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Effect of Parallel Gradations on Crushed Rock-Concrete Interface Behaviors*

ABSTRACT: The response of a soil-structure system subjected to monotonic or cyclic loading is significantly influenced by the mechanical behavior of its interface. In this paper, the friction characteristics of crushed rock-concrete interface and the role of the crushed rock deformability were examined through a series of monotonic and cyclic direct shear tests under the framework of parallel gradation technique. Two parallel gradation curves of the crushed rocks were used. All the tests were carried out in dry condition and with two initial void ratios representing both loose and dense states. Two concrete interfaces of different roughness were used in the interface direct shear tests. The static test results show that parallel gradation technique can be used to characterize the residual shear strength but not the volume change. Moreover, under cyclic loading, it can neither yield the same volume change, nor the same shear strength. The critical state of the soil interfaces was also studied, which shed some light on the mechanism of pile skin friction mobilization and modeling of soil-concrete interface behaviors.

KEYWORDS: parallel gradation, crushed rock-concrete interface, critical state, direct shear

Introduction

Parallel gradation modeling technique was first proposed by Love [1] and subsequently adopted by many researchers (Indraratna et al. [2] and Varadarajan et al. [3], and Kaya [4]). A material with a smaller grain size distribution, which is composed of the same material as the prototype, can be used to model the prototype material provided that their grain size distributions are parallel to each other. By using parallel gradation technique, Varadarajan, et al. [3] conducted drained triaxial tests on two modeled rockfill materials that provided good prediction of the behavior of the prototype.

Potyondy [5] did pioneering work on the maximum shear strength along the interface between soil and structures. Later on, as the testing technique advancing, different interfaces, dimensions of shear box and types of loading were investigated by many interface workers [6–9]. In the 1980s, simple shear device was developed and assumed to be a more appropriate apparatus for studying soil concrete interface [10–12]. Several factors such as normal stress, steel roughness, media diameter and material type were investigated that all significantly affect the frictional resistance at yielding [10]. After the development of the simple shear device, there was a trend that almost all the researchers switched from direct shear to simple shear on studying of the interface behaviors in their research; however, the simple shear test configuration was doubted that it can neither yield uniformly vertical stress nor shear stress [13–15]. Jewell [16] argued that symmetrical direct shear test with smooth end walls corresponds closely with simple shear and therefore provides a more reliable result. Because of relatively simple test set up and sample preparation procedures, direct shear test has still gained its weight in laboratory testing and has been commonly used in characterizing soil-structure interfaces. Almost at the same time, X-ray photography method has gradually matured and made it really simple and applicable to study the micro structure and topography of soil interfaces [17,18]. Hryiw and Ivsyan [19] discovered that surface topography is important to the characteristics of soil structure interfaces. The soil interface characteristics under cyclic shear loading have also been investigated in the past decades. Desai et al. [9] concluded that for cohesionless soils, interface responses become stiffer with an increase in the number of cycles and the rate of stiffening decreases as the number of cycles increase. Al-Douri and Poulos [20] carried out static and cyclic direct shear tests on carbonate sands concrete interfaces. Similar results were achieved and were proven that cyclic direct shear test is a very useful technique to study the soil interface characteristics.

Although great contributions were made to the understanding of characteristics of soil interface, few studies were focused on the effect of parallel gradation technique. In this study, monotonic and cyclic direct shear tests were carried out on different crushed rock-concrete interfaces. The main objectives of these tests were to study the effects of initial density, normal stress, grain size, number of cycles and surface roughness on the shear resistance and volume change characteristics of soil-concrete interface subjected to both static and cyclic loading; to validate and assess the parallel gradation technique in characterizing the soil-concrete interface behaviors and to study the friction mobilization and critical state of soil-concrete interface under monotonic and cyclic loading.

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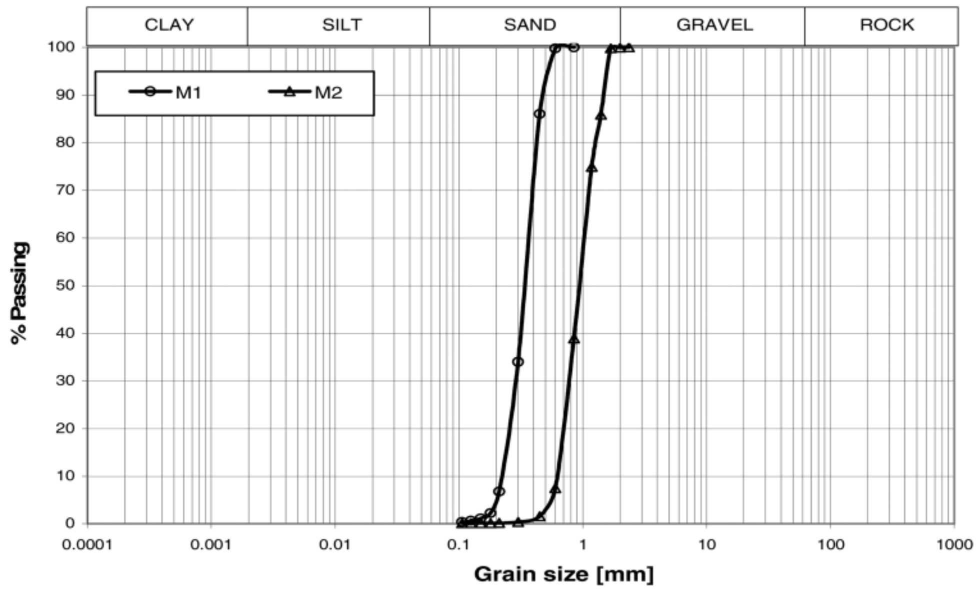

