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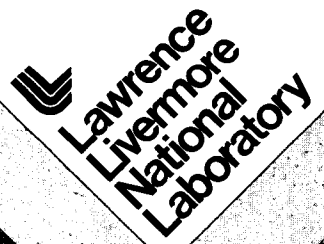
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# Effect of permeability on cooling of a magmatic intrusion in a geothermal reservoir

K. H. Lau

January 11, 1980



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K. H. Lau\*

Manuscript date: January 11, 1980

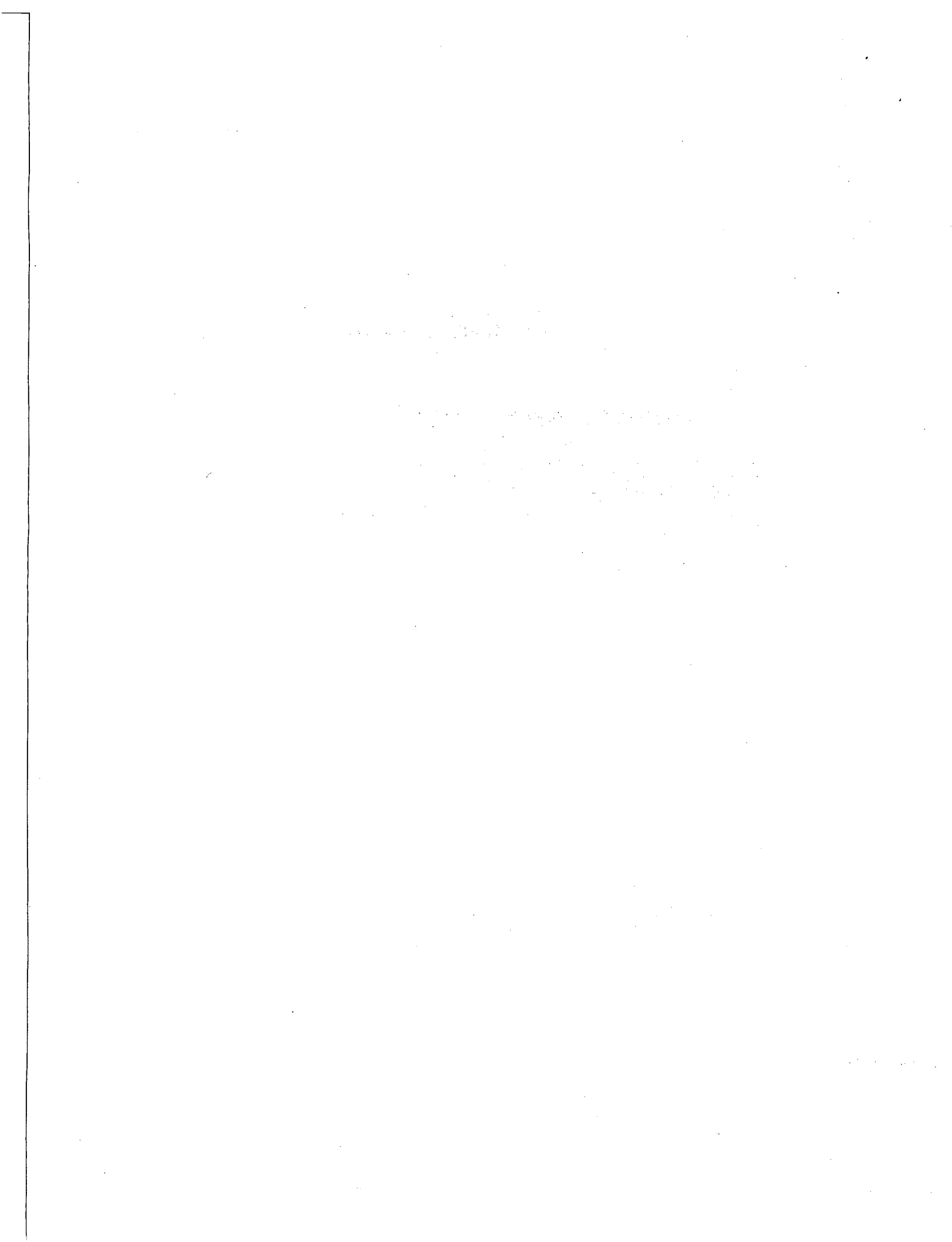
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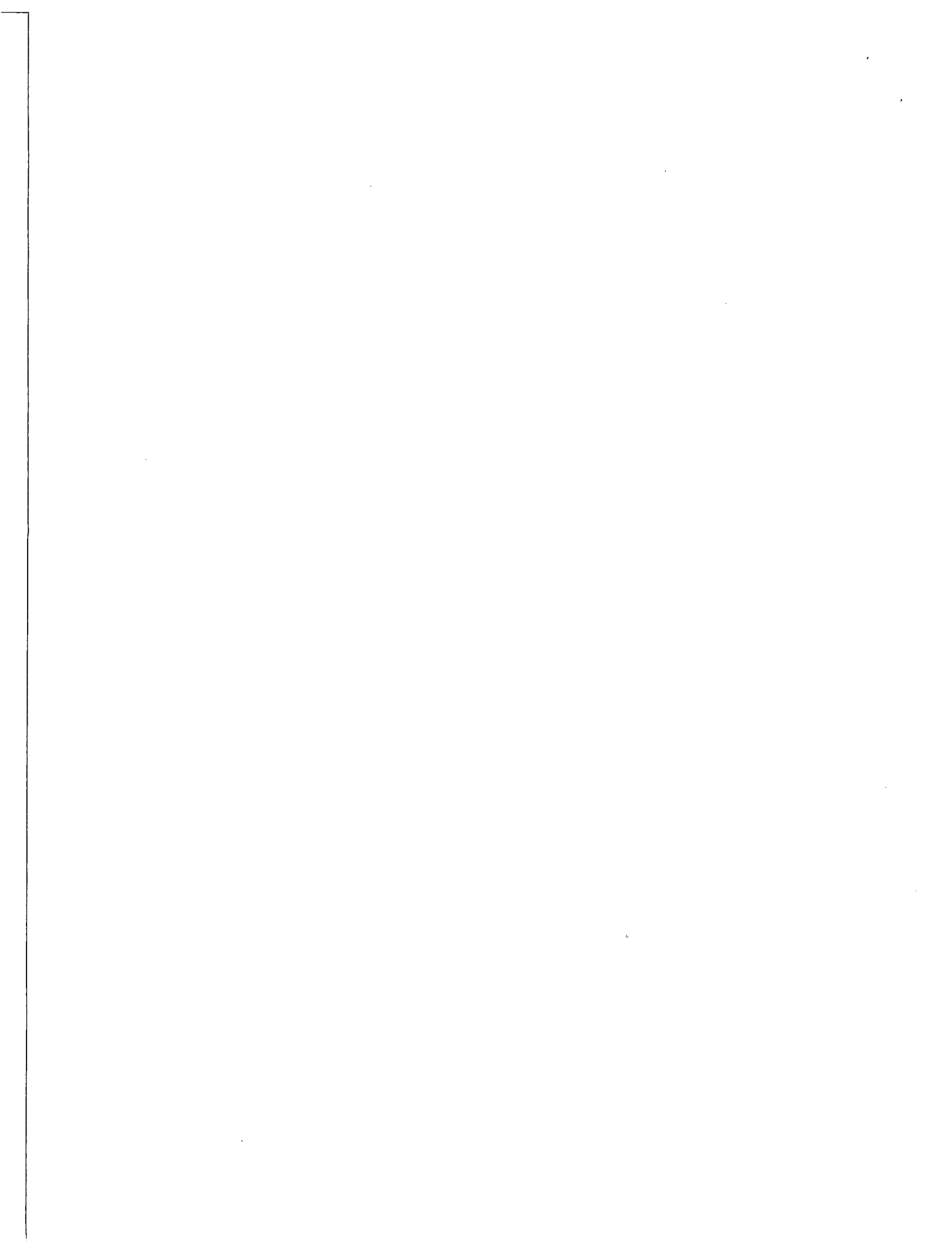
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## NOMENCLATURE

$u', v'$	velocity components in the x and y directions, respectively
$u, v$	dimensionless velocity component in the x and y directions, respectively
$g$	gravitational acceleration
$H$	depth of the reservoir floor
$L$	width of the reservoir
$\lambda_m$	thermal conductivity of the porous medium
$\lambda_{cap}$	thermal conductivity of the cap rock
$\psi'$	stream function
$\psi$	dimensionless stream function
$\theta'$	temperature referenced to $T_0$ ( $= T - T_0$ )
$\theta$	dimensionless temperature
$\Delta T$	maximum temperature referenced to $T_0$ ( $= T_{max} - T_0$ )
$x', y'$	Cartesian coordinates
$x, y$	dimensionless coordinates
$(\rho c)_m$	heat capacity of porous medium
$(\rho c)_f$	heat capacity of fluid
$Ra$	Rayleigh number
$K_x$	permeability in x direction
$K_y$	permeability in y direction
$\mu$	viscosity of fluid
$\rho$	density of fluid
$\beta$	thermal expansion coefficient
$\alpha_m$	thermal diffusivity of fluid porous medium
$t'$	time
$t$	dimensionless time
$\chi$	permeability ratio $K_y/K_x$
$\gamma$	heat capacity ratio $(\rho c)_f/(\rho c)_m$
$\eta$	dimensionless measurement from reservoir floor to cap rock
$\rho_0$	density of fluid at $T = T_0$
$Q$	surface heat flow
HFU	heat flow unit

### Subscripts

f	fluid
m	country rock
i	index in x direction
j	index in y direction

### Superscript

k	index in arbitrary time step k
2n + 1	index in (2n+1) <sup>th</sup> time step

EFFECT OF PERMEABILITY ON COOLING OF  
A MAGMATIC INTRUSION IN A GEOTHERMAL RESERVOIR

ABSTRACT

This report describes numerical modeling of the transient cooling of a magmatic intrusion in a geothermal reservoir that results from conduction and convection, considering the effects of overlying cap rock and differing horizontal and vertical permeabilities of the reservoir. These results are compared with data from Salton Sea Geothermal Field (SSGF). Multiple layers of convection cells are observed when horizontal permeability is much larger than vertical permeability. The sharp drop-off of surface heat flow experimentally observed at SSGF is consistent with the numerical results. We estimate the age of the intrusive body at SSGF to be between 6000 and 20,000 years.

INTRODUCTION

Because hydrothermal systems of a particular geothermal field are important in all aspects of geothermal power production, geophysicists and geothermal reservoir engineers are greatly interested in magmatic intrusions in the earth's crust. These intrusions, also known as plutons, are cooled by surrounding country rock. If the neighboring formations are permeable and saturated with ground water, then convective hydrothermal systems can result. The nature of these hydrothermal systems is determined by the physical properties of the surrounding formations.

Intrusive magma can take different forms or sizes. A sheet-like intrusive body--perpendicular to the stratification in the bedded rocks--is called a dike. Jaeger<sup>1</sup> and Horai<sup>2</sup> studied dike intrusion based on heat conduction alone. Recent studies<sup>3-5</sup> suggest that convection of ground water also plays an important role in heat transfer in geothermal fields.

Numerical modeling studies of dike-induced convection flow include the work of Lau and Cheng<sup>3</sup> on the effects of dike intrusion on steady-state temperature distribution, streamlines, and shape of water table in a volcanic

island aquifer. Norton and Knight<sup>4</sup> researched the time dependence of convective circulation and its influence on the cooling rate of massive plutons. Torrance and Sheu<sup>5</sup> studied the cooling of a pluton by assuming that the intrusion itself becomes permeable below a specified thermal stress-cracking temperature.

In all of these referenced studies, the permeability is assumed constant, and the existence of cap rock is not included in the analysis. Kasameyer and Younker<sup>6</sup> suggested that the cap rock and a large horizontal-to-vertical permeability ratio can be responsible for the dramatic reduction in geothermal gradient in the Salton Sea Geothermal Field (SSGF).

The present study of the cooling of a magmatic intrusion because of natural convection takes into account the effects of overlying cap rock of various thicknesses as well as of differing horizontal and vertical permeabilities in the reservoir. Results are specifically related to the SSGF. Figure 1 shows an idealized model.

