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Effect of Impurities on the Level Density of an Electron Gas in a Strong Magnetic Field

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The effect of impurities on the level density of an electron gas in a strong magnetic field is investigated. An approximate closed set of equations is given for the one-particle Green's function of the electron gas assuming the random distribution of the impurities. The self-energy included is the sum of multiple scattering from each impurity where the true Green's function is used for the propagator. The set of equations is solved numerically in a self-consistent way for the level density. Discussions are given for the validity of various approximations proposed in the past for this system.

§ 1. Introduction

The effect of impurities on an electron gas in an external magnetic field have been investigated by a number of authors.[1)~12)] It has been almost two decades since Dingle[1)] showed that the amplitude of the oscillatory part of the magnetic susceptibility is reduced when the assumption is made that the impurities cause the Lorentz-type broadening of the electronic state within the plane perpendicular to the magnetic field. Doman[7)] and Bychkov[5)] treated the problem by solving the equations for the one-particle Green's function of the electron gas. Brailsford[8)] extended his own formalism developed for the dilute solid solutions to include an external magnetic field, and obtained the reduction of the amplitude of the oscillatory part of the susceptibility and the change of the period of the oscillation.

The electrical conductivity of the ideal electron gas in the strong magnetic field shows a logarithmic divergence,[13)] which is connected with the divergence of the level density. It has been pointed out that this divergence will be removed under various assumptions, e.g., if one assumes finite level density at the Landau levels.[6)] In order to estimate the conductivity under the presence of impurities, Skobov,[3)] Bychkov,[4)] Kubo, Miyake and Hashitsume[6)] and Abrikosov[10)]

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investigated the one-particle Green's function of the electron gas in the strong magnetic field.

On the other hand, Hasegawa and Nakamura[11] and Saitoh, Fukuyama, Uemura and Shiba[12] have applied the formalism developed by Yonezawa and Matsubara[15] for the impurity conduction problem on the lattice to the investigation of the impurity band under the external magnetic field. Hasegawa and Nakamura solved the equations for the one-particle Green's function in the quantum limit, and Saitoh et al. treated the problem in the limit of weak magnetic field.

As stated above, various authors have investigated the one-particle Green's function of an electron gas in the strong magnetic field and introduced various approximations on it. But the limits of validity of their approximations have not been given clearly. In the present paper, we investigate the Green's function where we assume that the interaction of an electron with an impurity is of short range and that the distribution of the impurities is completely random. The self-energy included is the sum of multiple scattering from each impurity. We first assume the one-particle Green's function of an ideal gas for the propagators occurring in the self-energy diagrams, and show that the approximations proposed in the past[2)~7)] are mostly applicable only to the case when the impurity density is of rather high density. In the next place in § 3, we use the true renormalized Green's function for the propagators involved in the self-energy diagrams. This approximation is equivalent to the approximation adopted by Hasegawa and Nakamura[11] and Saitoh et al.[12] for the investigation of the impurity band problem under an external magnetic field. The improved approximation is considered to be applicable to lower densities as well. In § 4 the level density obtained by numerically solving the set of equations in the improved approximation is shown, and discussions are given on the results. Section 5 is for conclusion.

§ 2. Perturbation expansion

The level density of an electron gas is calculated with the aid of the one-particle Green's function. For an electron gas under an external magnetic field, we use the one-particle Green's function $G(p, p'; \omega)$ defined by

$$G(p, p'; \omega) = \int dt e^{i\omega t} \langle T(a_p(t) a_{p'}^*(0)) \rangle , \qquad (2\cdot1)$$

where a_p and a_p^* are the annihilation and creation operators for an electron in the Landau state p: p represents the set of three numbers n, p_y and p_z specifying the Landau state. $a_p(t)$ and $a_p^*(t)$ denote the operators in the Heisenberg representation and T is the time ordering operator. The spin of an electron is not considered. As no interaction is introduced between electrons of different spins, the following analysis applies to the set of electrons of each spin separately.

When no impurities are introduced, the Green's function $G(p, p'; \omega)$ reduces

to $G^{(0)}(p, \omega)\delta_{p,p'}$ where

$$G^{(0)}(p, \omega) = \frac{1}{\omega - \varepsilon(p)} \tag{2·2}$$

and

$$\varepsilon(p) = \left(n + \frac{1}{2}\right)\frac{eH}{mc} + \frac{p_z^2}{2m}. \tag{2·3}$$

Here H is the intensity of an external magnetic field, m and e are the electronic mass and charge, respectively, and c is the velocity of light ($\hbar$ being set equal to unity). The level density of the ideal system is given by

$$\begin{aligned}\rho^{(0)}(\omega) &= \sum_p \delta(\omega - \varepsilon(p)) \\ &= \frac{1}{\pi}\sum_p \lim_{\substack{\varepsilon \to 0 \\ (\varepsilon > 0)}} \mathrm{Im}\, G^{(0)}(p, \omega - i\varepsilon).\end{aligned} \tag{2·4}$$

For the system including impurities, the level density $\rho(\omega)$ is calculated by

$$\rho(\omega) = \frac{1}{\pi}\sum_p \lim_{\substack{\varepsilon \to 0 \\ (\varepsilon > 0)}} \mathrm{Im}\, G(p, \omega - i\varepsilon), \tag{2·5}$$

where

$$G(p, \omega) = G(p, p; \omega). \tag{2·6}$$

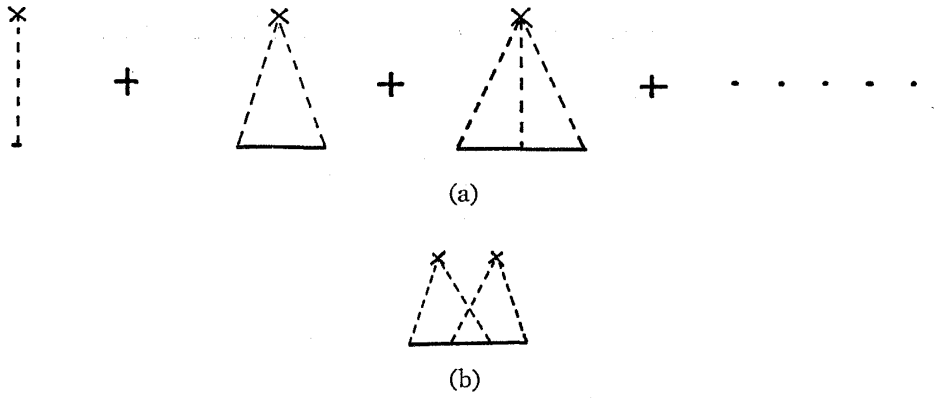
Following Edwards,[14] one can develop the perturbation expansion for the Green's function. For the present case one obtains

$$G(p, p'; \omega) = G^{(0)}(p, \omega)\left[\delta_{p,p'} + \sum_{p''} \Sigma(p, p''; \omega) G(p'', p'; \omega)\right], \tag{2·7}$$

where $\Sigma(p, p''; \omega)$ denotes the self-energy. $G(p, p'; \omega)$ and $\Sigma(p, p'; \omega)$ in (2·7) depend on the distribution of the impurities in the system. We consider those quantities averaged over the random distribution of the impurities in the continuous space and denote them by the same notation. After averaging, the equation for $G(p, p'; \omega)$ takes the same form as (2·7) where p, p' and p'' have the same values of p_y and p_z but may take different values of n. As a starting approximation, we shall consider the sum of the diagrams shown in Fig. 1(a) for the self-energy part.

