Full Length Research Paper

Effect of relaxation times and rotation on the propagation of plane waves in generalized magneto-thermo-viscoelasticity

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The problem of wave propagation in the generalized dynamical theory of thermo-elasticity proposed by Green and Lindsay is applied to study the propagation of harmonically time dependent thermo-visco-elastic plane waves of assigned frequency in an infinite visco-elastic solid in a magnetic field. A more general dispersion equation is deduced to determine the effects of rotation, viscoelasticity and relaxation time on the phase velocity of the coupled waves. The perturbation technique has been employed to obtain the phase velocity and attenuation coefficient for small thermo-elastic couplings. Taking an appropriate material, the numerical values of the phase velocity of the waves are computed and the results are shown graphically to illustrate the problem. The results are compared with those obtained earlier.

Key words: Plane waves, magneto-visco-elastic medium, phase velocity, attenuation coefficient perturbation technique.

INTRODUCTION

The study of plane wave propagation in presence of external magnetic field in an electrically conducting non-rotating medium was found in the works of many authors including Paria (1962), Willson (1963) and Dunkin and Eringen (1963). In the books of Parkus (1972, 1979) and Eringen and Maugin (1990), valuable information and developments in magnetoelasticity and magneto-thermo-elasticity in non-rotating elastic media are available. Using modified Fourier's law of heat conduction suggested by Kaliski (1965) and Lord and Shulman (1967) derived equations of generalized thermo-elasticity which was later applied by Nayfeh and Nasser (1973) to study plane waves of an infinite elastic body permeated by a primary magnetic field which was subjected to heating. Schoenberg and Censor (1973) investigated the propagation of elastic plane waves in a uniformly rotating medium and showed that the rotation causes the elastic waves to be dispersive and anisotropic. Afterwards, Bera (1998), Roy (1983) and Roy and Debnath (1983) discussed the problems of magneto-elastic and electro-magneto-thermoelastic plane waves in rotating medium. Since most large bodies like the earth, the moon and other planets have an angular velocity as Peltier (1974) discussed the visco-elastic nature of the earth, it would appear more realistic to consider the propagation of plane waves in a rotating magneto-thermo- visco-elastic medium. Agarwal (1978, 1979) considered, respectively, thermoelastic and magneto-thermoelastic plane wave propagation in an infinite elastic medium. Lateron, Mukhopadhyay and Bera (1989) applied the generalized dynamical theory of thermoelasticity to determine the distributions of temperature, deformation, stress and strain in an infinite isotropic visco-elastic solid of Kelvin-Voigt type permeated by a uniform magnetic field having

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distributed instantaneous and continuous heat sources.

Following Lord-Shulman's theory of generalized thermoelasticity, Puri (1976) and Roychoudhuri and Debnath (1983) studied plane wave propagation in infinite rotating elastic medium. Roy (1985) applied Green-Lindsay's theory of generalized thermoelasticity to study the effect of rotation and relaxation times on plane waves in the aforementioned medium. Recent works on 'magneto-thermo-visco-elasticity and magneto-thermo elasticity' are also available from the papers of Song et al. (2004), Othman (2005), Baksi and Bera (2006), Rakshit and Mukhopadhyay (2007), Mohamed and Yaqin (2008), Ezzat et al. (2009), Othman et al. (2009) and Abd-Alla et al. (2011). In the present paper, the linearized theory of Green and Lindsay in generalized thermo-visco-elasticity having two relaxation times is applied to study the propagation of harmonically time-dependent magneto-thermo-viscoelastic plane waves of assigned frequency in an infinite rotating viscoelastic solid of Kelvin-Voigt type. Using the 'perturbation technique', a dispersion relation for small thermoelastic coupling is obtained to determine the effects of rotation and relaxation times on the phase velocity of the waves in a visco-elastic medium permeated by a uniform magnetic field. Finally numerical values of the wave velocities at various frequencies are computed for an appropriate material and are presented graphically for the purpose of illustration.

FORMULATION OF THE PROBLEM

Let us consider the propagation of plane magneto-thermoelastic waves in a homogeneous viscoelastic medium with density ρ at uniform initial temperature. The medium is rotating with an angular velocity:

$$\bar{\Omega} = \Omega \bar{n}$$

Where $\bar{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in rotating frame of reference has two additional terms:

i) Centripetal acceleration $\bar{\Omega} \times (\bar{\Omega} \times \bar{u})$ due to the time varying motion only;

ii) The coriolis acceleration $2\bar{\Omega} \times \dot{\bar{u}}$.

The principle of balance of linear momentum leads to the equations of motion derived by Eringen and Maugin (1990).

$$\tau_{ij,j} + (\bar{J} \times \bar{B})_i = \rho[\ddot{u}_i + \{\bar{\Omega} \times (\bar{\Omega} \times \bar{u})\}_i + (2\bar{\Omega} \times \dot{\bar{u}})_i]$$
$$(i, j = 1,2,3) \qquad (1)$$

The variations of the magnetic and electric fields are given by Maxwell's equations (in absence of the displacement current and charge density):

$$\text{Curl } \bar{H} = \bar{J} \text{ , Curl } \bar{E} = -\frac{\partial \bar{B}}{\partial t}, \ \ \bar{B} = \mu'_e \bar{H}, \ \text{div } \bar{B} = 0 \qquad (2)$$

The modified Ohm's law is Roy and Debnath (1983):

$$\bar{J} = \sigma[\bar{E} + \{\frac{\partial \bar{u}}{\partial t} + (\bar{\Omega} \times \bar{u})\} \times \bar{B}], \qquad (3)$$

Assuming the small effect of temperature gradient on the electric current. Here, $\bar{H}$ = the total magnetic field vector, $\bar{B}$ = magnetic inductance vector, $\bar{E}$ = electric field vector, μ'_e = magnetic permeability of the medium, σ = electric conductivity of the medium, ρ = constant mass density, τ_{ij} = components of Cauchy's stress tensor, $\bar{u}$ = displacement vector, $\bar{\Omega}$ = rotation vector.