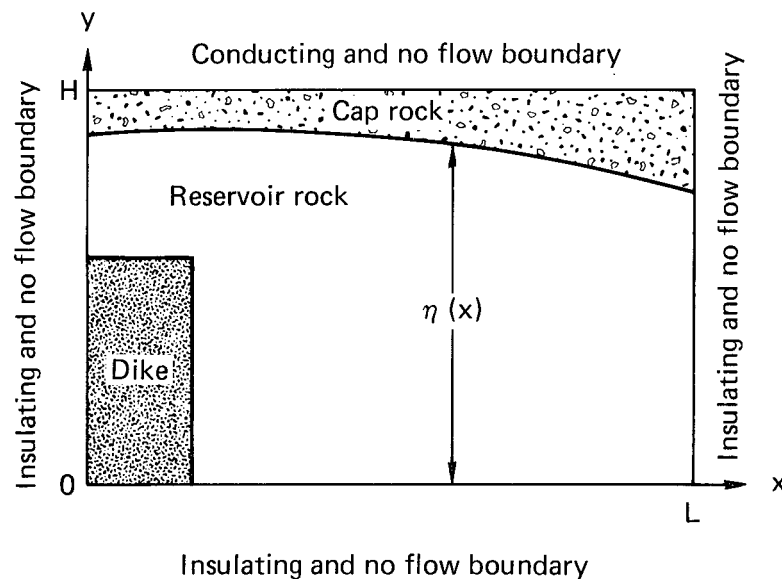


FIG. 1. Idealized model of a geothermal reservoir with dike intrusion.

## DESCRIPTION OF MODELING PROCESS

### GOVERNING EQUATIONS

The governing equations for the hydrothermal system in a porous medium are the continuity equation, Darcy's law, the energy equation, and the equation of state. With the Boussinesq approximation, these equations can be written as

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad , \quad (1)$$

$$u' = \left( \frac{-K_x}{\mu} \right) \left( \frac{\partial p'}{\partial x'} \right) \quad , \quad (2)$$

$$v' = \frac{-K_y}{\mu} \left( \frac{\partial p'}{\partial y'} + \rho g \right) \quad , \quad (3)$$

$$(\rho c)_m \frac{\partial \theta'}{\partial t'} + (\rho c)_f \left( u' \frac{\partial \theta'}{\partial x'} + v' \frac{\partial \theta'}{\partial y'} \right) = \left( \lambda_m \frac{\partial^2 \theta'}{\partial x'^2} + \frac{\partial^2 \theta'}{\partial y'^2} \right) \quad , \quad (4)$$

$$\rho = \rho_0 (1 - \beta \theta') \quad . \quad (5)$$

When one introduces the stream function  $\psi'$  and the following dimensionless variables,

$$u' = \frac{\partial \psi'}{\partial y'} \quad , \quad (6)$$

$$v' = \frac{\partial \psi'}{\partial x'} \quad , \quad (7)$$

$$t = \frac{\alpha_m}{H^2} t' \quad , \quad (8)$$

$$x = \frac{x'}{H} \quad , \quad (9)$$

$$y = \frac{y'}{H} \quad , \quad (10)$$

$$\theta = \frac{\theta'}{\Delta T} \quad , \quad (11)$$

$$u = \frac{u'H}{\alpha_m} , \quad (12)$$

$$v = \frac{v'H}{\alpha_m} , \quad (13)$$

$$\psi = \frac{\psi'}{\alpha_m} , \quad (14)$$

$$Ra = \frac{\rho_0 \beta g K_y H \Delta T}{\mu \alpha_m} , \quad (15)$$

the nondimensional form of the governing equations becomes

$$\frac{\partial \theta}{\partial t} + \gamma \left( \frac{\partial \psi}{\partial y} \right) \left( \frac{\partial \theta}{\partial x} \right) - \gamma \left( \frac{\partial \psi}{\partial x} \right) \left( \frac{\partial \theta}{\partial y} \right) = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} , \quad (16)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \chi \frac{\partial^2 \psi}{\partial y^2} = -Ra \frac{\partial \theta}{\partial x} . \quad (17)$$

#### BOUNDARY AND INITIAL CONDITIONS

The initial conditions of the problem are  $\psi = 0$  and  $\theta = 0$  everywhere in the region except in the intrusive area, where  $\theta = 1$ . The boundary condition at the surface is a constant temperature; i.e.,

$$\theta(x,1) = 0 . \quad (18)$$

The boundaries at  $x = 0$  and  $L/H$  are impermeable to flow and thermally nonconductive; i.e.,

$$\frac{\partial \psi}{\partial x} (0,y) = \frac{\partial \psi}{\partial x} \left( \frac{L}{H}, y \right) = 0 , \quad (19)$$

$$\psi(0,y) = \psi \left( \frac{L}{H}, y \right) = 0 . \quad (20)$$

The boundaries beneath the cap rock are impermeable to flow and thermally conductive; i.e.,

$$\psi(x, \eta) = 0, \quad (21)$$

$$\lambda_{\text{cap}} \frac{\partial \theta}{\partial y}(x, \eta) = \lambda_m \frac{\partial \theta}{\partial y}(x, \eta). \quad (22)$$

It is assumed that  $\lambda_{\text{cap}} = \lambda_m$  so that Eq. (16) applies to both the cap rock and the permeable regions.

The boundaries at  $y = 0$  are impermeable to flow and thermally nonconductive; i.e.,

$$\psi(x, 0) = 0, \quad (23)$$

and

$$\frac{\partial \theta}{\partial y}(x, 0) = 0. \quad (24)$$

#### NUMERICAL METHOD

The energy equation (16) is solved numerically by the Alternating Direction Implicit (ADI) method,<sup>7</sup> and the flow field equation (17) by the Gauss-Seidel iteration method. The region is divided into a uniform mesh, as shown in Fig. 2. The coordinates of the grid points are given by  $(x_i, y_j)$ , where  $x_i = (i-1)\Delta x$  and  $y_j = (j-1)\Delta y$ .

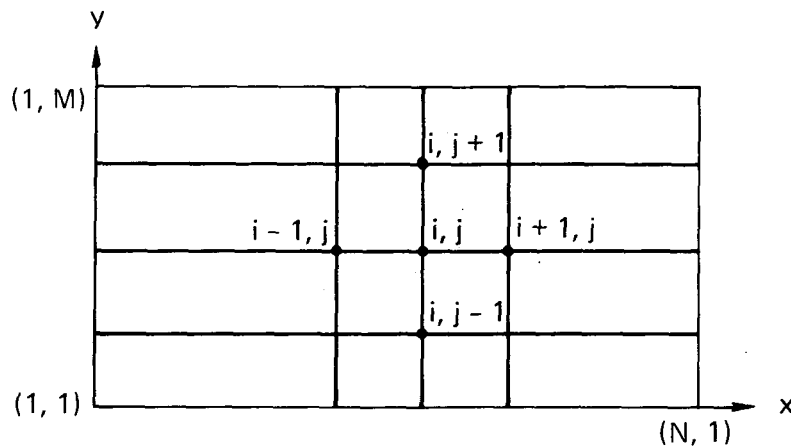


FIG. 2. Uniform mesh for the finite difference numerical solution.

A second-order finite-difference approximation formula is used for all spatial derivatives and a first-order finite-difference approximation for all time derivatives. The upwind scheme for the convection term is not used, but the numerical formulation can be easily adapted to the upwind scheme.

The ADI formulation of the energy equation (16) follows. First, the finite difference approximation for  $(2n+1)$ th time step is given as

$$\begin{aligned} \frac{\theta_{i,j}^{2n+1} - \theta_{i,j}^{2n}}{\Delta t} + u_{i,j}^{2n} \left( \frac{\theta_{i+1,j}^{2n+1} - \theta_{i-1,j}^{2n+1}}{2\Delta x} \right) + v_{i,j}^{2n} \left( \frac{\theta_{i,j+1}^{2n} - \theta_{i,j-1}^{2n}}{2\Delta y} \right) \\ = \frac{\theta_{i+1,j}^{2n+1} - 2\theta_{i,j}^{2n+1} + \theta_{i-1,j}^{2n+1}}{(\Delta x)^2} + \frac{\theta_{i,j+1}^{2n} - 2\theta_{i,j}^{2n} + \theta_{i,j-1}^{2n}}{(\Delta y)^2}, \end{aligned} \quad (25)$$

where

$$u_{i,j}^{2n} = \gamma \left( \frac{\psi_{i,j+1}^{2n} - \psi_{i,j-1}^{2n}}{2\Delta y} \right), \quad (26)$$

$$v_{i,j}^{2n} = -\gamma \left( \frac{\psi_{i+1,j}^{2n} - \psi_{i-1,j}^{2n}}{2\Delta x} \right). \quad (27)$$

Equation (25) can be rewritten as

$$\begin{aligned} \left( -\frac{u_{i,j}^{2n}}{2\Delta x} - \frac{1}{(\Delta x)^2} \right) \theta_{i-1,j}^{2n+1} + \left( \frac{1}{\Delta t} + \frac{2}{(\Delta x)^2} \right) \theta_{i,j}^{2n+1} \\ + \left( \frac{u_{i,j}^{2n}}{2\Delta x} - \frac{1}{(\Delta x)^2} \right) \theta_{i+1,j}^{2n+1} = \frac{1}{\Delta t} \theta_{i,j}^{2n} \\ - v_{i,j}^{2n} \left( \frac{\theta_{i,j+1}^{2n} - \theta_{i,j-1}^{2n}}{2\Delta y} \right) + \frac{\theta_{i,j+1}^{2n} - 2\theta_{i,j}^{2n} + \theta_{i,j-1}^{2n}}{(\Delta y)^2}. \end{aligned} \quad (28)$$

Equation (28) is valid for all grid points. At the boundary, both Eq. (28) and appropriate boundary conditions must be satisfied. We will now describe the finite difference equation for each boundary surface.



At the vertical boundary  $x = 0$  (i.e.,  $i = 1$ ), the condition  $\partial\theta/\partial x = 0$  requires that

$$\theta_{0,j}^k = \theta_{2,j}^k \quad (29)$$

Note that  $\theta_{0,j}^k$  is a grid point outside of the region of interest at any time step  $k$ . With the aid of Eq. (29), Eq. (28) becomes

$$\begin{aligned} \left( \frac{1}{\Delta t} + \frac{2}{(\Delta x)^2} \right) \theta_{1,j}^{2n+1} - \frac{2}{(\Delta x)^2} \theta_{2,j}^{2n+1} &= \frac{1}{\Delta t} \theta_{1,j}^{2n} \\ &- v_{1,j}^{2n} \left( \frac{\theta_{1,j+1}^{2n} - \theta_{1,j-1}^{2n}}{2\Delta y} \right) + \frac{\theta_{1,j+1}^{2n} - 2\theta_{1,j}^{2n} + \theta_{1,j-1}^{2n}}{(\Delta y)^2} \end{aligned} \quad (30)$$

for  $2 \leq j \leq M - 1$  .

At the vertical boundary  $x = L/H$  (i.e.,  $i = N$ ), the condition  $\partial\theta/\partial x = 0$  requires that

$$\theta_{N+1,j}^k = \theta_{N-1,j}^k \quad (31)$$

Combining Eqs. (31) and (28), we obtain

$$\begin{aligned} - \frac{2}{(\Delta x)^2} \theta_{N-1,j}^{2n+1} + \left( \frac{1}{\Delta t} + \frac{2}{(\Delta x)^2} \right) \theta_{N,j}^{2n+1} &= \frac{1}{\Delta t} \theta_{N,j}^{2n} \\ &- v_{N,j}^{2n} \left( \frac{\theta_{N,j+1}^{2n} - \theta_{N,j-1}^{2n}}{2\Delta y} \right) + \left( \frac{\theta_{N,j+1}^{2n} - 2\theta_{N,j}^{2n} + \theta_{N,j-1}^{2n}}{(\Delta y)^2} \right) \end{aligned} \quad (32)$$

for  $2 \leq j \leq M - 1$  .

At the lower boundary  $y = 0$  (i.e.,  $j = 1$ ), the boundary condition  $\partial\theta/\partial y = 0$  requires that

$$\theta_{i,0}^k = \theta_{i,2}^k \quad (33)$$

Combining Eqs. (33) and (28), we obtain

$$\begin{aligned} \left( -\frac{u_{i,1}^{2n}}{2\Delta x} - \frac{1}{(\Delta x)^2} \right) \theta_{i-1,1}^{2n+1} + \frac{1}{\Delta t} + \left( \frac{2}{(\Delta x)^2} \right) \theta_{i,1}^{2n+1} \\ + \left( \frac{u_{i,1}^{2n}}{2\Delta x} - \frac{1}{(\Delta x)^2} \right) \theta_{i+1,1}^{2n+1} = \frac{1}{\Delta t} \theta_{i,1}^{2n} \\ + \frac{2}{(\Delta y)^2} \left( \theta_{i,2}^{2n} - \theta_{i,1}^{2n} \right) \end{aligned} \quad (34)$$

for  $2 \leq i \leq N - 1$  .

