Effect of Slip Condition on Couple Stress Fluid Flow in a Channel with Mild Stenosis in the Presence of Uniform Magnetic Field

Gurju Awgichew[*], G. Radhakrishnamacharya and B. Shireesha

Department of Mathematics
National Institute of Technology
Warangal-506 004, India
gurjua@yahoo.com*, grk.nitw@yahoo.com, shireeshabajjuri@gmail.com

* Corresponding Author

Abstract

The steady flow of an incompressible couple stress fluid in a two dimensional uniform channel with stenosis under the influence of a magnetic field has been investigated. Assuming the stenosis to be mild, the flow equations have been analytically solved using the slip condition and expressions for the resistance to flow and wall shear stress have been derived. The effects of various parameters on these flow variables have been studied. It is found that the resistance to flow as well as the wall shear stress increase with the height of the stenosis and decrease with the couple stress and magnetic parameters. The effects of other parameters on resistance to flow and the wall shear stress have been considered.

1. Introduction

It is known that one of the major health hazards is atherosclerosis, which refers to narrowing of the arterial lumen, i.e., the inner open space or cavity of an artery, due to deposition of fatty substances. This may lead to hypertension, myocardial infarction, etc. The formation of stenosis, i.e., the abnormal and unnatural growth, disturbs the normal blood flow and there is considerable evidence to show that hydro dynamical factors such as wall shear stress, resistance to flow etc. can play a significant role in the development and progression of this pathological condition. Hence, a detailed knowledge of the flow field in a stenosed tube may help in the proper

understanding and prevention of arterial diseases.

In view of this, several authors have considered various mathematical models to study the flow of fluid in a stenosed artery. Young (1968) considered mild stenosis and analyzed the effect of stenosis growth on shear stress and resistance to flow. Tu and Deville (1996) presented a theoretical analysis of pulsatile flow of blood in stenotic arteries. Young and Tsai (1973) have experimentally investigated blood flow in the vicinity of stenosis in artery. Lee and Fung (1970) have obtained the numerical results for the streamlines and distribution of velocity, pressure, vorticity, and the shear stress for different Reynolds number in blood flow through locally constricted tube. Subsequently several researchers have considered fluid mechanical studies in stenotic region flow (Shukla et al., 1980; Forrester and Young, 1970; Misra and Chakravarty, 1986).

In most of the above studies, blood is considered as a Newtonian fluid, although it is well known that most of the physiological fluids including blood behave as a non-Newtonian fluid. The non-Newtonian behavior of blood is mainly due to the suspension of red blood cells in the plasma. When neutrally buoyant corpuscles are contained in a fluid in which exists a velocity gradient due to shearing stress, corpuscles have rotatory motion. Furthermore, it is observed that the corpuscles have spin angular momentum, in addition to orbital angular momentum. As a result, the symmetry of stress tensor is lost in the fluid motion that is subjected to spin angular momentum. The fluid that has neutrally buoyant corpuscles, when observed microscopically, exhibits non-Newtonian behavior, and its constitutive equation is expressed by Stokes (1966). Stokes (1966) represents the simplest generalization of the classical viscous fluid theory that sustains couple stresses and the body couples. The important feature of these fluids is that the stress tensor is not symmetric and their accurate flow behavior cannot be predicted by the classical Newtonian theory. The main effect of couple stresses will be to introduce a size dependent effect that is not present in the classical viscous theories. The importance of the consideration of couple stress effects in studies of physiological and some other fluids was indicated by Cowin (1974). Studies on the couple stress fluid behavior are very useful, because such studies bear the potential to better explain the behavior of rheologically complex fluids, such as liquid crystals, polymeric suspensions that have long-chain molecules, lubrication as well as human/sub-human blood (Stokes, 1966; Ghosh et al., 2010). Shehawey and Mekheimer (1994) analyzed the flow of a couple stress fluid for any arbitrary Reynolds' number and wave number, considering the wave amplitude to be small. Sankad and Radhakrishnamacharya (2011) discussed peristaltic transport of couple stress fluid in a channel with magnetic field and wall properties.