Let us take $\bar{H} = (0, 0, H_z)$ and also assume that the all field variables are supposed to be functions of x and t only. Using Equations 2 and 3, and neglecting higher order derivatives of H_z than the first, we get:

$$-\mu'_e \frac{\partial H_z}{\partial t} \approx \mu'_e H_z \frac{\partial^2 u}{\partial x \partial t} \qquad (4)$$

Taking $H_z = H_0 + h_z$ in (2.4) and neglecting small quantities of higher order, we obtain:

$$-\mu'_e \frac{\partial h_z}{\partial t} \approx \mu'_e H_0 \frac{\partial^2 u}{\partial x \partial t}$$

Integrating partially with respect to t, we get:

$$h_z = -H_0 \frac{\partial u}{\partial x}, \qquad (5)$$

Where it is assumed that as $x \to \infty$, both h_z and:

$$\frac{\partial u}{\partial x} \to 0 \qquad (6)$$

The constant of integration in Equation 5 has taken to be zero, to satisfy the regularity condition at infinity. The

stress displacement-temperature relation for the visco-elastic medium of Kelvin-Voigt type derived by Eringen (1967) is given:

$$\tau_{ij} = (\lambda_e + \lambda_v \frac{\partial}{\partial t})\Delta\delta_{ij} + 2(\mu_e + \mu_v \frac{\partial}{\partial t})e_{ij} - \gamma_e(\theta + \alpha_2'\dot{\theta})\delta_{ij}$$

(7)

Where e_{ij} is the components of Cauchy strain tensor, $\Delta = e_{ii}$ is the dilatation, λ_e, μ_e are Lame's elastic constants, λ_v, μ_v are Lame's viscous constants for the viscoelastic solid, $\gamma_e = (3\lambda_e + 2\mu_e)\alpha_t$, α_t being the coefficient of linear thermal expansion, θ is temperature change above reference temperature; θ_0, α_2' is the thermal relaxation time parameter and δ_{ij} is the Kronecker's delta.

Assuming that the rotation of the body has no effect on heat conduction of the medium, the heat conduction equation is:

$$K\nabla^2\theta = \rho c_v(\dot{\theta} + \alpha^*\ddot{\theta}) + \gamma_e\theta^*\dot{\Delta}, \quad i = 1, 2, 3$$

(8)

Where K is the thermal conductivity and c_v is the specific heat at constant volume of the medium and α^* is thermal relaxation time parameter.

PLANE WAVE SOLUTIONS AND DISPERSION RELATION

We consider the wave propagating in the x-direction and all the field variables are assumed to be function of x and time t only. We assume that $\bar{u} = (u, v, w)$ and $\bar{\Omega} = (0, 0, \Omega)$, where Ω is a constant. Equation 1 with Equation 7, then reduces to:

$$[(\lambda_e + 2\mu_e + \mu_e'H_0^2) + (\lambda_v + 2\mu_v)\frac{\partial}{\partial t}]\frac{\partial^2 u}{\partial x^2} - \gamma(\frac{\partial\theta}{\partial x} + \alpha\frac{\partial^2\theta}{\partial x\partial t}) =$$
$$\rho[\ddot{u} - u\Omega^2 - 2\Omega\dot{v}]$$

(9)

$$(\mu_e' + \mu_v\frac{\partial}{\partial t})(\frac{\partial^2 v}{\partial x^2}) = \rho[\ddot{v} - \Omega^2 v - 2\Omega\dot{u}]$$

(10)

$$(\mu_e' + \mu_v\frac{\partial}{\partial t})(\frac{\partial^2 w}{\partial x^2}) = \rho\ddot{w}$$

(11)

We introduce the following dimensionless quantities:

$$x^* = \frac{c_1}{k}x, \quad t^* = \frac{c_1^2}{k}t, \quad u^* = \frac{\rho c_1^3}{k\theta^*\gamma}u,$$

$$v^* = \frac{\rho c_1^3}{k\theta^*\gamma}v, \quad w^* = \frac{\rho c_1^3}{k\theta^*\gamma}w, \quad \theta = \theta_0\theta^*, \quad \Omega^* = \frac{k}{c_1^2}\Omega$$

After dropping star, the non-dimensional forms of the equations obtained from Equations 9, 10, 11 and 8 are:

$$[(1 + R_H) + M\frac{\partial}{\partial t}]\frac{\partial^2 u}{\partial x^2} - (\frac{\partial\theta}{\partial x} + \alpha'\frac{\partial^2\theta}{\partial x\partial t}) =$$
$$[\ddot{u} - u\Omega^2 - 2\Omega\dot{v}]$$

(12)

$$(1 + N\frac{\partial}{\partial t})(\frac{\partial^2 v}{\partial x^2}) = \beta^2[\ddot{v} - \Omega^2 v - 2\Omega\dot{u}]$$

(13)

$$(1 + N\frac{\partial}{\partial t})(\frac{\partial^2 w}{\partial x^2}) = \beta^2\ddot{w}$$

(14)

$$\frac{\partial\theta}{\partial t} + \alpha^{*'}\frac{\partial^2\theta}{\partial t^2} - \frac{\partial^2\theta}{\partial x^2} + \varepsilon_\theta\frac{\partial^2\theta}{\partial x\partial t} = 0,$$

(15)

Where,

$$c_1^2 = \frac{\lambda_e' + 2\mu_e'}{\rho}, \quad k = \frac{K}{\rho c_v}, \quad \varepsilon_\theta = \frac{\gamma^2\theta^*}{\rho^2 c_v c_1^2}, \quad \beta^2 = \frac{\lambda_e' + 2\mu_e'}{\mu_e'}, \quad M = \frac{\lambda_v + 2\mu_v}{\lambda_e' + 2\mu_e'}\cdot\frac{c_1^2}{k}$$

$$N = \frac{\mu_v}{\mu_e'}\cdot\frac{c_1^2}{k}, \quad \alpha' = \frac{\alpha c_1^2}{k}, \quad \alpha^{*'} = \frac{\alpha^* c_1^2}{k}, \quad R_H = \frac{\mu_e H_0^2}{\rho c_1^2}$$

Equations 15, 12 and 13 form a coupled system and represent coupled magneto-visco-thermal dilatational and shears waves, while Equation 14 uncouples form of the system. This coupling disappears when $\Omega = 0$. Thus the thermal field affects the dilatational and shear motion due to rotation.