At  $y = 1$ , the boundary condition is

$$\theta_{i,M}^k = 0 \quad (35)$$

for  $1 \leq i \leq N$  .

Equations (28), (29), and (33) lead to

$$\left( \frac{1}{\Delta t} + \frac{2}{(\Delta x)^2} \right) \theta_{1,1}^{2n+1} - \frac{2}{(\Delta x)^2} \theta_{2,1}^{2n+1} = \frac{1}{\Delta t} \theta_{1,1}^{2n} + \frac{2}{(\Delta y)^2} \left( \theta_{1,2}^{2n} - \theta_{1,1}^{2n} \right). \quad (36)$$

Equations (28), (31), and (33) lead to

$$\begin{aligned} -\frac{2}{(\Delta x)^2} \theta_{N-1,1}^{2n+1} + \left( \frac{1}{\Delta t} + \frac{2}{(\Delta x)^2} \right) \theta_{N,1}^{2n+1} = \frac{1}{\Delta t} \theta_{N,1}^{2n} \\ + \frac{2}{(\Delta y)^2} \left( \theta_{N,2}^{2n} - \theta_{N,1}^{2n} \right). \end{aligned} \quad (37)$$

Equations (28), (30), (32), (34), (36), and (37) consist of  $M - 1$  sets of  $N$  simultaneous equations of the form

$$B_{1,j}^{2n+1} + C_{1,j} \theta_{2,j}^{2n+1} = D_{1,j} \quad (38)$$

for  $1 \leq j \leq M - 1$  ,

$$A_{i,j} \theta_{i-1,j}^{2n+1} + B \theta_{i,j}^{2n+1} + C_{i,j} \theta_{i+1,j}^{2n+1} = D_{i,j} \quad (39)$$

for  $2 \leq i \leq N-1, 1 \leq j \leq M-1$  ,

$$A_{N,j} \theta_{N-1,j}^{2n+1} + B \theta_{N,j}^{2n+1} = D_{N,j} \quad (40)$$

for  $1 \leq j \leq M-1$  ,

where

$$B = \frac{1}{\Delta t} + \frac{2}{(\Delta x)^2} \quad , \quad (41)$$

$$C_{1,j} = \frac{2}{(\Delta x)^2} \quad (42)$$

for  $1 \leq j \leq M-1$  ,

$$C_{i,j} = \frac{u_{i,j}^{2n}}{2\Delta x} - \frac{1}{(\Delta x)^2} \quad (43)$$

for  $2 \leq i \leq N, 1 \leq j \leq M-1$  ,

$$A_{i,j} = -\frac{u_{i,j}^{2n}}{2\Delta x} - \frac{1}{(\Delta x)^2} \quad (44)$$

for  $1 \leq j \leq M-1, 1 \leq i \leq N-1$  ,

$$A_{N,j} = \frac{-2}{(\Delta x)^2} \quad (45)$$

for  $1 \leq j \leq M-1$  ,

$$D_{i,j} = \frac{1}{\Delta t} \theta_{i,j}^{2n} - v_{i,j}^{2n} \left( \frac{\theta_{i,j+1}^{2n} - \theta_{i,j-1}^{2n}}{2\Delta y} \right) + \frac{\theta_{i,j+1}^{2n} - 2\theta_{i,j}^{2n} + \theta_{i,j-1}^{2n}}{(\Delta y)^2} \quad (46)$$

for  $2 \leq j \leq M-1, 1 \leq i \leq N-1$  ,

$$D_{i,1} = \frac{1}{\Delta t} \theta_{i,1}^{2n} + \frac{2}{(\Delta y)^2} \left( \theta_{i,2}^{2n} - \theta_{i,1}^{2n} \right) \quad (47)$$

for  $1 \leq i \leq N - 1$  .

The solution of Eqs. (38), (39), and (40) can be obtained in a straightforward manner.<sup>7</sup> Let

$$w_1 = B \quad , \quad (48)$$

$$w_i = B - A_{i,j} B_{i-1} \quad (49)$$

for  $2 \leq i \leq N$ ,  $1 \leq j \leq M - 1$  ,

$$b_i = \frac{C_{i,j}}{w_i} \quad (50)$$

for  $1 \leq j \leq M - 1$ ,  $1 \leq i \leq N - 1$  ,

$$g_1 = \frac{D_{1,j}}{w_1} \quad (51)$$

for  $1 \leq j \leq M - 1$  ,

$$g_i = \frac{D_{i,j} - A_{i,j} g_{i-1}}{w_i} \quad (52)$$

for  $2 \leq i \leq N - 1$ ,  $1 \leq j \leq M - 1$  .

The solutions of the tridiagonal system are

$$\theta_{N,j}^{2n+1} = g_N \quad (53)$$

for  $1 \leq j \leq M - 1$  ,

$$\theta_{i,j}^{2n+1} = g_i - b_i \theta_{i+1,j}^{2n+1} \quad (54)$$

for  $1 \leq i \leq N - 1$ ,  $1 \leq j \leq M - 1$  .

The computational procedure used to obtain solutions of the tridiagonal system for each set of the N simultaneous equations is the following. For a given j (j<sup>th</sup> set of equations where j is from 1 to M - 1), Eqs. (48) through (54) are computed with ascending value of i from 1 to N. After Eqs. (48) through (54) are evaluated, proceed to evaluate Eqs. (54) and (55) with decreasing value of i from N to 1. The values of the temperature function are stored in temporary storage location to allow evaluation of Eqs. (48) through (54) at previous time step temperature values.

The difference equation for Eq. (16) at (2n+2)<sup>th</sup> time step is given as

$$\begin{aligned} \frac{\theta_{i,j}^{2n+2} - \theta_{i,j}^{2n+1}}{\Delta t} + u_{i,j}^{2n+1} \left( \frac{\theta_{i+1,j}^{2n+1} - \theta_{i-1,j}^{2n+1}}{2\Delta x} \right) \\ + v_{i,j}^{2n+1} \left( \frac{\theta_{i,j+1}^{2n+2} - \theta_{i,j-1}^{2n+2}}{2\Delta y} \right) = \frac{\theta_{i+1,j}^{2n+1} - 2\theta_{i,j}^{2n+1} + \theta_{i-1,j}^{2n+1}}{(\Delta x)^2} \\ + \frac{\theta_{i,j+1}^{2n+2} - 2\theta_{i,j}^{2n+2} + \theta_{i,j-1}^{2n+2}}{(\Delta y)^2} \end{aligned} \quad (55)$$

Equation (55) can be rewritten as

$$\begin{aligned} \left( -\frac{v_{i,j}^{2n+1}}{2\Delta y} - \frac{1}{(\Delta y)^2} \right) \theta_{i,j-1}^{2n+2} + \left( \frac{1}{\Delta t} + \frac{2}{(\Delta y)^2} \right) \theta_{i,j}^{2n+2} \\ + \frac{v_{i,j}^{2n+1}}{2\Delta y} - \frac{1}{(\Delta y)^2} \theta_{i,j+1}^{2n+2} = \frac{1}{\Delta t} \theta_{i,j}^{2n+1} - u_{i,j}^{2n+1} \left( \frac{\theta_{i+1,j}^{2n+1} - \theta_{i-1,j}^{2n+1}}{2\Delta x} \right) \\ + \frac{\theta_{i+1,j}^{2n+1} - 2\theta_{i,j}^{2n+1} + \theta_{i-1,j}^{2n+1}}{(\Delta x)^2} \end{aligned} \quad (56)$$

for  $2 \leq i \leq N - 1$ ,  $2 \leq j \leq M - 1$  .

Equation (56), when combined with boundary conditions (29), (31), (33), and (35), results in the following equations:

$$\begin{aligned} & \left( -\frac{v_{1,j}^{2n+1}}{2\Delta y} - \frac{1}{(\Delta y)^2} \right) \theta_{1,j-1}^{2n+2} + \left( \frac{1}{\Delta t} + \frac{2}{(\Delta y)^2} \right) \theta_{1,j}^{2n+2} \\ & + \left( \frac{v_{1,j}^{2n+1}}{2\Delta y} - \frac{1}{(\Delta y)^2} \right) \theta_{1,j+1}^{2n+2} = \frac{1}{\Delta t} \theta_{1,j}^{2n+1} + \frac{2}{(\Delta x)^2} \left( \theta_{2,j}^{2n+1} - \theta_{1,j}^{2n+1} \right) \end{aligned} \quad (57)$$

for  $2 \leq j \leq M - 1$  ,

$$\begin{aligned} & \left( -\frac{v_{N,j}^{2n+1}}{2\Delta y} - \frac{1}{(\Delta y)^2} \right) \theta_{N,j-1}^{2n+2} + \left( \frac{1}{\Delta t} + \frac{2}{(\Delta y)^2} \right) \theta_{N,j}^{2n+2} \\ & + \left( \frac{v_{N,j}^{2n+1}}{2\Delta y} - \frac{1}{(\Delta y)^2} \right) \theta_{N,j+1}^{2n+2} = \frac{1}{\Delta t} \theta_{N,j}^{2n+1} + \frac{2}{(\Delta x)^2} \left( \theta_{N-1,j}^{2n+1} - \theta_{N,j}^{2n+1} \right) \end{aligned} \quad (58)$$

for  $2 \leq j \leq M - 1$  ,

$$\begin{aligned} & \left( \frac{1}{\Delta t} + \frac{2}{(\Delta y)^2} \right) \theta_{i,1}^{2n+2} - \frac{2}{(\Delta y)^2} \theta_{i,2}^{2n+2} = \frac{1}{\Delta t} \theta_{i,1}^{2n+1} \\ & - u_{i,1}^{2n+1} \left( \frac{\theta_{i+1,1}^{2n+1} - \theta_{i-1,1}^{2n+1}}{2\Delta x} \right) + \frac{\theta_{i+1,1}^{2n+1} - 2\theta_{i,1}^{2n+1} \theta_{i-1,1}^{2n+1}}{(\Delta x)^2} \end{aligned} \quad (59)$$

for  $2 \leq i \leq N - 1$  ,

$$\begin{aligned} & \left( \frac{1}{\Delta t} + \frac{2}{(\Delta y)^2} \right) \theta_{1,1}^{2n+2} - \frac{2}{(\Delta y)^2} \theta_{1,2}^{2n+2} = \frac{1}{\Delta t} \theta_{1,1}^{2n+1} \\ & + \frac{2}{(\Delta x)^2} \left( \theta_{2,1}^{2n+1} - \theta_{1,1}^{2n+1} \right) , \end{aligned} \quad (60)$$

$$\begin{aligned} & \left( \frac{1}{\Delta t} + \frac{2}{(\Delta y)^2} \right) \theta_{N,1}^{2n+2} - \frac{2}{(\Delta y)^2} \theta_{N,2}^{2n+2} = \frac{1}{\Delta t} \theta_{N,1}^{2n+1} \\ & + \frac{2}{(\Delta x)^2} \left( \theta_{N-1,1}^{2n+1} - \theta_{N,1}^{2n+1} \right) . \end{aligned} \quad (61)$$

Equations (56) through (61) consist of N sets of (M - 1) simultaneous equations of the form

$$B_{i,1} \theta_{i,1}^{2n+2} + C_{i,1} \theta_{i,2}^{2n+2} = D_{i,1} \quad (62)$$

for  $1 \leq i \leq N$  ,

$$A_{i,j} \theta_{i,j-1}^{2n+2} + B \theta_{i,j}^{2n+2} + C_{i,j} \theta_{i,j+1}^{2n+2} = D_{i,j} \quad (63)$$

for  $1 \leq i \leq N, 2 \leq j \leq M - 2$  ,

$$A_{i,M-1} \theta_{i,M-2}^{2n+2} + B \theta_{i,M-1}^{2n+2} = D_{i,M-1} \quad (64)$$

for  $1 \leq i \leq N$  ,

where

$$B = \frac{1}{\Delta t} + \frac{2}{(\Delta y)^2} , \quad (65)$$

$$C_{i,1} = \frac{-2}{(\Delta y)^2} \quad (66)$$

for  $1 \leq i \leq N$  ,

$$C_{i,j} = \frac{v_{i,j}^{2n+1}}{2\Delta y} - \frac{1}{(\Delta y)^2} \quad (67)$$

for  $1 \leq i \leq N, 2 \leq j \leq M - 1$  ,

$$A_{i,j} = -\frac{v_{i,j}^{2n+1}}{2\Delta y} - \frac{1}{(\Delta y)^2} \quad (68)$$

for  $1 \leq i \leq N, 2 \leq j \leq M - 1$  ,

$$D_{i,j} = \frac{1}{\Delta t} \theta_{i,j}^{2n+1} - u_{i,j}^{2n+1} \left( \frac{\theta_{i+1,j}^{2n+1} - \theta_{i-1,j}^{2n+1}}{2\Delta x} \right) + \frac{\theta_{i+1,j}^{2n+1} - 2\theta_{i,j}^{2n+1} + \theta_{i-1,j}^{2n+1}}{(\Delta x)^2} \quad (69)$$

for  $2 \leq i \leq N - 1, 1 \leq j \leq M - 1$  ,

$$D_{1,j} = \frac{1}{\Delta t} \theta_{1,j}^{2n+1} + \frac{2}{(\Delta x)^2} \left( \theta_{2,j}^{2n+1} - \theta_{1,j}^{2n+1} \right) \quad (70)$$

for  $1 \leq j \leq M - 1$  ,

$$D_{N,j} = \frac{1}{\Delta t} \theta_{N,j}^{2n+1} + \frac{2}{(\Delta x)^2} \left( \theta_{N-1,j}^{2n+1} - \theta_{N,j}^{2n+1} \right) \quad (71)$$

for  $1 \leq j \leq M - 1$  .

