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EFFECT OF PRANDTL NUMBER ON HEAT TRANSFER IN FLOW PAST A HEATED CYLINDER

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ABSTRACT

The flow past a heated cylinder is studied numeri-cally in the steady separated buoyancy aided flow regime (Reynolds number < 47) by using a hybrid finite element finite volume (FEM-FVM) solver. In this study we eluci-date the physics behind the Prandtl number effects on heat transfer from the cylinder surface

NOMENCLATURE

α	Thermal diffusivity of the fluid
β	Coefficient of thermal expansion
D	Diameter of the cylinder; characteristic dimension
n	Normal direction to the cylinder surface
ν	Kinematic viscosity of the fluid
Nu	Nusselt number
Nu_θ	Local Nusselt number
Pr	Prandtl number
Re	Reynolds number
Ri	Richardson number
s	Direction along the cylinder surface
t	Non-dimensional time
T	Dimensional temperature
T_0	Dimensional temperature of free stream flow
T_w	Dimensional temperature on the cylinder surface
θ	Polar angle
Θ	Non-dimensional temperature

U_0	Free stream velocity
u	x-component of non-dimensional velocity
v	y-component of non-dimensional velocity
x, y	Co-ordinates in the 2 D Cartesian co-ordinate system

1. INTRODUCTION

The flow past a cylinder and the associated dynamics presents a classical problem in engineering with a myriad of applications. Flow over circular cylinders have been ex-tensively studied in the past with [1–4] reporting on the var-ious dynamics of the flow such as aerodynamics, boundary layer, wake dynamics etc. The present study deals with the flow past heated cylinder. Most of the previous works were concentrated on forced convection flow, [5, 6] how-ever mixed convection flow stands more closer to real life applications. In this paper, buoyancy effects are added to the flow in terms of Richardson number, Ri. Unlike forced convection flow, the mixed convection heat transfer char-acterizes mainly due to the interaction of viscous and the thermal boundary layers, which is the motivation of this study. Limited number of literature are found on the ef-fects of Prandtl number, Pr on flow past heated cylinders. Ref. [7, 8] are few among them. This study focuses on the mixed convection flow over a moderately heated cylinder and to elucidate numerically the effects of Pr on heat trans-fer behavior from the hot cylinder kept in a cold stream.

2. NUMERICAL METHOD

1. Governing Equations

The equations governing the flow under consideration are the two-dimensional incompressible Navier-Stokes and energy equations. The non-dimensional forms of the con-tinuity, momentum and energy equations in Cartesian co-ordinates are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+Ri.\Theta \quad (3)$$

$$\frac{\partial \Theta}{\partial t}+u\frac{\partial \Theta}{\partial x}+v\frac{\partial \Theta}{\partial y}=\frac{1}{RePr}\left(\frac{\partial^2 \Theta}{\partial x^2}+\frac{\partial^2 \Theta}{\partial y^2}\right) \qquad (4)$$

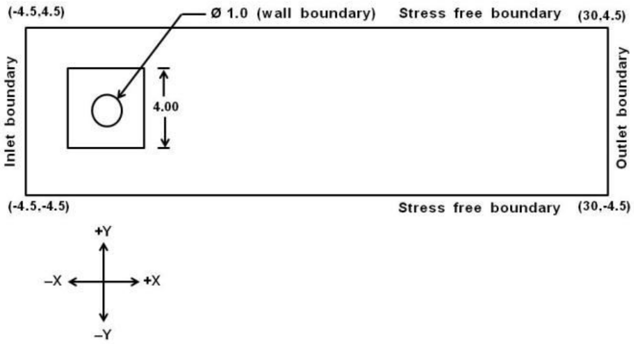

FIGURE 1. THE SCHEMATIC REPRESENTATION OF COMPUTATIONAL DOMAIN SHOWING THE CO-ORDINATES OF THE BOUNDARIES AND THE BOUND-ARY CONDITIONS USED.

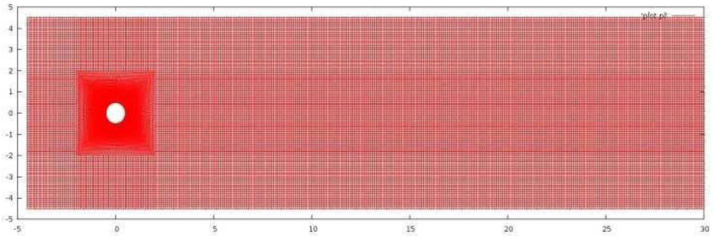

FIGURE 2. GRID USED IN THE PRESENT STUDY. ADAP-TIVE MESHING IS CREATED IN A SQUARE REGION AROUND THE CYLINDER.