DISPERSION EQUATION FOR THE SYSTEM

For harmonic solutions of the Equations 12, 13 and 15, we choose:

$$(u, v, \theta) = (u_0, v_0, \theta_0)\exp\{i(qx + \omega t)\}$$

(16)

Where u_0, v_0 and θ_0 are amplitude constants, ω is the prescribed frequency, q is the wave number, in general a

complex number. The phase velocity c and the attenuation coefficient are respectively given by:

$$c = \omega / R_e(q) \text{ and } a = -I_m(q) \tag{17}$$

Substituting Equation (16) into (12), (13) and (15), we get:

$$\alpha_1^* = \alpha' - i/\omega, \ \ \alpha_2^* = \alpha^{*\prime} - i/\omega, \ \ A_1 = 1 + R_H + iM\omega, \ \ A_2 = 1 + iN\omega \tag{21}$$

For the non-trivial solutions of u_0, v_0 and θ_0, the

$$(q^2 - \omega^2\alpha_2^*)\{(A_1q^2 - \Omega_0^2)(A_2q^2 - \beta^2\Omega_0^2) - 4\Omega^2\omega^2\beta^2\} - \varepsilon_\theta\alpha_1^*\omega^2q^2(A_2q^2 - \beta^2\Omega_0^2) = 0 \tag{22}$$

Where,

$$\Omega_0^2 = \Omega^2 + \omega^2.$$

In case $\Omega = 0$, the dispersion Equation 22 reduces to:

$$(q^2 - \omega^2\alpha_2^*)\{(A_1q^2 - \omega_0)(A_2q^2 - \beta^2\omega_0) - \varepsilon_\theta\alpha_1^*\omega^2q^2\} = 0 \tag{23}$$

Putting $M = N = R_H = 0$, equation numbers (22) and (23) reduce to the equation numbers (3.7) and (3.8) respectively in Roy (1985).

Equation 22 is therefore a more general dispersion equation in the sense that it incorporates the magneto-visco-elastic effect as well as the effects of rotation and relaxation parameters on the propagation of coupled waves. This wave may be called the Quasi-magneto-visco-elastic-thermal-shear wave.

$$(A_1q^2 - \omega^2 - \Omega^2)u_0 - 2i\omega\Omega v_0 - \alpha_1^*q\omega\theta_0 = 0 \tag{18}$$

$$2i\omega\Omega\beta^2 u_0 + (A_2q^2 - \omega^2\beta^2 - \Omega^2\beta^2)v_0 = 0 \tag{19}$$

$$-\varepsilon_\theta\omega qu_0 + (q^2 - \omega^2\alpha_2^*)\theta_0 = 0 \tag{20}$$

Where,

dispersion equation of the coupled wave is obtained from Equations 18 to 20 as:

PERTURBATION SOLUTION FOR SMALL ε_θ

To obtain the perturbation solution of the dispersion equation for small values of ε_θ, we first put $\varepsilon_\theta = 0$ in Equation 22 and get the following solutions:

$$q^2 = \omega^2\alpha_2^* \tag{24}$$

$$A_1A_2q^4 - q^2(A_1\beta^2\Omega_0^2 + A_2\Omega_0^2) + \beta^2\Omega_0^4 - 4\Omega^2\omega^2\beta^2 = 0$$

Which on solving, we get:

$$q_{2,1}^2 = J_{2,1}^2 \tag{25}$$

Where,

$$J_{2,1}^2 = \frac{(A_2 + A_1\beta^2)\Omega_0^2 \pm \{(A_2 + A_1\beta^2)^2\Omega_0^4 - 4A_1A_2\beta^2(\Omega^2 - \omega^2)^2\}^{\frac{1}{2}}}{2A_1A_2} \tag{26}$$

Let us write q^2 in the following forms:

$$q_u^2 = q_1^2 + n_u\varepsilon_\theta + o(\varepsilon_\theta^2) \tag{27}$$

$$q_v^2 = q_2^2 + n_v\varepsilon_\theta + o(\varepsilon_\theta^2) \tag{28}$$

$$q_\theta^2 = \omega^2\alpha_2^* + n_\theta\varepsilon_\theta + o(\varepsilon_\theta^2) \tag{29}$$

Substituting into Equation 22, comparing the lowest degree of ε_θ, and neglecting the terms of $o(\varepsilon_\theta^2)$, we obtain:

$$q_u^2 = J_1^2[1 + \frac{\omega^2\alpha_1^*\varepsilon_\theta(A_2J_1^2 - \beta^2\Omega_0^2)}{D_1}] \tag{30}$$

$$q_v^2 = J_2^2[1 + \frac{\omega^2\alpha_1^*\varepsilon_\theta(A_2J_2^2 - \beta^2\Omega_0^2)}{D_2}] \tag{31}$$

$$q_\theta^2 = \omega^2 \alpha_2^* [1 + \frac{\alpha_1^* \omega^2 \varepsilon_\theta (A_2 \alpha_2^* \omega^2 - \beta^2 \Omega_0^2)}{(A_1 \alpha_2^* \omega^2 - \Omega_0^2)(A_2 \alpha_2^* \omega^2 - \beta^2 \Omega_0^2) - 4\Omega^2 \beta^2 \omega^2}]$$ (32)

Where,

$$D_{1,2} = (J_{1,2}^2 - \omega^2 \alpha_2^*)[2A_1 A_2 J_{1,2}^2 - A_1 \beta^2 \Omega_0^2 - A_2 \Omega_0^2]$$
$$+ (A_1 J_{1,2}^2 - \Omega_0^2)(A_2 J_{1,2}^2 - \beta^2 \Omega_0^2) - 4\Omega^2 \beta^2 \omega^2$$ (33)

On putting $M = N = R_H = 0$, the results of Equations 26, 30 and 31 are in agreement with the corresponding results of Roy (1985).

DETERMINATION OF WAVE VELOCITIES AND ATTENUATION COEFFICIENT

From the aforementioned solution, we can observe that the dilatational, shear and thermal waves propagate in the magneto-visco-elastic medium and these waves are affected by the visco-elastic coupling coefficient ε_θ. Now we find out the wave 'velocity' and the attenuation coefficients of the waves for small ε_θ.