In a recent work, Abrikosov[10] investigated $\Sigma(p, p'; \omega)$ for the case when the interaction potential of an electron with impurities is described by the delta-function. In the corresponding analysis, Hasegawa and Nakamura[12] assumed that the matrix element of the interaction potential taken by two Landau states is a slowly varying function of the parameters describing the Landau states. They showed that, for such a case, Eq. (2·7) with (2·2) is reduced to

$$G(p, p'; \omega) = G(p, \omega)\delta_{p,p'} \tag{2·8}$$



(a)

(b)

Fig. 1. Diagrams for the self-energy part $\Sigma(p, \omega)$. (a) Considered diagrams, (b) an example of ignored diagrams.

and

$$G(p, \omega) = \frac{1}{\omega - \varepsilon(p) - \Sigma(p, \omega)}, \tag{2·9}$$

where $\Sigma(p, \omega) = \Sigma(p, p; \omega)$. When the self-energy diagrams shown in Fig. 1(a) are considered and the other "crossed diagrams" such as shown in Fig. 1(b) are neglected, $\Sigma(p, \omega)$ is a sum of contributions from each impurity. The self-energy part $\Sigma(p, \omega)$ is then expressed as

$$\begin{aligned} \Sigma(p, \omega) &= \rho[v + vF^{(0)}(\omega)v + \cdots] \\ &= \frac{\rho v}{1 - vF^{(0)}(\omega)}, \\ F^{(0)}(\omega) &= \sum_{p} G(p, \omega), \end{aligned} \tag{2·10}$$

where v is the magnitude of the interaction potential, the volume of the system is taken as unity and ρ is the total number or number density of the impurities in the system. It should be noted that this simple form (2·9) is achieved under the assumption that the interaction potential with an impurity is described by the delta-function. (2·10) shows that $\Sigma(p, \omega)$ is now independent of p. We denote the real part of $F^{(0)}(\omega - i\varepsilon)$ as $F_R^{(0)}(\omega)$. The imaginary part is $\pi\rho^{(0)}(\omega)$ as described by (2·4). Then one has

$$\Sigma(p, \omega) = \Delta(\omega) + i\Gamma(\omega), \tag{2·11}$$

$$\Delta(\omega) = \frac{\rho v[1 - vF_R^{(0)}(\omega)]}{[1 - vF_R^{(0)}(\omega)]^2 + v^2\pi^2\rho^{(0)}(\omega)^2}, \tag{2·12}$$

$$\Gamma(\omega) = \frac{\rho v^2\pi\rho^{(0)}(\omega)}{[1 - vF_R^{(0)}(\omega)]^2 + v^2\pi^2\rho^{(0)}(\omega)^2}. \tag{2·13}$$

Kahn[2] obtained a finite result for the conductivity by using (2·13) for the self-energy part where he has omitted $F_R^{(0)}(\omega)$. Abrikosov[10] mentioned the renor-

malization discussed in the following section but argued that it is not important for the problem of his interest in that work and adopted the same approximation as Kahn.

We define $t(\omega)$ by

$$t(\omega) = v + vF_R^{(0)}(\omega)v + \cdots$$

$$= \frac{v}{1 - vF_R^{(0)}(\omega)} . \tag{2·14}$$

Then Eqs. (2·12) and (2·13) for the real and imaginary parts, $\Delta(\omega)$ and $\Gamma(\omega)$, of the self-energy part $\Sigma(p, \omega)$ are expressed as follows:

$$\Delta(\omega) = \frac{\rho t(\omega)}{1 + t(\omega)^2\pi^2\rho^{(0)}(\omega)^2} , \tag{2·15}$$

$$\Gamma(\omega) = \frac{\rho t(\omega)^2\pi\rho^{(0)}(\omega)}{1 + t(\omega)^2\pi^2\rho^{(0)}(\omega)^2} . \tag{2·16}$$

The above-mentioned approximation adopted by Kahn[2] may be said to be the one where $t(\omega)$ in (2·16) is replaced simply by the magnitude of the impurity potential v. For the two extreme cases $t(\omega)\pi\rho^{(0)}(\omega) \gg 1$ and $t(\omega)\pi\rho^{(0)}(\omega) \ll 1$, $\Gamma(\omega)$ is evaluated as

$$\Gamma(\omega) \sim \rho/\pi\rho^{(0)}(\omega) \qquad \text{if} \quad t(\omega)\pi\rho^{(0)}(\omega) \gg 1 , \tag{2·17a}$$

$$\Gamma(\omega) \sim \rho t(\omega)^2\pi\rho^{(0)}(\omega) \qquad \text{if} \quad t(\omega)\pi\rho^{(0)}(\omega) \ll 1 . \tag{2·17b}$$

Thus $\mathrm{Im}\,\Sigma(p, \omega)$ is zero when $\rho^{(0)}(\omega) \sim \infty$ in (2·17a) and hence the infinity of the level density for such is not removed by the present approximation (2·10).

§ 3. Renormalization

Next we shall take account of the effect of the renormalization of the solid lines between dotted lines of Fig. 1, from $G^{(0)}(p, \omega)$ to $G(p, \omega)$. We proceed as in the preceding section, obtaining

$$\Sigma(p, \omega) = \rho[v + vF(\omega)v + \cdots]$$

$$= \frac{\rho v}{1 - vF(\omega)} \tag{3·1}$$

in place of (2·10), where

$$F(\omega) = \sum_p G(p, \omega) . \tag{3·2}$$

Equation (3·1) is written as

$$\Sigma(p, \omega) = \Delta(\omega) + i\Gamma(\omega) , \tag{3·3}$$

$$\Delta(\omega) = \frac{\rho v[1 - vF_R(\omega)]}{[1 - vF_R(\omega)]^2 + v^2\pi^2\rho(\omega)^2} , \tag{3·4}$$

$$\Gamma(\omega)=\frac{\rho v^2\pi\rho(\omega)}{[1-vF_R(\omega)]^2+v^2\pi^2\rho(\omega)^2}, \tag{3·5}$$

$$\lim_{\substack{\varepsilon\to 0\\(\varepsilon>0)}} F(\omega-i\varepsilon)=F_R(\omega)+i\pi\rho(\omega). \tag{3·6}$$

Substituting (2·9) into (3·2) and comparing with (3·6), one sees that

$$F_R(\omega)=\sum_p \frac{\omega-\varepsilon(p)-\Delta(\omega)}{[\omega-\varepsilon(p)-\Delta(\omega)]^2+\Gamma(\omega)^2}, \tag{3·7}$$

$$\rho(\omega)=\frac{1}{\pi}\sum_p \frac{\Gamma(\omega)}{[\omega-\varepsilon(p)-\Delta(\omega)]^2+\Gamma(\omega)^2}. \tag{3·8}$$