The solutions of the N sets of tridiagonal systems can be obtained in a straightforward manner. Let

$$w_1 = B \quad , \quad (72)$$

$$b_j = \frac{c_{i,j}}{w_j} \quad (73)$$

for  $1 \leq i \leq N, 1 \leq j \leq M - 2$  ,

$$w_j = B - A_{i,j} b_{j-1} \quad (74)$$

for  $1 \leq i \leq N, 2 \leq j \leq M - 1$  ,

$$g_1 = \frac{D_{i,1}}{w_i} \quad , \quad (75)$$

$$g_j = \frac{D_{i,j} - A_{i,j} g_{j-1}}{w_j} \quad (76)$$

for  $1 \leq i \leq N, 2 \leq j \leq M - 1$  .

The solutions are

$$\theta_{i,M-1}^{2n+2} = g_{M-1} \quad (77)$$

for  $1 \leq i \leq N$  ,

$$\theta_{i,j}^{2n+2} = g_j - b_j \theta_{i,j+1}^{2n+2} \quad (78)$$

for  $1 \leq j \leq M - 2, 1 \leq i \leq N$  .

A second-order finite-difference approximation for the stream function equation (17) is



$$\frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x)^2} + \chi \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y)^2} = -Ra \left( \frac{\theta_{i+1,j}^{2n+1} - \theta_{i-1,j}^{2n+1}}{2\Delta x} \right) \quad (79)$$

for  $1 \leq i \leq N - 1, 1 \leq j \leq M - 1$  .

Equation (79) can be rewritten as

$$\psi_{i,j} = \frac{1}{2(1 + \epsilon)} \left[ \psi_{i+1,j} + \psi_{i-1,j} + \epsilon\psi_{i,j+1} + \epsilon\psi_{i,j-1} + \frac{Ra (\Delta x)}{2} \left( \theta_{i+1,j}^{2n+1} - \theta_{i-1,j}^{2n+1} \right) \right] \quad (80)$$

for  $1 \leq i \leq N - 1, 1 \leq j \leq M - 1$  ,

where

$$\epsilon = \chi (\Delta y / \Delta x)^2 . \quad (81)$$

## NUMERICAL RESULTS

The flow field (stream function, velocity) is initialized to zero everywhere in the flow region. The temperature field is zero everywhere except in the region of an intrusive dike, where it is equal to 1. Figure 3 charts the numerical computation procedure, which is as follows:

1. Initial data values are set to conform with initial conditions of the problem.
2. Temperature field solutions are obtained for  $(2n+1)$ th time step using Eqs. (53) and (54).
3. The stream function equation (80) is solved by the Gauss-Seidel iteration method. The iteration is terminated when maximum change in stream function values is less than  $10^{-5}$  during two successive iteration cycles.
4. Velocity components are computed using Eqs. (26) and (27).
5. Temperature field solutions are obtained for  $(2n+2)$ th time step using Eqs. (77) and (78).
6. Steps 3 and 4 are performed again.
7. If desired, the temperature, stream function, velocity vector, and surface heat flow can be plotted.

8. If the maximum time step is reached, then the program is terminated. Otherwise a return to step 2 is required.

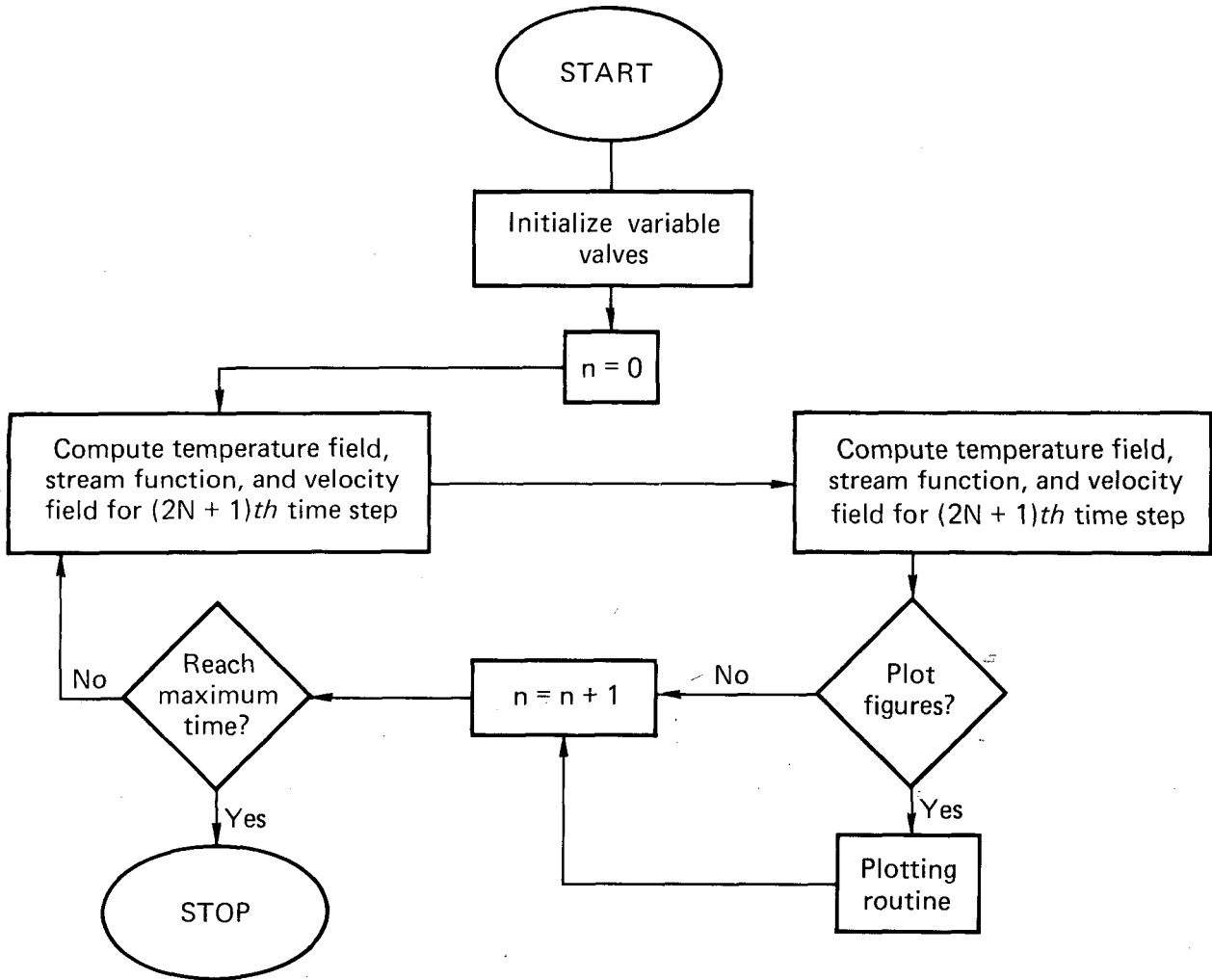


FIG. 3. Flowchart diagram of the numerical computation procedures.

The reservoir parameter values used in the numerical computation are

<u>Parameter</u>	<u>Value</u>
$K_x$ (permeability), mD	160
H (depth), m	6,000
L (width), m	12,000
$\lambda_m$ (conductivity), W/(m·K)	3.3
$\alpha_m$ (diffusivity), m <sup>2</sup> /s	$1.33 \times 10^{-6}$
$\Delta T$ (maximum temperature), K	700

The surface heat flow in terms of the dimensionless thermal gradient  $\partial\theta/\partial y$  is given by

$$Q = \lambda_m \frac{\partial\theta'}{\partial y'} = \lambda_m \frac{\Delta T}{H} \frac{\partial\theta}{\partial y} = 8.6 \frac{\partial\theta}{\partial y} \text{ (HFU)} \quad . \quad (82)$$

The relationship between real time ( $t'$ ) and dimensionless time ( $t$ ) is given by

$$t' = \frac{H^2}{\alpha_m} t = 870,000 t \text{ (in years)} \quad . \quad (83)$$

Figures 4 through 6 show the graphs of temperature, stream function, velocity vector, and surface heat flow produced by the cooling of an intrusive dike complex 1500 m in width and 3900 m in height located at the left boundary. All results were obtained with  $Ra = 200$  and time ( $t'$ ) = 10,400 y. Figure 4 is obtained with  $\chi = 2$ , Fig. 5 with  $\chi = 0.25$ , Fig. 6 with  $\chi = 0.5$ .

It is interesting to note from Figs. 4 and 5 that the surface heat flow is higher for the case of lower permeability ratio ( $\chi$ ). One can explain this by observing the flow patterns in these figures. For the case of the higher  $\chi$ , the flow is behaving like the flow near a vertical flat plate and therefore produces very little convection of heat from the top of the dike region to the surface. On the other hand the lower permeability ratio ( $\chi$ ) produces large convective flow on the top of the dike region. Figures 6 through 8 present

the effects of the dike's vertical dimension on surface heat flow. It is quite clear that the closer the top of the intrusion is to the surface, the higher the resulting surface heat flow.

Figure 9 presents the history of surface heat flow. The sharp drop-off of surface heat flow in the Salton Sea Geothermal Field (SSGF) as noted by Kasameyer and Younker<sup>6</sup> is consistent with these numerical results. Figure 10 presents the temperature contour plots at various time steps. A simple analytic model by Hanson<sup>8</sup> involving horizontal convection transport beneath a conductive cap suggests that the age of the intrusive body is between 6000 and 20,000 y, based on field data from the SSGF. Figure 9 provides more data substantiating this estimate of the age of the intrusive dike. In Fig. 11, the results indicate that when  $\chi$  is very small, multilayer convective cells exist.

The appendix contains the finite-difference heat and mass transport computer program used for the above calculations.