FIG. 1—*Grain Size Distribution of the Tested Materials.*

Testing Program

Materials

The crushed rock was shipped from Iron Mountain Trap Rock Company, MO, which provided 3 and 4 A mainline ballast to rail road industry. The downsized crushed rock is gray to brown, the grain shape is angular, the hardness is about 7 and the specific gravity is 2.67. Figure 1 shows the grain size distribution curves, where M1 and M2 are parallel to each other. Both M1 and M2 are classified as poorly graded sand (ASTM D 2487 [21]).

Test Set Up

GCTS Servo-Controlled direct shear device operated by the Computer Aided Testing Software (C.A.T.S.) was used to conduct the whole tests. Firstly, the direct shear box (6.40 cm D × 3.14 cm H) was assembled and mounted on the direct shear machine. Adjusts were made to the gap between the two parts of the shear box by turning the set-screws. Secondly, the plate (rough/smooth, consists the same thickness of the half box) were placed into the bottom box, and then the sands were slowly poured in and compacted to obtain the target density in three layers. Sand samples were prepared by the dry tamping method. Compaction was performed by applying the static weight of a 0.5 in. diameter aluminum temper 12 times on the surface of each layer. The weight of the tempers were 0.3 and 2.7 kg so that to achieve the desired loose and dense density. After that the dial gage transducers were attached to the shear box and the C.A.T.S. program keyin were set up, the loading then started and the shear force, time and