Quasi-magneto-visco-thermal wave

$$R_e(q_\theta) = \omega \sqrt{\frac{L + \alpha^{*/}}{2} + \frac{\varepsilon_\theta}{2} \cdot \frac{\omega^3}{\sqrt{2}} \cdot \frac{1}{(D_v^2 + D_m^2)}}$$
$$\{\sqrt{L + \alpha^{*/}}(N_v D_v + N_m D_m) - \sqrt{L - \alpha^{*/}}(N_m D_v - N_v D_m)\}$$ (34)

$$I_m(q_\theta) = \omega \sqrt{\frac{L - \alpha^{*/}}{2} + \frac{\varepsilon_\theta}{2} \cdot \frac{\omega^3}{\sqrt{2}} \cdot \frac{1}{(D_v^2 + D_m^2)}}$$
$$\{\sqrt{L + \alpha^{*/}}(N_m D_v - N_v D_m) - \sqrt{L - \alpha^{*/}}(N_v D_v + N_m D_m)\}$$ (35)

Where,

$$N_v = \alpha^/ (\alpha^{*/} \omega^2 + N\omega^2 - \Omega_0^2 \beta^2) - (1 - \alpha^{*/} N\omega^2)$$ (36)

$$N_m = -(\alpha^{*/} \omega + N\omega - \frac{\Omega_0^2 \beta^2}{\omega}) - \alpha^/ \omega(1 - \alpha^{*/} N\omega^2)$$ (37)

$$D_v = (1 + R_H)(\alpha^{*/} \omega^2 + M\omega^2 - \Omega_0^2)(\alpha^{*/} \omega^2 + N\omega^2 - \Omega_0^2 \beta^2)$$
$$- \omega^2(1 - \alpha^{*/} M\omega^2)(1 - \alpha^{*/} N\omega^2) - 4\omega^2 \Omega^2 \beta^2$$ (38)

$$D_m = -\omega[(1 - \alpha^{*/} M\omega^2)(\alpha^{*/} \omega^2 + N\omega^2 - \Omega_0^2 \beta^2)]$$

$$+ (1 - \alpha^{*/} N\omega^2)\{(1 + R_H)\alpha^{*/} \omega^2 + M\omega^2 - \Omega_0^2\}]$$ (39)

$$L = \frac{\sqrt{\{(\alpha^{*/} \omega)^2 + 1\}}}{\omega}$$

and

$$\sqrt{\alpha_2^*} = \frac{1}{\sqrt{2}}[\sqrt{L + \alpha^{*/}} + i\sqrt{L - \alpha^{*/}}]$$ (40)

Therefore, the thermal wave velocity $c_\theta = \omega / R_e(q_\theta)$ and the attenuation coefficient:

$$a_\theta = -I_m(q_\theta),$$

Where, $R_e(q_\theta)$ and $I_m(q_\theta)$ are obtained as earlier mentioned.

Detailed calculation is given in the Appendix.

Quasi-magneto-visco-dilatational wave

Using Equation 26 in 30 for small ε_θ, the Quasi-magneto-visco-elastic dilatational wave 'velocity' $c_e = \omega / R_e(q_u)$ and the attenuation coefficient $a_e = -I_m(q_u)$, where:

$$R_e(q_u) = A_3 + \frac{\omega \varepsilon_\theta}{2} \cdot \frac{1}{P^2 + Q^2} \cdot [A_3\{P(\alpha^/ \omega k_3 + F_4) + Q(\alpha^/ \omega F_4 - k_3)\}$$

$$- B_3\{P(\alpha^/ \omega F_4 - k_3) - Q(\alpha^/ \omega k_3 + F_4)\}]$$ (41)

$$I_m(q_u) = B_3 + \frac{\omega \varepsilon_\theta}{2} \cdot \frac{1}{P^2 + Q^2} \cdot [B_3\{P(\alpha^/ \omega k_3 + F_4) + Q(\alpha^/ \omega F_4 - k_3)\}$$

$$- A_3\{P(\alpha^/ \omega F_4 - k_3) - Q(\alpha^/ \omega k_3 + F_4)\}]$$ (42)

Where,

$$P_1 = \{1 + (1 + R_H)\beta^2\}\Omega_0^4 - \omega^2(N + \beta^2 M)^2 \Omega_0^4 - 4\beta^2(\Omega^2 - \omega^2)^2(1 + R_H - \omega^2 MN)\Omega_0^4$$

$$P_2 = \omega[2(N + \beta^2 M)\{1 + (1 + R_H)\}\beta^2\}\Omega_0^4 - 4\beta^2(\Omega^2 - \omega^2)^2\{(1 + R_H)N + M\})]$$

$$A = \left\{\frac{\sqrt{P_1^2 + P_2^2} + P_1}{2}\right\}^{\frac{1}{2}}$$

$$B = \frac{P_2}{|P_2|}\left\{\frac{\sqrt{P_1^2 + P_2^2} - P_1}{2}\right\}^{\frac{1}{2}}$$

$$R_3 = \frac{1}{2} \cdot \frac{\{(1+R_H) - \omega^2 MN\}\{(1+(1+R_H)\beta^2)\Omega_0^2 - A\} + \omega\{(1+R_H)N + M\}\{\omega(N + \beta^2 M)\Omega_0^2 - B\}}{\{(1+R_H) - \omega^2 MN\}^2 + \omega^2\{(1+R_H)N + M\}^2}$$

$$R_4 = \frac{1}{2} \cdot \frac{\{(1+R_H) - \omega^2 MN\}\{\omega(N + \beta^2 M)\Omega_0^2 - B\} - \omega\{(1+R_H)N + M\}\{(1+(1+R_H)\beta^2)\Omega_0^2 - A\}}{\{(1+R_H) - \omega^2 MN\}^2 + \omega^2\{(1+R_H)N + M\}^2}$$

Quasi-magneto-visco-shear wave

Using Equation 26, we get for small ε_θ, the Quasi-magneto-visco-shear wave 'velocity' $c_s = \omega / R_e(q_v)$ and the attenuation coefficient $a_s = -I_m(q_v)$ where:

$$A_3 = \frac{1}{\sqrt{2}}[\sqrt{R_3^2 + R_4^2} + R_3]^{\frac{1}{2}}$$

$$B_3 = \frac{1}{\sqrt{2}}\frac{R_4}{|R_4|}[\sqrt{R_3^2 + R_4^2} - R_3]^{\frac{1}{2}}$$

$$R_e(q_u) = A_1 + \frac{\omega\varepsilon_\theta}{2} \cdot \frac{1}{P'^2 + Q'^2} \cdot [A_1\{P'(\alpha'\omega k_1 + F_2) + Q'(\alpha'\omega F_2 - k_1)\}$$

$$E_3 = 2R_3(1+R_H - \omega^2 MN) - 2\omega R_4\{(1+R_H)N+M\} - \{(1+R_H)\beta^2 + 1\}\Omega_0^2$$

$$E_4 = 2R_3\omega\{(1+R_H)N+M\} + 2R_4(1+R_H - \omega^2 MN) - \omega(N+\beta^2 M)\Omega_0^2$$

