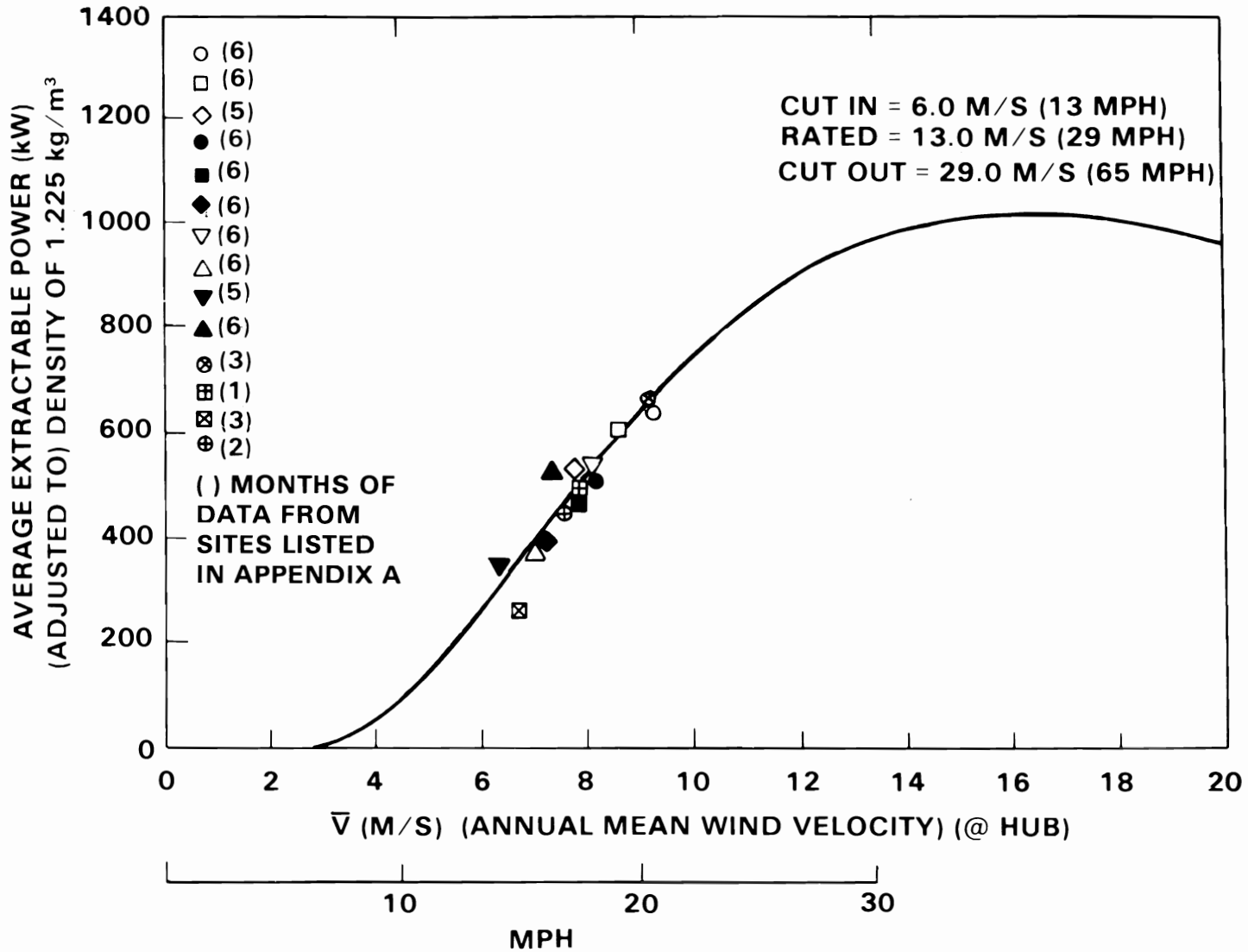


FIGURE 7. Example 1 Wind Turbines' Average Extractable Power

the anomalous wind condition produced a bimodal frequency distribution when coupled with a few months of its usual high wind environment.

To examine the effect that the cut-out velocity has on the example wind turbine's performance, the process above is repeated for various cut-out velocities. The results of such an exercise are presented in Figure 8. It may be beneficial to the designer to perform such studies knowing the wind conditions of the area for which he is designing his machine.

Equation 11 may now be used to estimate the present worth of the example wind turbine. The effect of changes in mean wind speed (e.g., from 8 m/s to 9 m/s) on the present worth of the machine may also be estimated. For our example let's assume that the wind turbine has a 30-year life and the interest rate is 12% and the worth of a kW-hr from the wind turbine is 4.5 cents. Then from Equation 11 and Table 4,

For  $\bar{V} = 8$  m/s the present worth of the wind turbine would be  $\approx$  \$1.66 million.

For  $V = 9$  m/s the present worth of the wind turbine would be  $\approx$  \$2.04 million.

For this example a 1 m/s mean wind difference would result in a present worth difference of  $\approx$  \$0.38 million.

Use of caution is imperative when applying the above analysis because currently the interest percentage and the worth of a kW-hr from a wind turbine may be difficult, if not impossible, to determine.

#### EXAMPLE 2

Assume that there is a wind turbine whose characteristics are as follows:

Cut-in velocity (at hub height) = 4.47 m/s (10 mph)  
Rated velocity (at hub height) = 10.73 m/s (24 mph)  
Blade diameter - 38.1 m (125 ft)  
Rated power = 200 kW

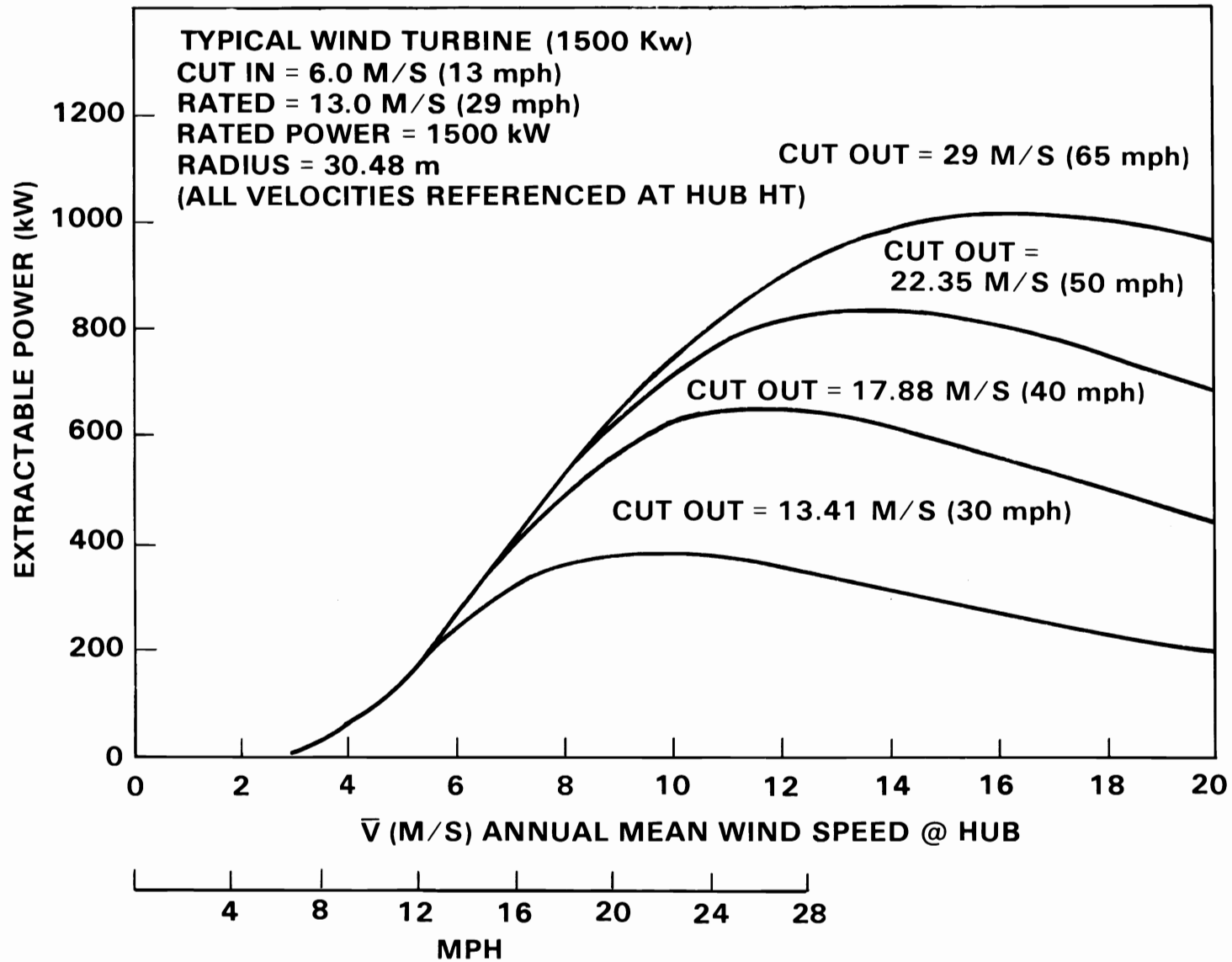


FIGURE 8. Example 1 Wind Turbines' Performance as a Function Cut-out Velocity

This example is given in addition to Example 1 to demonstrate the effect of using a lower cut-out velocity and of using an overall smaller system. Following the same procedure outlined in Example 1, the output parameters for this example turbine are given in Table 5.

Figures 9 and 10 graphically present the results indicated in Table 5. Figure 9 presents the wind turbine's percent down time and percent running at rated while Figure 10 presents the average expected output as a function of the long term average mean wind speed. Data from the 16 high wind sites were again used to compute the Example 2 wind turbine's performance characteristics. These data are also presented in Figures 9 and 10.

The Example 2 estimates computed using the Rayleigh distribution are not quite as reliable as the Example 1 estimates. The primary reason for this is that the Example 2 machine operates at a rated power over a much more limited wind speed range. This range is quite insensitive to the shape of  $P(v)$ . Thus, the shorter this range, the greater the error one would expect from using a single frequency distribution in computing estimates of machine performance. However, the greatest discrepancies usually result from the sites with 1 or 2 months of data, while those sites with 6 months of data provide more reasonable estimates. This provides an indication of the effect of the record length on the variability of the mean statistics.

The lower cut-out speed causes the maximum average output to occur near an average wind speed of 11 m/s. Therefore, a site with a mean speed greater than 11 m/s would not improve the average output of this machine.

The present worth analysis would be performed identically to Example 1 except Table 5 would be used for the necessary values for the Example 2 machine.

