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Effect of gravity on visco-elastic surface waves in solids involving time rate of strain and stress of first order

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Abstract. The aim of the present paper is to study the effect of gravity on visco-elastic surface waves in solids. The wave velocity equations are deduced from Biot's theory of initial stress on the assumption that gravity creates a type of initial stress – hydrostatic in nature. Resulting equations are used to investigate surface waves of the Rayleigh, Love and Stoneley types. Results are in good agreement with corresponding classical results when gravity and viscosity are neglected.

Keywords. Visco-elastic; first order; surface waves; gravity.

1. Introduction

Usefulness of surface waves is well recognised in the study of earthquakes, seismology, geophysics and geodynamics. The theory of surface waves has been widely developed by Rayleigh (1885), Voigt (1887), Stoneley (1924), Ewing *et al* (1957, pp. 257–259, 311), Hunter (1960, pp. 1–57), Bland (1960, pp. 30–75), Flugge (1967, pp. 3–21) and Jeffreys (1959, pp. 35–38).

Effects of gravity, curvature and viscosity are not considered in detail. Considering the effect of gravity in the problem of propagation of waves in solids, in particular, on an elastic globe has been discussed first by Bromwich (1898). Subsequently, the investigations of the effects of gravity was considered by Love (1911, pp. 144–178) in his text, *Some problems of geodynamics*, wherein he exhibited that the velocity of Rayleigh waves is increased to a significant extent by the gravitational field when wavelengths are large. Biot (1965, pp. 44–45) investigated the effect of gravity on Rayleigh waves by assuming gravity to create a type of initial stress of hydrostatic nature and the medium to be incompressible. Adapting the same theory of initial stress and using the dynamical equations of motion for a homogeneous isotropic elastic solid medium under the initial stress, some problems of waves and vibrations have been studied by different investigators. De & Sengupta (1973) studied the effects of gravity on elastic waves and vibrations and also on the propagation of waves in an elastic layer (De & Sengupta 1974). Das & Sengupta (1990a) considered problems of surface waves in general visco-elastic media of higher order and also surface waves in thermo-visco-elastic media considering time rate of stress and strain of higher order (Das & Sengupta 1990b). Roy & Sengupta (1983a) investigated the rotatory vibration of a general visco-elastic solid sphere and also the radial vibration of a general visco-elastic solid sphere (Roy & Sengupta 1983b). The details are found in the work of Eringen & Suhubi (1975, pp. 518, 524–530, 622, 830).

In this paper an attempt has been made to formulate the equations of motion in visco-elastic media under the influence of gravity. Starting from the dynamical equations of motion for a homogeneous isotropic elastic solid medium under initial stress, as presented by Biot (1965, pp. 273–281), the authors have derived the wave velocity equations satisfied by displacement potentials ϕ and ψ to account for gravity and viscosity. This theory is then applied to the particular examples of Rayleigh waves, Love waves and Stoneley waves. Final wave velocity equations in each case are in good agreement with the corresponding classical results when the gravity field and the viscous field are neglected.

2. Formulation of the problem

Consider two homogeneous semi-infinite visco-elastic solid media M_1 and M_2 welded in contact in the influence of gravity (figure 1). Suppose that the media are separated by a plane horizontal boundary, extending to infinitely great distance from the origin, M_2 being above M_1. As a reference co-ordinate system we consider a set of orthogonal cartesian axes $0x_1x_2x_3$, the origin 0 being any point on the boundary and $0x_3$ pointing normally to M_1. Consider the possibility of a type of wave travelling in the positive x_1-direction in such a manner that the disturbance is largely confined to the neighbourhood of the boundary and at any instant all particles in any line parallel to the x_3-axis have equal displacements. These two assumptions conclude that the

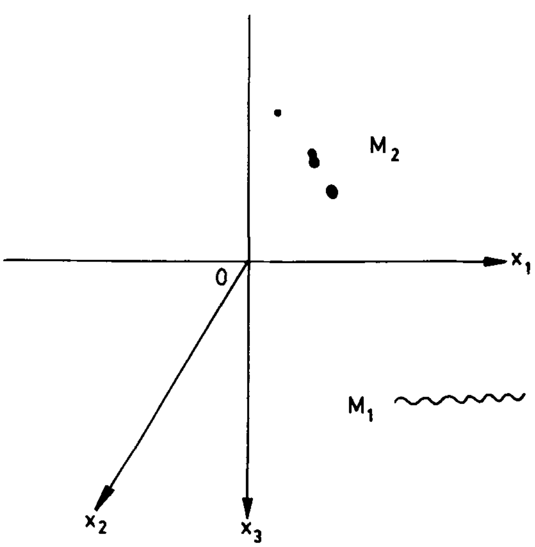

Figure 1. Interface geometry.

wave is a surface wave and all partial derivatives with respect to x_2 are zero.