$$+ B_1\{P'(\alpha'\omega F_2 - k_1) - Q'(\alpha'\omega k_1 + F_2)\}] \tag{43}$$

$$F_3 = (1+R_H)R_3 - \omega MR_4 - \Omega_0^2$$

$$I_m(q_v) = B_1 + \frac{\omega\varepsilon_\theta}{2} \cdot \frac{1}{P'^2 + Q'^2} \cdot [B_1\{P'(\alpha'\omega k_1 + F_2) + Q'(\alpha'\omega F_2 - k_2)\}$$

$$F_4 = R_4 + \omega NR_3$$

$$k_3 = R_3 - \omega NR_4 - \beta^2\Omega_0^2$$

$$- A_1\{P'(\alpha'\omega F_2 - k_1) - Q'(\alpha'\omega k_1 + F_2)\}] \tag{44}$$

$$k_4 = R_4(1+R_H) + \omega MR_3$$

$$A_1 = \left\{ \frac{\sqrt{R_1^2 + R_2^2} + R_1}{2} \right\}^{\frac{1}{2}}$$

$$P = E_3(R_3 - \omega^2\alpha^*) - E_4(R_4 + \omega) + F_3 k_3 - F_4 k_4 - 4\Omega^2\beta^2\omega^2$$

$$Q = E_3(R_4 + \omega) + E_4(R_3 - \omega^2\alpha^{*/}) + k_3 k_4 + F_3 F_4$$

$$B_1 = \frac{R_2}{|R_2|}\left\{ \frac{\sqrt{R_1^2 + R_2^2} - R_1}{2} \right\}^{\frac{1}{2}}$$

$$R_1 = \frac{1}{2} \cdot \frac{\{(1+R_H) - \omega^2 MN\}\{(1+(1+R_H)\beta^2)\Omega_0^2 + A\} + \omega\{(1+R_H)N + M\}\{\omega(N + \beta^2 M)\Omega_0^2 + B\}}{\{(1+R_H) - \omega^2 MN\}^2 + \omega^2\{(1+R_H)N + M\}^2}$$

$$R_2 = \frac{1}{2} \cdot \frac{\{(1+R_H) - \omega^2 MN\}\{\omega(N + \beta^2 M)\Omega_0^2 + B\} - \omega\{(1+R_H)N + M\}\{(1+(1+R_H)\beta^2)\Omega_0^2 + A\}}{\{(1+R_H) - \omega^2 MN\}^2 + \omega^2\{(1+R_H)N + M\}^2}$$

$$E_1 = 2R_1(1+R_H - \omega^2 MN) - 2\omega R_2\{(1+R_H)N+M\} - \{(1+R_H)\beta^2 + 1\}\Omega_0^2$$

$$k_1 = R_1 - \omega NR_2 - \beta^2\Omega_0^2$$

$$E_2 = 2R_1\omega\{(1+R_H)N+M\} + 2R_2(1+R_H - \omega^2 MN) - \omega(N+\beta^2 M)\Omega_0^2$$

$$k_2 = R_2(1+R_H) + \omega MR_1$$

$$F_1 = (1+R_H)R_1 - \omega MR_2 - \Omega_0^2$$

$$P' = E_1(R_1 - \omega^2\alpha^*) - E_2(R_2 + \omega) + F_1 k_1 - F_2 k_2 - 4\Omega^2\beta^2\omega^2$$

$$F_2 = R_2 + \omega NR_1$$

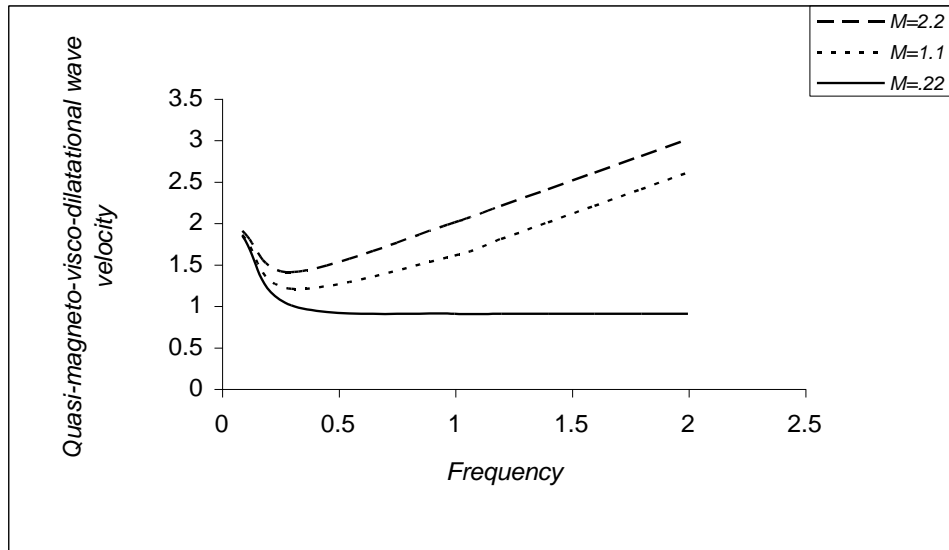$$Q' = E_1(R_2 + \omega) + E_2(R_1 - \omega^2\alpha^{*/}) + k_1 k_2 + F_1 F_2$$

