Effect of photon statistics on phase conjugation by four-wave mixing

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The effects of fluctuation associated with chaotic fields on degenerate four-wave mixing are studied. The chaotic field phase conjugates more efficiently than the single-mode coherent light for both weak and strong fields, the reflectivity being enhanced by a factor of 2 in the former case. The fluctuations are also shown to have a significant effect on reflectivity as a function of detuning.

Degenerate four-wave mixing has become one of the most important techniques to generate phase-conjugate signals.[1–3] This process involves the interaction of two counterpropagating pump waves with a weak signal wave to produce a phase-conjugate wave that propagates in a direction opposite that of the signal wave. The reflection coefficient for the phase-conjugate wave is given by[3]

$$R = \frac{|\kappa \sin \Omega L|^2}{|\Omega \cos \Omega L + \alpha_R \sin \Omega L|^2}, \qquad (1)$$

with

$$\alpha = \alpha_0 \frac{(1 - i\Delta)\ (1 + 2I/I_s)}{(1 + \Delta^2)(1 + 4I/I_s)^{3/2}} = \alpha_R - i\alpha_I,$$

$$\kappa^* = i\alpha_0 \frac{(1 - i\Delta) \quad 2I/I_s}{(1 + \Delta^2)(1 + 4I/I_s)^{3/2}},$$

$$\Omega = (|\kappa|^2 - \alpha_R{}^2)^{1/2}, \qquad (2)$$

where it is assumed that the two levels between which the atom makes a transition are connected by a single photon transition. In the above expression, I is the pump intensity, I_s is the frequency-dependent saturation intensity $[= I_s{}^0 (1 + \Delta^2)]$, Δ is the normalized pump atom detuning, i.e., T_2 (− atomic frequency + pump frequency), α_0 is the unsaturated absorption coefficient, and L is the interaction length. This result has been derived for a monochromatic, coherent pump laser. As is evident from Eq. (1), this phenomenon is a nonlinear optical process, i.e., R is a nonlinear function of I; its properties are expected to depend on the photon-correlation (coherence) properties of the exciting field. Earlier studies of multiphoton absorption processes[4,5] have shown that the yield or efficiency with chaotic light is larger by a factor of $N!$ compared with the case when N photons are absorbed from a coherent source.

In this Letter, we present the results of a study of degenerate four-wave mixing when the temporal fluctuations of the pump field are taken into account. We assume that the pump field is a chaotic field. The general problem of a chaotic field with finite bandwidth is an extremely complicated one. In the context of phase conjugacy this becomes even more complex because of resonances, saturation effects, and boundary conditions. Thus, to illustrate the effect of the statistics of the pump field, we consider the case when the bandwidth of the chaotic field is much smaller than the natural width of the atomic transition, although this is a somewhat idealized situation. Since the pump field in the problem is a stochastically fluctuating field, the atomic variables also acquire a stochastic character and must be averaged over the fluctuations of the pump field. In the limit of long coherence time (zero bandwidth), it is well known that the averaging of the atomic variables with respect to the fluctuations in the chaotic field reduces to an averaging with respect to the intensity-distribution function of the chaotic field.[6] The chaotic field undergoes Gaussian amplitude fluctuations, and this corresponds to the following probability distribution for the fluctuating intensity:

$$p(I) = \frac{1}{\langle I \rangle} \exp(-I/\langle I \rangle), \qquad (3)$$

where $\langle I \rangle$ is the average intensity of the fluctuating field. Hence the reflection coefficient for the phase-conjugate wave using chaotic light would be given by

$$R_{\mathrm{CH}} = \frac{1}{\langle I \rangle} \int_0^\infty \mathrm{d}I \exp(-I/\langle I \rangle) R_{\mathrm{COH}}(I), \qquad (4)$$

where R_{CH} and R_{COH} represent, respectively, the reflection coefficient obtained for chaotic and coherent fields, $R_{\mathrm{COH}}(I)$ being given by Eq. (1). In general, R_{CH} can only be obtained numerically. However, analytical formulas can be obtained in the limiting cases of weak and strong fields. For simplicity, we consider the on-resonance case ($\Delta = 0$).