Let us assume that u_1, u_2 and u_3 are the components of displacement at any point (x_1, x_2, x_3) at time t. We may separate out the purely dilatational and purely rotational disturbances associated with the components u_1 and u_3 by introducing two displacement potentials ϕ and ψ in the form

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, \quad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}, \tag{1}$$

where ϕ and ψ are the functions of the co-ordinates x_1, x_3, and t and

$$\nabla^2\phi = \Delta, \quad \nabla^2\psi = \frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3},$$

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}, \quad \Delta = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}.$$

The component u_2 is associated with purely distortional movement. We mark that ϕ, ψ and u_2 are respectively associated with *P*-waves, *SV*-waves and *SH*-waves (Bullen 1965, pp. 252–265). The symbols have their usual meanings.

The dynamical equations of motion for the three-dimensional problem under the state of initial stress and gravity (Biot 1965, pp. 44–45, 273–281) are

$$\frac{\partial\sigma_{11}}{\partial x_1} + \frac{\partial\sigma_{12}}{\partial x_2} + \frac{\partial\sigma_{13}}{\partial x_3} + \rho g\frac{\partial u_3}{\partial x_1} = \rho\frac{\partial^2 u_1}{\partial t^2},$$

$$\frac{\partial\sigma_{21}}{\partial x_1} + \frac{\partial\sigma_{22}}{\partial x_2} + \frac{\partial\sigma_{23}}{\partial x_3} + \rho g\frac{\partial u_3}{\partial x_2} = \rho\frac{\partial^2 u_2}{\partial t^2},$$

$$\frac{\partial\sigma_{31}}{\partial x_1} + \frac{\partial\sigma_{32}}{\partial x_2} + \frac{\partial\sigma_{33}}{\partial x_3} - \rho g\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x^2}\right) = \rho\frac{\partial^2 u_3}{\partial t^2}, \tag{2}$$

where ρ is the density of the homogeneous medium, g is the acceleration due to gravity and σ_{ij} are the stress components.

The stress–strain relation according to Voigt (1887) in an isotropic visco-elastic solid medium of first order is

$$\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\sigma_{ij} = \left(\lambda_1 + \lambda_2\frac{\partial}{\partial t}\right)\Delta\delta_{ij} + 2\left(\mu_1 + \mu_2\frac{\partial}{\partial t}\right)e_{ij}, \tag{3}$$

where η_1, λ_1, μ_1 are elastic constants with η_2, λ_2 and μ_2 accounting for viscosity, e_{ij} is the strain tensor and δ_{ij} is the Kronecker symbol.

Substituting (3) in (2), we obtain displacement equations of motion in a first-order visco-elastic medium under the influence of gravity as

$$\left[(\lambda_1 + \mu_1) + (\lambda_2 + \mu_2)\frac{\partial}{\partial t}\right]\frac{\partial\Delta}{\partial x_1} + \left(\mu_1 + \mu_2\frac{\partial}{\partial t}\right)\nabla^2 u_1 + \rho g\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\frac{\partial u_3}{\partial x_1} = \rho\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\frac{\partial^2 u_1}{\partial t^2},$$

$$\left(\mu_1 + \mu_2\frac{\partial}{\partial t}\right)\nabla^2 u_2 = \rho\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\frac{\partial^2 u_2}{\partial t^2},$$

$$\left[(\lambda_1+\mu_1)+(\lambda_2+\mu_2)\frac{\partial}{\partial t}\right]\frac{\partial\Delta}{\partial x_3}+\left(\mu_1+\mu_2\frac{\partial}{\partial t}\right)\nabla^2 u_3$$
$$-\rho g\left(\eta_1+\eta_2\frac{\partial}{\partial t}\right)\frac{\partial u_1}{\partial x_1}=\rho\left(\eta_1+\eta_2\frac{\partial}{\partial t}\right)\frac{\partial^2 u_3}{\partial t^2}, \quad (4)$$

where ρ, η_1, η_2, λ_1, λ_2, μ_1, μ_2 denote the properties of the medium M_1 and those with dashes the properties of the medium M_2.