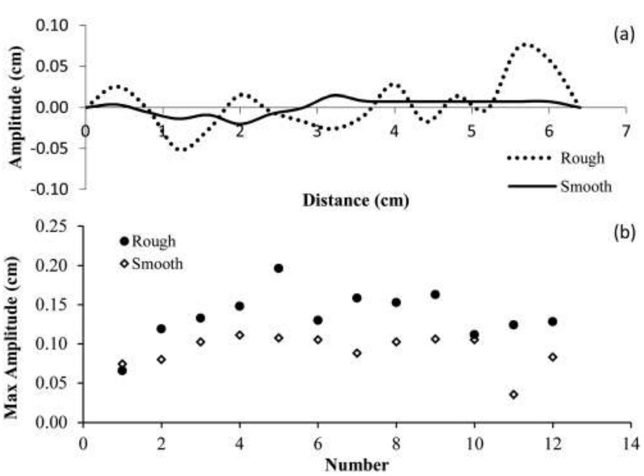

FIG. 2—*Typical interface profiles of the concrete plate.*

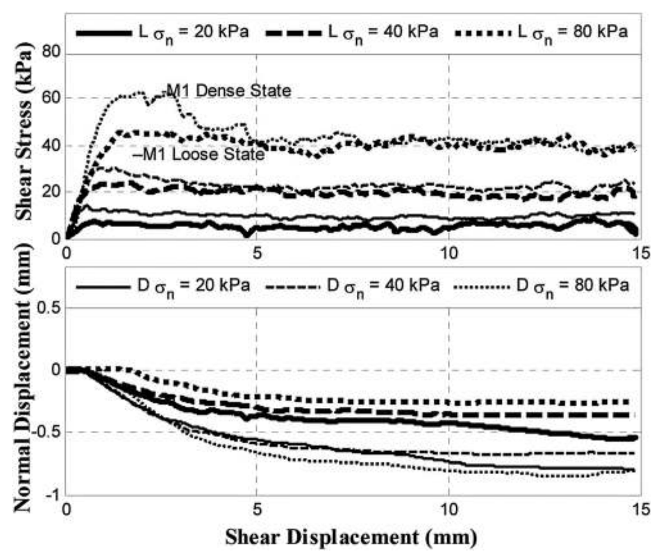

FIG. 3—*M1 on rough interface.*

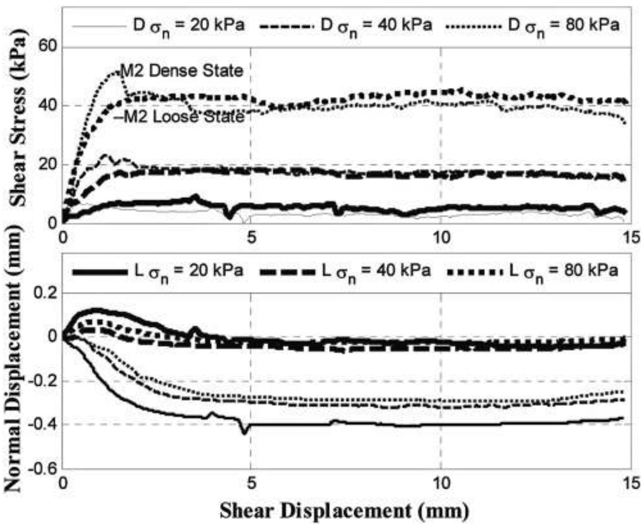

FIG. 4—*M2 on rough interface.*

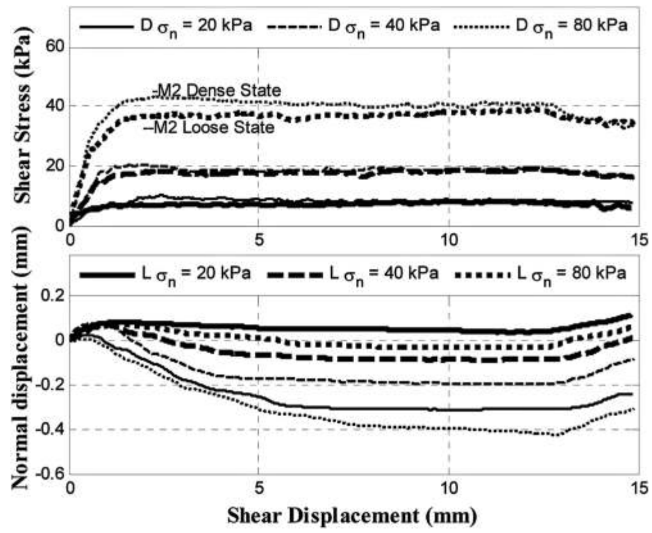

FIG. 6—*M2 on smooth interface.*

displacement were measured at convenient time intervals. Finally, the tests were not stopped until the shear force became constant (ASTM D 3080-04 [22]).

The tests were conducted on rough and smooth concrete interfaces, respectively. Typical interface profiles are shown in Fig. 2(*a*). The concrete plate roughness R is defined as the relative height between the highest peak and lowest trough along a surface profile [21]. Twelve interface roughness profiles were obtained on each concrete plate and the maximum amplitude of each measurement of both rough and smooth interfaces are plotted in Fig. 2(*b*). The maximum R values of the rough and smooth concrete interfaces in this study are 0.19 cm and 0.10 cm respectively. On each concrete plate, two parallel gradation materials in both dense and loose soil states were tested. Specimens were prepared in two initial void ratios 0.77 and 1.16, respectively, representing dense and loose packing states.

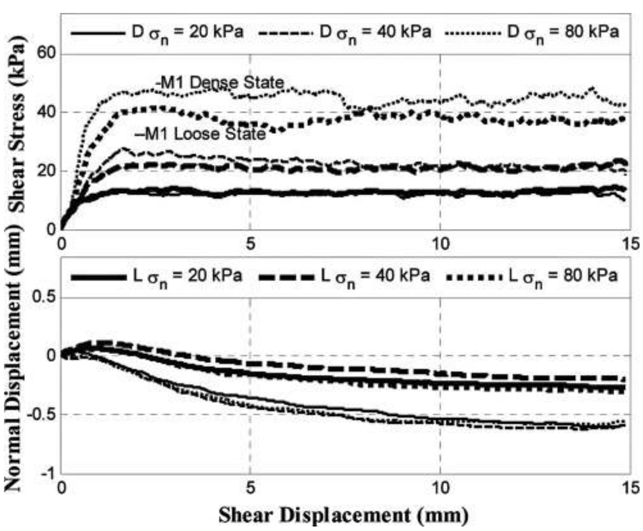

FIG. 5—*M1 on smooth interface.*

Test Results

Monotonic Tests

Figures 3–6 show the results of the direct shear tests performed on different concrete interface roughness. The stress-strain response of both dense and loose state of the same material (M1 or M2) under constant normal stress 20, 40, and 80 kPa, was plotted respectively in each figure. In general, for the dense and loose crushed rock-concrete interfaces, both of their peak shear strength was mobilized at a small displacement level, typically ranging from 2 to 3 mm. For dense soil interface, the post peak stress-strain behavior was very clear, and after 5 mm shear deformation it almost reached its residual state. Frost et al. [22] sheared Ottawa 20–20 sand against a rough finished concrete interface and found that the peak shear strength occurred at a deformation of about 2 mm. Similar phenomenon was also observed by Gomez et al. [23], where a large displacement shear box was used to investigate the response of a variety of interfaces, including clay geo-membrane interfaces, sand to concrete interfaces and sand to steel interfaces. The peak, residual and ultimate interface friction angles Φ_{peak}, $\Phi_{ultimate}$ and $\Phi_{residual}$ were listed in Table 1 and the critical state lines on rough and smooth interfaces were drawn in Fig. 7.