Weak Fields, i.e., $I/I_s{}^0 \ll 1$

From Eqs. (1) and (2), we find that $\alpha_R \sim$ constant, $\Omega \sim$ constant, and $\kappa \sim I/I_s{}^0 \equiv \mathcal{I}$. Hence

$$R_{\mathrm{COH}} \sim |\kappa|^2 \Rightarrow R_{\mathrm{COH}} \sim \mathcal{I}^2. \qquad (5)$$

Now the average value of the nth power of the instantaneous intensity is given by

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$$\langle \mathcal{I}^n \rangle = \frac{1}{\langle \mathcal{I} \rangle} \int_0^\infty d\mathcal{I} \exp(-\mathcal{I}/\langle \mathcal{I} \rangle) \times \mathcal{I}^n = n! \langle \mathcal{I} \rangle^n. \tag{6}$$

Thus, if both fields have the same mean intensity $\langle \mathcal{I} \rangle$, then it follows from Eqs. (5) and (6) that in the weak-field limit

$$\langle R \rangle_{CH} / \langle R \rangle_{COH} = 2!, \tag{7}$$

so that the efficiency for obtaining the phase-conjugate reflected wave by four-wave mixing with chaotic light is twice that obtained with purely coherent beams.

Intense Fields, i.e., $I/I_s^0 \gg 1$

From Eqs. (2), we have

$$\Omega = \frac{i\alpha_0}{1 + 4\mathcal{I}} \cdot \tag{8}$$

We obtain the following expression for the reflection coefficient with coherent light:

$$R_{COH} = \left[\frac{(1 - \alpha_0 L x)\sinh(1/x)}{(4\alpha_0 L x)^{1/2}\cosh(1/x) + (1 + \alpha_0 L x)\sinh(1/x)} \right]^2, \quad x = \frac{1 + 4\mathcal{I}}{\alpha_0 L} \cdot \tag{9}$$

At large intensities, $1/x \ll 1$, so that we may use the asymptotic expansion of the hyperbolic functions. Retaining only linear terms in $1/x$, we find that Eq. (9) reduces to

$$R_{COH} \cong \frac{\alpha_0 L}{4x} \cdot \tag{10}$$

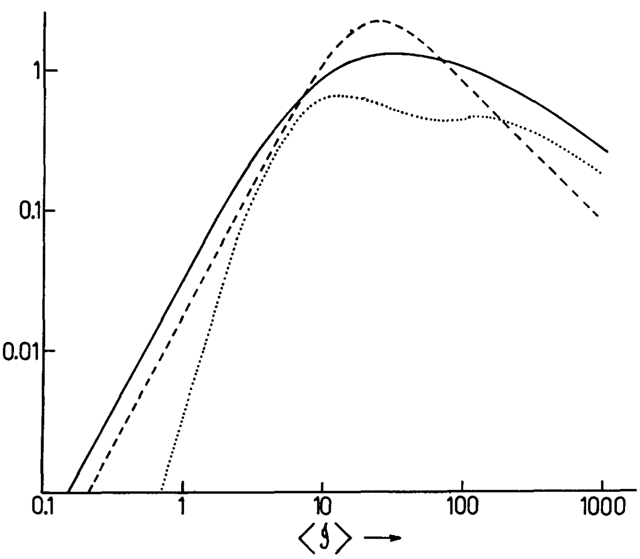

Fig. 1. Reflectivity versus average pump intensity for $\Delta = 6$, $\alpha_0 L = 40$. The solid curve corresponds to a chaotic field and the dashed curve to coherent field. The dotted curve, on the same scale, shows the variation of the intensity fluctuations $\langle I^2 \rangle - \langle I \rangle^2 \propto \langle R^2 \rangle - \langle R \rangle^2$ of the phase-conjugate wave with the pump intensity for the same set of parameters.