The dynamics of the flow past the circular cylinder can be represented by the non-dimensional parameter Reynolds number (Re), based on the diameter of the cylinder, defined as $Re = \frac{U_0 D}{\nu}$. The energy equation is coupled with the y - momentum equation through the coupling term Ri, the Richardson number, the non dimensional form of the buoy-ancy, defined as the ratio of buoyancy force to the inertia force $(\frac{\beta g \Delta T D^2}{U_0 \nu})$. The Pr in the energy equation manifests the relative dominance of viscous diffusion over the ther- mal diffusion $(Pr = \frac{\nu}{\alpha})$. In other words, it is defined as the ratio of the viscous and thermal boundary layer thicknesses.

A hybrid FEM-FVM solver is used to solve the governing equations [1-4] of the flow. Details of the solver and algorithm is given [9]. A rectangular domain is selected around the cylinder for the computational analysis as shown in Fig.1.

The size of the domain is selected (34.5D× 9D) after domain independence test and is discretized using quadrilateral elements. An adaptive type of meshing is created in the square region around the cylinder as shown in Fig.2 to accurately capture the flow physics and the Nusselt number (Nu) which is the non dimensional heat transfer coefficient defined later in this paper. A grid independence test is carried out to get the optimum value of the grid size. 50000 quadrilateral cells are used in the present computation with 50575 nodal points as shown in Fig.2. The details of the domain independence and the grid independence tests are not given in this paper due to page constraints and will be detailed at the time of conference.

Boundary Conditions: A uniform velocity boundary condition of u = 1.0 with cold flow T = 0.0 is specified on the inlet boundary. Zero-stress boundary conditions are specified on the top and bottom boundaries. The exit boundary is specified by a zero normal gradient boundary condition for both velocities and temperature. No slip boundary condition is imposed on the cylinder with isothermal conditions (T = 1.0) on the cylinder surface.

Validation: The code is validated for the present case and compared with previous results and consolidated in Tab. 1 . The value of *Nu* obtained is also verified by computing it from its correlation with *Ri* proposed by [15, 16] (Eqn. 5),

$$\frac{Nu}{Nu_{Ri=0}} = (1+Ri)^{0.2133} \qquad (5)$$

3. RESULTS AND DISCUSSIONS

It is fundamental in fluid dynamics that a flow separation occurs and recirculation bubble appears behind the cylinder when flow takes place over it at a Reynolds number 4 < *Re* < 47. This recirculation bubble contains twin vortices, one rotating in clockwise direction and the other in anticlockwise direction. In flow past heated cylinder, the changes in the recirculation bubble and the associated wake structure have a major role in determining the heat transferred from the cylinder surface. The *Pr* also has high impacts on the wake structure developed behind the cylinder since it is a measure of the two boundary layer interaction. In the present study, effects of *Pr* on heat transfer from a heated cylinder is carried out for a steady laminar flow (4 < *Re* < 47). The representative *Re* selected for the study is 25, well within the steady separated region. The three dimensional instability called the mode-E, will be dominating for buoyancy condition above *Ri* < 2. Hence in the present 2D analysis, we restrict ourselves to a less buoyancy condition. The range of non dimensional parameters that governs the flow are, 0.25 < *Pr* < 100 and 0.25 < *Ri* < 2 for a representative *Re* of 25. Nusselt number (*Nu*) describes the convection heat transferred from the heated cylinder into the flow and is estimated locally as

$$Nu_{\theta} = -\frac{\partial \Theta}{\partial n}\bigg|_{S}, \qquad (6)$$

The distribution of *Nu* against the polar angle q for *Re* = 25 and *Ri* = 1.0 is plotted in Fig. 3 for different *Pr*. Polar angle q varies along the surface of the cylinder from the rear stagnation point (q = 0deg to q = 360deg in counter-clockwise direction). From the graph it is seen that the maximum value of *Nu*q occurs at the front stagnation point for all the *Pr* and the minimum values are at the separation points on either sides of the rear stagnation point and then increases towards the rear stagnation point. The maximum *Nu*q occurs at the front stagnation point since the normal gradient of temperature at this point is the highest as the cold free stream flow at T = 0 contacts the heated cylinder first at this point. Thereafter, the fluid gets heated as it flows along the surface of the cylinder, decreasing the local temperature gradient thereby decreasing the *Nu*q on either side of the front stagnation point as shown in Fig. 3. After the separation point, an increase in *Nu*q can be seen in Fig.3, which shows the presence of the recirculation bubble. Inside this bubble, heat is drained continuously from the cylinder due to the recirculation happening inside the bubble. As the *Pr* is increased, local Nusselt number also increases, without losing the characteristic behavior of the curve as can be seen in Fig.3. The reason for this improvement in heat transfer is elucidated in the next paragraph.