Now the problem is to solve the set of Eqs. (3·4), (3·5), (3·7) and (3·8) for $\Delta(\omega)$, $\Gamma(\omega)$, $F_R(\omega)$ and $\rho(\omega)$. We introduce $t'(\omega)$ by

$$\begin{aligned} t'(\omega)&=v+vF_R(\omega)v+\cdots\\ &=\frac{v}{1-vF_R(\omega)}. \end{aligned} \tag{3·9}$$

Then Eqs. (3·4) and (3·5) for $\Delta(\omega)$ and $\Gamma(\omega)$ are expressed as follows:

$$\Delta(\omega)=\frac{\rho t'(\omega)}{1+t'(\omega)^2\pi^2\rho(\omega)^2}, \tag{3·10}$$

$$\Gamma(\omega)=\frac{\rho t'(\omega)^2\pi\rho(\omega)}{1+t'(\omega)^2\pi^2\rho(\omega)^2}. \tag{3·11}$$

By (3·11) the imaginary part $\Gamma(\omega)$ is estimated as

$$\Gamma(\omega)\sim\rho/\pi\rho(\omega) \qquad \text{if} \quad t'(\omega)\pi\rho(\omega)\gg 1, \tag{3·12a}$$

$$\Gamma(\omega)\sim\rho t'(\omega)^2\pi\rho(\omega) \qquad \text{if} \quad t'(\omega)\pi\rho(\omega)\ll 1. \tag{3·12b}$$

If the ω-dependence of $F_R(\omega)$ and hence that of $t'(\omega)$ related with $F_R(\omega)$ by (3·9) could be ignored, $\rho(\omega)$ would be now obtained by solving (3·8) with (3·10) and (3·11). In fact, Bychkov[5] assumed this and gave an argument to show that Im $\Sigma(p,\omega)$ may be estimated by the value for the case when no external field is applied. A detailed argument on this problem was given by Kubo et al.[6] Kubo et al.[6] and Doman[7] solved by taking $\rho t'(\omega)^2\pi\rho(\omega)$ for $\Gamma(\omega)$. Equation (3·12b) shows that their result applies to the case $t'(\omega)\pi\rho(\omega)\ll 1$. For sufficiently large ρ, the resulting $\pi\rho(\omega)$ is expressed to be always smaller than $1/t'$, and the assumption is consistent. However for a small ρ, $\pi\rho(\omega)$ will become larger than $1/t'$, and we have estimate $\Gamma(\omega)\sim\rho/\pi\rho(\omega)$ by (3·12a). Since $\rho(\omega)$ is expected to be finite, $\Gamma(\omega)$ becomes finite and we expect to have a consistent finite result for $\rho(\omega)$ even for small ρ; cf. (2·17a) where $\Gamma(\omega)$ is zero at the ω where $\rho^{(0)}(\omega)=\infty$. In order to obtain reasonable results for all ρ, we calculate the level density by using the full expression (3·5) for $\Gamma(\omega)$ in the subsequent section.

§ 4. Calculation of the level density

In the approximation proposed in the preceding section, the level density of the electron gas in the strong magnetic field in the presence of impurities is obtained by solving the following set of coupled equations:

$$F_R(\omega) = \sum_p \frac{\omega - \varepsilon(p) - \Delta(\omega)}{[\omega - \varepsilon(p) - \Delta(\omega)]^2 + \Gamma(\omega)^2}, \tag{4·1}$$

$$\rho(\omega) = \frac{1}{\pi} \sum_p \frac{\Gamma(\omega)}{[\omega - \varepsilon(p) - \Delta(\omega)]^2 + \Gamma(\omega)^2}, \tag{4·2}$$

$$\Delta(\omega) = \frac{\rho t'(\omega)}{1 + t'(\omega)^2 \pi^2 \rho(\omega)^2}, \tag{4·3}$$

$$\Gamma(\omega) = \frac{\rho t'(\omega)^2 \pi \rho(\omega)}{1 + t'(\omega)^2 \pi^2 \rho(\omega)^2}, \tag{4·4}$$

$$t'(\omega) = \frac{v}{1 - v F_R(\omega)}. \tag{4·5}$$

The summation with respet to p_y and p_z are performed in (4·1) and (4·2). Then they are reduced to

$$F_R(\omega) = \frac{eHm^{1/2}}{2\pi c} \sum_{n=0}^{\infty} \left\{ \frac{E(\omega) + [E(\omega)^2 + \Gamma(\omega)^2]^{1/2}}{E(\omega)^2 + \Gamma(\omega)^2} \right\}^{1/2} \times \{E(\omega) - [E(\omega)^2 + \Gamma(\omega)^2]^{1/2}\} / \Gamma(\omega), \tag{4·6}$$

$$\rho(\omega) = \frac{eHm^{1/2}}{2\pi^2 c} \sum_{n=0}^{\infty} \frac{\{E(\omega) + [E(\omega)^2 + \Gamma(\omega)^2]^{1/2}\}^{1/2}}{[E(\omega)^2 + \Gamma(\omega)^2]^{1/2}}, \tag{4·7}$$

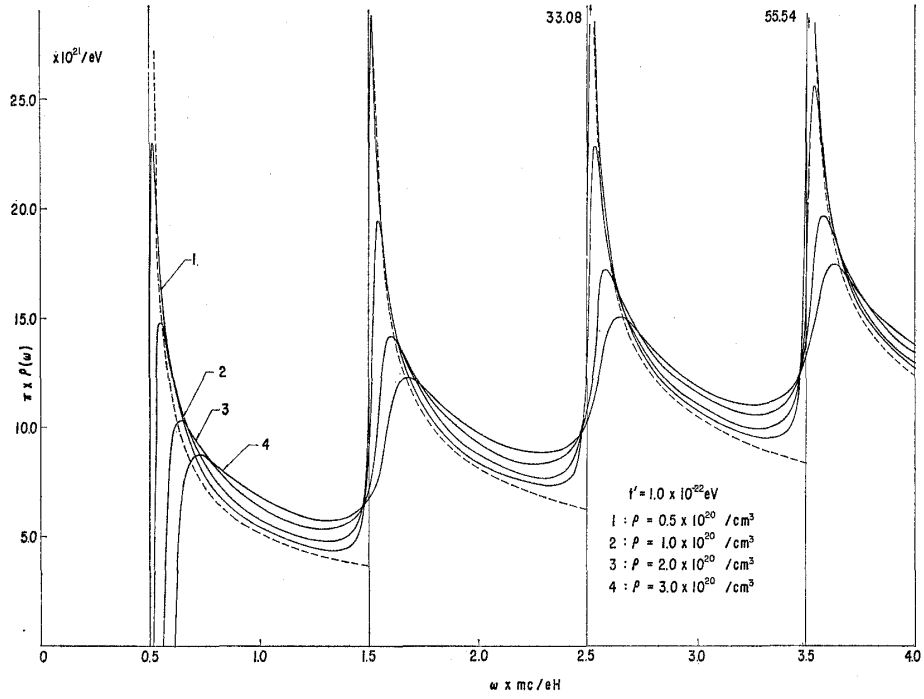
$$E(\omega) = \omega - \frac{eH}{mc}\left(n + \frac{1}{2}\right) - \Delta(\omega). \tag{4·8}$$

We shall ignore the ω-dependence of $t'(\omega)$ in the first set of calculations, in order to save computer time. Then we have