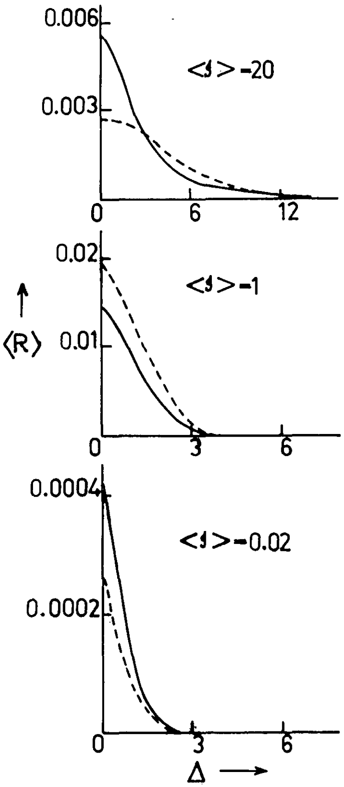

Fig. 2. Reflectivity versus atom pump detuning for various pump intensities. The parameter $\alpha_0 L$ is set equal to unity.

Substituting expression (10) into Eq. (4) and simplifying, we find that the reflection coefficient for chaotic light in this limit reduces to

$$R_{CH} = \frac{\alpha_0^2 L^2}{16\langle \mathcal{I} \rangle} \exp\left(\frac{1}{4\langle \mathcal{I} \rangle}\right) E_1\left(\frac{1}{4\langle \mathcal{I} \rangle}\right), \tag{11}$$

where E_1 is the exponential integral defined by

$$E_1(a) = \int_a^\infty \frac{dz}{z} e^{-z}. \tag{12}$$

Figure 1 shows a plot of the reflectivity as obtained by numerical integration of Eq. (4). This is studied as a function of the normalized mean intensity $\langle \mathcal{I} \rangle$ for $\Delta = 6$ and $\alpha_0 L = 40$. The graph shows that for weak fields the reflectivity that is due to monochromatic chaotic field (solid curve) approaches twice the monochromatic coherent field values (dashed curve), so that the generation of a phase-conjugate reflected wave is more efficient with chaotic light than with coherent light. However, the maximum reflectivity that can be obtained by such a process is greater in the case for monochromatic coherent excitation. At large field strength, the chaotic field once again yields a higher reflectivity. We also find that the numerical values calculated from Eq. (4) are in good agreement with those calculated by the approximate analytical expression (11). In the same figure, we have also shown the intensity fluctuations of the phase-conjugate wave, i.e., $\langle R^2 \rangle - \langle R \rangle^2$, as a function of the pump intensity. This shows that the fluctuations are quite significant near the maximum of the curve. These could be of the same order of magnitude as $\langle R \rangle^2$. Note also that, even in the limit of extremely low intensities, the fluctuations associated with the chaotic pump are important since $\langle R^2 \rangle - \langle R \rangle^2 \propto \langle I^4 \rangle - \langle I^2 \rangle^2 = 24\langle I \rangle^4 - 4\langle I \rangle^4 = 5\langle I^2 \rangle^2$. Figure 2 shows the dependence of the reflectivity on the detuning of the pump field for various pump intensities. The reflectivity curve is symmetric about $\Delta = 0$ and peaks on resonance with a width determined by the statistical nature of the pump. For weak fields, the width of the resonance is considerably less than the natural linewidth and is practically independent of the fluctuations in the pump field. Higher values of the pump intensity lead to power broadening of the resonance, the chaotic field yielding a sharper resonance than the coherent field.

In conclusion, we have shown the significance of the statistical fluctuations on the efficiency of the generation of phase-conjugate signals. A similar situation is expected to prevail for degenerate four-wave mixing when the atomic transition involves a coherent two-photon process.[7]

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