TABLE 1—*Interface friction angles.*

	Interface Friction Angle		
	Dense		Loose
Materials	Φ_{peak}	$\Phi_{residual}$	$\Phi_{ultimate}$
M1_Roughness	39.9°	29.6°	30.1°
M2_Roughness	37.0°	28.4°	27.2°
M1_Smoothness	33.3°	25.5°	28.0°
M2_Smoothness	33.2°	26.7°	24.5°

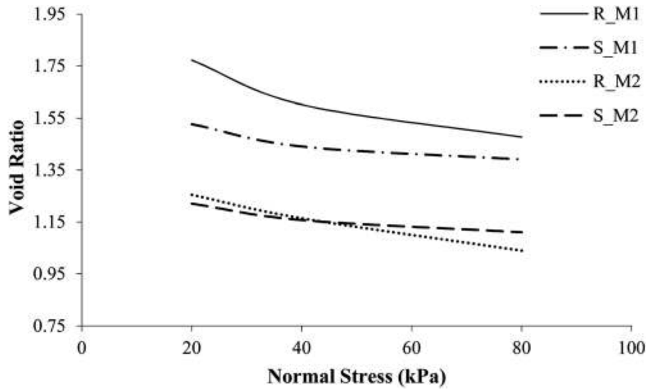

FIG. 7—*Critical state line of M1 and M2 on rough and smooth interface.*

Cyclic Tests

Figures 8–9 and Figs. 11–12 are the cyclic direct shear test results of M1 and M2 on rough and smooth concrete interfaces. The normal stress was set at a constant value of 40 kPa and the total number of cycles was 30 at a frequency of 1/10 Hz. The shear displacement amplitude was set at 1 mm that is smaller than the total shear displacement at the peak shear strength in the monotonic shearing on the same interface. Figures 10 and 13 are the plots of maximum shear strength mobilization and vertical displacement (volume) changing versus number of cycles.

Discussions

Monotonic Tests

Figures 3–4 display the stress-strain behaviors of M1 and M2 against rough concrete interfaces. The peak shear strength increased as the normal stress increased as well as the mobilization of internal friction angle before reaching the 'plateau.' Dense crushed rock-concrete interfaces all exhibited post-peak strain softening behaviors and their normal displacements were all distinctively higher than those of loose crushed rock-concrete interfaces. As expected, no peak shear stress was observed on loose crushed rock-concrete interface and it yielded all the way to the critical state as shearing continues. Although the peak shear strengths of M1 and M2 on rough interfaces were slightly different, the residual shear strengths were almost the same (Figs. 3–4).

A distinctive difference was observed in Figs. 5 and 6. Both M1 and M2 in dense state did not show any peak shear stresses but behaved like loose granular materials, which yielded all the way to the same critical state. However, by carefully examining their volumetric behaviors, all the crushed rock-concrete interfaces experienced a small contraction at the beginning then followed by dilation.

As for the interface friction angles, although peak and residual interface friction angles of M1 are slightly larger than those of M2, the differences are negligible. However, the volumetric behavior (normal displacement) and their shear-dilatancy behaviors are inconsistent. This might hint that the physical modeling technique of parallel gradation is only applicable and practical in terms of strength behavior.

Compared to the internal friction angle of the pure crushed rock, these interface friction angles are relatively high. This might be due to the particles lodge at the interface, particles bunch up while sliding and changing surface roughness during shear. Before shearing, small particles will be squeezed into the voids under high normal stress; once shearing onset, particles tend to slide on the concrete interface and when meeting with the lodged particles they tend to bunch up; thus increases the interface resistance to the shear load. Similar observation and explanation were given by Gomez et al. [23].

Critical density (critical void ratio) theory which formed as a corner stone for critical state soil mechanics was first developed by Casagrande [24]. Loose and dense sands samples were sheared in the direct shear apparatus and he found that under large strains the volume of the sample tended to stay constant. That is to say,