## CONCLUSIONS

A method to provide rule-of-thumb curves for estimating wind turbine performance using only the long term average wind speed has been presented. The results of this technique have been compared with the results obtained

TABLE 5. Example 2 Wind Turbines' Performance Characteristics

EXAMPLE 2  
WIND TURBINE DESIGN VALUES

CUT IN WIND SPEED = 4.47 M / S (10 mph)  
 RATED WIND SPEED = 10.73 M / S (24 mph)  
 CUT OUT WIND SPEED = 17.88 M / S (40 mph)  
 RATED POWER = 200 kW  
 BLADE RADIUS = 19.95 M (62.5 ft)

MEAN VELOCITY (M / S)	EXTRACTABLE POWER (KW)	TOTAL POWER (KW)	CP	PERCENT DOWN TIME	PERCENT TIME RATED	T1	T2
3.50	12.22	57.19	0.214	72.2	0.1	12.10	0.12
4.00	20.52	85.36	0.240	62.5	0.4	19.82	0.70
4.50	30.24	121.54	0.249	53.9	1.1	27.94	2.30
5.00	40.78	166.72	0.245	46.6	2.7	35.42	5.36
5.50	51.60	221.91	0.233	40.5	5.0	41.58	10.02
6.00	62.20	288.09	0.216	35.4	8.0	46.16	16.04
6.50	72.20	366.29	0.197	31.3	11.5	49.20	23.00
7.00	81.29	457.48	0.178	28.0	15.2	50.89	30.40
7.50	89.26	562.68	0.159	25.5	18.9	51.49	37.77
8.00	95.98	682.89	0.141	23.7	22.4	51.24	44.73
8.50	101.41	819.10	0.124	22.6	25.5	50.39	51.02
9.00	105.58	972.31	0.109	22.1	28.2	49.09	56.48
9.50	108.56	1143.54	0.095	22.2	30.5	47.51	61.05
10.00	110.48	1333.76	0.083	22.6	32.4	45.70	64.73
10.50	111.45	1544.00	0.072	23.5	33.8	43.89	67.56
11.00	111.61	1775.24	0.063	24.7	34.8	42.00	69.62
11.50	111.09	2028.49	0.055	26.2	35.5	40.11	70.99
12.00	110.01	2304.75	0.048	27.8	35.9	38.25	71.76
12.50	108.47	2605.01	0.042	29.6	36.0	36.45	72.02
13.00	106.57	2930.28	0.036	31.5	35.9	34.72	71.86
13.50	104.40	3281.56	0.032	33.5	35.7	33.06	71.34
14.00	102.02	3659.85	0.028	35.5	35.3	31.48	70.54
14.50	99.48	4066.15	0.024	37.5	34.8	29.98	69.50
15.00	96.85	4501.46	0.022	39.5	34.1	28.56	68.29

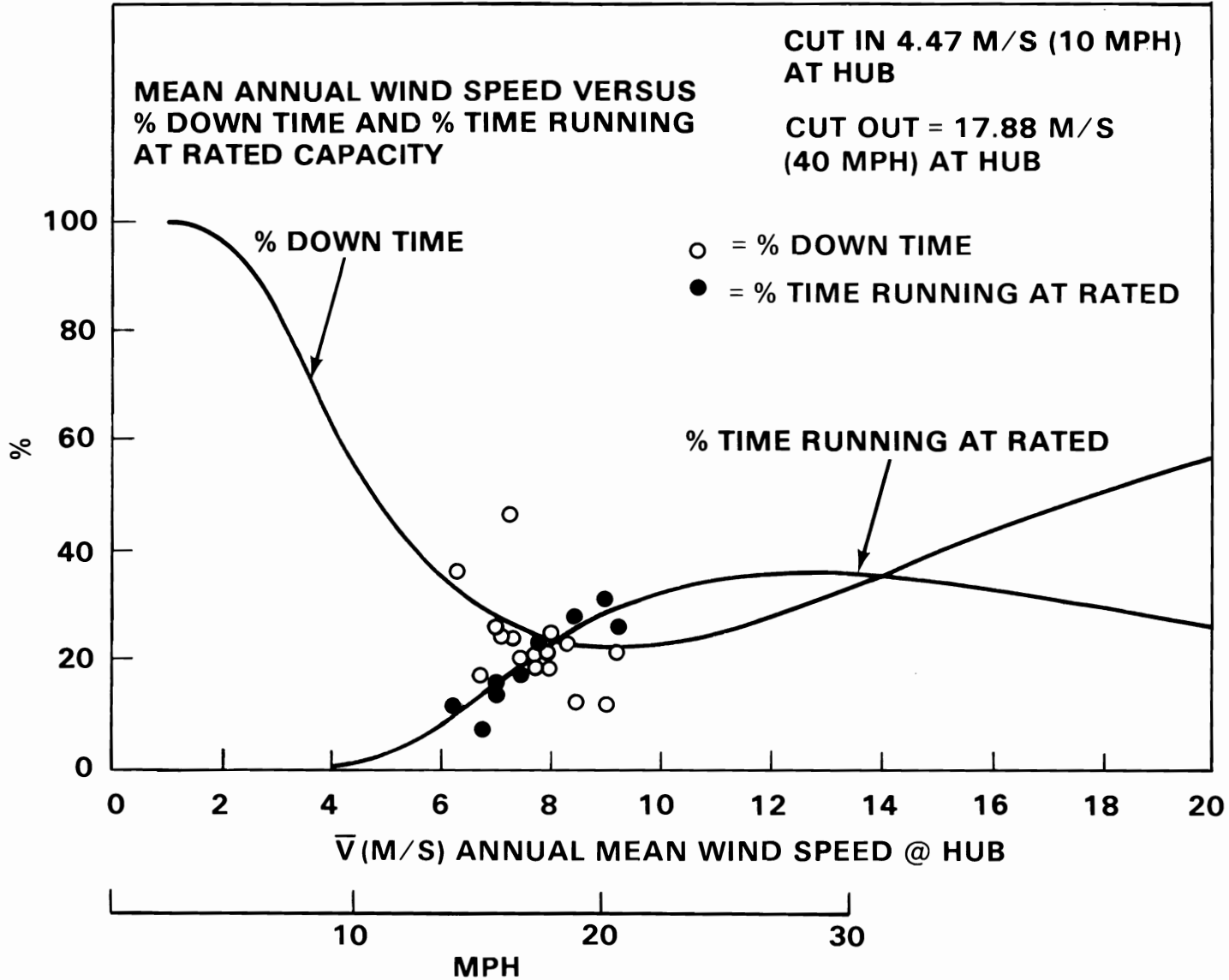


FIGURE 9. Example 2 Wind Turbines' Percent Down Time and Percent Running at Rated (dots represent data from sites listed in Appendix A)

# TYPICAL WIND TURBINE (200 Kw)

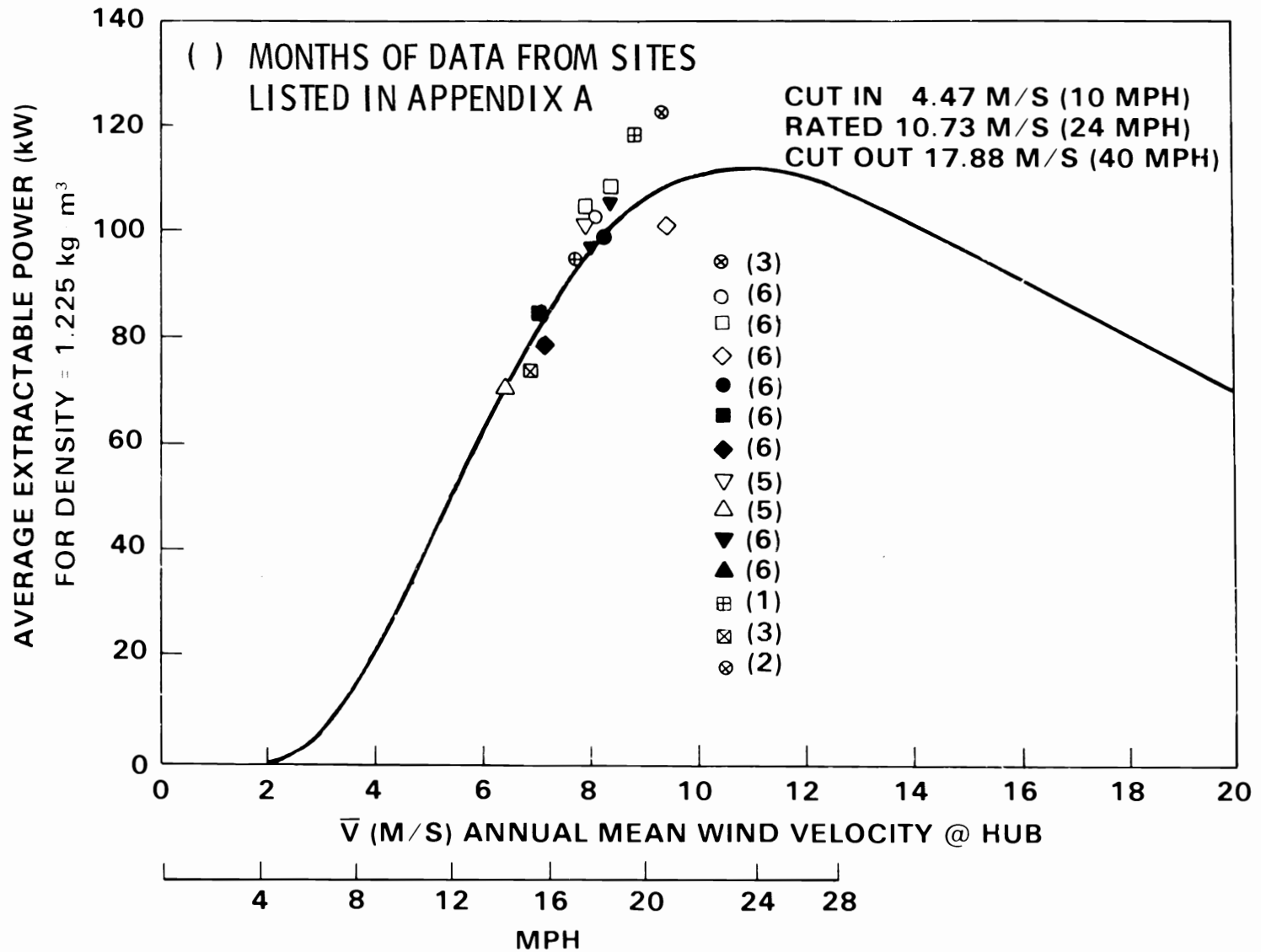


FIGURE 10. Example 2 Wind Turbines' Average Extractable Power

using data from 16 sites scattered throughout the United States and Puerto Rico. The comparisons are extremely favorable. When the data record length is 4 to 6 months, the errors are confined to 10% or less. If the data record is 1 to 3 months, the errors may be as high as 25%, but are generally less than 15%. The estimate of the wind speed frequency distribution is assumed to have a Rayleigh shape which is a single parameter frequency distribution requiring only the mean wind speed as input. The technique used is suggested only for sites with long term mean wind speeds greater than 4.5 m/s (10 mph).