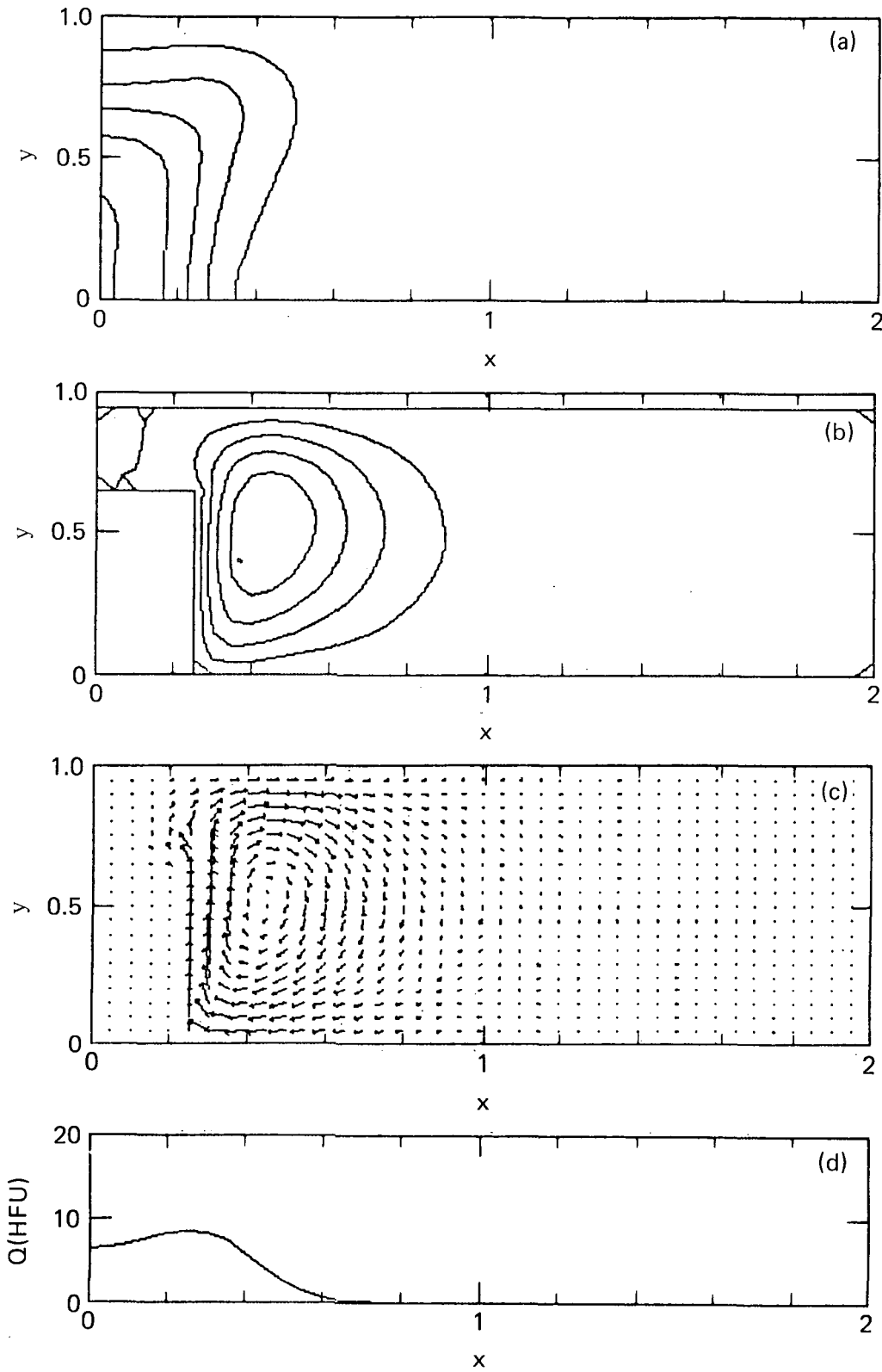


FIG. 4. Temperature (a), stream function (b), velocity (c), and surface heat flow (d) produced by cooling of intrusive dike located at left boundary.  $Ra = 200$ ,  $\chi = 2.0$ ,  $\eta = 0.9$ , and  $t = 0.012$ .

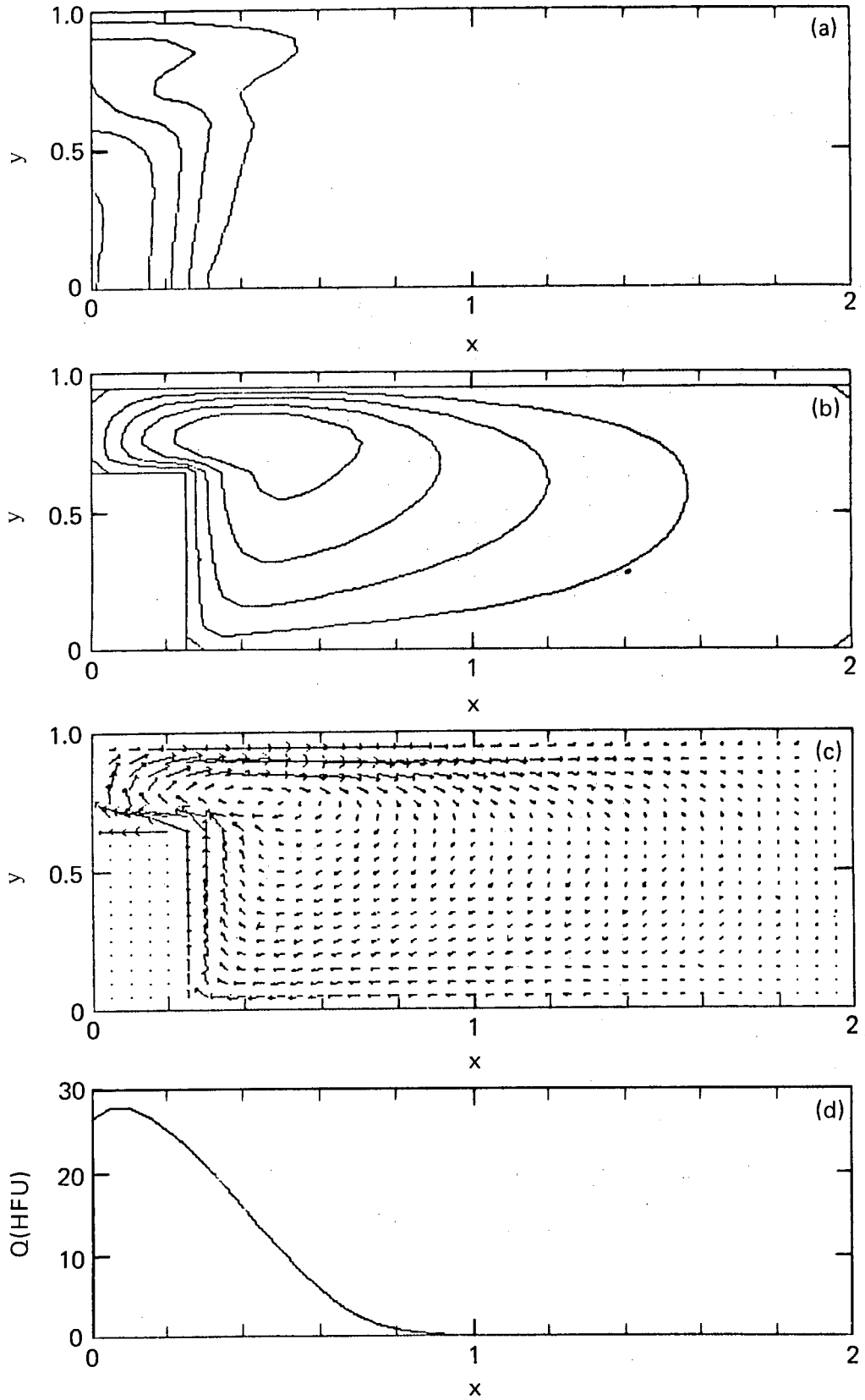


FIG. 5. Temperature (a), stream function (b), velocity (c), and surface heat flow (d) produced by cooling of intrusive dike located at left boundary.  $Ra = 200$ ,  $\chi = 0.25$ ,  $\eta = 0.9$ , and  $t = 0.012$ .

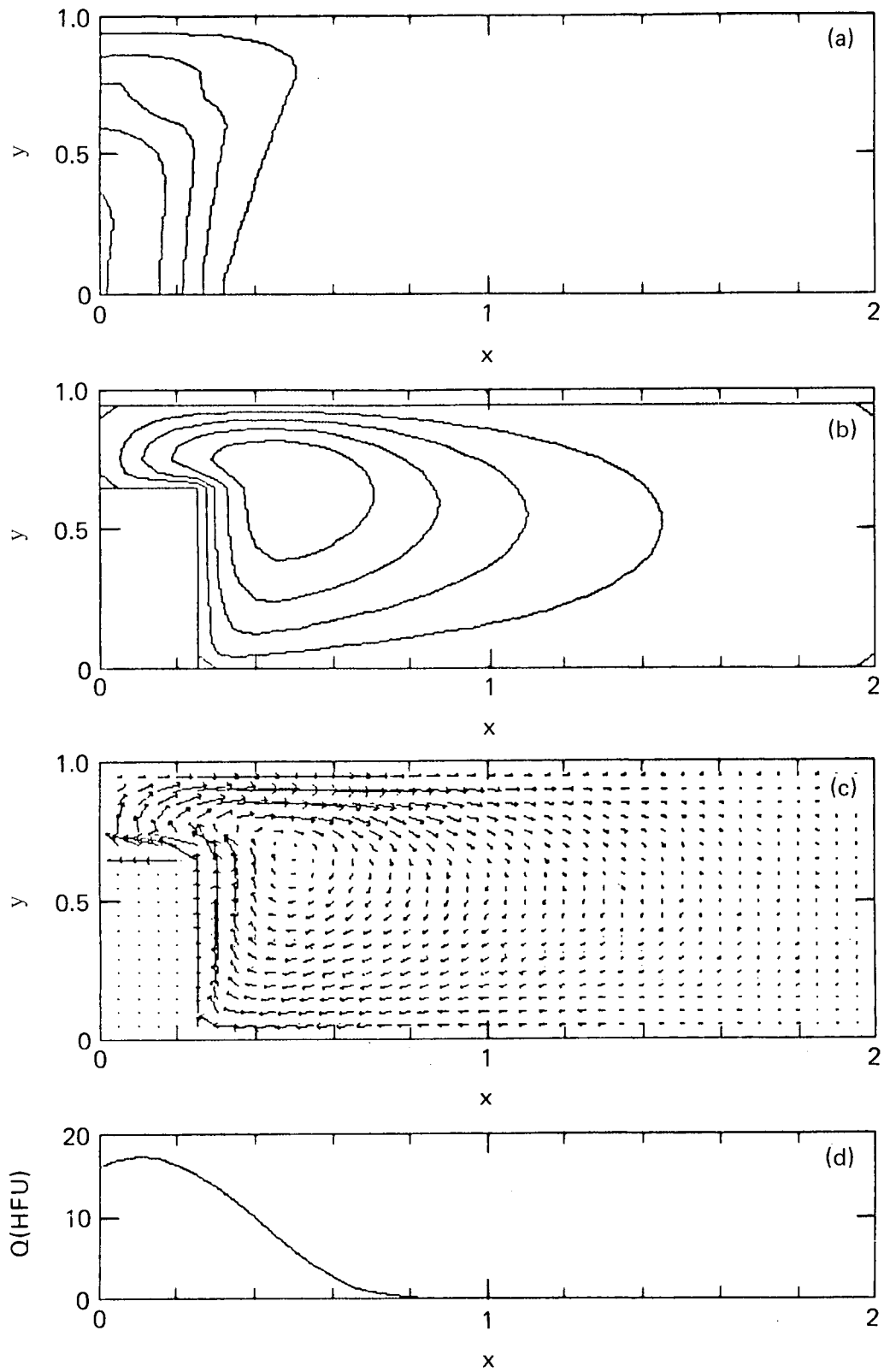


FIG. 6. Effect of the vertical dimension on temperature (a), stream function (b), velocity (c), and surface heat flow (d) during cooling of intrusive dike located at left boundary.  $Ra = 200$ ,  $\chi = 0.5$ ,  $\eta = 0.9$ , and  $t = 0.012$ .