Magneto hydrodynamics (MHD) is the science which deals with the motion of highly conducting fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid (Mekheimer, 2008). As early as 1937, Hartmann and Lazarus (1937) studied the influence of a uniform transverse magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite parallel thermally insulated plates that are in a stationary state. Since then, this work on magneto hydrodynamic (MHD) flow has received much attention. This is due to the fact that such studies are useful particularly for getting a proper understanding

of the functioning of different machines used by clinicians for pumping blood (Misra et al., 2008). Misra et al. (2008) pointed out that theoretical researches with an aim to explore the effect of a magnetic field on the flow of blood in atherosclerotic vessels also find application in a blood pump used by cardiac surgeons during the surgical procedure. The MHD principles may be used to reaccelerate the flow of blood in a human artery system and thereby it is useful in the treatment of certain cardiovascular disorders (Korchevskii and Marochunik, 1965) and in the diseases with accelerated blood circulation like hemorrhages and hypertension, etc.

The study of fluid flow in confined geometry requires specification of the boundary condition for the fluid velocity at the solid wall. No slip boundary condition is the core concept in fluid dynamics. The no slip condition for a viscous fluid states that the fluid velocity at all fluid boundary is equal to that of the solid boundary i.e., the fluid and the boundary move with the same velocity. Although this assumption is successful in describing fluid flow on macroscopic length scales, it needs a revision for the micro and nano-scales due to possible slip of the fluid relative to the wall. Interaction of couple stresses and slip flow on peristaltic transport in uniform and non-uniform channels has been considered by Sobh (2008). Srinivas et al. (2009) attempted to study the influence of slip conditions, wall properties and heat transfer on MHD peristaltic transport. Ramakrishnan and Shailendhra (2011) investigated the combined effects of the magnetic field, permeable walls and slip parameter on the steady flow of a fluid in a channel.

In the present paper, the effect of slip condition on steady couple stress fluid flow in a two dimensional uniform channel with stenosis is considered. Assuming that a uniform magnetic field is applied perpendicular to the wall of the channel and the stenosis is to be mild, closed-form solution has been obtained and expressions for resistance to flow and shear stress at the wall have been derived. The effects of various relevant parameters on these flow variables have been studied. Since this study has been carried out for a situation when the channel is subjected to an external magnetic field, it bears the promise of significant application in magnetic/electromagnetic therapy, which has gained enough popularity today.

2. Formulation of the Problem

We consider the steady, incompressible couple stress fluid flow through a two dimensional uniform channel with local stenosis. The Cartesian coordinate system (x, y) is chosen so that the x-axis coincides with the center line of the channel and the y-axis normal to it. Further, it is assumed that a uniform magnetic field B_0 is applied to the fluid normal to the walls of the channel. The stenosis is supposed to be mild and develops in a symmetric manner. The boundary of the channel is taken as (Shukla et al., 1980)

$$\eta(x) = \begin{cases} d_0 - \dfrac{\delta}{2}\left(1 + cos\dfrac{2\pi}{L_0}(x - d_1 - \dfrac{L_0}{2})\right), & d_1 \leq x \leq d_1 + L_0, \\ d_0, & \text{otherwise}, \end{cases} \tag{1}$$

where d_0 is the mean half width of the non-stenotic region of the channel, L_0 is the length of