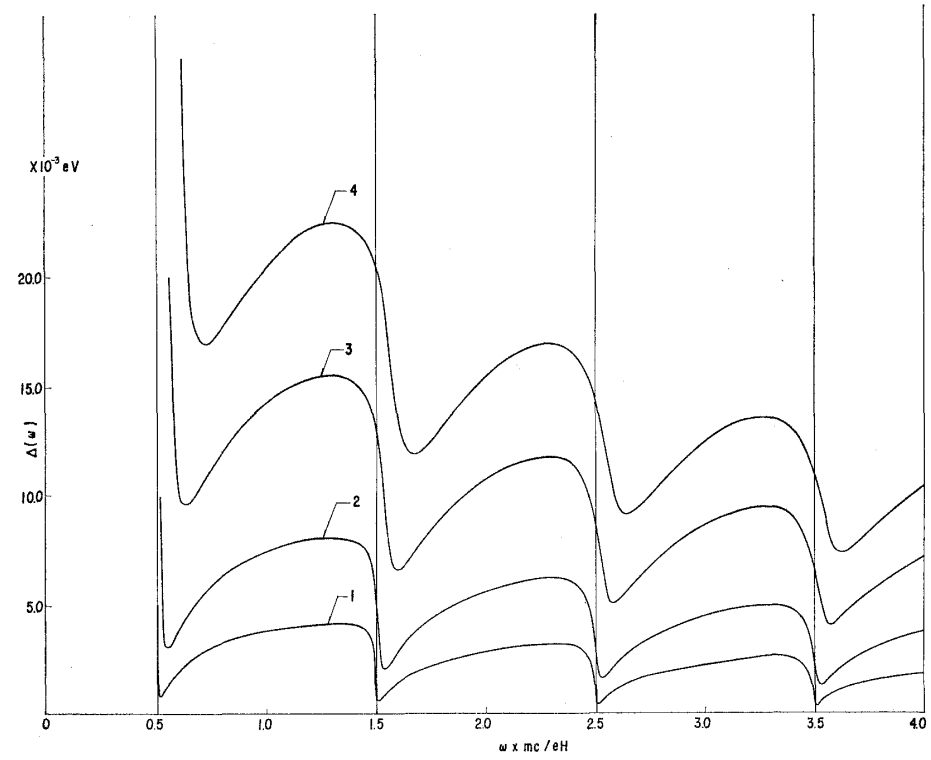
$$\Delta(\omega) = \frac{\rho t'}{1 + t'^2 \pi^2 \rho(\omega)^2}, \tag{4·9}$$

$$\Gamma(\omega) = \frac{\rho t'^2 \pi \rho(\omega)}{1 + t'^2 \pi^2 \rho(\omega)^2} \tag{4·10}$$

in place of (4·3) and (4·4). We have now solved the set of Eqs. (4·7), (4·8), (4·9) and (4·10) in a self-consistent way. The procedure we used is as follows: For a given ω, we choose a value of $\rho(\omega)$ and evaluated the right-hand side of (4·7) with the aid of (4·8), (4·9) and (4·10) and check whether it gives the assumed value. We continue this trial until a consistent result is achieved. This procedure is repeated for a number of values of ω. The calculation was performed by using IBM 7040/1401 system in College of Science and Engineering,

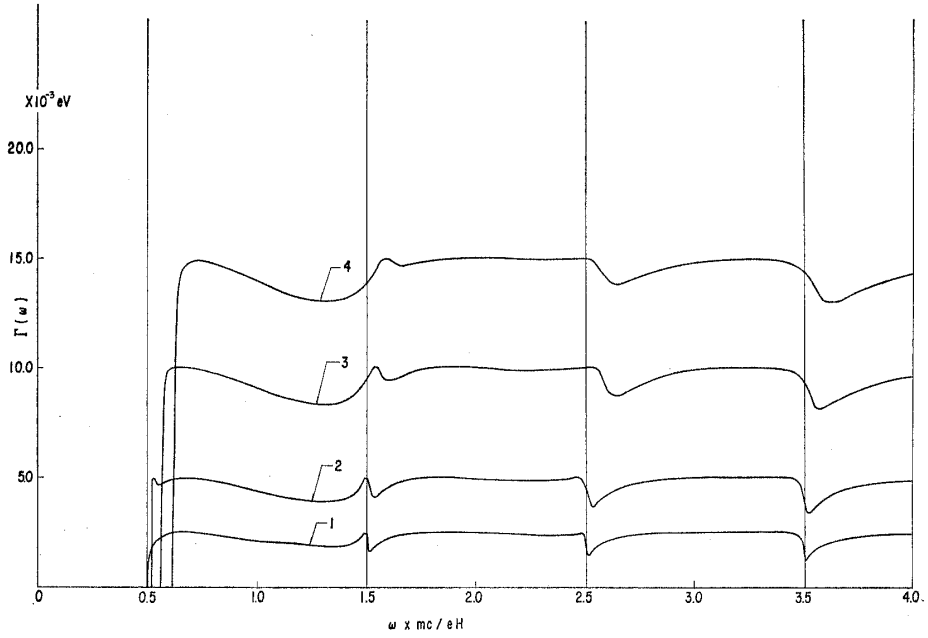


(a)



(b)

Fig. 2. (Figure captions are printed on the next page below.)



(c)

Fig. 2. (a) Curves for the level density $\rho(\omega)$, (b) curves for the shift $\Delta(\omega)$ and (c) curves for the width $\Gamma(\omega)$. The parameters are $t'=1.0\times10^{-22}$ eV and the impurity density $\rho=0.5\times10^{20}$ (curve 1), 1.0×10^{20} (curve 2), 2.0×10^{20} (curve 3) and 3.0×10^{20} (curve 4) per unit volume, respectively. Broken line represents the level density of the electron gas in the magnetic field without impurities.

Aoyama Gakuin University. In calculation of $F_R(\omega)$ and $\rho(\omega)$ by (4·6) and (4·7), we have summed up to $n=100$. This arbitrary cut-off is required by our unrealistic choice of the delta-function type potential. The curves obtained for $\rho(\omega)$, $\Delta(\omega)$ and $\Gamma(\omega)$ are shown in Figs. 2~5.

Figure 2 represents the curves for $t'=1.0\times10^{-22}$ eV and $\rho=0.5\times10^{20}$, 1.0×10^{20}, 2.0×10^{20} and 3.0×10^{20} per unit volume, respectively.

Figure 3 represents the curves for $\rho=1.0\times10^{20}$ per unit volume and $t'=0.5\times10^{-22}$, 1.0×10^{-22} and 2.0×10^{-22} eV, respectively.