Introducing (1) into (4), we get the following wave equations in M_1 satisfied by ϕ, ψ and u_2

$$\frac{\partial^2\phi}{\partial t^2}=\left(V_{1T}^2+V_{2T}^2\frac{\partial}{\partial t}\right)\nabla^2\phi/L+g\frac{\partial\psi}{\partial x_1},$$
$$\frac{\partial^2\psi}{\partial t^2}=\left(V_{1S}^2+V_{2S}^2\frac{\partial}{\partial t}\right)\nabla^2\psi/L-g\frac{\partial\phi}{\partial x_1},$$
$$\frac{\partial^2 u_2}{\partial t^2}=\left(V_{1S}^2+V_{2S}^2\frac{\partial}{\partial t}\right)\nabla^2 u_2/L, \quad (5)$$

where

$$V_{1T}^2=(\lambda_1+2\mu_1)/\rho, \quad V_{2T}^2=(\lambda_2+2\mu_2)/\rho,$$
$$V_{1S}^2=\mu_1/\rho, \quad V_{2S}^2=\mu_2/\rho, \quad L=\eta_1+\eta_2\frac{\partial}{\partial t},$$

and similar relations in M_2 with ρ, η_1, η_2, λ_1, λ_2, μ_1, μ_2 replaced by ρ^1, η_1^1, η_2^1, λ_1^1, λ_2^1, μ_1^1, μ_2^1 and so on.

2.1 *Boundary conditions*

The boundary conditions are:

(i) The component of displacement at the interface between the media M_1 and M_2 must be continuous at all points and times.

(ii) The stress components σ_{31}, σ_{32}, σ_{33} must be continuous across the interface, i.e.

$$L\sigma_{31}=\left(\mu_1+\mu_2\frac{\partial}{\partial t}\right)\left(2\frac{\partial^2\phi}{\partial x_1\partial x_3}+\frac{\partial^2\psi}{\partial x_1^2}-\frac{\partial^2\psi}{\partial x_3^2}\right),$$
$$L\sigma_{32}=\left(\mu_1+\mu_2\frac{\partial}{\partial t}\right)\frac{\partial u_2}{\partial x_3},$$
$$L\sigma_{33}=\left(\lambda_1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2\phi+2\left(\mu_1+\mu_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\phi}{\partial x_3^2}+\frac{\partial^2\psi}{\partial x_1\partial x_3}\right), \quad (6)$$

and similar expressions for M_2 across the interface between M_1 and M_2 must be continuous at all points and times.

3. Solutions of the problem

Let us take the solutions of (5) in the exponential form

$$(\phi, \psi, u_2) = [\hat{\phi}(x_3), \hat{\psi}(x_3), \hat{u}_2(x_3) \exp[i(\eta x_1 - \omega t)], \tag{7}$$

for the medium M_1 and similar solutions for M_2, the functions $\hat{\phi}, \hat{\psi}, \hat{u}_2$ being replaced by $\hat{\phi}', \hat{\psi}', \hat{u}_2^1$.

Putting (7) in (5), we get a set of differential equations for the medium M_1 as follows

$$\left[\frac{d^2}{dx_3^2} - (\eta^2 - \omega^2 \eta_K^*/V_{KT}^2)\right]\hat{\phi} = -ig\eta\hat{\psi} \times \eta_K^*/V_{KT}^2,$$

$$\left[\frac{d^2}{dx_3^2} - (\eta^2 - \omega^2 \eta_K^*/V_{KS}^2)\right]\hat{\psi} = ig\eta\hat{\phi} \times \eta_K^*/V_{KS}^2,$$

$$\left[\frac{d^2}{dx_3^2} - (\eta^2 - \omega^2 \eta_K^*/V_{KS}^2)\right]\hat{u}_2 = 0, \tag{8}$$

where

$$\eta_K^* = (\eta_1 - i\omega\eta_2), \quad V_{KT}^2 = V_{1T}^2 - i\omega V_{2T}^2, \quad V_{KS}^2 = V_{1S}^2 - i\omega V_{2S}^2.$$

Similar relations for M_2 can be obtained replacing $\hat{\phi}$, $\hat{\psi}$, $\hat{u}_2$, η_1, η_2, V_{1T}, V_{2T}, V_{1S}, V_{2S}, η_K^*, V_{KT}, V_{KS}, λ_1, μ_1, λ_2, μ_2, ρ by dashes.