stenosis, L is the length of the channel and δ is the maximum height of the stenosis (Figure 1).
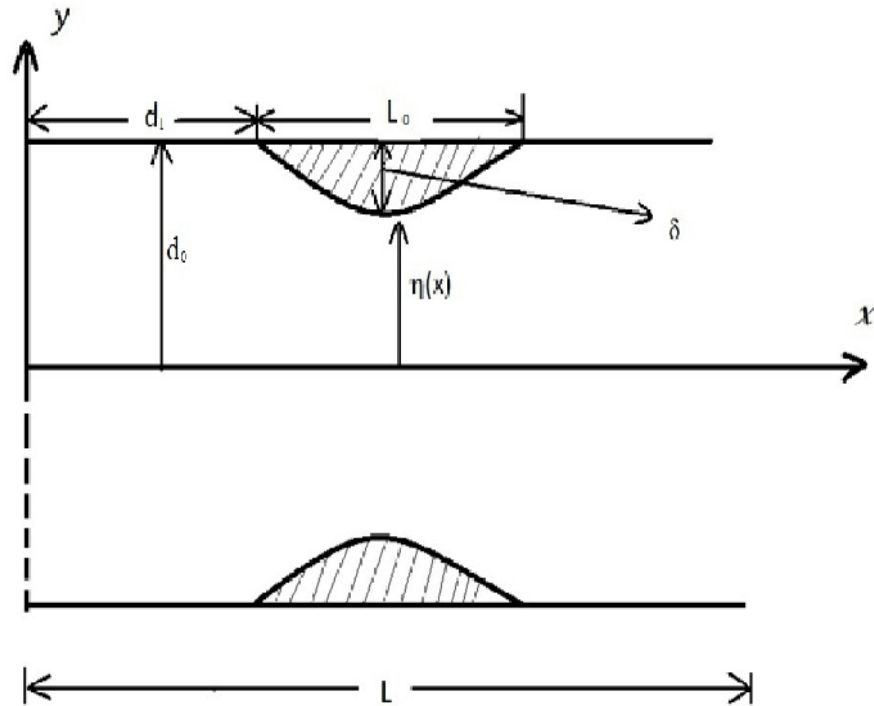

Figure 1. Geometry of the channel with local stenosis

It is also assumed that the fluid is of small electrical conductivity with magnetic Reynolds number much less than unity, so that the induced magnetic field can be neglected in comparison with applied magnetic field. The equations governing the motion of the incompressible couple stress fluid for the present problem by neglecting body forces and body couples are given as (Alemayehu and Radhakrishnamacharya, 2011)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \eta' \frac{\partial^4 u}{\partial y^4} - \sigma B_0^2 u = 0, \tag{3}$$

$$-\frac{\partial p}{\partial y} = 0, \tag{4}$$

where u and v are the velocity components along the x and y directions respectively, p, the pressure, σ, the electric conductivity of the fluid, μ, the viscosity coefficient of the classical fluid dynamics and η', the couple stress fluid viscosity.
The boundary conditions are given by

$$u = \frac{-d_0\sqrt{Da}}{\alpha}\frac{\partial u}{\partial y} \qquad at \quad y = \pm\eta, \tag{5}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad at \quad y = \pm\eta. \tag{6}$$

Here, (5) is the slip boundary condition (Bahtt and Sacheti, 1979) and (6) indicates the vanishing of couple stress. Further, Da is the permeability parameter (or Darcy number) and α is the slip parameter.

Taking the restriction for mild stenosis (Young, 1968) and solving equations (2-4) under the boundary conditions equation (5) and equation (6), the velocity is given as

$$u(y) = -\frac{1}{\sigma B_0^2}\frac{\partial p}{\partial x}\left[S_2^* \cosh(m_1^* y) - S_1^* \cosh(m_2^* y) + 1\right], \tag{7}$$

where

$$m_1^* = \sqrt{\frac{\mu}{2\eta'}\left(1 + \sqrt{1 - \frac{4\sigma B_0^2 \eta'}{\mu^2}}\right)}, m_2^* = \sqrt{\frac{\mu}{2\eta'}\left(1 - \sqrt{1 - \frac{4\sigma B_0^2 \eta'}{\mu^2}}\right)},$$