## REFERENCES

1. Putnam, P. C., Power From the Wind, Van Nostrand Reinhold Company, 1948.
2. Justus, C. G., W. R. Hargraves and A. Mikhail, Reference Wind Speed Distributions and Height Profiles for Wind Turbine Design and Performance Evaluation Applications, ORO/5108-76/4, August 1976.



APPENDIX A

FREQUENCY DISTRIBUTIONS OF HIGH WIND SITES  
AROUND THE UNITED STATES AND PUERTO RICO

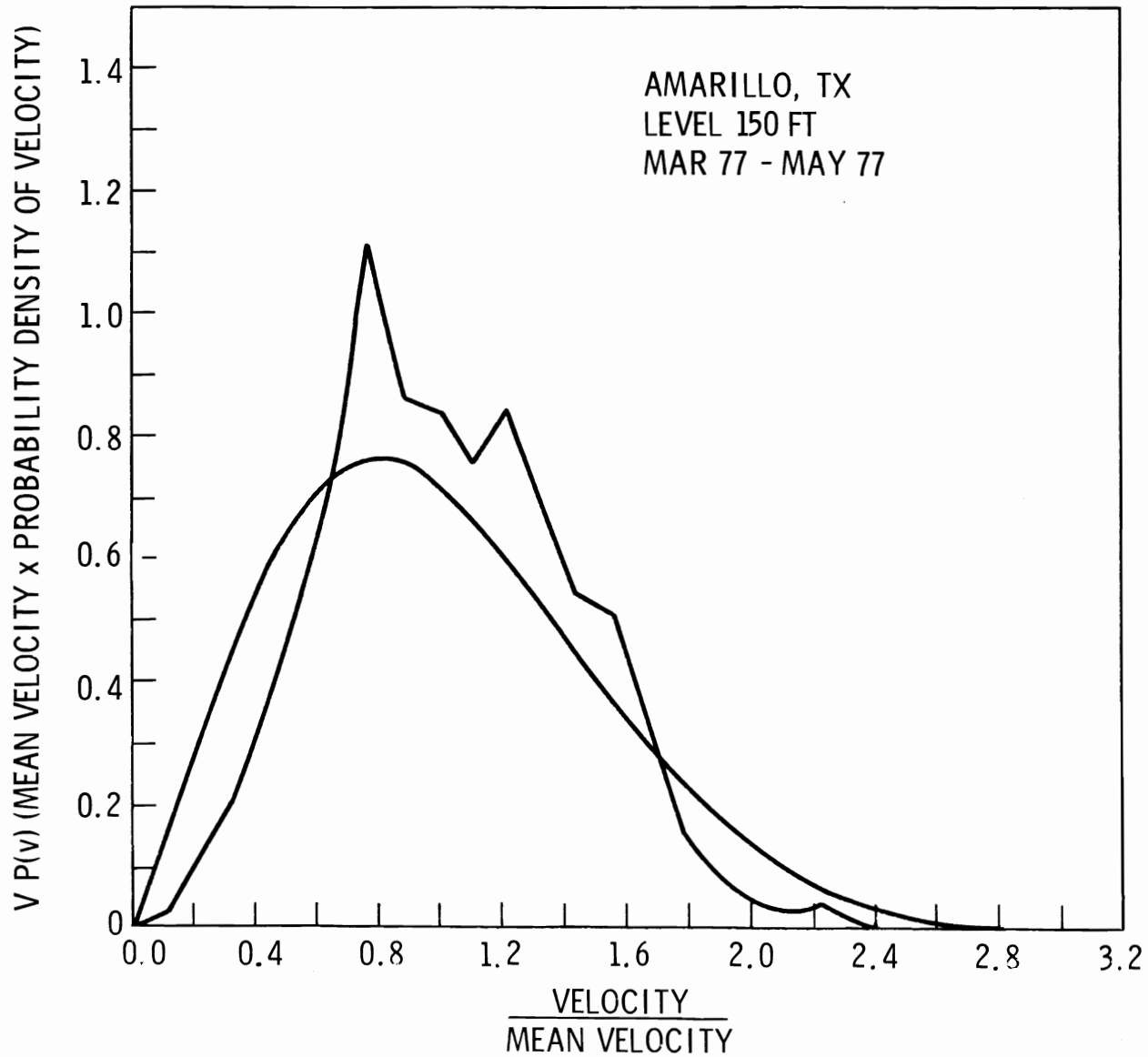


FIGURE A-1. Wind Speed Frequency Distribution from Amarillo, TX  
(The smooth curve is the normalized Rayleigh distribution  
while the jagged curve is the normalized frequency  
distribution obtained from data)

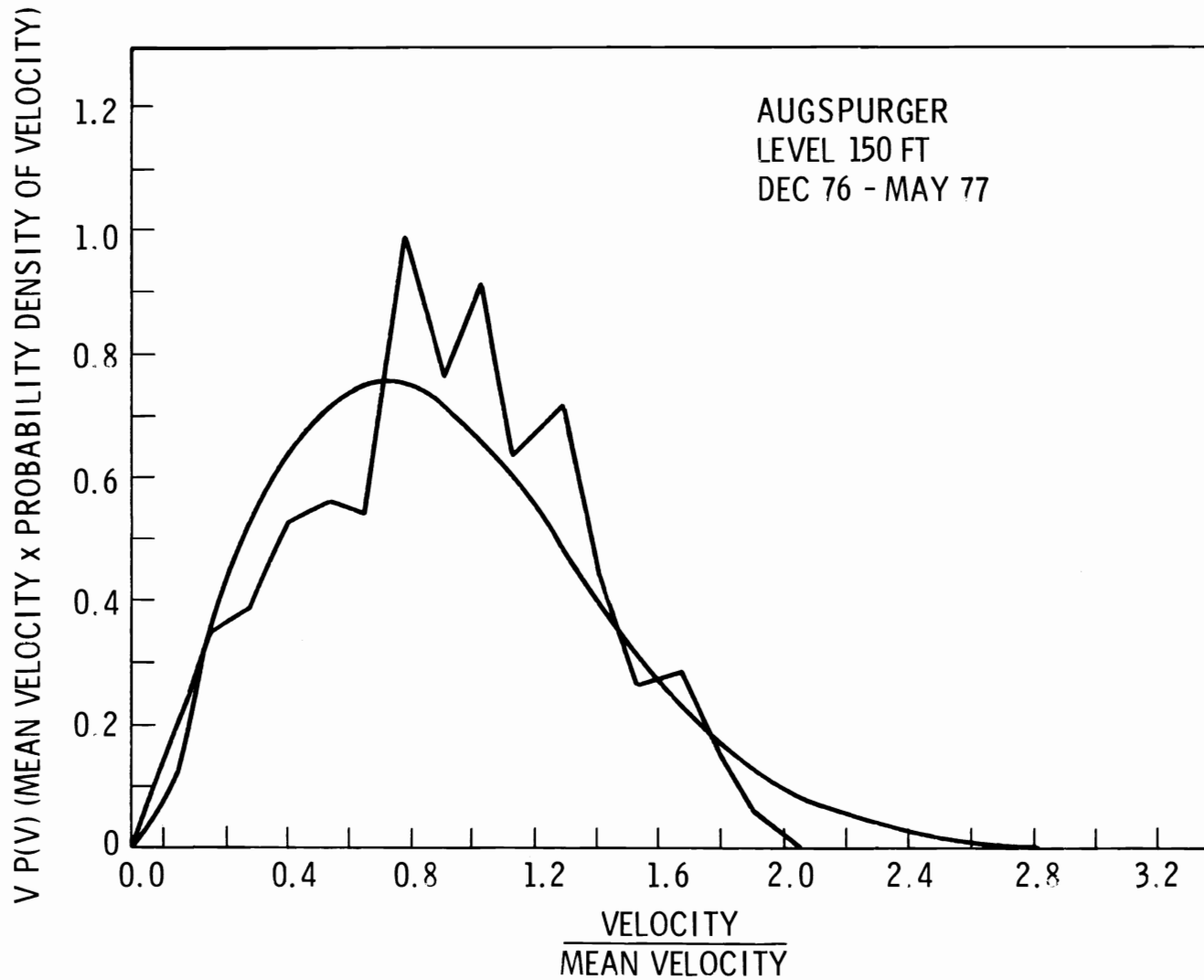


FIGURE A-2. Wind Speed Frequency Distribution from Augspurgen, WA  
(See Figure A-1 for details)