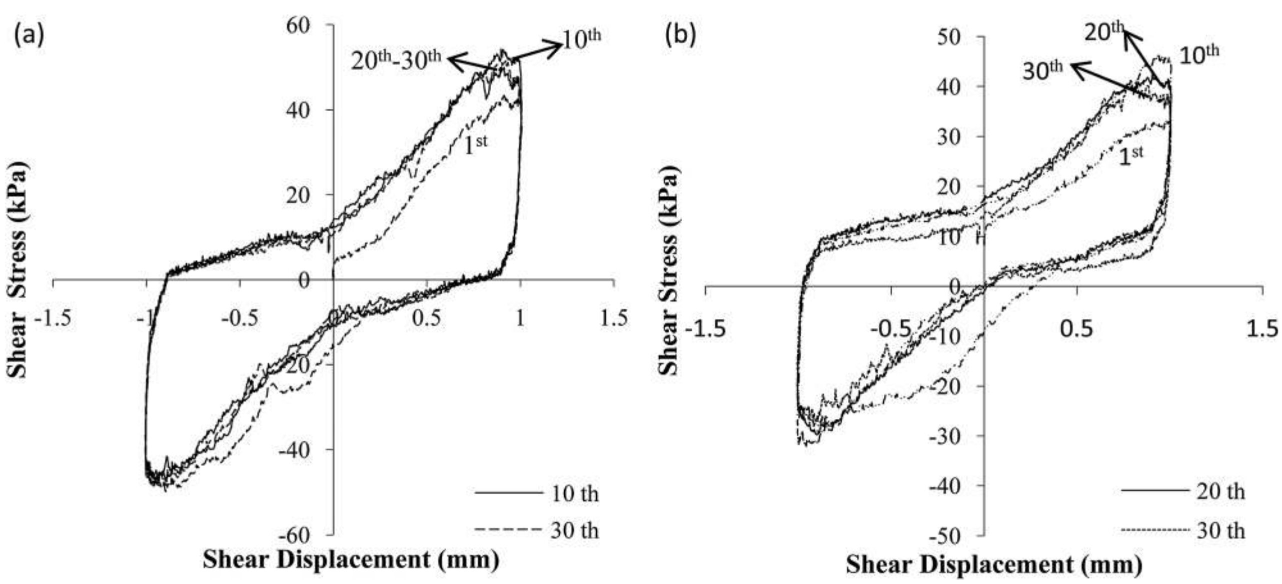

FIG. 8—*Shear stress versus shear displacement of M1 (a) and M2 (b) in dense state on roughness interface.*

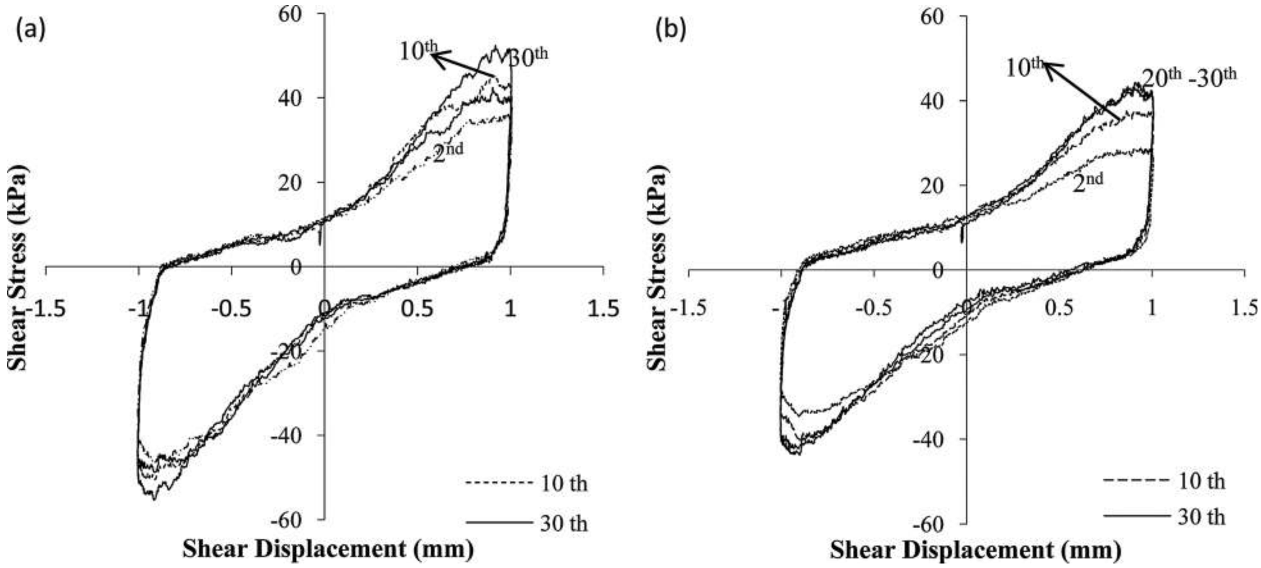

FIG. 9—*Shear stress versus shear displacement of M1 (a) and M2 (b) in loose state on roughness interface.*

the dense specimen will dilate and its initial void ratio will increase to the critical void ratio during shearing; on the other hand, the loose specimen will undergo contraction and its initial void ratio will decrease to the critical void ratio in the end. The critical void ratio is also found to be only a function of the effective confining pressure and is stress path independent. Later, the critical void ratio theory was furthered by Roscoe et al. [25] who concluded that it can be applied to all kinds of soils, including glass beads and steel balls. Castro [26] carried on Casagrande's work and developed a liquefaction evaluation criterion which is called "steady state" (actually is found to be the same as critical state). Since then, critical void ratio study has gained its weight in liquefaction evaluation and many researchers have carried out extensive experiments on studying the critical void ratio on different materials.