$$a_1^* = \left(m_1^*\right)^2 \cosh(m_1^* \eta)\left[\cosh(m_2^* \eta) + \frac{d_0\sqrt{Da}}{\alpha}m_2^* \sinh(m_2^* \eta)\right],$$

$$a_2^* = \left(m_2^*\right)^2 \cosh(m_2^* \eta)\left[\cosh(m_1^* \eta) + \frac{d_0\sqrt{Da}}{\alpha}m_1^* \sinh(m_1^* \eta)\right],$$

and

$$S_1^* = \frac{(m_1^*)^2 \cosh(m_1^* \eta)}{a_1^* - a_2^*}, S_2^* = \frac{(m_2^*)^2 \cosh(m_2^* \eta)}{a_1^* - a_2^*}.$$

3. Analysis

The flux Q of the fluid is given by

$$Q = 2\int_0^\eta u\,dy = \frac{-2}{\sigma B_0^2}\frac{\partial p}{\partial x}\left[S_2^*\frac{sinh(m_1^*\eta)}{m_1^*} - S_1^*\frac{sinh(m_2^*\eta)}{m_2^*} + \eta\right]. \tag{8}$$

Introducing the following dimensionless quantities

$$\delta^* = \frac{\delta}{d_0}, x^* = \frac{x}{L}, d_1^* = \frac{d_1}{L}, L_0^* = \frac{L_0}{L}, H = \frac{\eta}{d_0}, p^* = \frac{p}{(\sigma B_0^2 UL)}, Q^* = \frac{Q}{Ud_0} \tag{9}$$

in equation (1) and equation (8), we get (after dropping asterisks)

$$Q = -2\frac{\partial p}{\partial x}\left[S_2\frac{sinh(m_1 H)}{m_1} - S_1\frac{sinh(m_2 H)}{m_2} + H\right], \tag{10}$$

where $m = d_0 m^* = d_0(\mu/\eta')^{\frac{1}{2}}$ is a couple stress parameter, $M = B_0 d_0\left(\frac{\sigma}{\mu}\right)^{1/2}$ is called the Hartman number (magnetic parameter),

$$m_1 = m_1^* d_0 = \sqrt{\frac{m^2}{2}\left(1 + \sqrt{1 - \frac{4M^2}{m^2}}\right)}, \quad m_2 = m_2^* d_0 = \sqrt{\frac{m^2}{2}\left(1 - \sqrt{1 - \frac{4M^2}{m^2}}\right)},$$

$$a_1 = m_1^2 cosh(m_1 H)\left[cosh(m_2 H) + \frac{\sqrt{Da}}{\alpha}m_2 sinh(m_2 H)\right],$$

$$a_2 = m_2^2 cosh(m_2 H)\left[cosh(m_1 H) + \frac{\sqrt{Da}}{\alpha}m_1 sinh(m_1 H)\right],$$

and

$$S_1 = \frac{m_1^2 cosh(m_1 H)}{a_1 - a_2}, S_2 = \frac{m_2^2 cosh(m_2 H)}{a_1 - a_2}.$$

From equation (10), we obtain

$$\frac{\partial p}{\partial x} = -\frac{Q}{2}\left(\frac{1}{S_2\frac{sinh(m_1 H)}{m_1} - S_1\frac{sinh(m_2 H)}{m_2} + H}\right). \tag{11}$$

Integrating equation (11) with respect to x, we get the pressure difference Δp along the total length of the channel as

$$\Delta p = \frac{Q}{2} \int_0^1 \left(\frac{1}{S_2 \dfrac{sinh(m_1 H)}{m_1} - S_1 \dfrac{sinh(m_2 H)}{m_2} + H} \right) dx .$$ (12)

The resistance to flow, denoted by λ, is defined by

$$\lambda = \frac{\Delta p}{Q} .$$ (13)

Using equation (12) in equation (13), we get

$$\lambda = \frac{1}{2} \int_0^1 \left(\frac{1}{S_2 \dfrac{sinh(m_1 H)}{m_1} - S_1 \dfrac{sinh(m_2 H)}{m_2} + H} \right) dx .$$ (14)