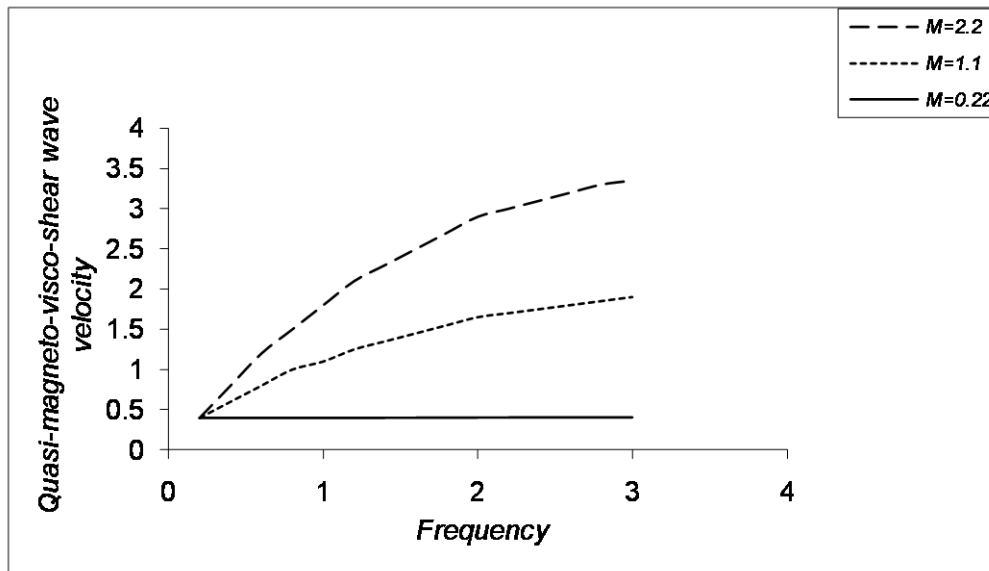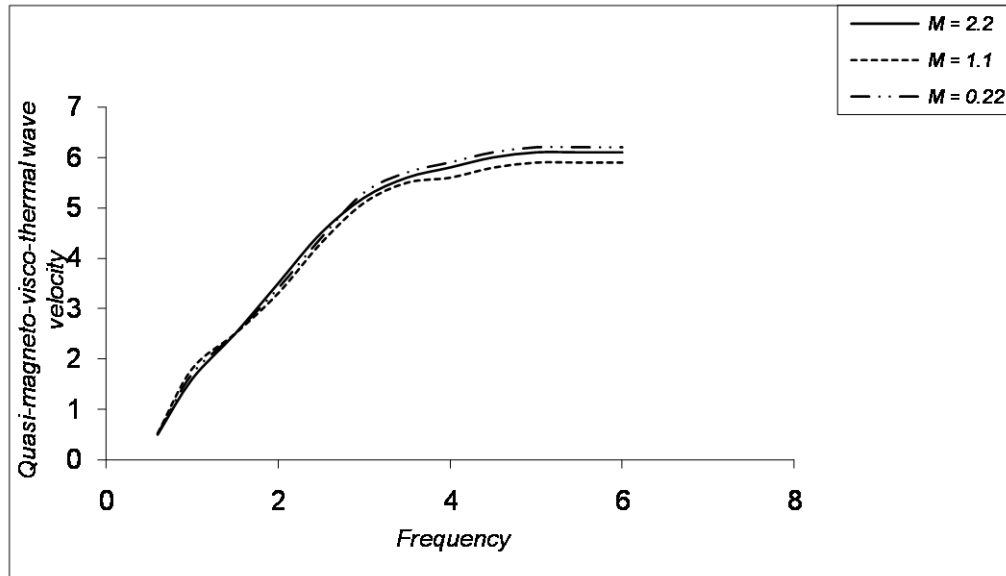**Figure 1.** Quasi-magneto-visco-dilatational wave velocity against real frequency ω

Figure 2. Quasi-magneto-visco-shear wave velocity against real frequency ω .

NUMERICAL RESULTS

For numerical computation, we take copper as the working substance for which $\lambda_e = 1.387 \times 10^{12}$ Dyne/cm^2, $\mu_e = 0.448 \times 10^{12}$ dyne/cm^2, $\rho = 8.93$ g/cm^3, $k = 10^{-11}$ s, $\alpha^* = 2 \times 10^{-11}$ s. We take $\Omega = 0.1$.

The numerical computations of the Quasi-magneto-visco-dilatational wave velocity, the Quasi-magneto-visco-shear wave velocity, and the Quasi-magneto-visco-thermal wave velocity for small values of ε_θ are done

and the corresponding graphs are plotted. The Quasi-magneto-visco-dilatational wave velocity is drawn against the real frequency ω for three different values of viscoelastic parameter M in Figure 1. The Quasi-magneto-visco-shear wave velocity is plotted against ω for different values of M in Figure 2 and the Quasi-magneto-visco-thermal wave velocity is drawn against ω for same values of M in Figure 3. The variation of different wave velocity with the visco-elastic parameter M can be seen from the graph. It is observed that although in the case of Quasi-magneto-visco-thermal wave

Figure 3. Quasi-magneto-visco-thermal wave velocity against real frequency ω.
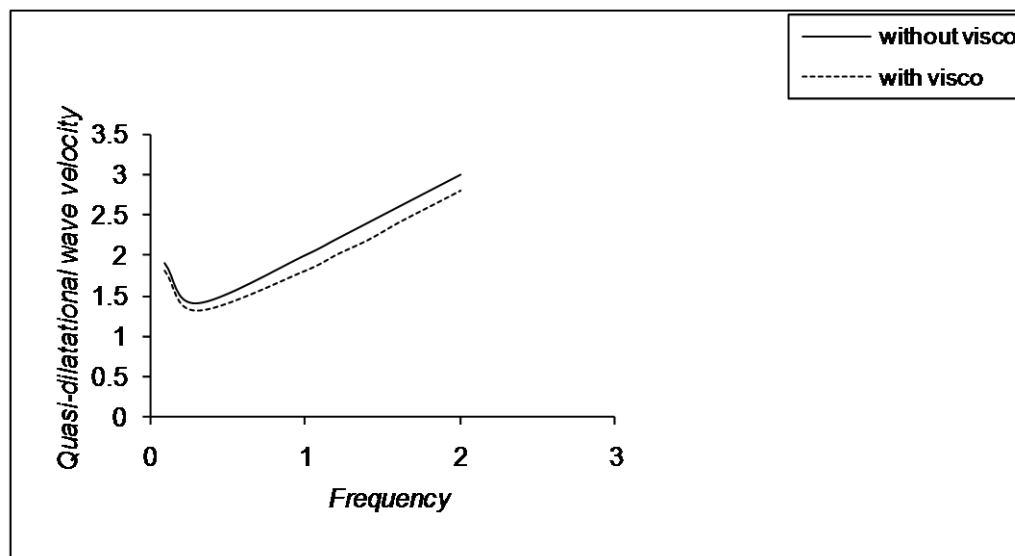
Figure 4. Comparison of Quasi-magneto-visco-dilatational wave velocity in the presence and absence of viscoelastic parameter

velocity obtained in Figure 3, the variation is not pronounced appreciably, in case of the Quasi-magneto-visco-thermal-shear wave velocity obtained in Figure 2, the variations are more pronounced for higher values of ω. It is clear from the Figures 4, 5 and 6 that in the viscoelastic medium, the amplitude of dilatational wave velocity, shear wave velocity and thermal wave velocity decreases. In Figure 7, comparison of Quasi-magneto-visco-dilatational wave velocity in the presence and

absence of magnetic field has been shown to show their respective effects.