Clearly, these equations (8) have exponential solutions and in order that ϕ, ψ and u_2 describe surface waves, they must become vanishingly small as $x_3 \to \infty$.

Hence for the medium M_1, we have

$$\phi = [A_1 \exp[-x_3](\eta^2 - \zeta_1^2)^{\frac{1}{2}} + A_2 \exp[-x_3](\eta^2 - \zeta_2^2)^{\frac{1}{2}}]\exp[i(\eta x_1 - \omega t)], \tag{9a}$$

$$\psi = [B_1 \exp[-x_3](\eta^2 - \zeta_1^2)^{\frac{1}{2}} + B_2 \exp[-x_3](\eta^2 - \zeta_2^2)^{\frac{1}{2}}]\exp[i(\eta x_1 - \omega t)], \tag{9b}$$

$$u_2 = [C \exp[-x_3](\eta^2 - \omega^2 \eta_K^*/V_{KS}^2)^{\frac{1}{2}}]\exp[i(\eta x_1 - \omega t)]. \tag{9c}$$

We have similarly those for the medium M_2 as

$$\phi' = [A_1' \exp[x_3](\eta^2 - \zeta_1'^2)^{\frac{1}{2}} + A_2' \exp[x_3](\eta^2 - \zeta_2'^2)^{\frac{1}{2}}]\exp[i(nx_1 - \omega t)], \tag{9d}$$

$$\psi' = [B_1' \exp[x_3](\eta^2 - \zeta_1'^2)^{\frac{1}{2}} + B_2' \exp[x_3](\eta^2 - \zeta_2'^2)^{\frac{1}{2}}]\exp[i(\eta x_1 - \omega t)], \tag{9e}$$

$$u_2' = [C' \exp[x_3](\eta^2 - \omega^2 \eta_K^{*\prime}/V_{KS}'^2)^{\frac{1}{2}}]\exp[i(\eta x_1 - \omega t)], \tag{9f}$$

where ζ_j^2 and $\zeta_j'^2$ $(j = 1, 2)$ are respectively the roots of the equations

$$[\omega^2 - \zeta^2 \times V_{KS}^2/\eta_K^*][\omega^2 - \zeta^2 \times V_{KT}^2/\eta_K^*] - g^2\eta^2 = 0, \tag{10a}$$

$$[\omega^2 - \zeta'^2 \times V_{KS}'^2/\eta_K^{*\prime}][\omega^2 - \zeta'^2 \times V_{KT}'^2/\eta_K^{*\prime}] - g^2\eta^2 = 0, \tag{10b}$$

and

$$B_1 = \alpha_1 A_1, \quad B_2 = \alpha_2 A_2, \quad B'_1 = \alpha'_1 A'_1, \quad B'_2 = \alpha'_2 A'_2,$$

with

$$\alpha_j = ig\eta/(\omega^2 - \zeta_j^2 \times V_{KS}^2/\eta_K^*), \quad \alpha'_j = ig\eta/(\omega^2 - \zeta_j'^2 \times V_{KS}'^2/\eta_K^{*\prime}), \quad (j = 1, 2).$$

In evaluating

$$(\eta^2 - \zeta^2)^{\frac{1}{2}}, \quad (\eta^2 - \zeta'^2)^{\frac{1}{2}}, \quad (\eta^2 - \omega^2 \times \eta_K^*/V_{KS}^2)^{\frac{1}{2}}, \quad (\eta^2 - \omega^2 \times \eta_K^{*\prime}/V_{KS}'^2)^{\frac{1}{2}},$$

the root with positive real part will be taken.