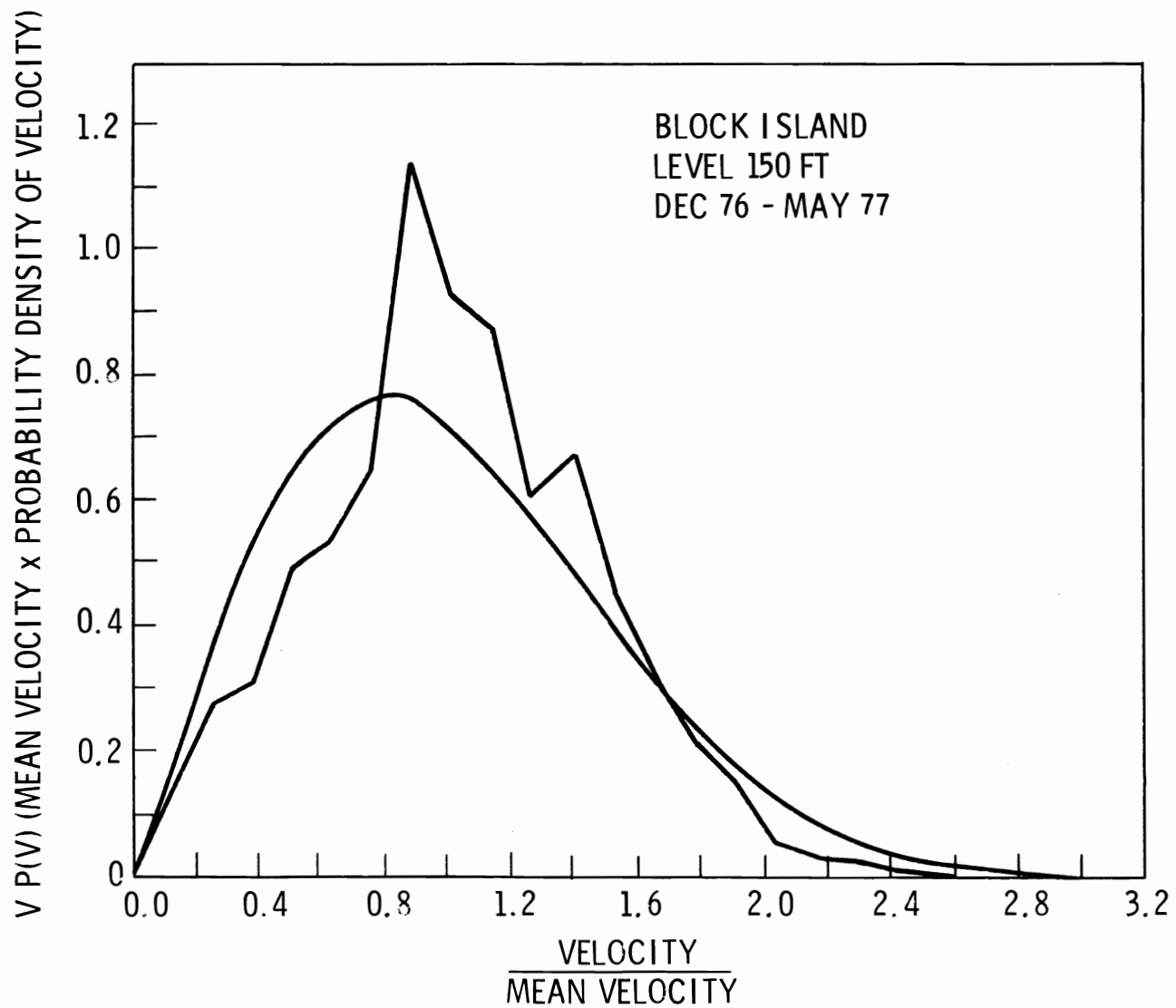


FIGURE A-3. Wind Speed Frequency Distribution from Block Island, RI  
(See Figure A-1 for details)

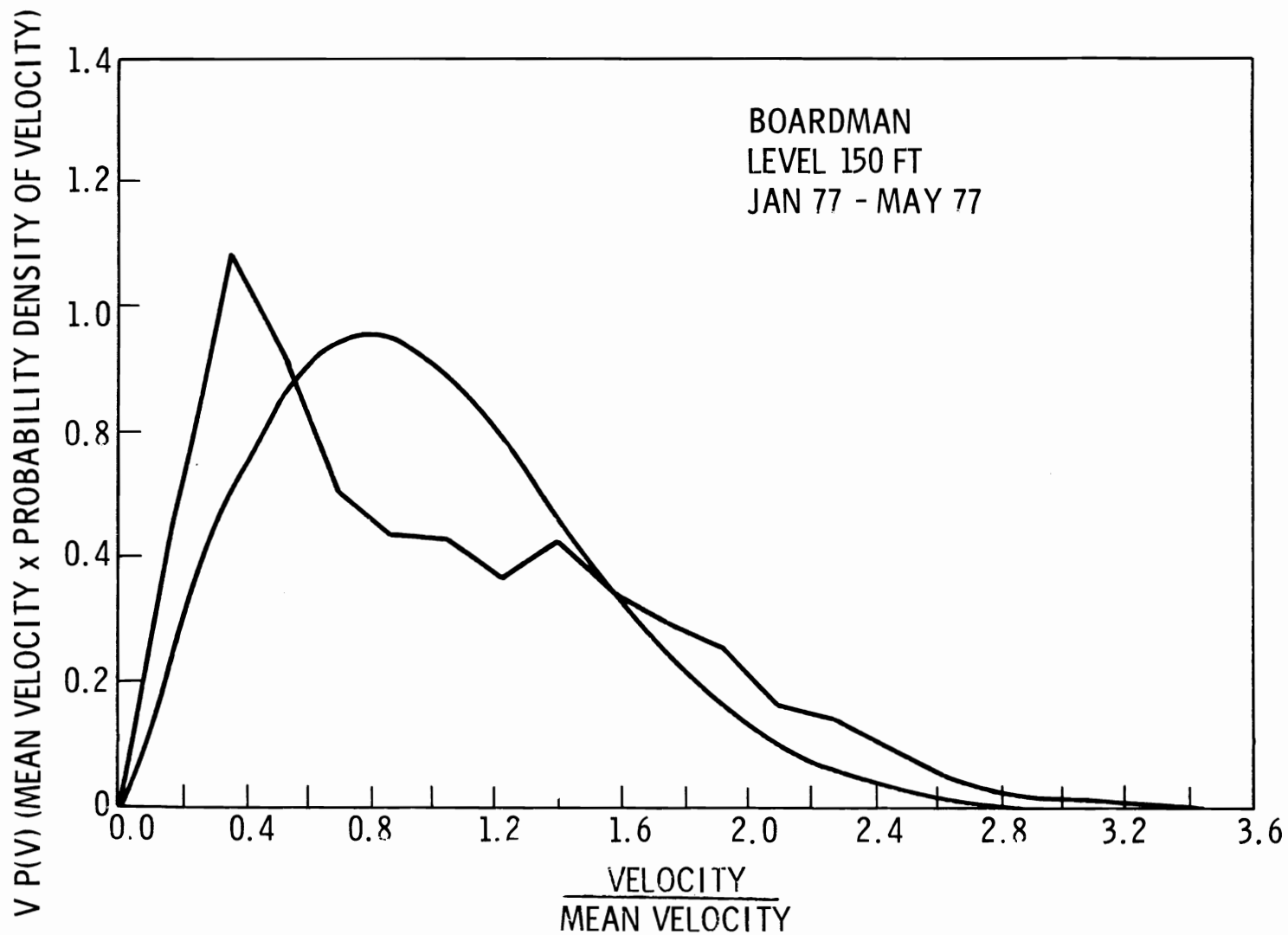


FIGURE A-4. Wind Speed Frequency Distribution from Boardman, OR  
(See Figure A-1 for details)

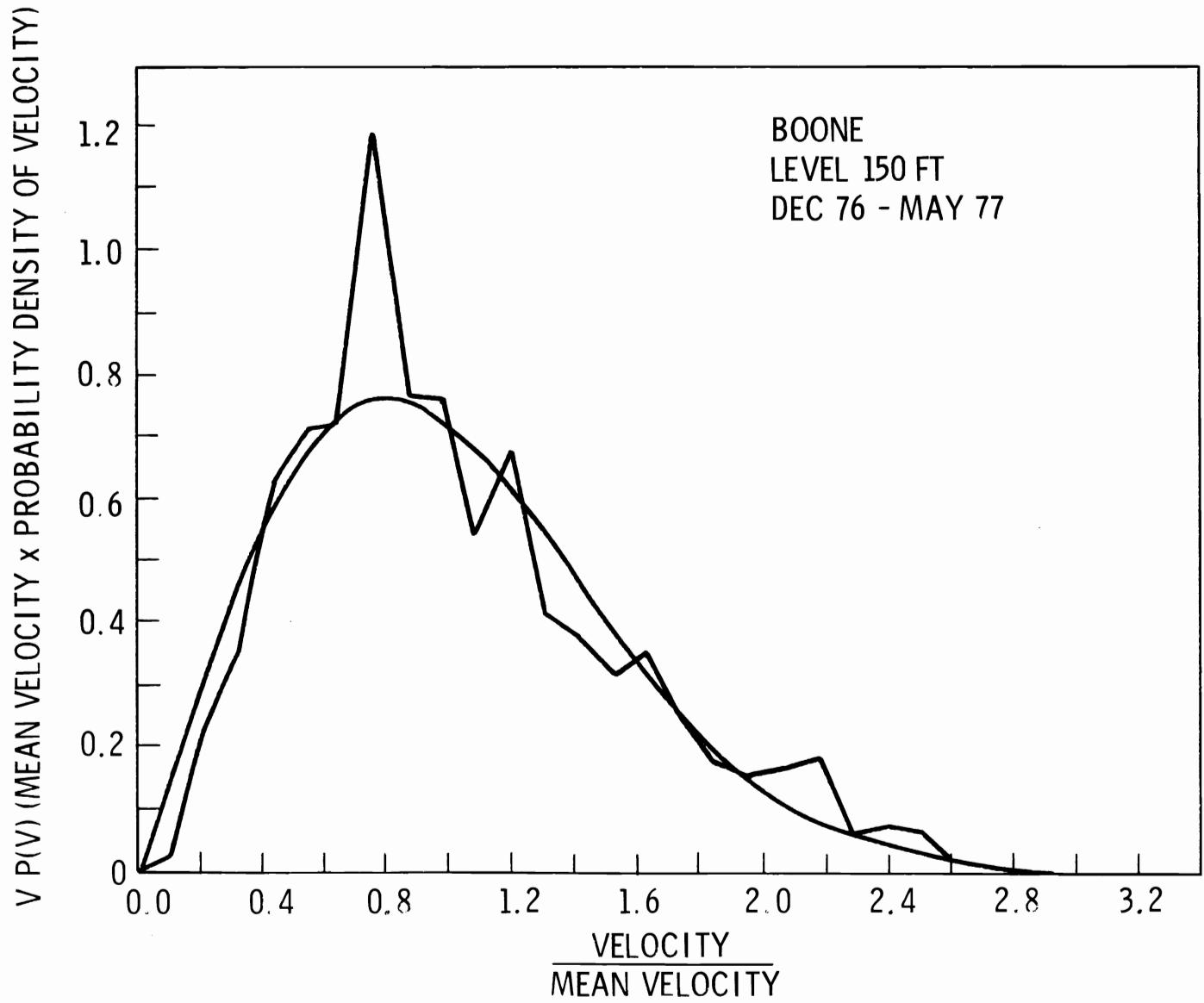


FIGURE A-5. Wind Speed Frequency Distribution from Boone, NC  
(See Figure A-1 for details)