The pressure drop in the case of no stenosis ($H = 1$), denoted by Δp_n, is obtained from equation (12) as

$$\Delta p_n = \frac{Q}{2} \int_0^1 \left(\frac{1}{S_{22} \dfrac{sinh(m_1)}{m_1} - S_{11} \dfrac{sinh(m_2)}{m_2} + 1} \right) dx ,$$ (15)

where

$$S_{11} = \frac{m_1^2 \, cosh(m_1)}{a_{11} - a_{22}}, \quad S_{22} = \frac{m_2^2 \, cosh(m_2)}{a_{11} - a_{22}},$$

$$a_{11} = m_1^2 \, cosh(m_1) \left[cosh(m_2) + \frac{\sqrt{Da}}{\alpha} m_2 \, sinh(m_2) \right],$$

and

$$a_{22} = m_2^2 \, cosh(m_2) \left[cosh(m_1) + \frac{\sqrt{Da}}{\alpha} m_1 \, sinh(m_1) \right].$$

The resistance to flow in the absence of stenosis, λ_n, is obtained from equation (13) as

$$\lambda_n = \frac{\Delta p_n}{Q}.$$

(16)

Using equation (15) in equation (16), we get

$$\lambda_n = \frac{1}{2}\frac{1}{\int_0^1}\left(\frac{1}{S_{22}\dfrac{sinh(m_1)}{m_1} - S_{11}\dfrac{sinh(m_2)}{m_2} + 1} \right) dx.$$

(17)

The normalized resistance to the flow, $\overline{\lambda}$, is given by

$$\overline{\lambda} = \frac{\lambda}{\lambda_n}.$$

(18)

The shear stress acting on the wall of the channel is given by

$$\tau_w = -\mu \frac{\partial u}{\partial y}\bigg|_{y=\eta}.$$

(19)

Using

$$\tau_w^* = \frac{\tau_w}{\left(\dfrac{\mu U}{d_0} \right)}$$

and non-dimensional quantities given by equation (9) in equation (7) and equation (19), we get (after dropping asterisks)

$$\tau_w = -\frac{Q}{2}\left(\frac{S_2 m_1 \, sinh(m_1 H) - S_1 m_2 \, sinh(m_2 H)}{S_2 \dfrac{sinh(m_1 H)}{m_1} - S_1 \dfrac{sinh(m_2 H)}{m_2} + H} \right).$$

(20)

The shear stress at the wall in the absence of stenosis ($H = 1$), denoted by $(\tau_w)_n$, can be obtained from equation (20) as

$$(\tau_w)_n = -\frac{Q}{2}\left(\frac{S_{22} m_1 \, sinh(m_1) - S_{11} m_2 \, sinh(m_2)}{S_{22} \dfrac{sinh(m_1)}{m_1} - S_{11} \dfrac{sinh(m_2)}{m_2} + 1} \right).$$

(21)

The normalized shear stress at the wall is, $\overline{\tau}_w$, given by

$$\overline{\tau}_w = \frac{\tau_w}{(\tau_w)_n} \;.$$
(22)

4. Results and Discussion

The resistance to flow and the wall shear stress are two important characteristics in the study of blood flow through a stenosed artery. The expressions for resistance to the flow and wall shear stress, given by equation (18) and equation (22) respectively have been numerically evaluated using MATHEMATICA software for different values of the relevant parameters and presented graphically. Tabular representations are also used for those parameters whose effects cannot be seen from the graphs.

Figures 2-6 show the effects of various parameters on the resistance to flow in a uniform channel with mild stenosis in the presence of a uniform magnetic field. It can be observed that the resistance to flow increases with the height of the stenosis (Figures 2-6). This result agrees with the previous results obtained by Young (1968), Shukla et al. (1980), Forrester and Young (1970). Further, it can be noticed that the resistance to flow increases with the length of the stenosis (Figures 2 and 3) and the slip parameter (Figure 4), but decreases with the couple stress parameter (Figures 2 and 3), the Darcy number (Figure 5) and the magnetic parameter (Figure 6).