Based on our study, soil concrete interfaces also exhibited a kind of critical state under large strains. Since the void ratio used in calculation is actually a mean value of the sample, "bulk critical void ratio" is recommended in this paper as a proper name for describing soil concrete interfaces. Although M1 and M2 were packed into dense and loose state, compared to the corresponding bulk critical void ratios, they are all plotted under critical state lines and they all experienced a dilative behavior under shearing. In Fig. 7, it is clearly displayed that no matter on rough plate or smooth plate, M1 always had a larger "bulk critical void ratio" than M2. The "bulk critical void ratio" decreases as the confining pressure increases. "Bulk critical void ratio lines of M1 and M2 are almost parallel to each other and they exhibited a linear relationship corresponding to the normal confining stresses. For M1, the bulk critical state line on rough interface is a little higher than that on smooth interface; however, for M2 the bulk critical state line on both interfaces are almost the same. This demonstrated that the bulk critical state line is largely controlled by the gradation, but slightly affected by the interface roughness.

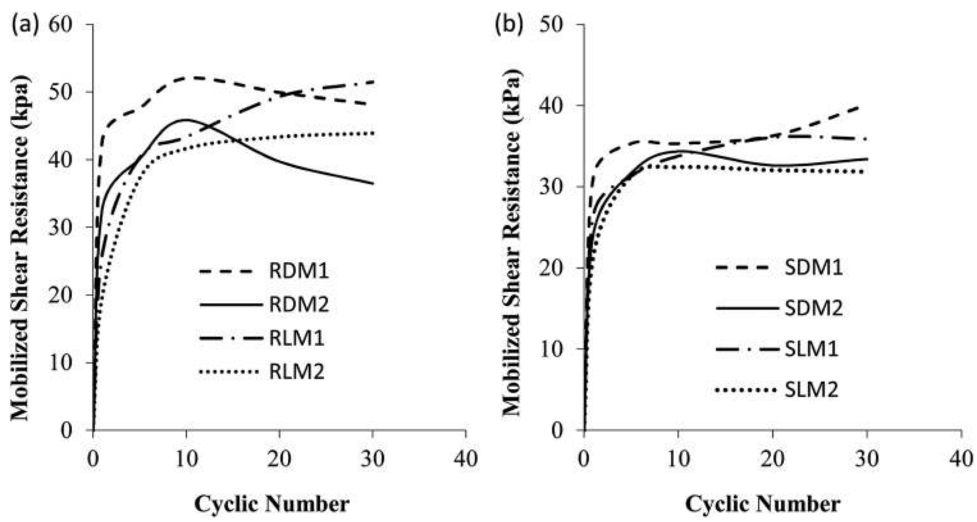

FIG. 10—*Maximum shear resistance versus number of cycles of M1 and M2 (Notation: R_Rough, S_Smooth, D_Dense and L_Loose).*

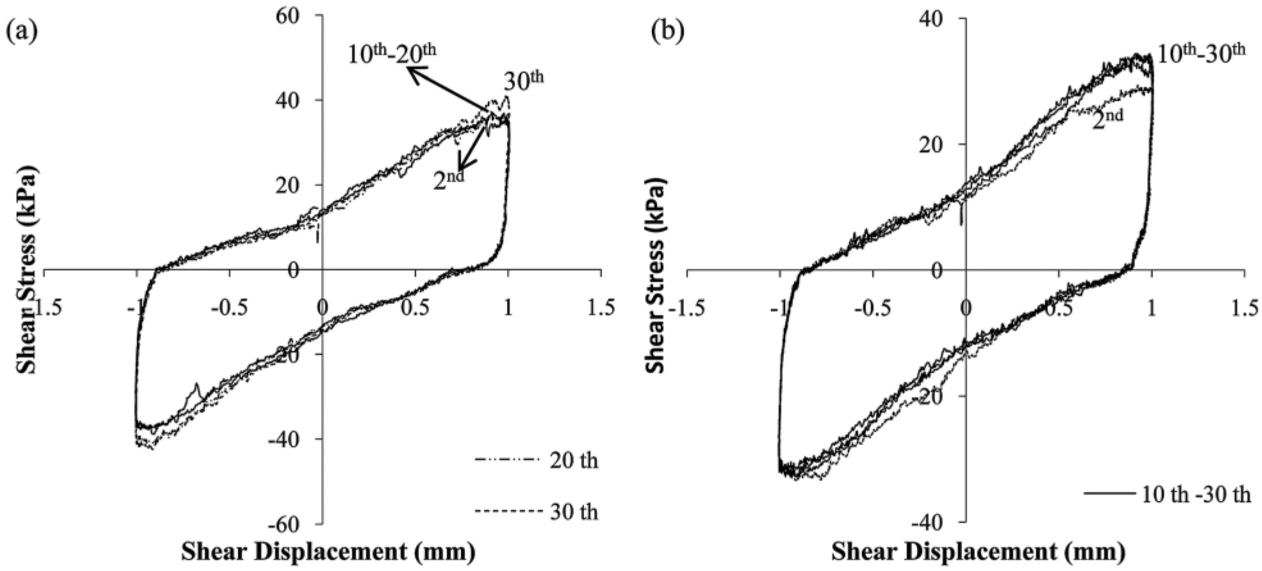

FIG. 11—*Shear stress versus shear displacement of M1 (a) and M2 (b) in dense state on smooth interface.*

It is anticipated that for a given material, no matter how rough or smooth the interface is, its bulk critical state line is always the same.