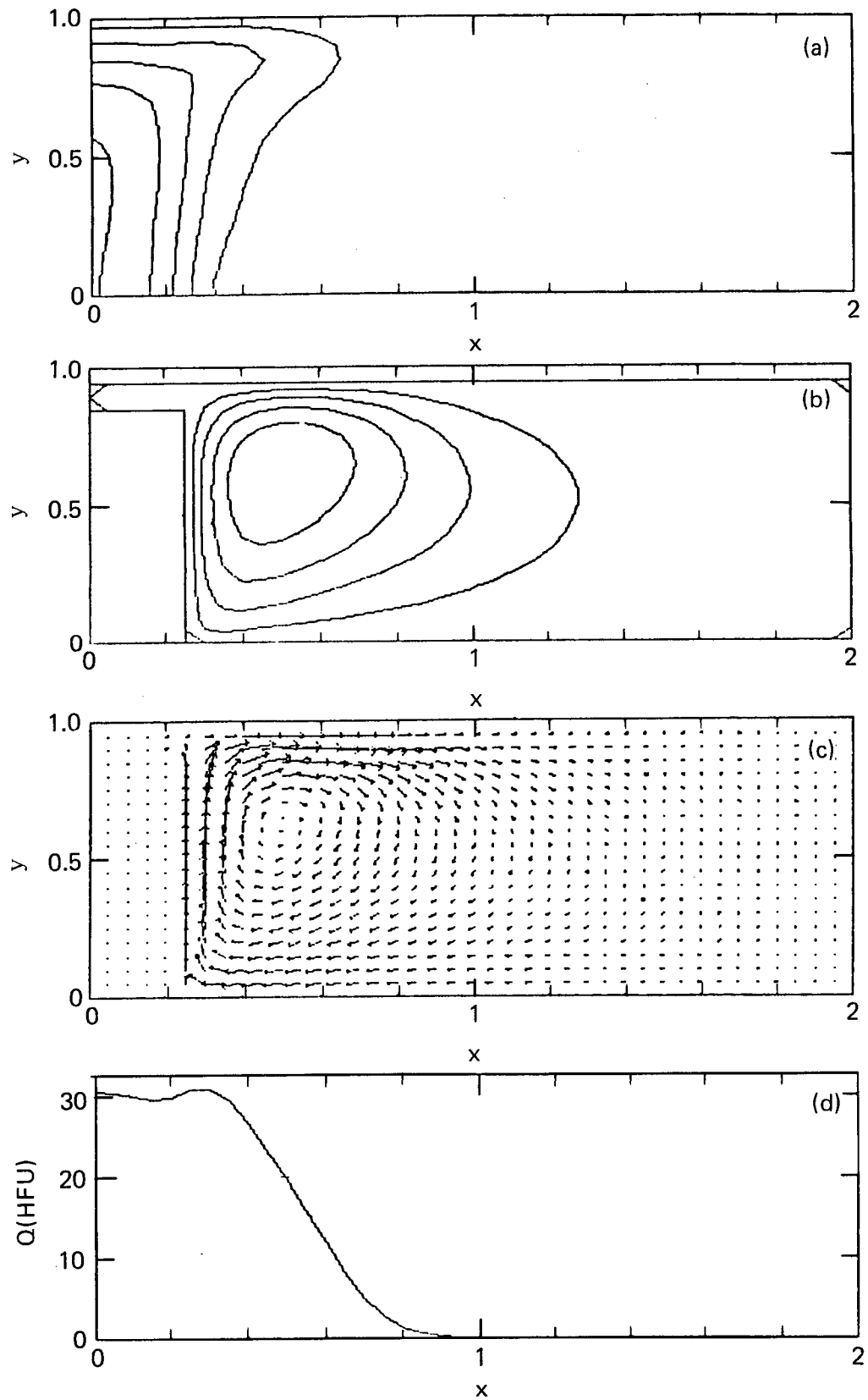


FIG. 7. Effect of the vertical dimension on temperature (a), stream function (b), velocity (c), and surface heat flow (d) during cooling of intrusive dike located at left boundary.  $Ra = 200$ ,  $\chi = 0.5$ ,  $\eta = 0.9$ , and  $t = 0.012$ . Note change in size of dike.



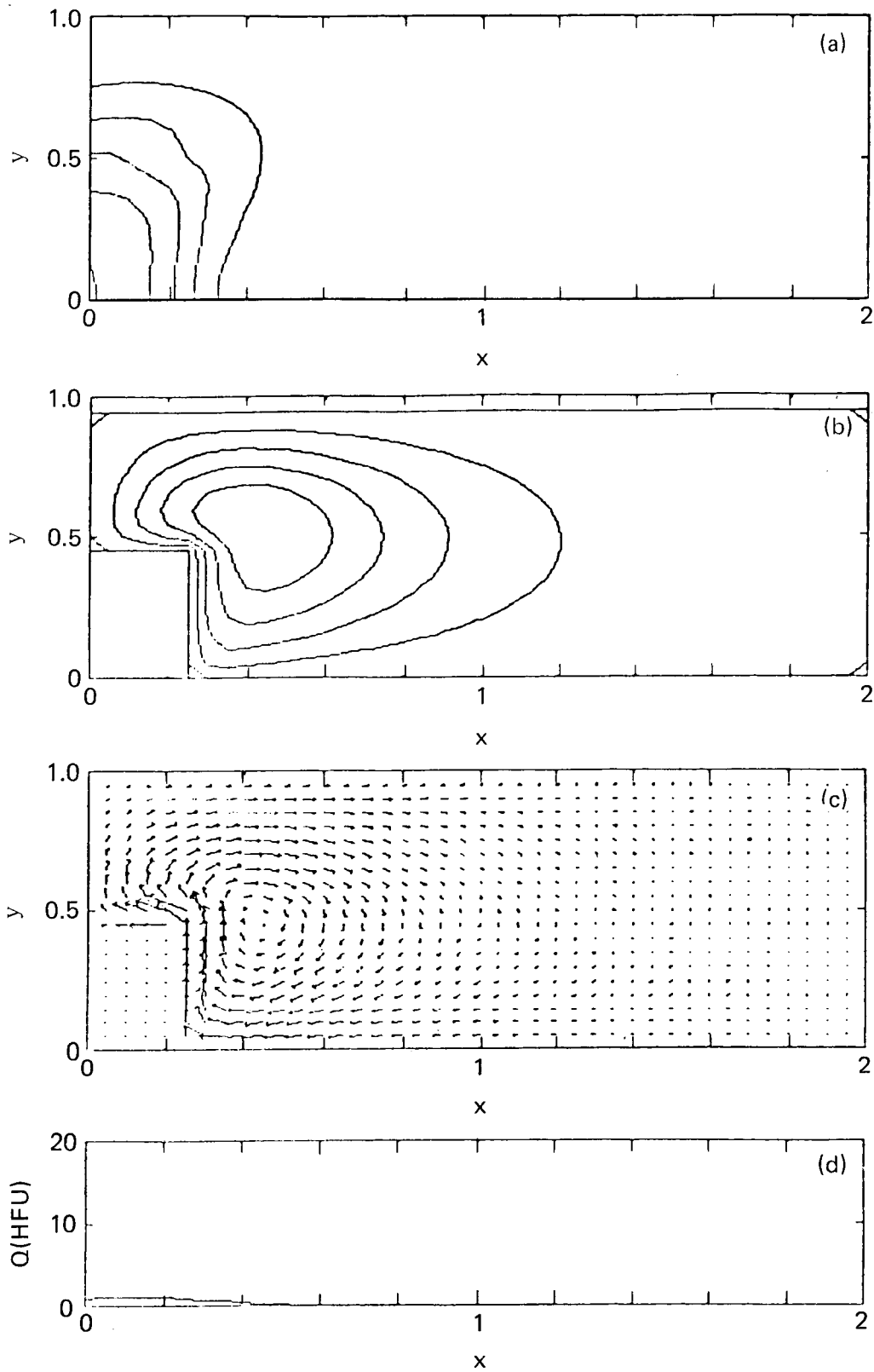


FIG. 8. Effect of the vertical dimension on temperature (a), stream function (b), velocity (c), and surface heat flow (d) during cooling of intrusive dike located at left boundary.  $Ra = 200$ ,  $\chi = 0.5$ ,  $\eta = 0.9$ , and  $t = 0.012$ . Note change in size of dike.

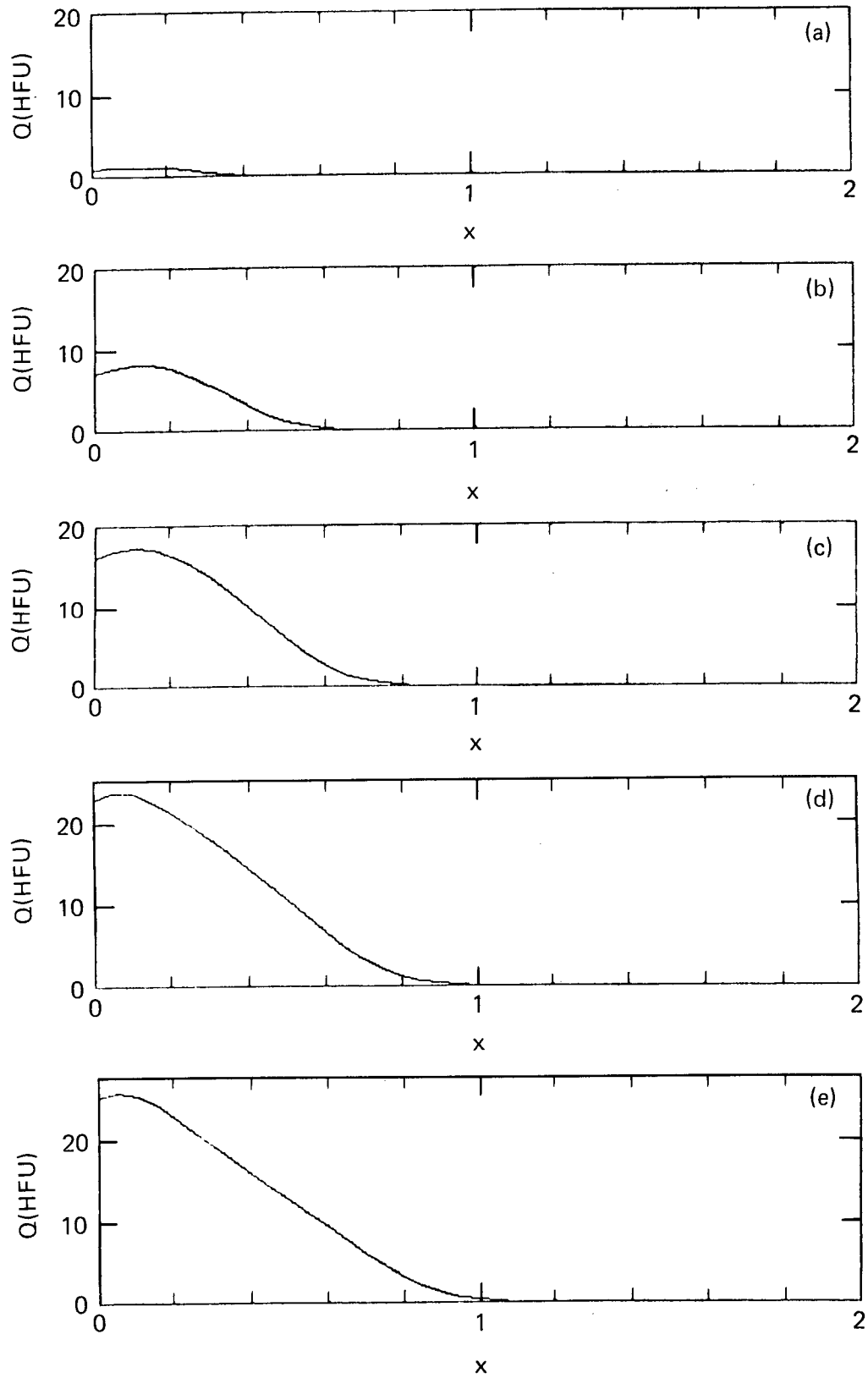


FIG. 9. History of surface heat flow for dike located at left boundary.  $Ra = 200$ ,  $\chi = 0.5$ ,  $\eta = 0.9$  with  $t = 0.004$  at (a),  $t = 0.008$  at (b),  $t = 0.012$  at (c),  $t = 0.016$  at (d), and  $t = 0.020$  at (e).

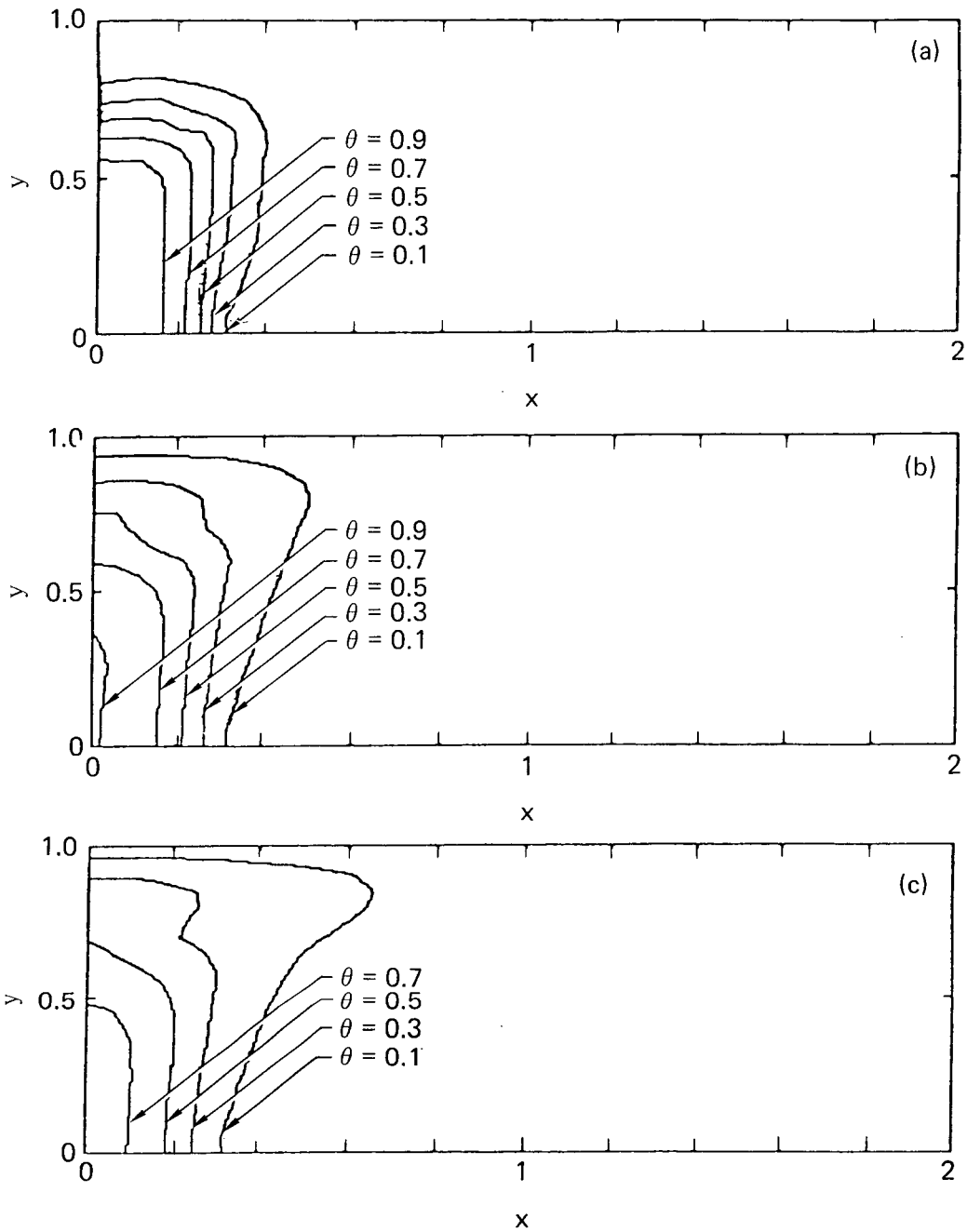


FIG. 10. Temperature contour plots for dike located at left boundary.  $Ra = 200$ ,  $\chi = 0.5$ ,  $\eta = 0.9$  with  $t = 0.004$  at (a),  $t = 0.012$  at (b), and  $t = 0.02$  at (c).