Figures 7-8 and Tables 1-2 show the effects of various parameters on the wall shear stress. The wall shear stress increases with the height of stenosis (Figures 7 and 8) and (Tables 1 and 2). This result agrees with previous results obtained by Young (1968), Lee and Fung (1970). Moreover, the wall shear stress increases with the slip parameter (Figure 7), but decreases with the Darcy number (Figure 8), the couple stress parameter (Table 1) and the magnetic parameter (Table 2). It can also be noticed that the effect of different parameters is not very significant on the wall shear stress compared to that of the resistance to flow (Figures 2-8 and Tables (1 and 2)).
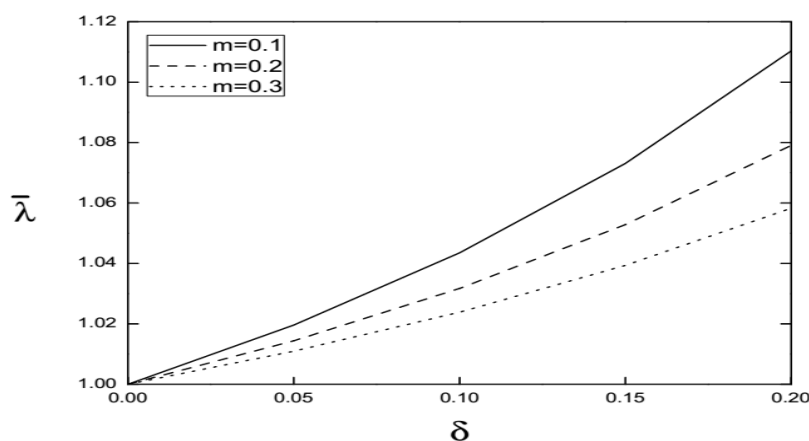

Figure 2. Effect of $m \, on \; \overline{\lambda} \, (L_0 = 0.2, d_1 = 0.4, \alpha = 0.02, Da = 0.002, M = 5)$
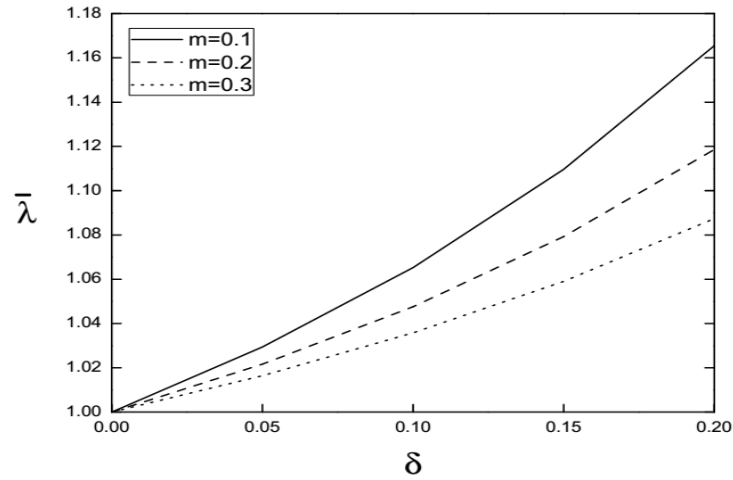
Figure 3. Effect of m on $\bar{\lambda}$ ($L_0 = 0.3, d_1 = 0.4, \alpha = 0.02, Da = 0.002, M = 5$)
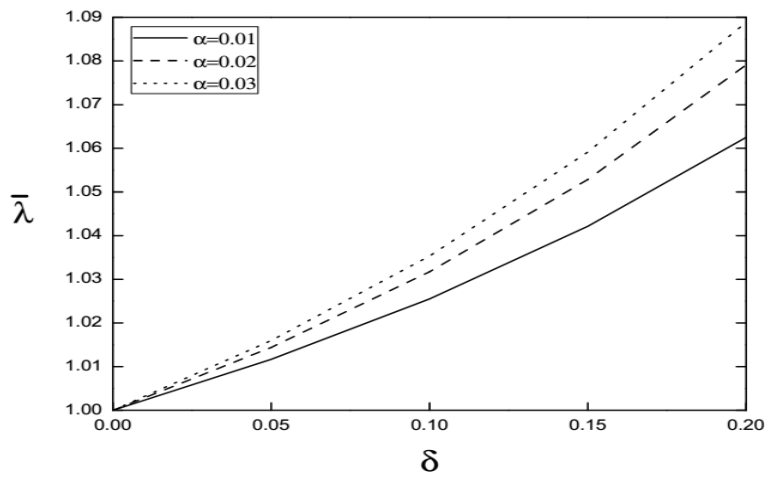