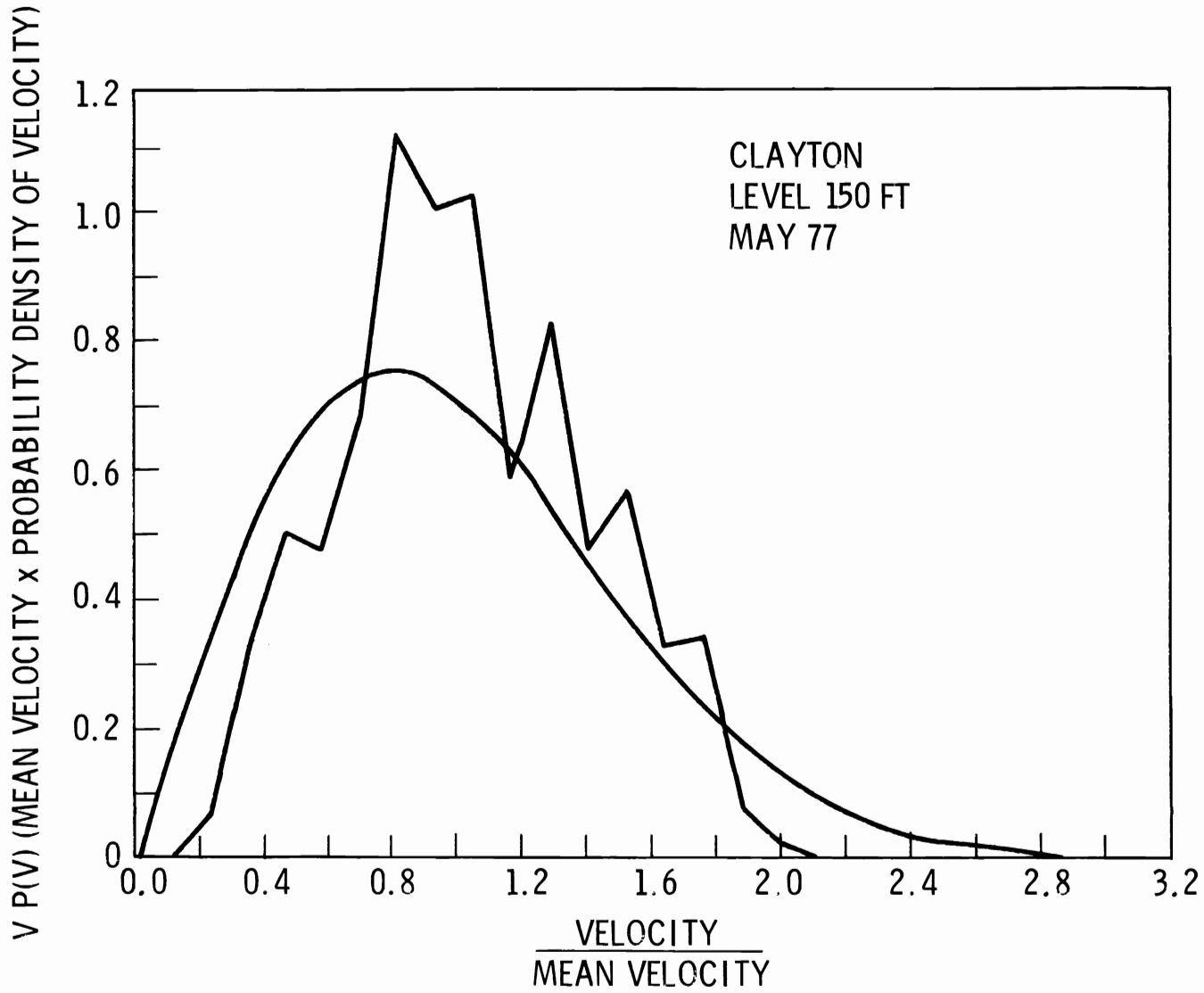


FIGURE A-6. Wind Speed Frequency Distribution from Clayton, NM  
(See Figure A-1 for details)

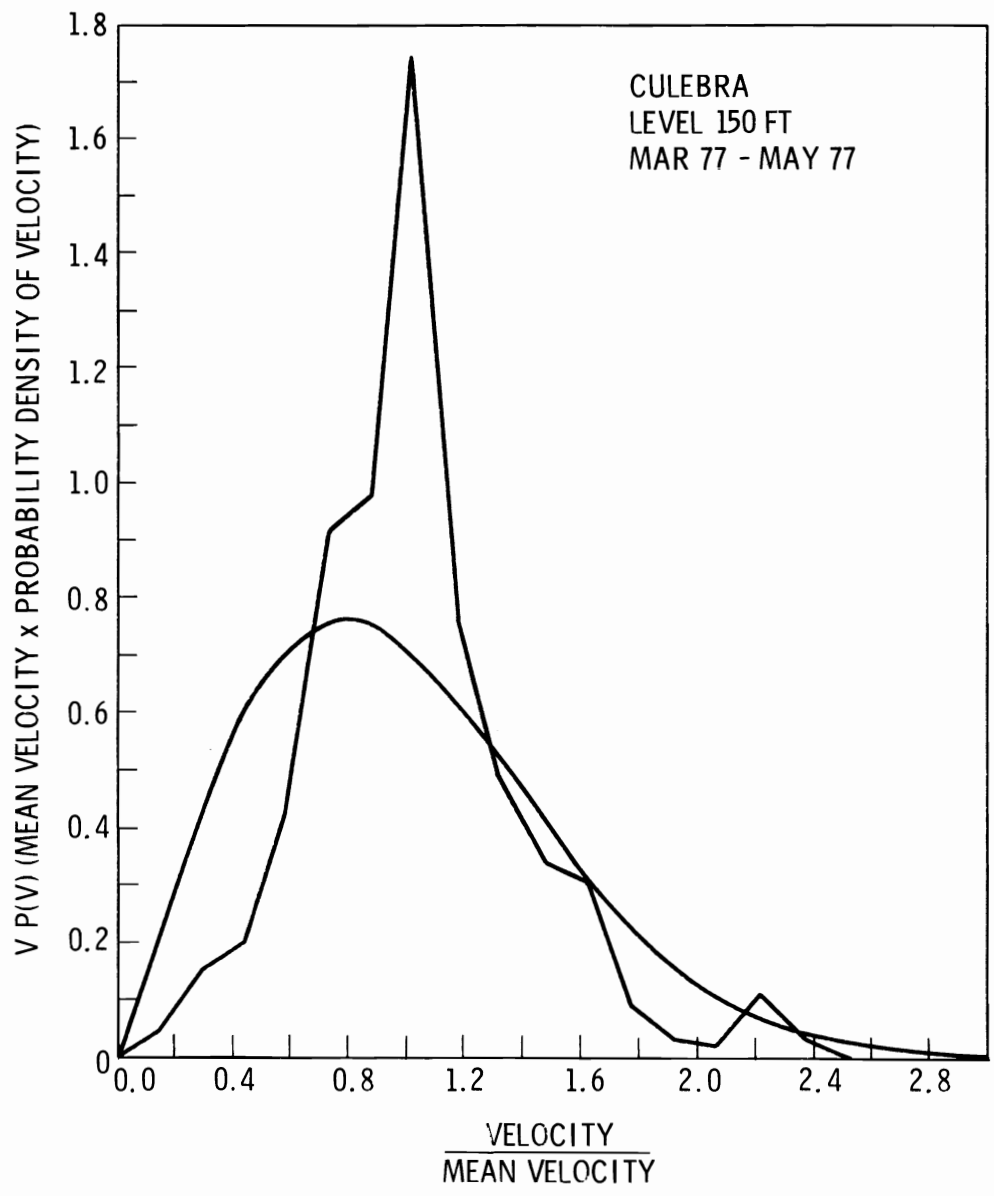


FIGURE A-7. Wind Speed Frequency Distribution from Culebra, PR  
(See Figure A-1 for details)

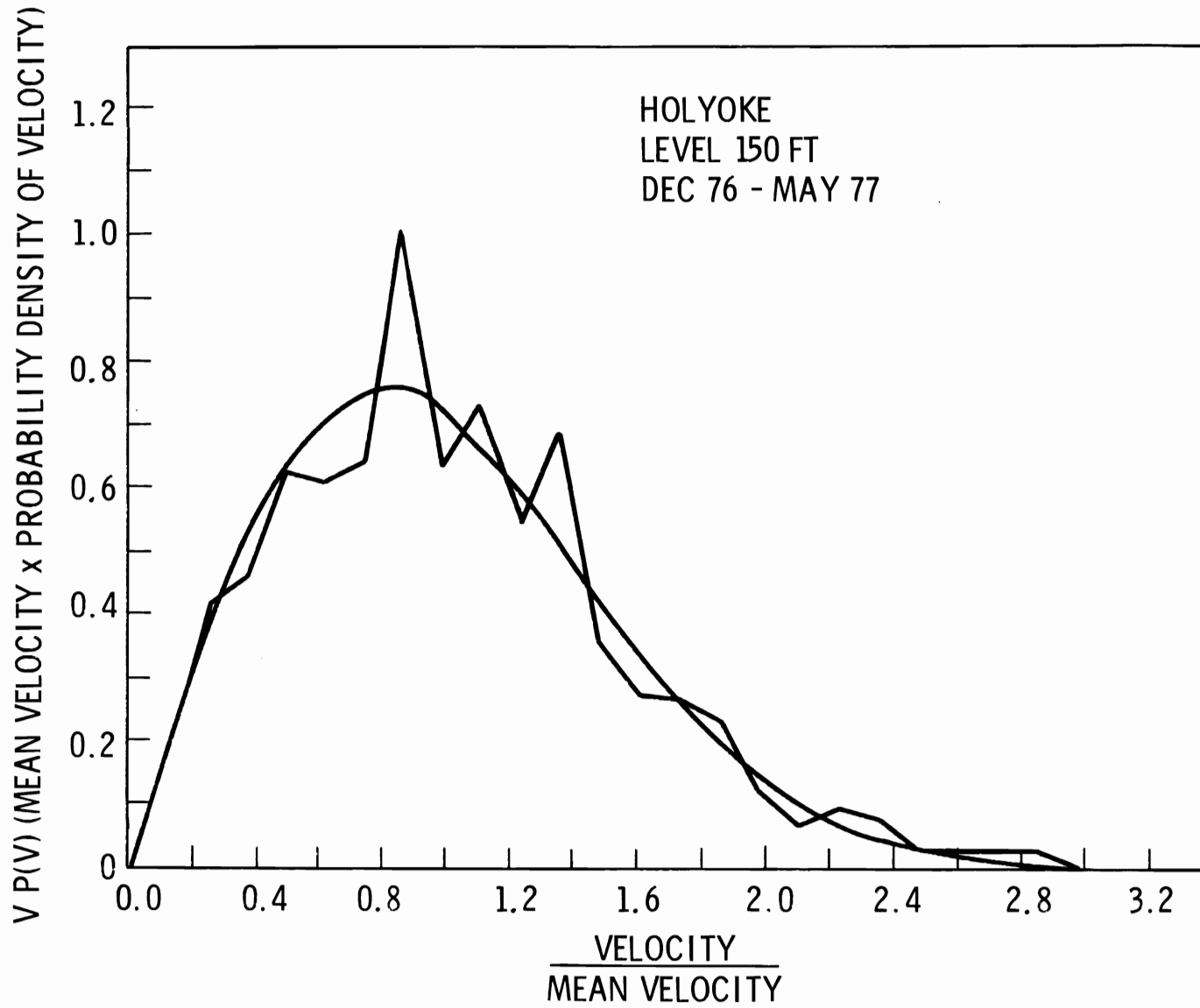


FIGURE A-8. Wind Speed Frequency Distribution from Holyoke, MA  
(See Figure A-1 for details)