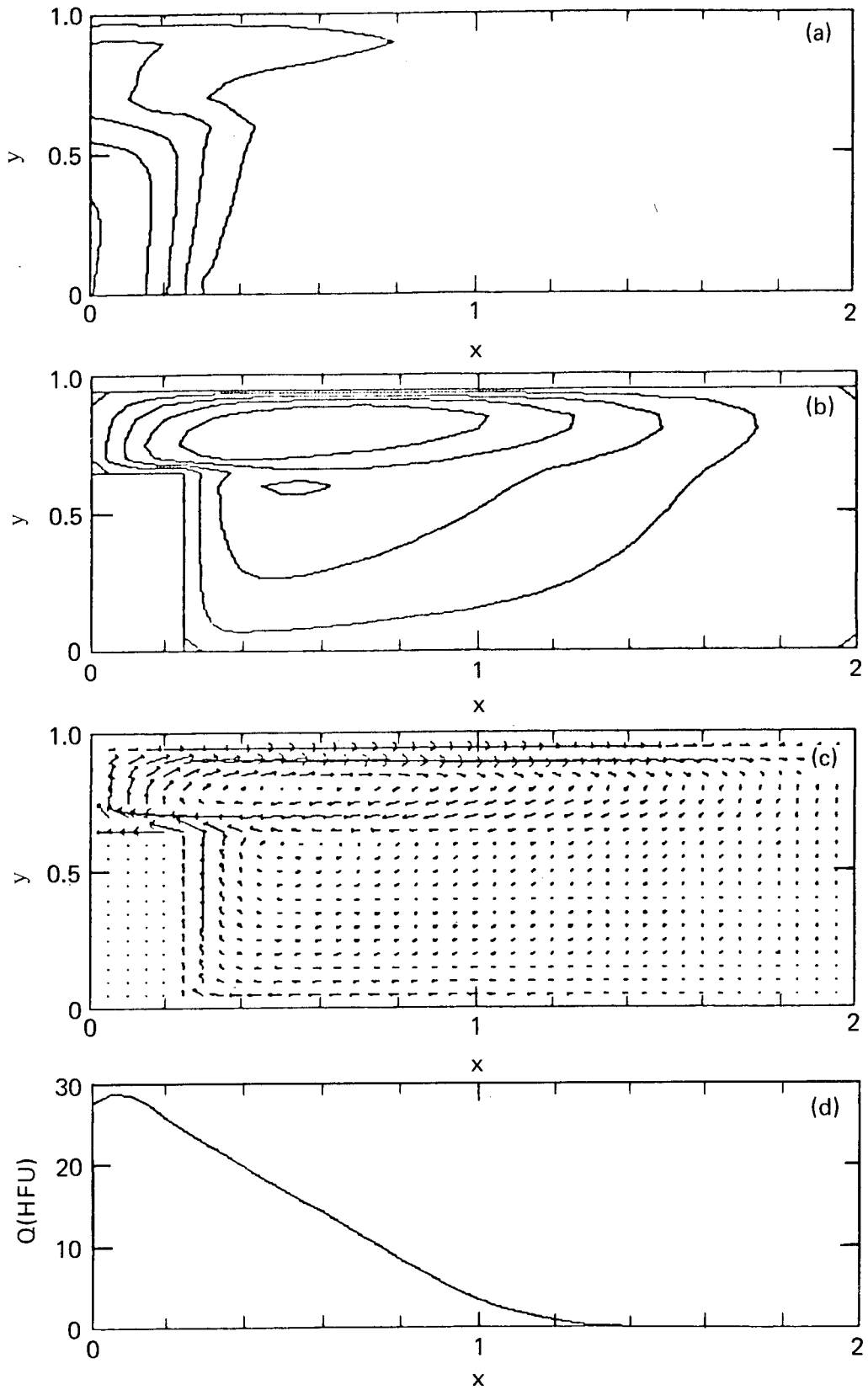


FIG. 11. Multilayer convective cells at low  $\chi$  ratio.

#### ACKNOWLEDGMENTS

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APPENDIX:  
COMPUTER PROGRAM

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VARIABLES DESCRIPTIONS
-----
T(I,J) TEMPERATURE VALUE AT GRID POINT (I,J)
S(I,J) STREAM FUNCTION VALUE AT GRID POINT (I,J)
U(I,J) VELOCITY COMPONENT IN X-DIRECTION AT GRID POINT (I,J)
V(I,J) VELOCITY COMPONENT IN Y-DIRECTION AT GRID POINT (I,J)
KRATIO VERTICAL AND HORIZONTAL PERMEABILITY RATIO
IPWS LEFT BOUNDARY OF DIKE
IPW RIGHT BOUNDARY OF DIKE
IPHI HEIGHT OF THE DIKE
IPH HEIGHT OF THE DIKE
IUH LOCATION OF THE CAP ROCK
IMAX MAXIMUM NUMBER OF POINT IN X-DIRECTION
JMAX MAXIMUM NUMBER OF POINT IN Y-DIRECTION
DELT INCREMENTAL VALUE OF EACH TIME STEP
DELX INCREMENTAL VALUE OF EACH GRID POINT IN X-DIRECTION
DELY INCREMENTAL VALUE OF EACH GRID POINT IN Y-DIRECTION
RELAX RELAXATION FACTOR USED IN STREAM FUNCTION ITERATION
R RAYLEIGH NUMBER
CYMAX MAXIMUM NUMBER OF TIME STEPS DESIRED
IPL0T NUMBER OF TIME STEPS BETWEEN TWO RJET PLOTS
-----
PROGRAM GEOTHERMAL(TAPE59,TAPE61)
REAL KRATIO
DIMENSION T(61,21),S(61,21),U(61,21),V(61,21)
DIMENSION CL(6)
DIMENSION CS(6)
DIMENSION X(61),Y(61),W(61),G(61),B(61),TS(61)
DATA T/1281*0./
DATA S/1281*0./
DATA U/1281*0./
DATA V/1281*0./
-----
STATEMENT FUNCTION USED BY ADI SOLUTION
-----
D(I,J)=DELTIN*T(I,J)-V(I,J)*(T(I,J+1)-T(I,J-1))/(2.*DELY)+
1 DELY2I*(T(I,J+1)-2.*T(I,J)+T(I,J-1))
D1(I,J)=DELTIN*T(I,J)-U(I,J)*(T(I+1,J)-T(I-1,J))/(2.*DELX) +
1 DELX2I*(T(I+1,J)-2.*T(I,J)+T(I-1,J))
-----
PROGRAM STARTS HERE
-----
CALL CHANGE("+GEOTH1")
CALL ASSIGN(61,6HPRINT1)
CALL RJETID
-----
TEMPERATURE FIELD PLOTTING LEVEL VALUES
-----

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C

CL(1)=0.1  
DO 50 I=2,6  
CL(I)=CL(I-1)+0.2  
CONTINUE

-----  
PARAMETERS OF THE PROBLEM  
-----

IPWS=1  
IPW=6  
CYMAX=20  
IPL0T=4  
IUH=20  
IPHI=14  
IPH=IPH1  
DELT=0.001  
IMAX=41  
JMAX=21  
R=200.  
KRATIO=0.01

-----  
OTHER COMPUTATIONAL CONSTANTS  
-----

IMAX1=IMAX-1  
JMAX1=JMAX-1  
IMAX2=IMAX-2  
JMAX2=JMAX-2  
TIME=0.  
ICYC=0  
DELY=1.0/FLOAT(JMAX1)  
DELX=DELY  
XMAX=IMAX1\*DELX  
DELTIN=1./DELT  
DELX2I=1./((DELX\*DELX)  
DELY2I=1./((DELY\*DELY)  
IPH1=IPH+1  
RELAX=0.8  
IUH1=IUH-1  
EPSI=KRATIO\*DELX\*DELY2I\*DELX  
RX=R\*DELX\*DELX  
SMAX=0.  
SDEL=0.  
X(1)=0.  
Y(1)=0.  
DO 11 I=2,IMAX  
X(I)=X(I-1)+DELX  
CONTINUE  
DO 12 I=2,JMAX  
Y(I)=Y(I-1)+DELY  
CONTINUE

-----  
SET INITIAL TEMPERATURE FIELD VALUES  
-----



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```
000115 C -----
000116
000117 DO 10 I=1PWS,1PW
000118 DO 10 J=1,1PHI
000119 T(I,J)=1.0
000120 10 CONTINUE
000121
000122 C -----
000123 C ITERATION LOOP STARTS HERE
000124 C -----
000125
000126 1000 CONTINUE
000127
000128 C -----
000129 C START ADI ITERATION FOR TEMPERATURE FIELD
000130 C ADI IN X-DIRECTION
000131 C FOR Y=0
000132 C -----
000133
000134 ITIME=1
000135 W(1)=DELTIN + 2.*DELX2I
000136 B(1)= -2.*DELX2I/W(1)
000137 G(1)= DELTIN * T(1,1) + 2.*DELY2I*(T(1,2)-T(1,1))
000138 G(1)=G(1)/W(1)
000139 DO 100 I=2,IMAX
000140 A=U(I,1)/(2.*DELX) + DELX2I
000141 W(1)=W(1) + A*B(I-1)
000142 B(1)=(U(I,1)/(2.*DELX)-DELX2I)/W(1)
000143 G1=DELTIN*T(I,1) + 2.*DELX2I*(T(I,2)-T(I,1))
000144 G(1)=(G1+A*G(I-1))/W(1)
000145 100 CONTINUE
000146
000147 C -----
000148 C SOLUTION FOR TEMPERATURE FIELD AT Y=0
000149 C STORE SOLUTION IN TEMPORARY STORAGE
000150 C -----
000151
000152 TS(IMAX)=G(IMAX)
000153 DO 101 I=1,IMAX1
000154 I1=IMAX-I
000155 TS(I1)=G(I1)-B(I1)*TS(I1+1)
000156 101 CONTINUE
000157
000158 C -----
000159 C SOLUTION FOR TEMPERATURE FIELD AT ALL OTHER Y
000160 C IMPLICIT SOLUTION FOR ALL OTHER Y IN X-DIRECTION
000161 C -----
000162
000163
000164 DO 105 J=2,JMAX1
000165 G(1)=D(1,J)/W(1)
000166 DO 106 I=2,IMAX
000167 A=U(I,J)/(2.*DELX) +DELX2I
000168 W(1)=W(1) + A*B(I-1)
000169 B(1)=(U(I,J)/(2.*DELX)-DELX2I)/W(1)
000170 G(1)=(D(1,J)+A*G(I-1))/W(1)
000171 106 CONTINUE
*****
```

\*\*\*\*\*

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MAIN.