In these calculations, the following numerical values are used for the external field H and the electronic mass m:

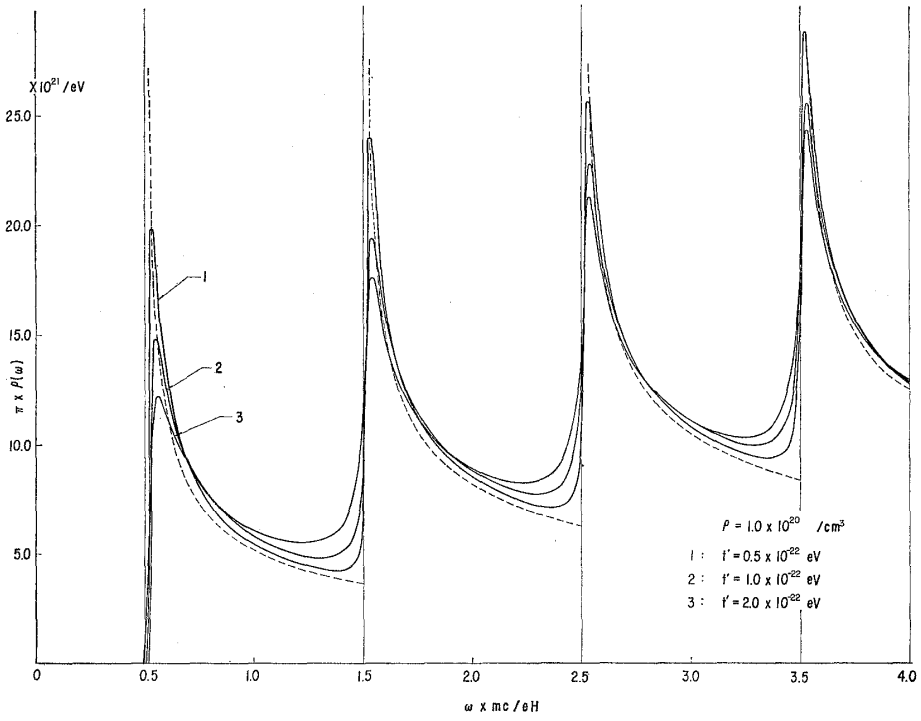
$$eH/mc=1.1576\times10^{-1}\,\text{eV}\,,\quad (H=10^{7}\text{G})$$

$$m=9.1091\times10^{-28}\text{g}\,.\quad \text{(free electron mass)}$$

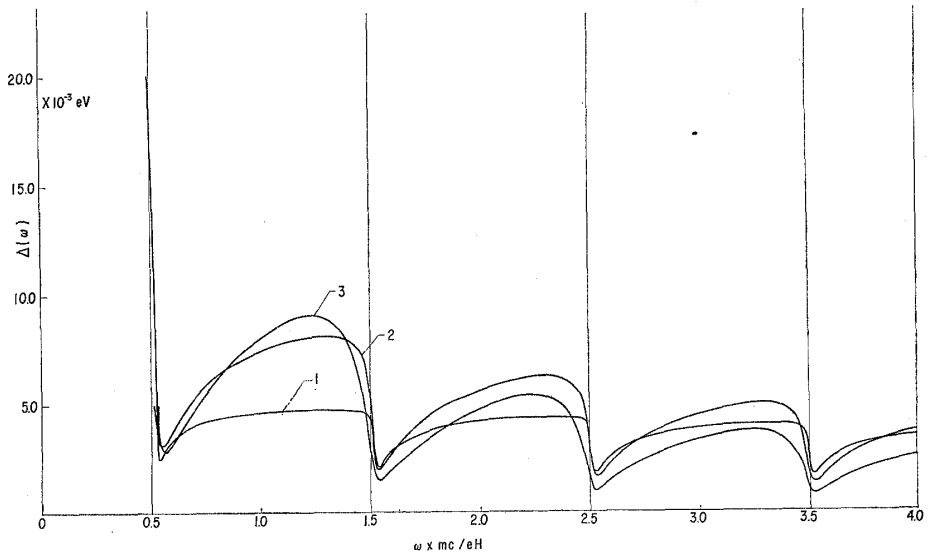
Figures 2(a) and 3(a) show that the level density $\rho(\omega)$ obtained in the present approximation is finite at the Landau levels which are shifted by $\Delta(\omega)$, and that the heights of the peaks decrease with increasing the impurity density or increasing the magnitude of t'. $\Delta(\omega)$ and $\Gamma(\omega)$ are obtained as a function of ω. These curves must be compared with the assumption of constant Δ and Γ, which had been sometimes adopted: See e.g., references 5) and 6). From

Figs. 2(b) and 3(b), one sees that $\Delta(\omega)$ has minima at ω which corresponds to the peaks of $\rho(\omega)$. A sharp drop in $\Gamma(\omega)$ is observed at the same ω in Figs. 2(c) and 3(c). It should be noted that this behavior is connected with the estimate (3·12a) at the peaks of $\rho(\omega)$.

The results for the case with a rather high density of impurities are shown

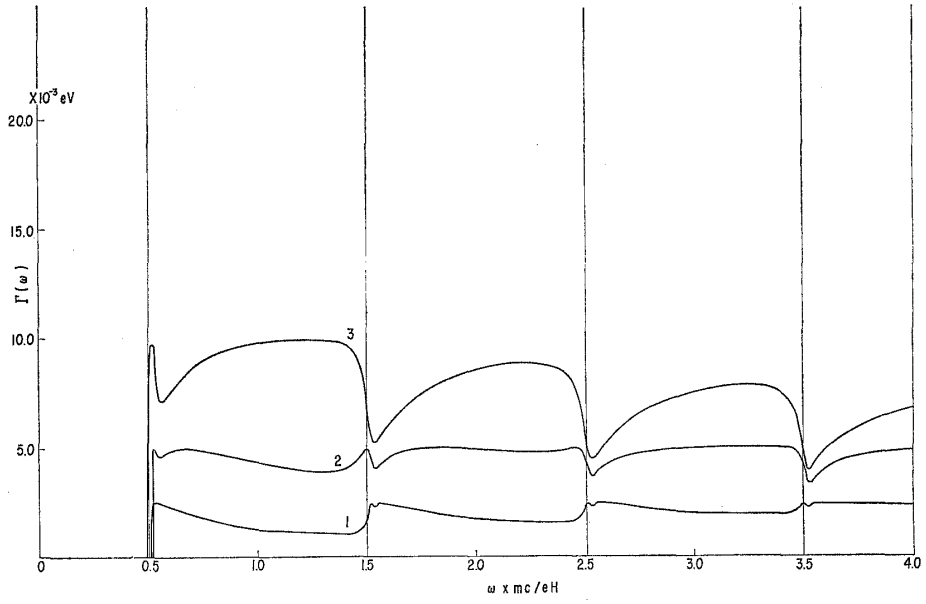


(a)



(b)

Fig. 3. (Figure captions are printed on the next page below.)