Conclusion

It is observed that although in the case of Quasi-thermal wave velocity in respect of change of frequency, the variation is not pronounced appreciably; but in case of
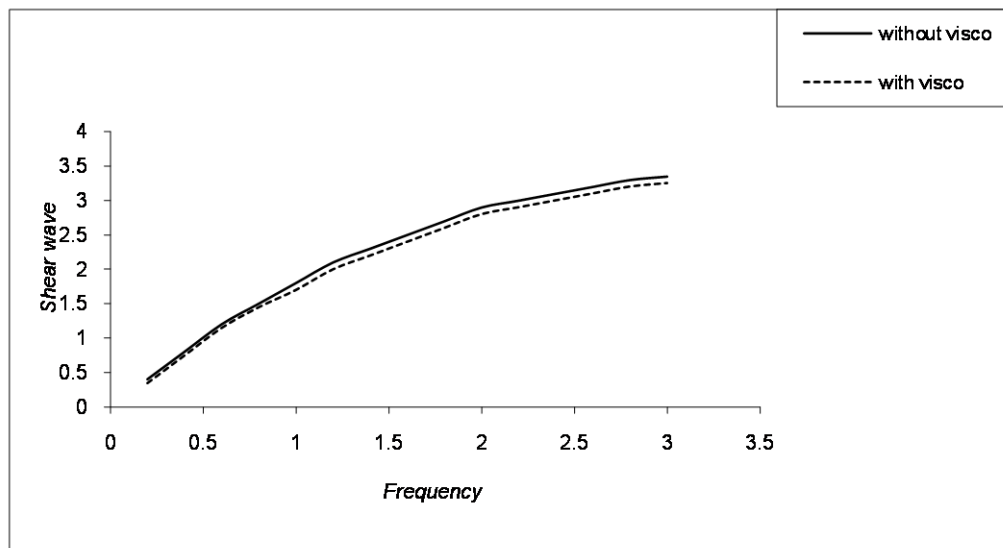
Figure 5. Comparison of Quasi-magneto-shear wave velocity in the presence and absence of viscoelastic parameter.
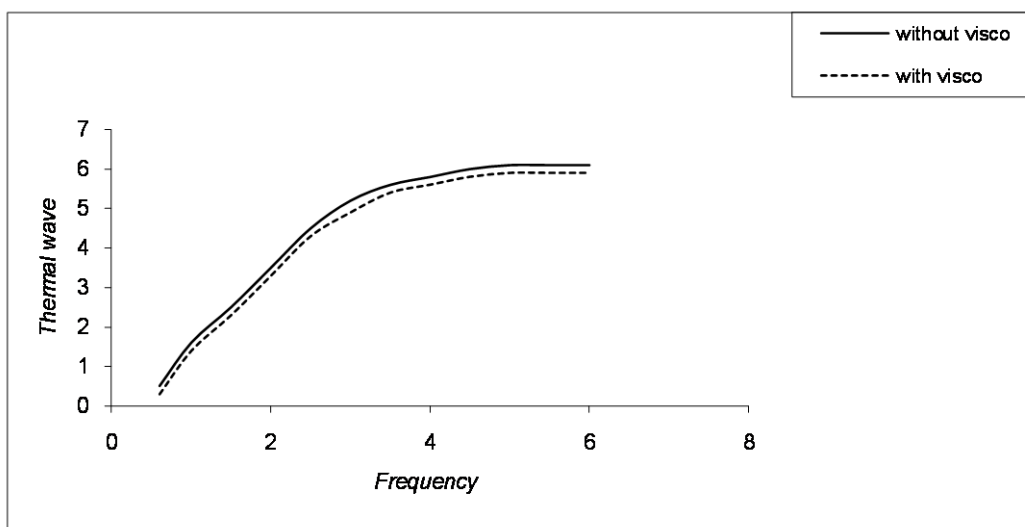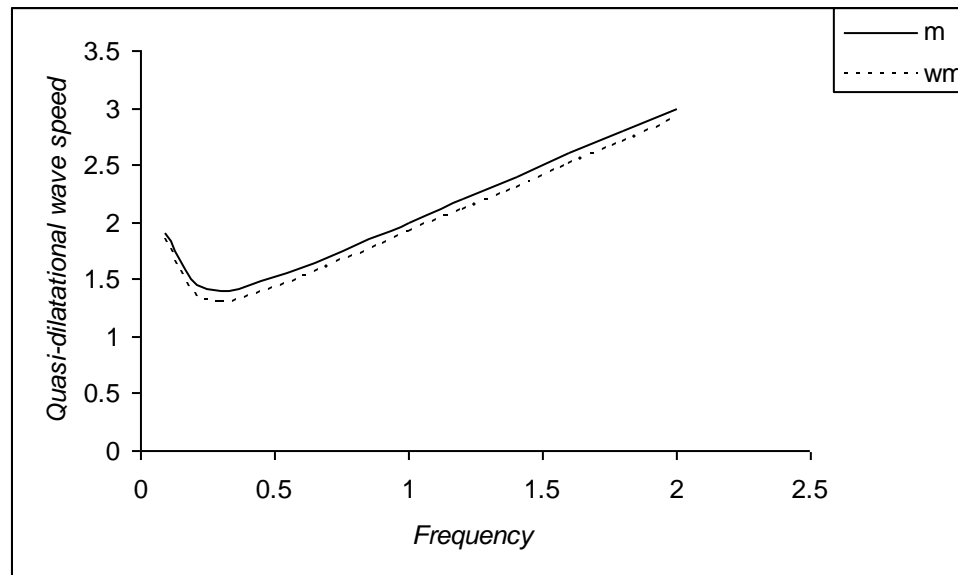

Figure 6. Comparison of Quasi-magneto-thermal wave velocity in the presence and absence of viscoelastic parameter.

the Quasi-magneto-visco-thermal-shear wave velocity and dilatational wave velocity, the variations are more pronounced for higher values of ω and due to rotational effect. A comparison has been made of dilatational wave velocities in the presence and absence of viscoelastic parameter. Similar comparisons have been made for shear wave velocity and thermal wave velocity under the influence of rotation and magnetic field. All the discussions have been represented in the enclosed graphs. Finally, it may be recalled that in the viscoelastic medium, the amplitude of dilatational wave velocity,

shear wave velocity and thermal wave velocity decreases. The effect of rotation in the magneto-thermo-visco-elastic medium have been shown in the respective graphs for finding the wave velocities.