```
000172
000173 C -----
000174 C STORE SOLUTION FROM TEMPORARY STORAGE IN TEMPERATURE FIELD
000175 C -----
000176
000177 DO 107 I=1,IMAX
000178 T(I,J-1)=TS(I)
000179 107 CONTINUE
000180
000181 C -----
000182 C SOLUTION IS STORED IN TEMPORARY STORAGE
000183 C -----
000184
000185 TS(IMAX)=G(IMAX)
000186 DO 108 I=1,IMAX1
000187 I1=IMAX-I
000188 TS(I1)=G(I1)-B(I1)*TS(I1+1)
000189 108 CONTINUE
000190 105 CONTINUE
000191
000192 C -----
000193 C STORE SOLUTION FROM TEMPORARY STORAGE
000194 C INTO TEMPERATURE FIELD
000195 C -----
000196
000197 DO 109 I=1,IMAX
000198 T(I,JMAX1)=TS(I)
000199 109 CONTINUE
000200 TIME=TIME+DELT
000201 ICYC=ICYC+1
000202 GO TO 305
000203
000204
000205 C -----
000206 C SECOND HALF OF ADI SOLUTION STARTS HERE
000207 C -----
000208
000209
000210 C -----
000211 C SOLUTION FOR X=0
000212 C -----
000213
000214 180 CONTINUE
000215 ITIME=2
000216 W(1)=DELTIN + 2.*DELY2I
000217 B(1)=-2.*DELY2I/W(1)
000218 G(1)=DELTIN*T(1,1) + 2.*DELY2I*(T(2,1)-T(1,1))
000219 G(1)=G(1)/W(1)
000220 DO 200 J=2,JMAX1
000221 A=V(1,J)/(2.*DELY) + DELY2I
000222 W(J)=W(1) + A*B(J-1)
000223 B(J)=(V(1,J)/(2.*DELY)-DELY2I)/W(J)
000224 G1=DELTIN*T(1,J) + 2.*DELY2I*(T(2,J)-T(1,J))
000225 G(J)=(G1 + A*G(J-1))/W(J)
000226 200 CONTINUE
000227
000228 C -----
*****
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*****      CHAT  170A   BOX W42  07:51:30 08/08/79R           MAIN.
000229      C          STORE SOLUTION IN TS TEMPORARY
000230      C          -----
000231
000232          TS(JMAX1)=G(JMAX1)
000233          DO 201 J=1,JMAX2
000234          J1=JMAX1-J
000235          TS(J1)=G(J1)-B(J1)*TS(J1+1)
000236      201      CONTINUE
000237
000238      C          -----
000239      C          SOLUTION FOR 0<X<L
000240      C          -----
000241
000242          DO 205 I=2,IMAX1
000243          G(1)=D1(I,1)/W(1)
000244          DO 206 J=2,JMAX1
000245          A=V(I,J)/(2.*DELY) + DELY2I
000246          W(J)=W(1) + A*B(J-1)
000247          B(J)=(V(I,J)/(2.*DELY)-DELY2I)/W(J)
000248          G(J)=(D1(I,J) + A*G(J-1))/W(J)
000249      206      CONTINUE
000250
000251      C          -----
000252      C          STORE SOLUTION INTO T
000253      C          -----
000254
000255          DO 207 J=1,JMAX1
000256          T(I-1,J)=TS(J)
000257      207      CONTINUE
000258
000259      C          -----
000260      C          STORE SOLUTION TEMPORARY IN TS
000261      C          -----
000262
000263          TS(JMAX1)=G(JMAX1)
000264          DO 208 J=1,JMAX2
000265          J1=JMAX1-J
000266          TS(J1)=G(J1)-B(J1)*TS(J1+1)
000267      208      CONTINUE
000268      205      CONTINUE
000269
000270      C          -----
000271      C          SOLUTION FOR X=L
000272      C          -----
000273
000274          G(1)=DELTIN*T(IMAX,1)+2.*DELY2I*(T(IMAX1,1)-T(IMAX,1))
000275          G(1)=G(1)/W(1)
000276          DO 210 J=2,JMAX1
000277          A=V(IMAX,J)/(2.*DELY) + DELY2I
000278          W(J)=W(1) + A*B(J-1)
000279          B(J)=(V(IMAX,J)/(2.*DELY) - DELY2I)/W(J)
000280          G1=DELTIN*T(IMAX,J) + 2.*DELY2I*(T(IMAX1,J)-T(IMAX,J))
000281          G(J)=(G1+A*G(J-1))/W(J)
000282      210      CONTINUE
000283
000284      C          STORE SOLUTION INTO T
000285
*****

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```
000286      DO 211 J=1,JMAX1
000287      T(IMAX1,J)=TS(J)
000288      211  CONTINUE
000289
000290
000291      TS(JMAX1)=G(JMAX1)
000292      DO 212 J=1,JMAX2
000293      J1=JMAX1-J
000294      TS(J1)=G(J1)-B(J1)*TS(J1+1)
000295      212  CONTINUE
000296
000297      C      -----
000298      C      SOLUTION OBTAINED FOR X=L
000299      C      -----
000300
000301      DO 213 J=1,JMAX1
000302      T(IMAX,J)=TS(J)
000303      213  CONTINUE
000304      TIME=TIME+DELT
000305      ICYC=ICYC+1
000306
000307      C      -----
000308      C      ENTER STREAM FUNCTION ITERATION LOOP
000309      C      -----
000310
000311      305  DELS=0.
000312      SMAX=0.
000313      SMIN=0.
000314      DO 310 I=2,IMAX1
000315      IF(I.LT.IPWS)GO TO 311
000316      IF(I.GT.IPW)GO TO 311
000317      JSTART=IPH1
000318      GO TO 312
000319      311  JSTART=2
000320      312  DO 315 J=JSTART,IUH1
000321      ST=S(I+1,J)+S(I-1,J)+EPSI*(S(I,J+1)+S(I,J-1))
000322      ST=ST+RX*(T(I+1,J)-T(I-1,J))/(2.*DELS)
000323      ST=ST/(2*(1.+EPSI))
000324      ST1=S(I,J)
000325      S(I,J)=RELAX*ST + (1.-RELAX)*S(I,J)
000326      IF(SMAX.GT.S(I,J))GO TO 316
000327      SMAX=S(I,J)
000328      316  CONTINUE
000329      IF(SMIN.LT.S(I,J))GO TO 317
000330      SMIN=S(I,J)
000331      317  CONTINUE
000332      DELS1=ABS(ST1-S(I,J))
000333      IF(DELS1.LT.DEELS)GO TO 315
000334      DELS=DELS1
000335      315  CONTINUE
000336      310  CONTINUE
000337
000338      C      -----
000339      C      CHECK TO SEE IF ITERATION CONVERGENT CRITERIA WERE MET
000340      C      -----
000341
000342      IF(DELS.GT.1.0E-5)GO TO 305
*****
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MAIN.

```
000343
000344 C -----
000345 C PLOT ON RJET
000346 C PLOT TEMPERATURE
000347 C -----
000348
000349 IF(MOD(ICYC,IPL0T).NE.0)GO TO 4000
000350 CALL MAPS(0.,XMAX,0.,1.,0.11,1.0,0.11,0.43)
000351 CALL SETLCH(0.5,1.5,0,0,2,0)
000352 WRITE(100,3)TIME
000353 3 FORMAT("TEMPERATURE AT TIME = ",F7.3)
000354 CALL RCONTR(6,CL,0,T,61,X,1,IMAX,1,Y,1,JMAX,1)
000355 CALL FRAME
000356
000357 C -----
000358 C PLOT STREAM FUNCTION
000359 C -----
000360
000361 CS(1)=SMIN
000362 CS(6)=0.
000363 DS=(SMAX-SMIN)/5.0
000364 DO 360 I=2,5
000365 CS(I)=CS(I-1)+DS
000366 360 CONTINUE
000367 CALL MAPS(0.,XMAX,0.,1.,0.11,1.0,0.11,0.43)
000368 CALL SETLCH(0.5,1.5,0,0,2,0)
000369 WRITE(100,4)
000370 4 FORMAT("STREAM FUNCTION")
000371 CALL RCONTR(6,CS,0,S,61,X,1,IMAX,1,Y,1,JMAX,1)
000372 CALL FRAME
000373
000374 C -----
000375 C COMPUTE VELOCITY
000376 C -----
000377
000378 4000 CONTINUE
000379 SMAX=0.
000380 DO 400 I=1,IMAX
000381 DO 400 J=1,IUH
000382 U(I,J)=(S(I,J+1)-S(I,J-1))/(2.*DELY)
000383 V(I,J)=(S(I-1,J)-S(I+1,J))/(2.*DELX)
000384 IF(ABS(U(I,J)).LE.SMAX)GO TO 410
000385 SMAX=ABS(U(I,J))
000386 410 IF(ABS(V(I,J)).LE.SMAX)GO TO 400
000387 SMAX=ABS(V(I,J))
000388 400 CONTINUE
000389
000390 C -----
000391 C PLOT VELOCITY
000392 C -----
000393
000394 IF(MOD(ICYC,IPL0T).NE.0)GO TO 4010
000395 CALL MAPS(0.,XMAX,0.,1.,0.11,1.0,0.11,0.43)
000396 CALL SETLCH(0.5,1.5,0,0,2,0)
000397 WRITE(100,5)
000398 5 FORMAT("VELOCITY")
000399 DO 430 I=2,IMAX1
*****
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000400      DO 430 J=2, JMAX1
000401      XS=X(I)
000402      YS=Y(J)
000403      FAC=2.5
000404      XE=XS+FAC*U(I, J)*DELX/SMAX
000405      YE=YS+FAC*V(I, J)*DELY/SMAX
000406      CALL PLOTV(XS,YS,XE,YE)
000407 430  CONTINUE
000408      CALL FRAME
000409
000410
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DO 430 J=2, JMAX1
XS=X(I)
YS=Y(J)
FAC=2.5
XE=XS+FAC*U(I, J)*DELX/SMAX
YE=YS+FAC*V(I, J)*DELY/SMAX
CALL PLOTV(XS,YS,XE,YE)
CONTINUE
CALL FRAME

```

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C -----
C PLOT TEMPERATURE PROFILES
C -----

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```

CALL MAPS(0., XMAX, 0., 1., 0.11, 1.0, 0.11, 0.43)
CALL SETLCH(0.5, 1.5, 0, 0, 2, 0)
WRITE(100, 6)
FORMAT("TEMPERATURE PROFILES")
6 DO 450 I=1, IMAX1, 10
IF(I.EQ.1)GO TO 451
XS=X(I)
CALL LINE(XS, 1.0, XS, 0., 0)
451 DO 460 J=1, JMAX
TS(J)=T(I, J)*0.5+X(I)
460 CONTINUE
CALL TRACE(TS, Y, JMAX)
450 CONTINUE
CALL FRAME

```

```

C -----
C PLOT SURFACE HEAT FLOW
C -----

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```

HMAX=0.
CALL MAPS(0., XMAX, 0., 20., 0.11, 1.0, 0.11, 0.3)
CALL SETLCH(0.5, 24., 0, 0, 2, 0)
WRITE(100, 7)
7 FORMAT("SURFACE HEAT FLOW")
DO 500 I=1, IMAX
W(I)=(T(I, JMAX2)-T(I, JMAX))/(2.*DELY)
W(I)=W(I)*8.6
500 CONTINUE
CALL TRACE(X, W, IMAX)

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```

C -----
C PLOT TEMPERATURE BENEATH THE CAP
C -----

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CALL MAPS(0., XMAX, 0., 0.5, 0.11, 1.0, 0.61, 0.8)
DO 510 I=1, IMAX
W(I)=T(I, IUH)
510 CONTINUE
CALL SETLCH(0.5, 1.0, 0, 0, 2, 0)
WRITE(100, 8)
8 FORMAT("TEMPERATURE BENEATH THE CAP")
CALL TRACE(X, W, IMAX)
CALL FRAME

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C -----  
C LOOP BACK FOR NEXT TIME STEP  
C -----  
4010 CONTINUE  
IF(I TIME.EQ.1)GO TO 180  
IF(ICYC.LE.CYMAX)GO TO 1000  
C -----  
C PROGRAM END. WILL PRINT THE LAST TIME STEP TEMPERATURE AND  
C STREAM FUNCTION VALUES.  
C -----  
PRINT 9000  
9000 FORMAT("1 THIS IS THE TEMPERATURE DATA")  
DO 20 I=1,IMAX  
PRINT 9001,(T(I,J),J=1,JMAX)  
9001 FORMAT(1H,11F10.5)  
20 CONTINUE  
PRINT 9002  
9002 FORMAT(1H1,"STREAM FUNCTION")  
DO 21 I=1,IMAX  
PRINT 9001,(S(I,J),J=1,JMAX)  
21 CONTINUE  
CALL EXIT  
END
```