(c)

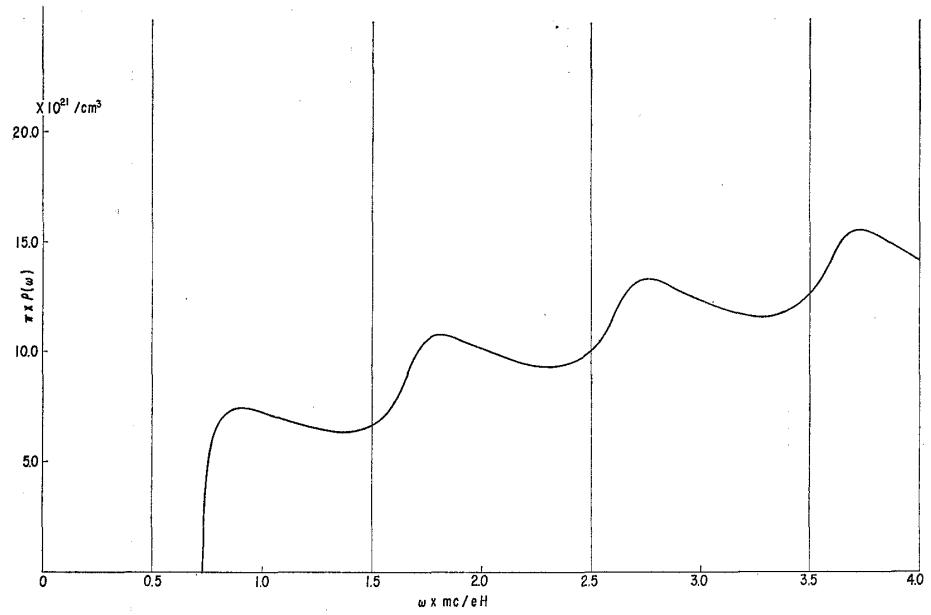
Fig. 3. (a) Curves for the level density $\rho(\omega)$, (b) curves for the shift $\Delta(\omega)$ and (c) curves for the width $\Gamma(\omega)$. The parameters are $\rho=1.0\times10^{20}$ per unit volume and $t'=0.5\times10^{-22}$eV (curve 1), 1.0×10^{-22}eV (curve 2), and 2.0×10^{-22} (curve 3)eV, respectively.

in Fig. 4 where $\rho=5.0\times10^{20}$ per unit volume and $t'=1.0\times10^{-22}$ eV. In Fig. 5 the results for rather small t' are given where $\rho=1.0\times10^{20}$ per unit volume and $t'=1.0\times10^{-23}$ eV. For these cases, $\Gamma(\omega)$ roughly follows the behavior of $\rho(\omega)$. In particular, for the case of rather high impurity density (Fig. 4(a)), the estimate (3·12b) $t'\pi\rho(\omega)<1$ is valid even at the peaks of $\rho(\omega)$. As the impurity density is reduced, this inequality becomes invalid near the top of the peaks (see Fig. 2(a), curve 1).

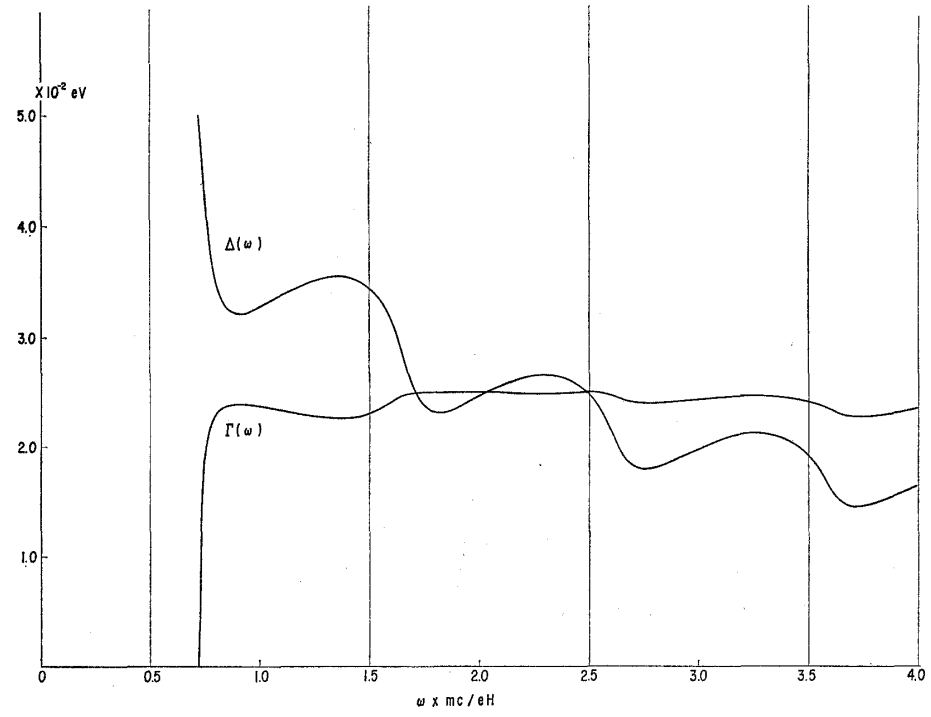
Finally we present a set of curves obtained by solving the set of Eqs. (4·1) $\sim$(4·5) self-consistently, for $\rho(\omega)$ and $t'(\omega)$. The parameters chosen are $v=1.0\times10^{-22}$ eV and $\rho=1.0\times10^{20}$ and 3.0×10^{20} per unit volume. The curves for $\rho(\omega)$, $t'(\omega)$, $\Delta(\omega)$ and $\Gamma(\omega)$ are given in Figs. 6(a)$\sim$6(d). Figure 6(b) shows that $t'(\omega)$ is nearly constant and has dips at the ω corresponding to the positions of the peaks of $\rho(\omega)$.

The behavior of $t'(\omega)$ for two curves of Fig. 6(b) are almost the same except for the dips at ω corresponding to the peaks of $\rho(\omega)$, and it seems that $t'(\omega)$ is not sensitive to the impurity density.

As seen from Fig. 6(b), the magnitude of $t'(\omega)$ is of order of 10^{-23} eV where parameters are chosen as $v=1.0\times10^{-22}$ eV and $\rho=1.0\times10^{20}$ per unit volume for curve 1. This result may be compared with that shown in Fig. 5 where t' is set to be constant and $t'=1.0\times10^{-23}$ eV and with the same impurity density as the one above. The comparison is represented in Fig. 7. In Fig. 7, the both curves for $\rho(\omega)$ are almost coincident, but the behavior of $\Delta(\omega)$ and

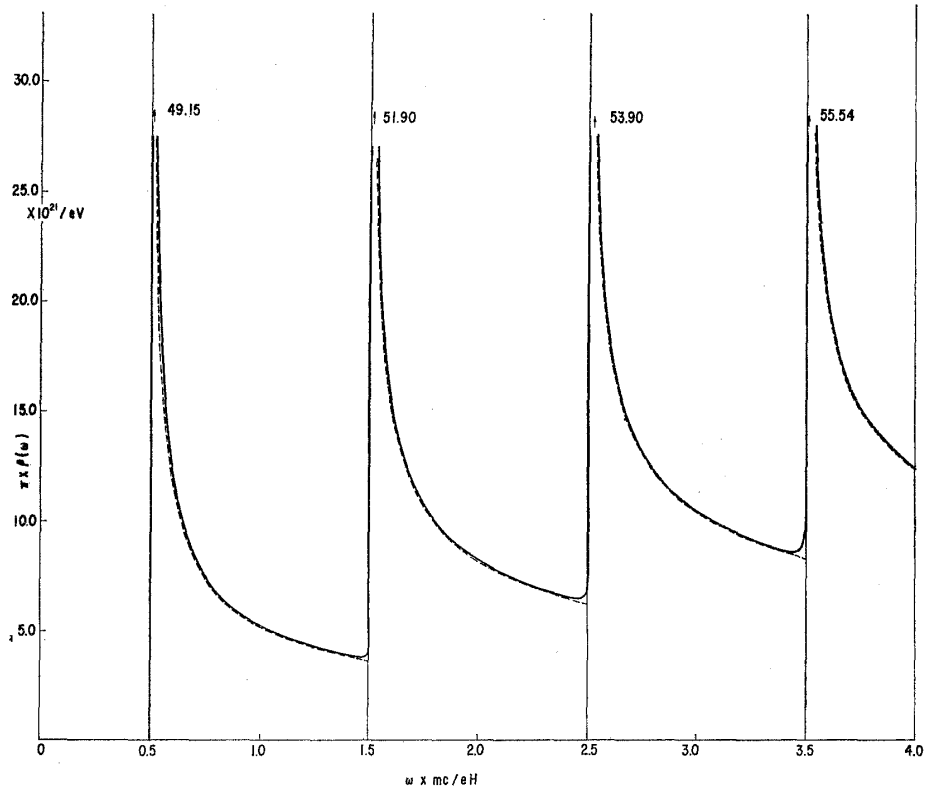


(a)