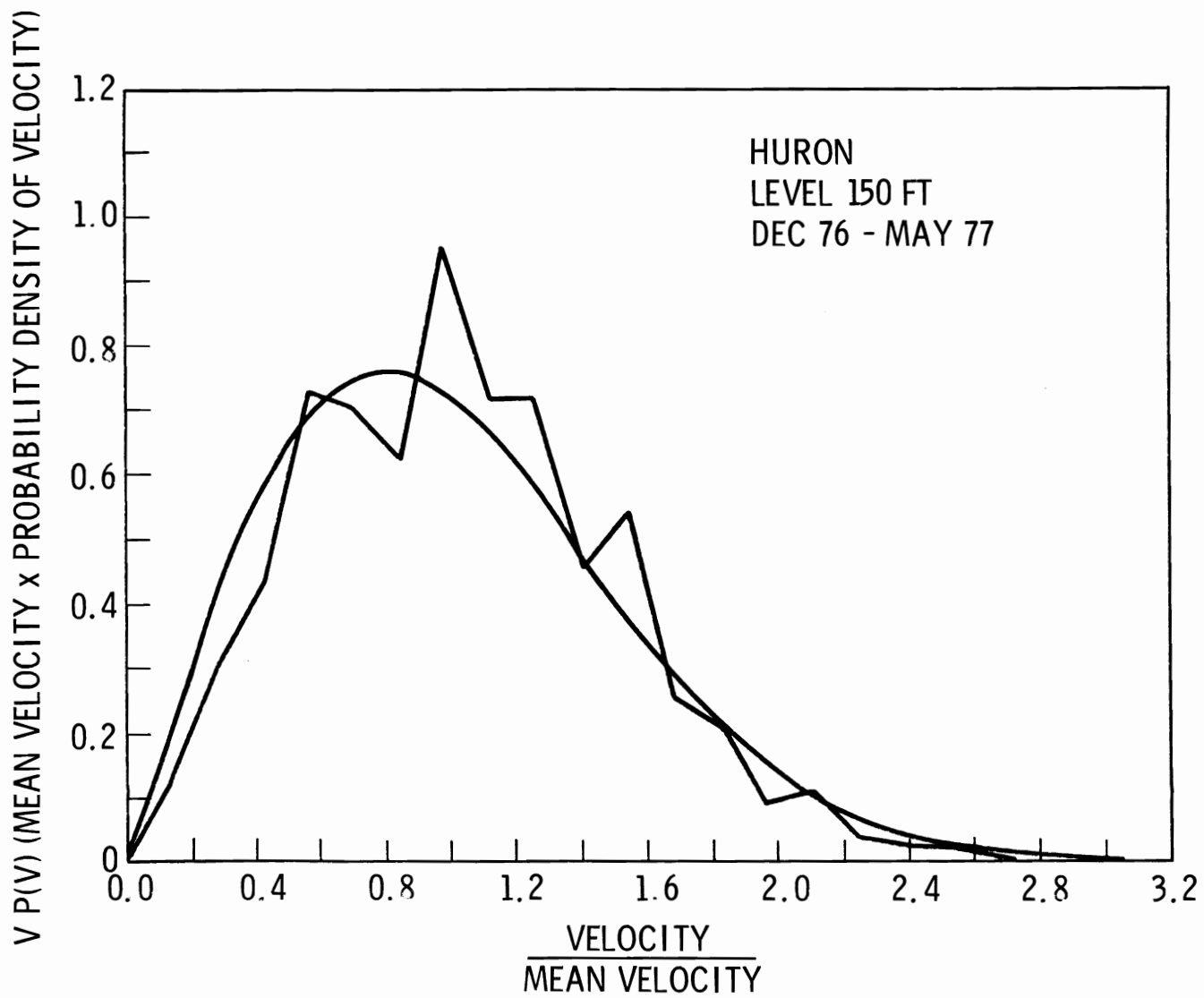


FIGURE A-9. Wind Speed Frequency Distribution from Huron, SD  
(See Figure A-1 for details)

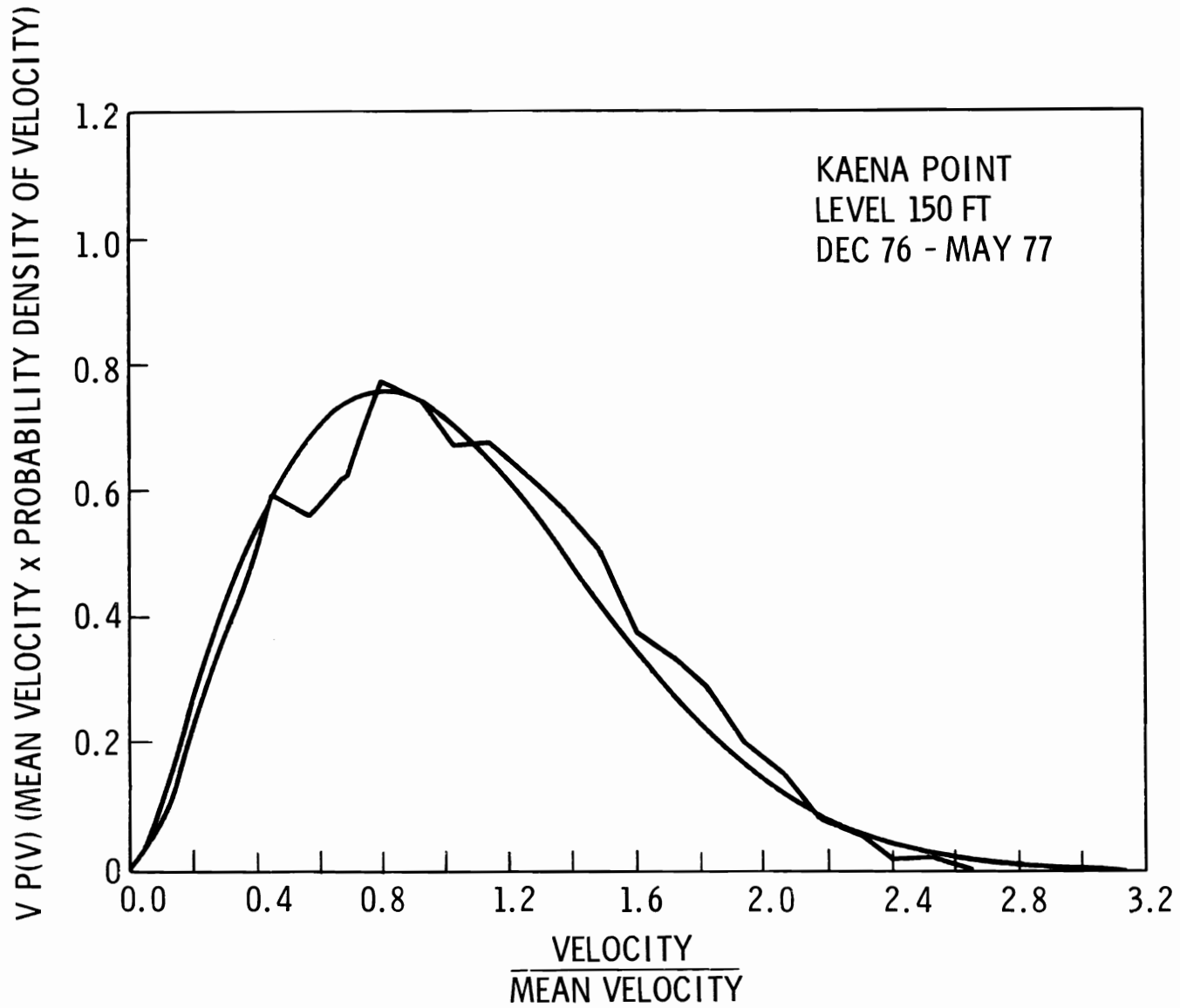


FIGURE A-10. Wind Speed Frequency Distribution from Kaena Pt., HI  
(See Figure A-1 for details)