Cyclic Tests

Stress-strain behaviors on rough interface of dense state displayed a stress increasing in the first a few cycles (Fig. 8). For M1 (Fig. 8(*a*)), the shear stress was small in the first cycle and increased to the peak at a value of 56.7 kPa (tenth cycle), after that it began to soften and all the way dropped to 51.4 kPa in the 30th cycle. For M2 (Fig. 8(*b*)), the peak shear strength was 46.6 kPa and the corresponding cycle was tenth. Figures 9(*a*) and (*b*) shows the cyclic stress-strain behaviors of M1 and M2 on rough interface in loose state. It is fairly clear that both materials displaced a stress hardening during shear. The loose interfaces hardened as the number of cycles increased and finally reached to a ‘plateau’ where shear resistance became constant. The maximum shear resistances versus number of cycles of M1 and M2 under rough interfaces were plotted in Fig. 10(*a*) which clearly showed that their behaviors were exactly the same where both of them underwent a post peak softening as the cyclic number increased. In Fig. 10(*b*), the loose interface hardening trend is more explicitly displayed; M1 and M2 exhibited almost the same shear resistance within the first ten cycles. After that, shear resistance of M1 increased slightly as the number of cycles increased, but shear resistance of M2 stayed constant.

Figure 11 displays the cyclic stress strain behavior of M1 and M2 on smooth interface under dense packing state. Different

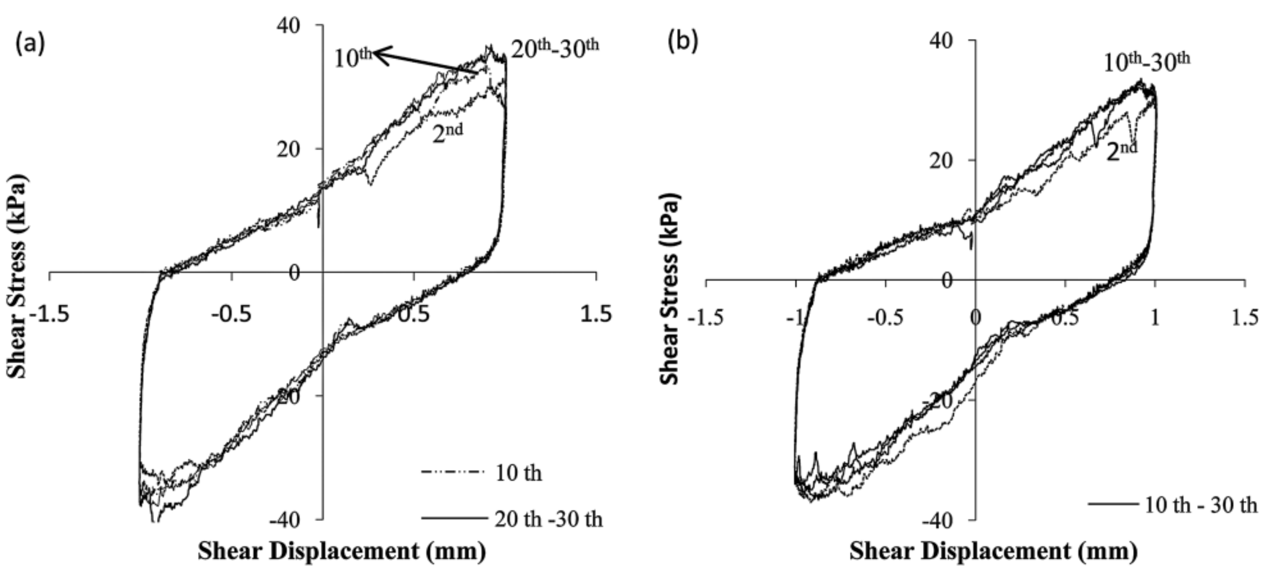

FIG. 12—*Shear stress versus hear displacement of M1 (a) and M2 (b) in loose state on smooth interface.*