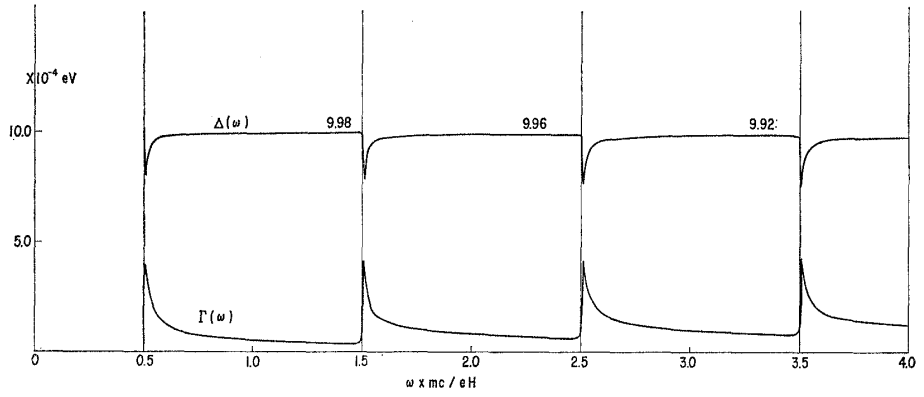


(b)

Fig. 4. (a) Curve for $\rho(\omega)$ and (b) curves for $\Delta(\omega)$ and $\Gamma(\omega)$. The parameters are the impurity density $\rho=5.0\times10^{20}$ per unit volume and $t'=1.0\times10^{-22}$ eV.



(a)



(b)

Fig. 5. (a) Curve for $\rho(\omega)$ and (b) curves for $\Delta(\omega)$ and $\Gamma(\omega)$. The parameters are $\rho=1.0\times10^{20}$ per unit volume and $t'=1.0\times10^{-23}$ eV.

$\Gamma(\omega)$ are rather different. Apart from $\Delta(\omega)$ and $\Gamma(\omega)$, it is expected that the level density $\rho(\omega)$ may be replaced by that obtained by setting $t'=$constant.

When we approximate $t'(\omega)$ by $t'=$constant, it is necessary to evaluate the values of t' from the magnitude of the impurity potential v. From Eq. (4·5), $t'(\omega)$ is reduced from v by a factor of denominator which is positive and greater than 1. Hence, as adopted by Kahn[2], Doman[7] and Kubo et al.,[6] the ap-

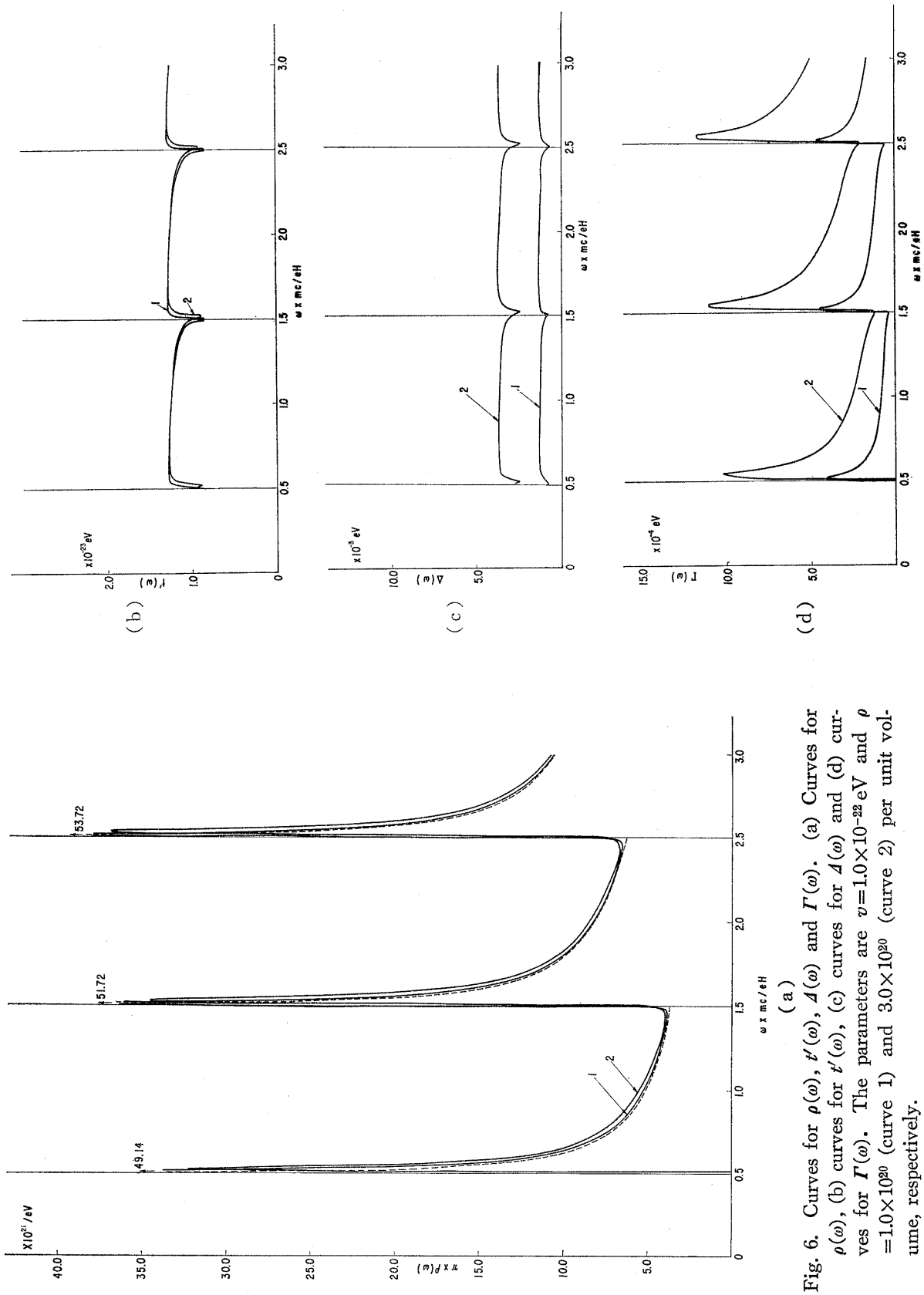
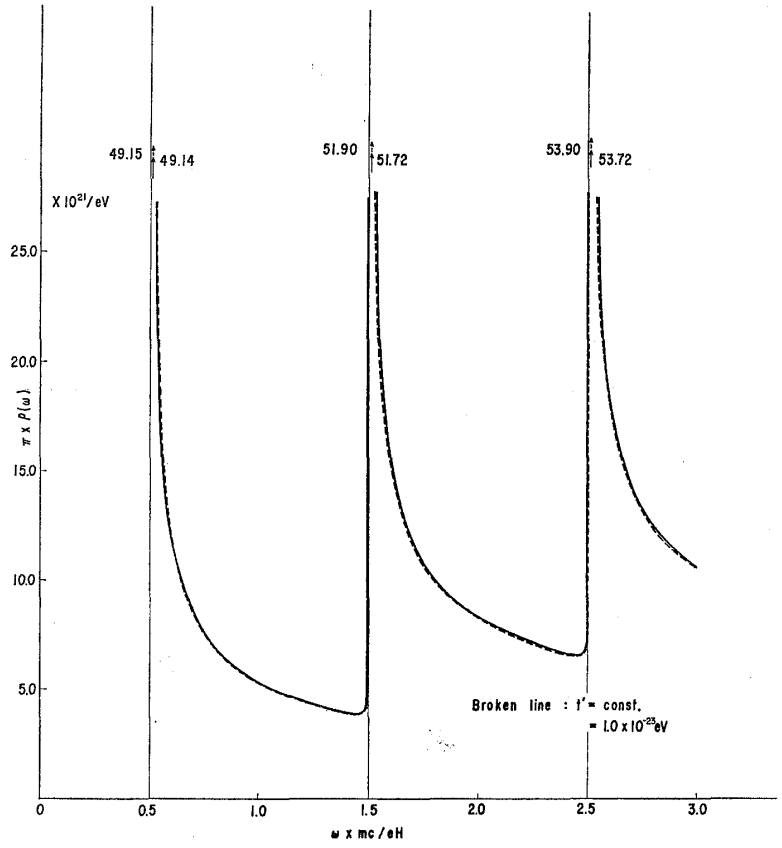
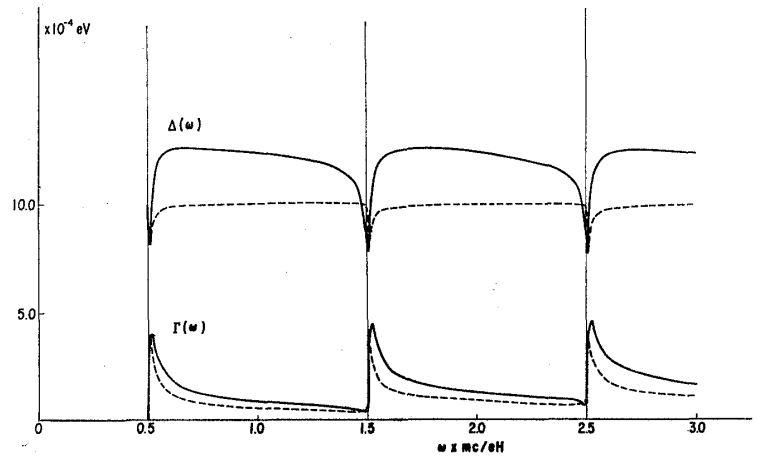