Applying boundary conditions (i) and (ii), we obtain

$$[1 - i\alpha_1 Q_1]A_1 + [1 - i\alpha_2 Q_2]A_2 = [1 + i\alpha'_1 Q'_1]A'_1 + [1 + i\alpha'_2 Q'_2]A'_2, \tag{11a}$$

$$C = C', \tag{11b}$$

$$[\alpha_1 + iQ_1]A_1 + [\alpha_2 + iQ_2]A_2 = [\alpha'_1 - iQ'_1]A'_1 + [\alpha'_2 - iQ'_2]A'_2, \tag{11c}$$

$$\begin{aligned}&\rho \times (V_{KS}^2/\eta_K^*)[\{2iQ_1 + (1 + Q_1^2)\alpha_1\}A_1 + \{2iQ_2 + (1 + Q_2^2)\alpha_2\}A_2] \\ &\quad = \rho' \times (V_{KS}'^2/\eta_K^{*\prime})[\{-2iQ'_1 + (1 + Q_1'^2)\alpha'_1\}A'_1 + \{-2iQ'_2 + (1 + Q_2'^2)\alpha'_2\}A'_2],\end{aligned} \tag{11d}$$

$$\begin{aligned}&-\rho \times (V_{KS}^2/\eta_K^*)(\eta^2 - \omega^2\eta_K^*/V_{KS}^2)^{\frac{1}{2}} \times C \\ &\quad = \rho' \times (V_{KS}'^2/\eta_K^{*\prime})(\eta^2 - \omega^2\eta_K^{*\prime}/V_{KS}'^2)^{\frac{1}{2}} \times C',\end{aligned} \tag{11e}$$

$$\begin{aligned}&(\rho/\eta_K^*)[\{V_{KT}^2(Q_1^2 - 1) + 2V_{KS}^2(1 - i\alpha_1 Q_1)\}A_1 + \{V_{KT}^2(Q_2^2 - 1) + \\ &\qquad + 2V_{KS}^2(1 - i\alpha_2 Q_2)\}A_2] \\ &= (\rho'/\eta_K^{*\prime})[\{V_{KT}'^2(Q_1'^2 - 1) + 2V_{KS}'^2(1 + i\alpha'_1 Q'_1)\}A'_1 + \\ &\qquad + \{V_{KT}'^2(Q_2'^2 - 1) + 2V_{KS}'^2(1 + i\alpha'_2 Q'_2)\}A'_2].\end{aligned} \tag{11f}$$

From (11b) and (11e) we find that only possible values of C and C' are zeros. Hence there is no propagation of displacement u_2. Thus no *SH*-waves occur in this case.

Eliminating the constants A_1, A_2, A'_1, A'_2 from (11a)–(11d) we obtain the wave velocity equation finally in determinant form as

$$|M_{ij}| = 0, \quad (i, j = 1, 2, 3, 4), \tag{12}$$

where

$$M_{1K} = [1 - i\alpha_K Q_K], \quad M_{1K+2} = -[1 + i\alpha'_K Q'_K]; \quad [\text{when } K = 1, 2]$$

$$M_{2K} = [\alpha_K + iQ_K], \quad M_{2K+2} = -[\alpha'_K - iQ'_K];$$

$$M_{3K} = \rho \times (V_{KS}^2/\eta_K^*)[2iQ_K + (1 + Q_K^2)\alpha_K],$$

$$M_{3K+2} = -\rho' \times (V_{KS}'^2/\eta_K^{*\prime})[-2iQ'_K + (1 + Q_K'^2)\alpha'_K];$$

$$M_{4K} = (\rho/\eta_K^*)[V_{KT}^2(Q_K^2 - 1) + 2V_{KS}^2(1 - i\alpha_K Q_K)],$$

$$M_{4K+2} = (-\rho'/\eta_K^{*\prime})[V_{KT}'^2(Q_K'^2 - 1) + 2V_{KS}'^2(1 + i\alpha'_K Q'_K)];$$

and

$$Q_K = (1 - \zeta_K^2/\eta^2)^{\frac{1}{2}}, \quad Q'_K = (1 - \zeta_K'^2/\eta^2)^{\frac{1}{2}}, \quad (\text{when } K = 1, 2).$$

From (12) we get the wave velocity of surface waves in the common boundary under consideration in the presence of gravity and viscosity where viscosity is of first order including strain rate and stress rate simultaneously. Although, effects of viscosity and gravity are small, the present analysis should prove to be useful in circumstances where these influences cannot be neglected.

4. Particular cases

4.1 *Rayleigh waves*

To start with a particular case of surface waves, we consider visco-elastic Rayleigh waves under the influence of gravity in which the plane boundary is a free surface such that M_2 is replaced by vacuum. Here we also note that there can be no *SH*-waves.