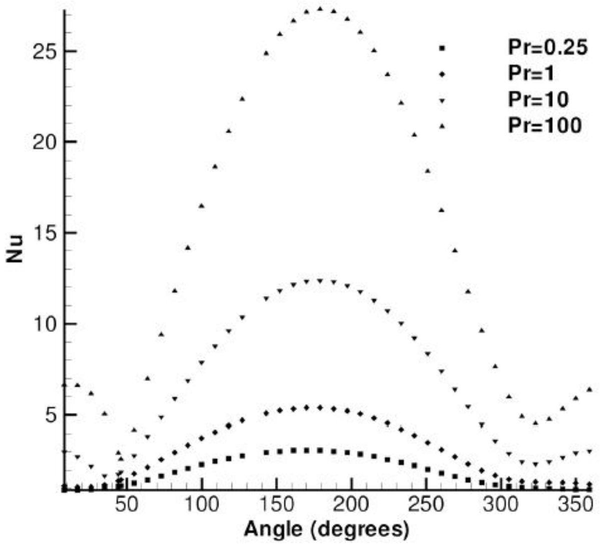

FIGURE 3. THE Nu DISTRIBUTION AROUND THE HEATED CYLINDER FOR Re = 25 AND Ri = 1.0 COM-PUTED FOR DIFFERENT Pr. ANGLES 0 deg AND 360 deg REPRESENTS THE REAR STAGNATION POINT AND AN-GLE 180 deg REPRESENTS FRONT STAGNATION POINT.

The variation of the isotherms with Ri and Pr for Re = 25 is plotted and consolidated in Fig. 4. In this, the first column of figures are plots obtained for Ri = 0.50 and the second column of figures for Ri = 2.00. The three rows of plots represents the three different Pr values, 0.50, 5 and 50 respectively. In Fig. 4, isotherms with same contour values are taken for comparison, in all the plots. As Pr increases, (top to bottom in Fig. 4) the isotherms are found to be crowding around the cylinder for both Ri =0.50 and 2.00, reducing the thermal boundary layer thickness since an in-crease in Pr would decrease the thermal diffusivity α . The reduction in thermal boundary layer thickness decreases the local temperature gradient $\frac{\partial \Theta}{\partial n}$, thereby decreasing the local Nusselt number. This is the reason for enhancement in heat transfer from the cylinder, as Pr is increased.

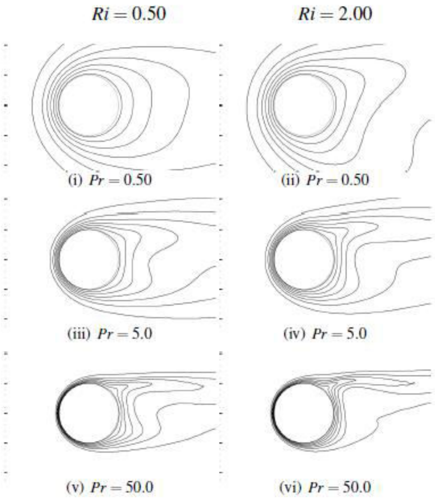

FIGURE 4. ISOTHERM CONTOURS FOR Re = 25 AND FOR VARIOUS VALUES OF Ri AND Pr. IT SHOULD BE NOTED THAT ISOTHERMS WITH THE SAME CONTOUR VALUES ARE TAKEN FOR COMPARISON IN ALL CASES. THE FIRST COLUMN OF PLOTS REPRESENTS Ri = 0.50 AND SECOND COLUMN REPRESENTS PLOTS FOR Ri = 2.00.

4. CONCLUSION

Cold flow past a heated horizontal cylinder, kept at isothermal condition is analyzed numerically using a hybrid finite element-finite volume (FEM-FVM) solver. We have considered the steady laminar flow for the present analysis for a representative Reynolds number of 25 and $0.25 < Pr < 100$ and $0.25 < Ri < 2$. The effect of the buoyancy, which is considered negligible in most of the previous works, was included in terms of the non-dimensional parameter Ri. The study presents Pr of the cold fluid, flowing over the heated cylinder as an effective heat transfer enhancement strategy. It is understood from the isotherm plots that an increase in Pr would result in the reduction in the thermal boundary layer thickness, in-creasing Nu_θ thereby increasing heat transferred from the cylinder surface. The variation of Nu_θ over the cylinder is plotted and found that the maximum heat transfer occurs at the front stagnation point for all the cases considered here. Then it decreasing up to the separation point on either side of the rear stagnation point and then increases towards the rear stagnation point, which shows the presence of the re-circulation bubble.

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