ACKNOWLEDGEMENT

Figure 7. Comparison of Quasi-magneto-visco-dilatational wave **velocity** in the presence and absence of magnetic field

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APPENDIX

Calculation of $R_e(q_\theta)$ and $I_m(q_\theta)$

$$q_\theta^2 = \omega^2 \alpha_2^*[1 + \frac{\alpha_1^* \omega^2 \varepsilon_\theta (A_2 \alpha_2^* \omega^2 - \beta^2 \Omega_0^2)}{(A_1 \alpha_2^* \omega^2 - \Omega_0^2)(A_2 \alpha_2^* \omega^2 - \beta^2 \Omega_0^2) - 4\Omega^2 \beta^2 \omega^2}]$$

$$(A_1 \alpha_2^* \omega^2 - \Omega_0^2)(A_2 \alpha_2^* \omega^2 - \beta^2 \Omega_0^2) - 4\Omega^2 \beta^2 \omega^2$$

$$= [\{(1+R_H) + i\omega M\}(\alpha^{*/} - \frac{i}{\omega})\omega^2 - \Omega_0^2].[(1+i\omega N)(\alpha^{*/} - \frac{i}{\omega})\omega^2] - 4\Omega^2 \beta^2 \omega^2$$

$$= [\{(1+R_H)\alpha^{*/}\omega^2 + M\omega^2 - \Omega_0^2\}\{\alpha^{*/}\omega^2 + N\omega^2 - \beta^2 \Omega_0^2\} - \omega^2(\alpha^{*/}\omega^2 M - 1)(\alpha^{*/}\omega^2 N - 1)$$
$$- 4\Omega^2 \beta^2 \omega^2]$$

$$+ i[\omega(\omega^2 N \alpha^{*/} - 1)\{(1+R_H)\alpha^{*/}\omega^2 + M\omega^2 - \Omega_0^2\} + \omega(\omega^2 M \alpha^{*/} - 1)(\alpha^{*/}\omega^2 + N\omega^2 - \beta^2 \Omega_0^2)]$$
$$= D_v + iD_m$$

where $D_v =$

$$[\{(1+R_H)\alpha^{*/}\omega^2 + M\omega^2 - \Omega_0^2\}\{\alpha^{*/}\omega^2 + N\omega^2 - \beta^2 \Omega_0^2\} - \omega^2(\alpha^{*/}\omega^2 M - 1)(\alpha^{*/}\omega^2 N - 1)$$

$$- 4\Omega^2 \beta^2 \omega^2] \text{ and } D_m =$$

$$i[\omega(\omega^2 N \alpha^{*/} - 1)\{(1+R_H)\alpha^{*/}\omega^2 + M\omega^2 - \Omega_0^2\} + \omega(\omega^2 M \alpha^{*/} - 1)(\alpha^{*/}\omega^2 + N\omega^2 - \beta^2 \Omega_0^2)]$$

$$\alpha_1^* \omega^2 \varepsilon_\theta (A_2 \alpha_2^* \omega^2 - \beta^2 \Omega_0^2)$$

$$= (\alpha^{/} - \frac{i}{\omega})\omega^2 \varepsilon_\theta \{(1+i\omega N)(\alpha^{*/} - \frac{i}{\omega})\omega^2 - \beta^2 \Omega_0^2\}$$

$$= \omega^2 \varepsilon_\theta [\{\alpha^{/}(\omega^2 \alpha^{*/} + N\omega^2 - \beta^2 \Omega_0^2) + (\omega^2 N \alpha^{*/} - 1)\} + i\{\alpha^{/}(\omega^2 N \alpha^{*/} - 1)\omega - \frac{1}{\omega}(\omega^2 \alpha^{*/} + N\omega^2 - \beta^2 \Omega_0^2)\}]$$

$$= \omega^2 \varepsilon_\theta (N_v + N_m)$$

where $N_v = \omega^2 \varepsilon_\theta [\{\alpha^{/}(\omega^2 \alpha^{*/} + N\omega^2 - \beta^2 \Omega_0^2) + (\omega^2 N \alpha^{*/} - 1)\}$

and $N_m = \{\alpha^{/}(\omega^2 N \alpha^{*/} - 1)\omega - \frac{1}{\omega}(\omega^2 \alpha^{*/} + N\omega^2 - \beta^2 \Omega_0^2)\}]$

$$q_\theta^2 = \omega^2 \alpha_2^*[1 + \frac{\alpha_1^* \omega^2 \varepsilon_\theta (A_2 \alpha_2^* \omega^2 - \beta^2 \Omega_0^2)}{(A_1 \alpha_2^* \omega^2 - \Omega_0^2)(A_2 \alpha_2^* \omega^2 - \beta^2 \Omega_0^2) - 4\Omega^2 \beta^2 \omega^2}]$$

$$= \omega^2(\alpha^* - \frac{i}{\omega})[1 + \frac{\omega^2\varepsilon_\theta(N_\nu + N_m)}{D_\nu + iD_m}]$$

$$= \omega^2(\alpha^* - \frac{i}{\omega})[1 + \omega^2\varepsilon_\theta \cdot \frac{(N_\nu D_\nu + N_m D_m) + i(N_m D_\nu - N_\nu D_m)}{D_\nu^2 + D_m^2}]$$

$$R_e(q_\theta) = \omega\sqrt{\frac{L + \alpha^{*/}}{2}} + \frac{\varepsilon_\theta}{2} \cdot \frac{\omega^3}{\sqrt{2}} \cdot \frac{1}{(D_\nu^2 + D_m^2)} \cdot$$

$$\{\sqrt{L + \alpha^{*/}}(N_\nu D_\nu + N_m D_m) - \sqrt{L - \alpha^{*/}}(N_m D_\nu + N_\nu D_m)\}$$

$$I_m(q_\theta) = \omega\sqrt{\frac{L - \alpha^{*/}}{2}} + \frac{\varepsilon_\theta}{2} \cdot \frac{\omega^3}{\sqrt{2}} \cdot \frac{1}{(D_\nu^2 + D_m^2)}$$

$$\{\sqrt{L + \alpha^{*/}}(N_m D_\nu + N_\nu D_m) - \sqrt{L - \alpha^{*/}}(N_\nu D_\nu + N_m D_m)\}$$