Hence in view of (11d) and (11f), we obtain

$$[2iQ_1 + (1+Q_1^2)\alpha_1]A_1 + [2iQ_2 + (1+Q_2^2)\alpha_2]A_2 = 0, \tag{13}$$

$$[V_{KT}^2(Q_1^2 - 1) + 2V_{KS}^2(1 - i\alpha_1 Q_1)]A_1 + + [V_{KT}^2(Q_2^2 - 1) + 2V_{KS}^2(1 - i\alpha_2 Q_2)]A_2 = 0. \tag{14}$$

After elimination of A_1 and A_2 from (13) and (14), we have

$$|M'_{ij}| = 0, \quad (i,j = 1,2), \tag{15}$$

where

$$M'_{1m} = [2iQ_m + (1+Q_m^2)\alpha_m]; \qquad \text{(when } m = 1,2\text{)},$$

$$M'_{2m} = [V_{KT}^2(Q_m^2 - 1) + 2V_{KS}^2(1 - i\alpha_m Q_m)].$$

Equation (15) describes Rayleigh waves in a visco-elastic solid medium of the Voigt (1887) type under the influence of gravity including strain rate and stress rate. In the absence of viscous and gravitational fields, this equation tallies with the corresponding classical result (Bullen 1965, pp. 252–265).

4.2 *Love waves*

For Love type surface waves on the surface of the earth, we assume only the non-zero component of displacement u_2 which is a function of x_1, x_3 and t. All other components of displacement are zero. Let us assume that medium M_2 is obtained by two horizontal plane surfaces at a finite distance H apart, the upper plane surface being free while the lower plane surface forms the medium M_1 and extends to an infinitely great distance.

For medium M_2 we must retain the full solution, since displacement no longer diminishes with increasing distance from the boundary surface of the two media and for medium M_1, solutions are the same as it is in the general case.

Therefore, for medium M_1, we write

$$u'_2 = [C'_1 \exp[x_3](\eta^2 - \omega^2\eta_K^{*\prime}/V'^2_{KS})^{\frac{1}{2}} + + C'_2 \exp[-x_3](\eta^2 - \omega^2\eta_K^{*\prime}/V'^2_{KS})^{\frac{1}{2}}]\exp[i(\eta x_1 - \omega t)], \tag{16}$$

where the restriction that the real part of $(\eta^2 - \omega^2\eta_K^{*\prime}/V'^2_{KS})^{\frac{1}{2}}$ be positive is not required for M_2.

In the present case boundary conditions are

(i) u_2 and σ_{32} are continuous at $x_3 = 0$
(ii) $\sigma'_{32} = 0$, at $x_3 = -H$.

Employing boundary conditions (i) and (ii) and using (9c) and (16), we get the following equations

$$C = C'_1 + C'_2, \tag{17}$$

$$-\rho \times (V_{KS}^2/\eta_K^*)(\eta^2 - \omega^2\eta_K^*/V_{KS}^2)^{\frac{1}{2}} \times C = \rho' \times (V_{KS}'^2/\eta_K^*) \times \times (\eta^2 - \omega^2\eta_K^{*\prime}/V_{KS}'^2)^{\frac{1}{2}}[C'_1 - C'_2], \tag{18}$$

$$C'_1 \exp[-H(\eta^2 - \omega^2\eta_K^*/V_{KS}'^2)^{\frac{1}{2}}] - C'_2 \exp[H(\eta^2 - \omega^2\eta_K^*/V_{KS}'^2)^{\frac{1}{2}}] = 0. \tag{19}$$

Eliminating C, C'_1 and C'_2 from (17), (18) and (19), we get

$$\rho \times (V_{KS}^2/\eta_K^*)(1 - c^2\eta_K^*/V_{KS}^2)^{\frac{1}{2}} + \rho' \times (V_{KS}'^2/\eta_K^{*\prime})((c^2\eta_K^{*\prime}/V_{KS}'^2) - 1)^{\frac{1}{2}} \times \times \tan\{\eta H((c^2\eta_K^{*\prime}/V_{KS}'^2) - 1)^{\frac{1}{2}}\} = 0 \tag{20}$$

(where $c = \omega/\eta$), which is the required wave velocity equation for Love waves under the influence of gravity in a visco-elastic solid medium of first order including strain and stress rates. It is seen from (20) that Love waves do not depend upon the gravity field, although they depend upon the viscous field.