Fig. 6. Curves for $\rho(\omega)$, $t'(\omega)$, $\Delta(\omega)$ and $\Gamma(\omega)$. (a) Curves for $\rho(\omega)$, (b) curves for $t'(\omega)$, (c) curves for $\Delta(\omega)$ and (d) curves for $\Gamma(\omega)$. The parameters are $v=1.0\times10^{-22}$ eV and $\rho=1.0\times10^{20}$ (curve 1) and 3.0×10^{20} (curve 2) per unit volume, respectively.



(a)



(b)

Fig. 7. Comparison of curves for constant t' (Fig. 5) and curves obtained for an ω-dependent $t'(\omega)$ (Fig. 6 curve 1) for $\rho(\omega)$ (b) curves for $\Delta(\omega)$ and $\Gamma(\omega)$.

proximation in which $t'(\omega)$ is replaced by v could overestimate the effect of impurities on the level density.

§ 5. Conclusion

The approximations discussed in the preceding sections are summarized in Fig. 8 and Table I for the self-energy part Σ or its imaginary part Γ. In the approximation of Bychkov,[5] f is the scattering matrix of an electron gas when no external field is applied. Bychkov replaced f by a constant and hence the approximation is equivalent to Kubo et al.[6] and Doman's[7] approximation, except for the fact that an effective constant f occurs in place of v.

Table I. Comparison of various approximations for the self-energy part.

Kahn	$\Gamma=\mathrm{Im}\,\Sigma=\dfrac{\rho v^2\pi\rho_0(\omega)}{1+v^2\pi^2\rho_0(\omega)^2}$
Bychkov	$\Sigma\sim\rho f^2 G\sim\dfrac{i\pi}{2\tau_0}$
Kubo, Miyake and Hashitsume Doman	$\Sigma\sim\rho v^2 G$
Abrikosov (∼Kahn)	$\Sigma\sim\dfrac{\rho v}{1-i\pi\rho_0(0)v}$ [a]
Hasegawa and Nakamura, Saitoh et al., Present work	$\Sigma\sim\dfrac{\rho v}{1-Gv}$

a) Under the condition $\omega\ll p_z^2/2m$, $\rho(0)\sim\rho_0(0)$.

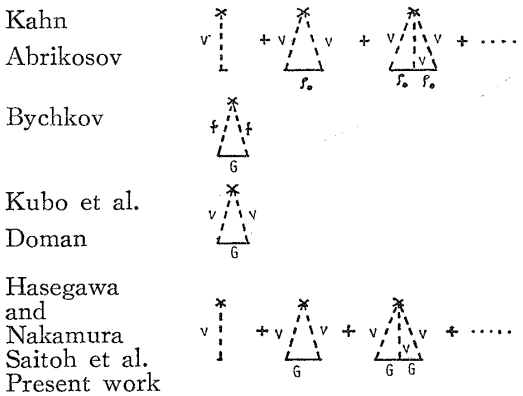


Fig. 8. Self-energy diagrams taken into account in the approximation proposed by various authors.

It is pointed out at the end of § 2 that the divergence of the level density at the Landau levels will not be removed by Kahn's[2] approximation. In § 3 Kubo et al.[6] and Doman's[7] approximation is found to be applicable only when the impurity density is large. The situation is the same also for Bychkov's approximation.[5] The present approximation which takes into account the renormalization of the one-particle Green's function in the self-energy part will be reasonable even at the peaks of the Landau levels for the system of low impurity density as well as for high impurity density. Hasegawa and Nakamura[12] solved analytically the set of equations analogous to (2·9) and (3·1) with (3·2) in the quantum limit and investigated the behavior of roots for the impurity band.

Further calculation of $\rho(\omega)$ is in progress by taking account of the ω-dependence of $t'(\omega)$, and the relation between v and $t'(\omega)$ will be investigated in more

detail. Application of our results to the electrical conductivity in the strong magnetic field and to the de Haas-van Alphen effect are now under consideration.

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