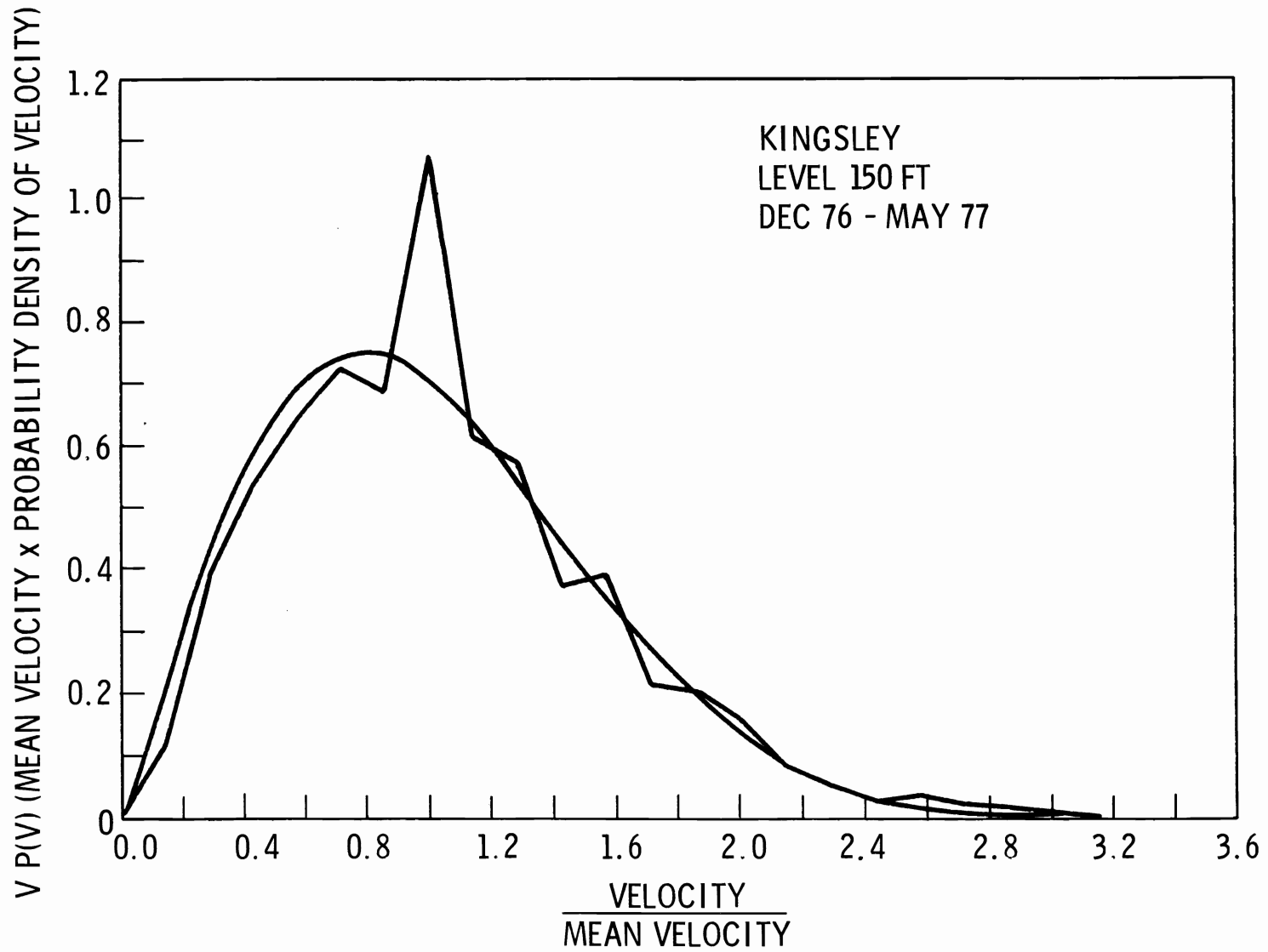


FIGURE A-11. Wind Speed Frequency Distribution from Kingsley, NB  
(See Figure A-1 for details)

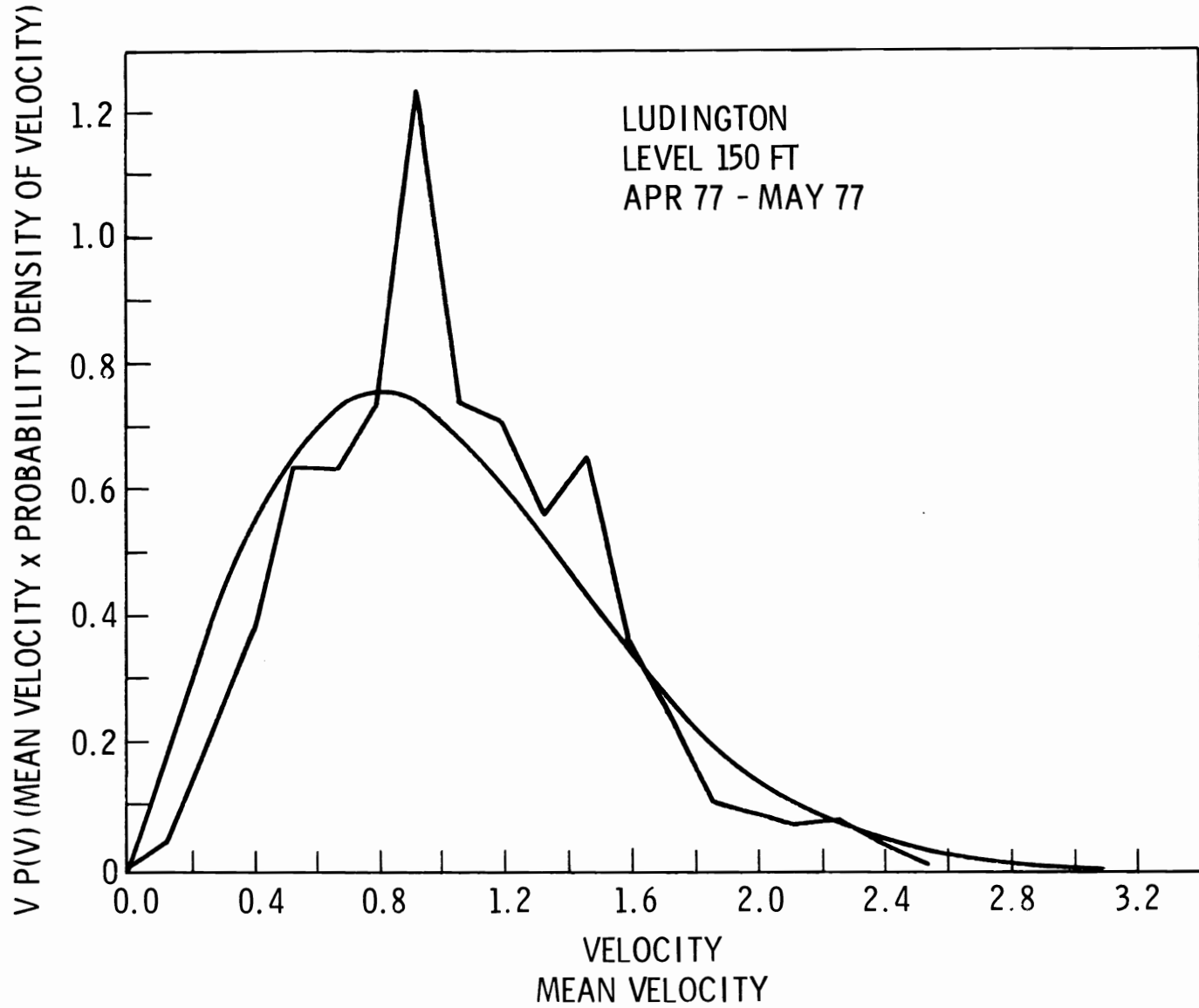


FIGURE A-12. Wind Speed Frequency Distribution from Ludington, MI  
(See Figure A-1 for details)

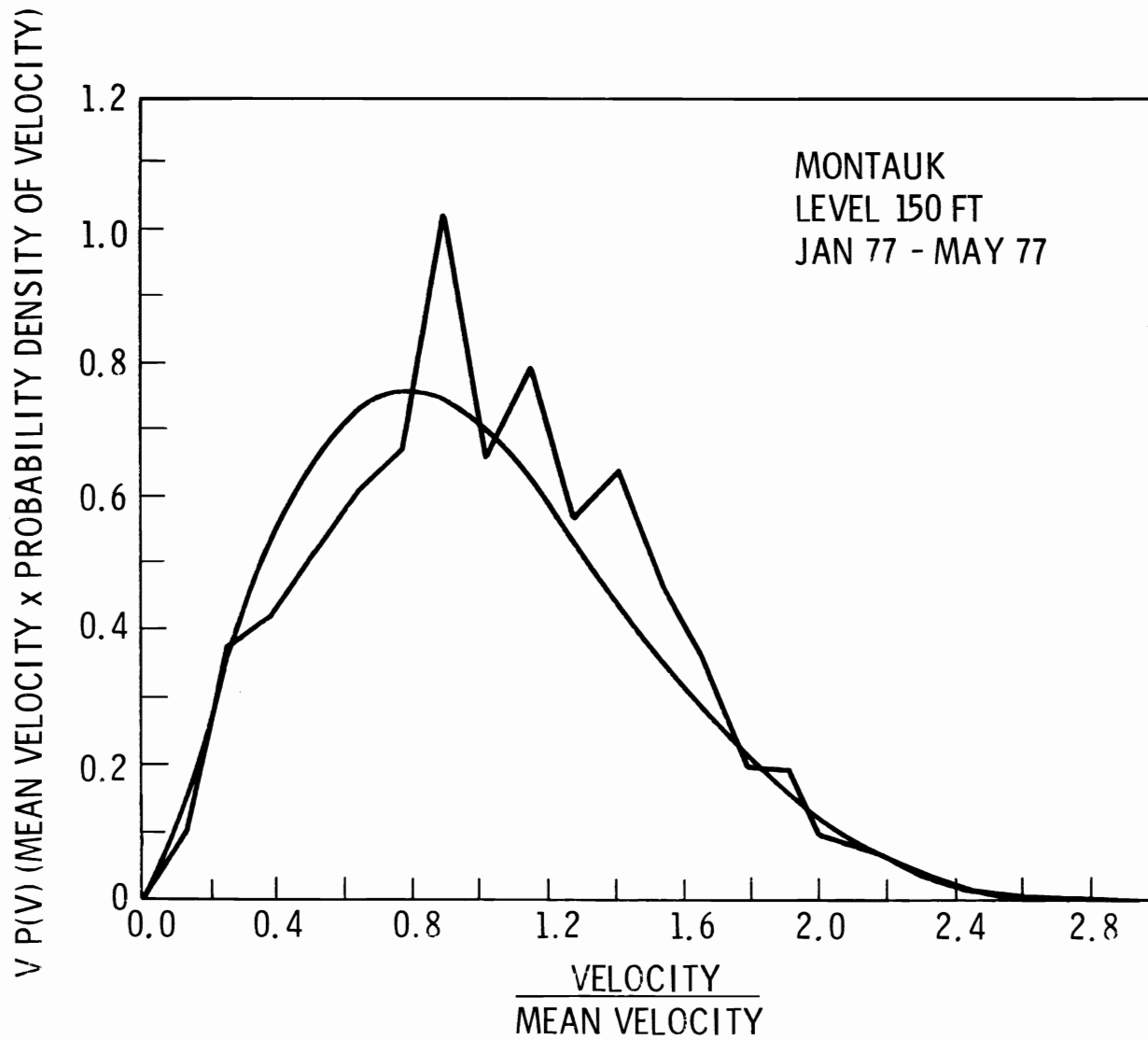


FIGURE A-13. Wind Speed Frequency Distribution from Montauk Pt., NY  
(See Figure A-1 for details)