Taking $\eta_0 = 1$ and $\eta_1 = \eta'_1 = \lambda_1 = \lambda'_1 = \mu_1 = \mu'_1 = 0$, (20) reduces to the corresponding result (Bullen 1965, pp. 252–265) in perfectly elastic medium.

4.3 *Stoneley waves*

The generalized forms of Rayleigh waves are Stoneley waves in which we assume that waves are propagated in the vicinity of interface of two semi-infinite media M_1 and M_2. Wave velocity of Stoneley waves in the presence of viscosity and gravity effects is determined by the roots of the equation (12) in the case of a visco-elastic medium of first order including strain and stress rates simultaneously. This equation of course reduces once more to the classical result in the absence of these effects.

5. Discussion and conclusions

It is clear from the above investigation that visco-elastic surface waves are affected by the time rate of strain and stress parameters. These parameters influence the wave velocity to an extent depending on the corresponding constants characterizing the visco-elasticity of the material. Further, the effect of gravity is significant when the wavelength is large, while the effect is small when the wavelength is small. However, there is always dispersion of waves due to gravity.

It is noted that Love waves are not affected by the gravity field in any way as is evident from the equations of motion. As regards Rayleigh waves in visco-elastic solid medium of the Voigt type under the influence of gravity, we find that the wave velocity equation proves that there is dispersion of waves in presence of gravity and viscosity.

The results are in agreement with corresponding classical results when gravity and viscosity are neglected. This wave velocity equation is useful for numerical work.

The Stoneley wave velocity equation is very similar to the corresponding problem in the classical theory of elasticity. Here also there is dispersion of waves due to the presence of the gravity field and visco-elastic nature of the solid. This generalized type of surface wave in visco-elastic infinite solids accounting for gravity reduces to the classical Stoneley wave when gravity is absent.

References

Biot M A 1965 *Mechanics of incremental deformation* (New York: Wiley)

Bland D R 1960 *The theory of linear visco-elasticity* (London: Pergamon) (Monograph on the subject contains many cases of stress analysis)

Bromwich T J I A 1898 On the influence of gravity on elastic waves and, in particular on the vibrations of an elastic globe. *Proc. London Math. Soc.* 30: 98–120

Bullen K E 1965 *An introduction to the theory of seismology* (London: Cambridge University Press)

Das T K, Sengupta P R 1990a Surface waves in general visco-elastic media of higher order. *Indian J. Pure Appl. Math.* 21(7): 661–675

Das T K, Sengupta P R 1990b Surface waves in thermo-visco-elastic media considering time rate of stress and strain of higher order. *Gerlands Beitr. Geophys. Leipzig* 99: 337–448

De S N, Sengupta P R 1973 Plane Lamb's problem under the influence of gravity. *Gerlands Beitr. Geophys. Leipzig* 82: 421–426

De S N, Sengupta P R 1974 Influence of gravity on wave propagation in an elastic layer. *J. Acoust. Soc. Am.* 55: 919–921

Eringin A C, Suhubi E S 1975 *Linear theory in elasto-dynamics* (New York, London: Academic Press) vol. 2

Ewing W M, Jardetzky W S, Press F 1957 *Elastic waves in layered media* (London: McGraw-Hill)

Flugge W 1967 *Visco-elasticity* (London: Blaisdell)

Hunter S C 1960 Visco-elastic waves. In *Progress in solid mechanics* (eds) I N Sneddon, R Hill (Amsterdam, New York: North Interscience)

Jeffreys H 1959 *The earth* 4th edn (Cambridge: University Press)

Love A E H 1911 *Some problems of geodynamics* (New York: Dover)

Rayleigh Lord 1885 On waves propagated along the plane surface of an elastic solid. *Proc. London Math. Soc.* 17: 4–11

Roy S K, Sengupta P R 1983a Rotatory vibration of a sphere of general visco-elastic solid. *Gerlands Beitr. Geophys., Leipzig* 92: 70–76

Roy S K, Sengupta P R 1983b Radial vibration of a sphere of general visco-elastic solid. *Gerlands Beitr. Geophys., Leipzig* 92: 435–442

Stoneley R 1924 Elastic waves at the surface of separation of two solids. *Proc. R. Soc., London* 106: 416–428

Voigt W 1987 Theoretische studien über die Elasticitäts verhältnisse der Krystalle. *Abh. Ges. Wiss. Göttingen* 34