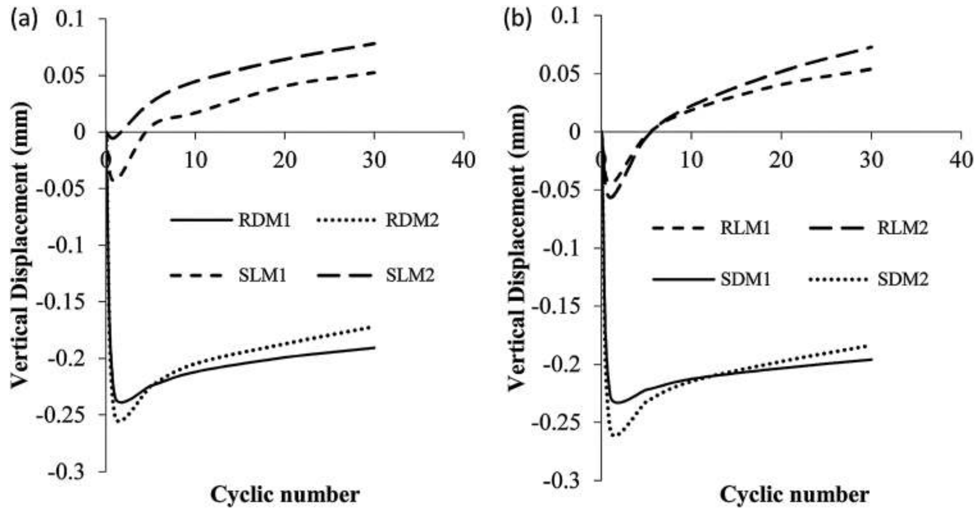

FIG. 13—*Vertical displacement versus number of cycles of M1 and M2 (Notation: R_Rough, S_Smooth, D_Dense and L_Loose).*

behaviors were observed from the rough interface. Both of M1 and M2 did not show any post peak shear stress point but exhibited a strain hardening as the number of cycles increased. Figure 12 shows the cyclic stress strain behavior of M1 and M2 on the smooth interface under loose state. Shear strength increased as the number of the cycles increased and the ultimate shear strength of M1 and M2 were 37.5 kPa and 35.2 kPa, respectively. The strain hardening trend under the dense packing state on the smooth interface was more clearly in previous Fig. 10(*b*). The shear resistances increased as the number of cycles increased and almost after fourth-fifth cycle they stayed constant where all the curves extended flatly. In Figs. 13(*a*) and (*b*), vertical displacement versus number of the cycles of M1 and M2 were plotted. The interfaces under dense packing state experienced a significant dilation in the beginning and then evolved into a stable state; the loose interface; on the other hand, after a little bit of dilation, contracted all the way until shearing stopped. The volumetric behavior is almost the same for both M1 and M2 with the only difference being how much dilation they underwent. For the dense interface, it dilated significantly and the volumetric strain was negative; for loose interface, although it dilated in the beginning, it contracted significantly in the subsequent cycles and its volumetric strain was positive.

On the rough interface, the major difference between M1 and M2 is the cyclic shear strength, no matter they underwent a strain hardening or a strain softening process, M1 always showed higher peak and ultimate cyclic shear strength, the volumetric behaviors were almost the same. However, on smooth interface, M1 and M2 exhibited almost the same ultimate shear strength and the same stress strain behaviors under both dense and loose state where only contractive behaviors were observed during shear. Although the difference between M1 and M2 is not so big and all the specimens underwent the same type of stress strain behaviors under the same soil state, at least, it does show that there exists a defect in the parallel gradation technique in modeling rough soil interface under cyclic loading.

Based on the micro-structure analysis published previously by (Mortara et al. [27] and Uesugi [18]), an explanation for the curves (behavior) in Figs. 10 and 13 was postulated, it counts for the mechanism of critical state interface characteristics and has important implications for the skin friction developed in crushed rock materials. Under cyclic shear force, once the shear is started, the lodged particles are forced to move and bunched up with other particles (dilation), as the shear deformation increasing, grains inside the specimen will slid and overcome the resistances around by rotating and rearranging themselves continuously. Gradually, all the particles tend to evolve into a minimum resistance structure so that the shear force becomes constant; then when the shear force reversed, the new structure is being rearranged again and all the particles tend to form a new minimum resistance structure in the reverse direction. The soil structure will be changed again and again during the shearing reversal and the old structure (before shearing) will be totally destroyed and all the memories will be lost. After several cycles, all the grains will form into a new flexible structure whose shear resistance will not change under the future stress reversal.

Conclusions

Interface monotonic and cyclic direct shear tests were carried out to investigate the frictional behavior of crushed rock-concrete interfaces. Parallel gradation physical modeling technique was used and the test results were compared. Two sets of materials at both loose and dense states, and rough and smooth concrete interfaces were investigated. Critical state of soil interface was studied and the corresponding critical state lines were obtained.

Shear strength of crushed rock-rough concrete interface was relatively higher than the one with the smooth interface. M1 and M2 that under the same initial packing state, same normal stress and same roughness concrete interfaces tended to exhibit similar peak and residual shear strength under monotonic shearing. Under cyclic loading, both of their peak shear resistances were fully mobilized in the first few cycles and after that came into constant resistance. Compared to rough interfaces, smooth concrete interfaces exhibited little peak shear stress but more residual behavior (contractive behavior) when shear strength is fully mobilized. Results from monotonic tests suggested that parallel gradation physical modeling technique is applicable in characterization of residual shear strength but not volume change. Under cyclic loading condition, the peak and residual shear strength of M1 and M2 were not the same, but the volumetric behaviors exhibited exactly the same trend. This indicates that the parallel gradation technique might not work for characterizing the cyclic resistance and volume change behaviors. More attention and study are needed in validating this technique under shear-dilatancy and various stress paths. The "bulk critical void ratio" decreases as the confining pressure increases. The position of the interface critical state line is more affected by the gradation where less influenced by the interface roughness. More research should be done to quantify the roughness and gradation influences on the position of interface critical state line.

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