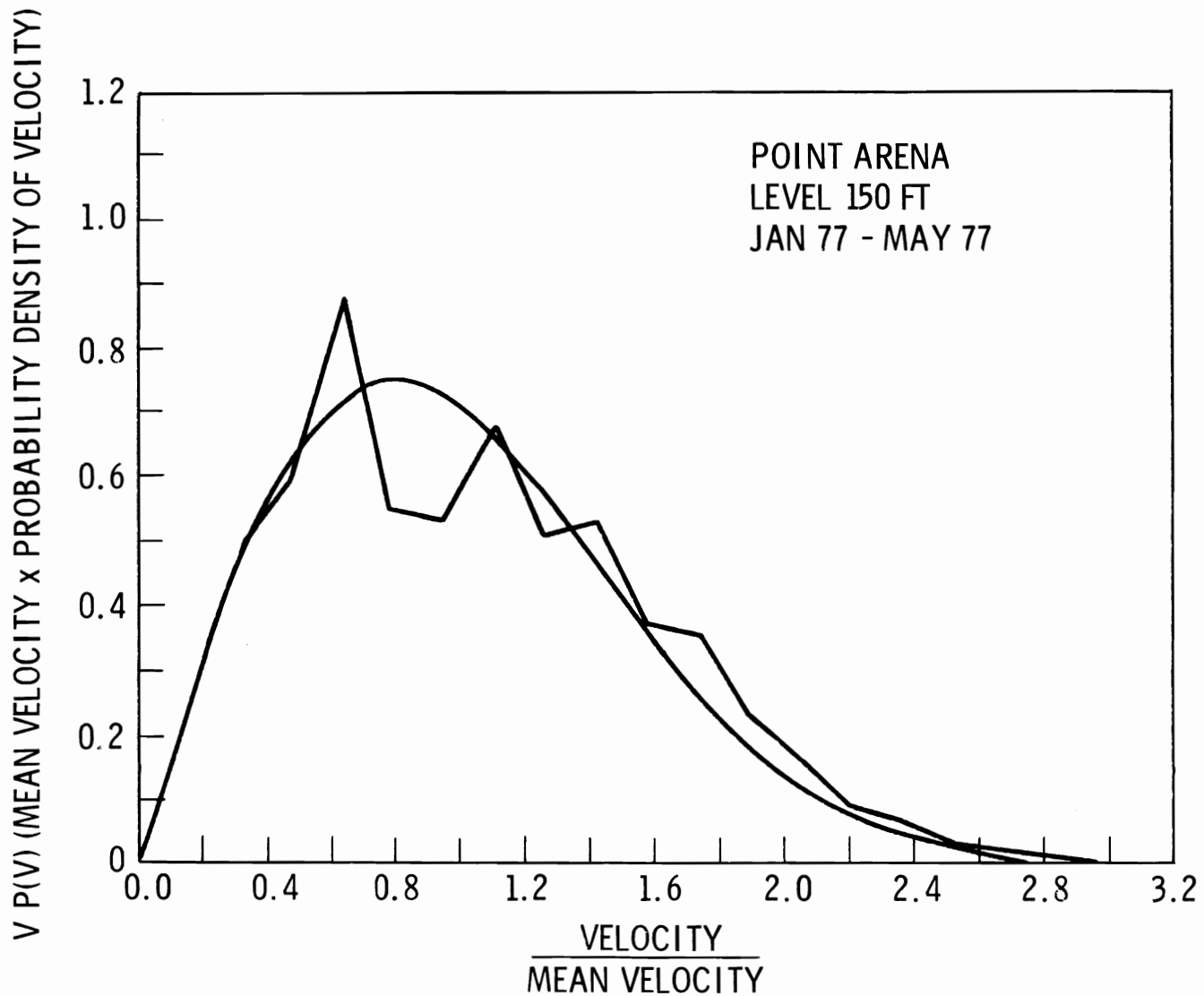


FIGURE A-14. Wind Speed Frequency Distribution from Point Arena, CA  
(See Figure A-1 for details)

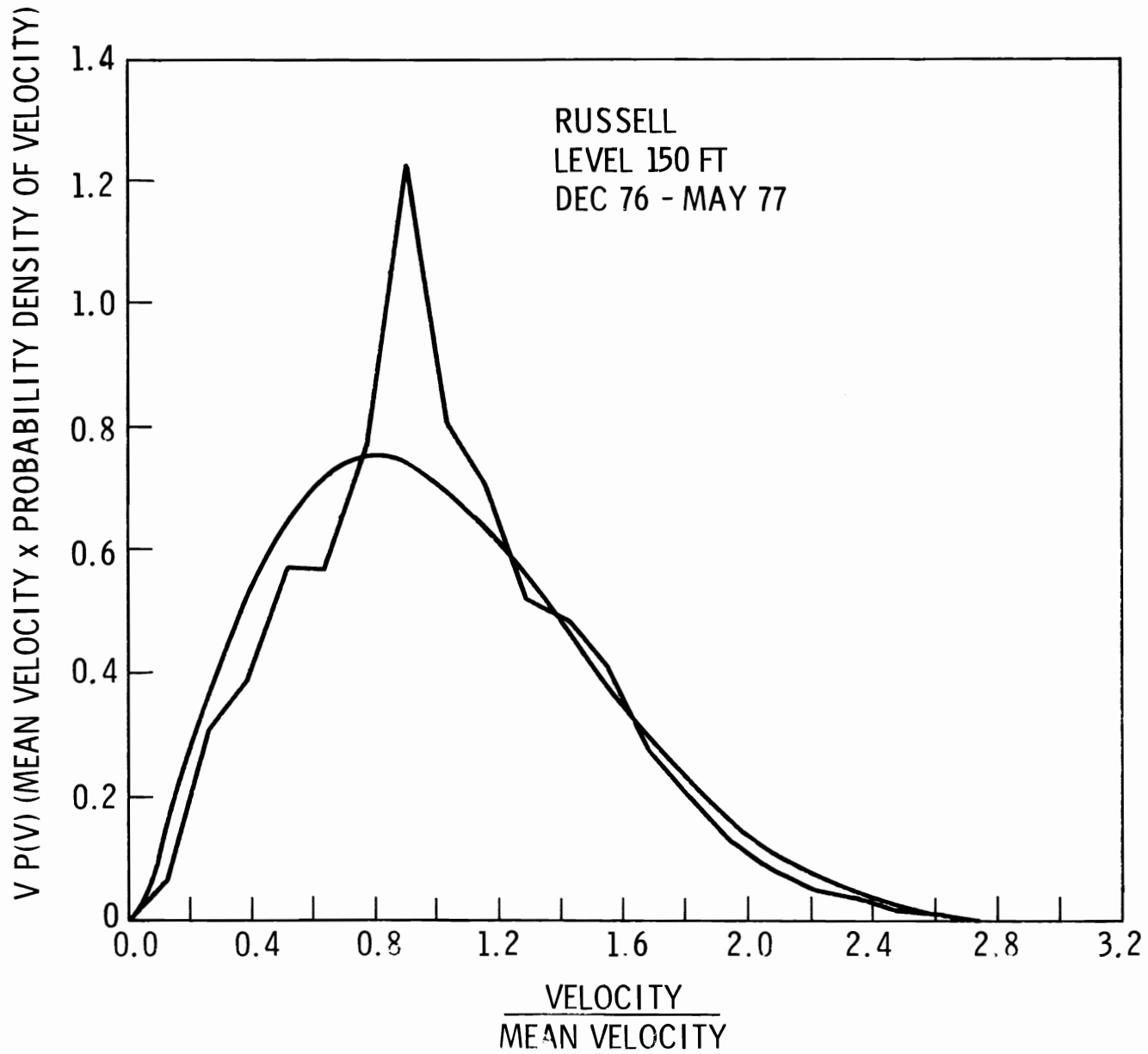


FIGURE A-15. Wind Speed Frequency Distribution from Russell, KS  
(See Figure A-1 for details)

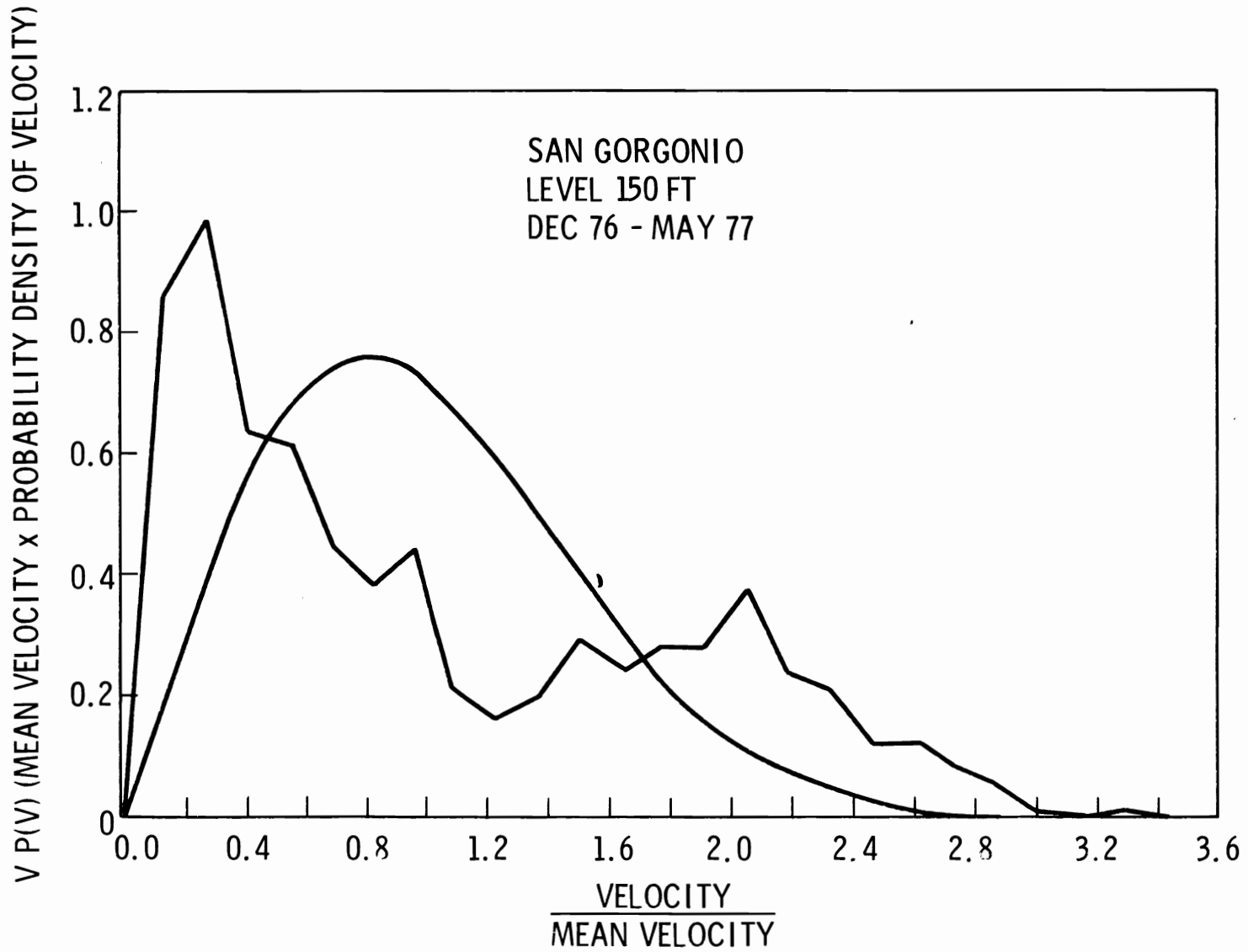


FIGURE A-16. Wind Speed Frequency Distribution from San Gorgonio, CA  
(See Figure A-1 for details)

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