Figure 4. Effect of α on $\bar{\lambda}$ ($L_0 = 0.2, d_1 = 0.4, Da = 0.002, m = 0.2, M = 5$)
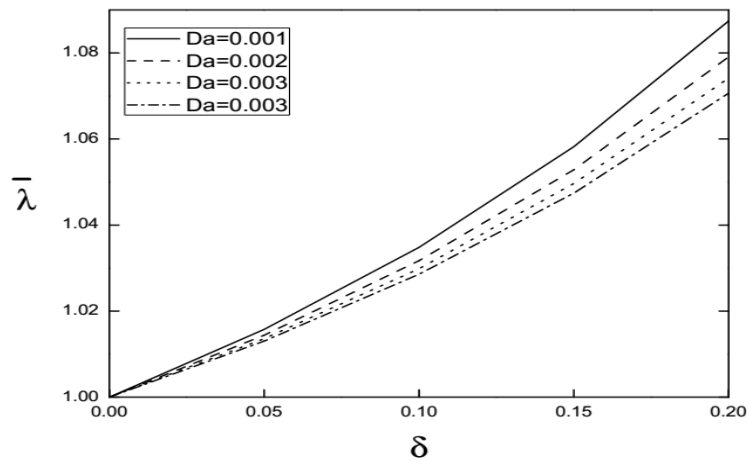

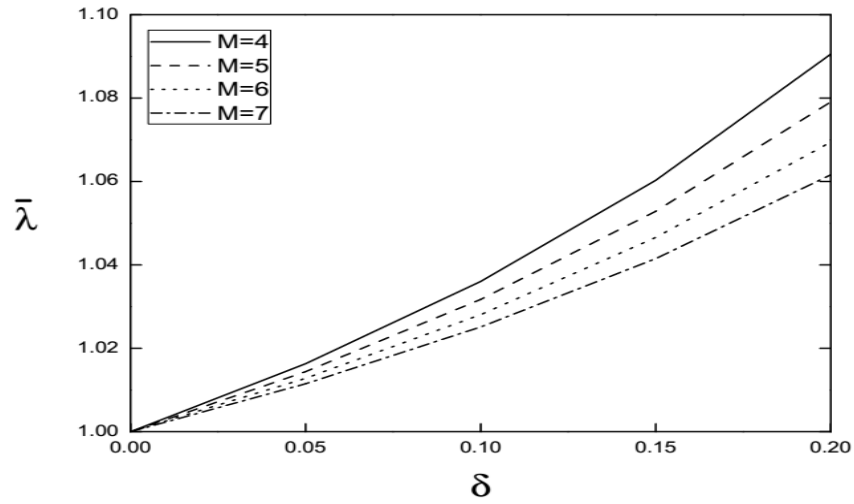**Figure 5**. Effect of Da on $\bar{\lambda}$ ($L_0 = 0.2, d_1 = 0.4, \alpha = 0.02, m = 0.2, M = 5$)

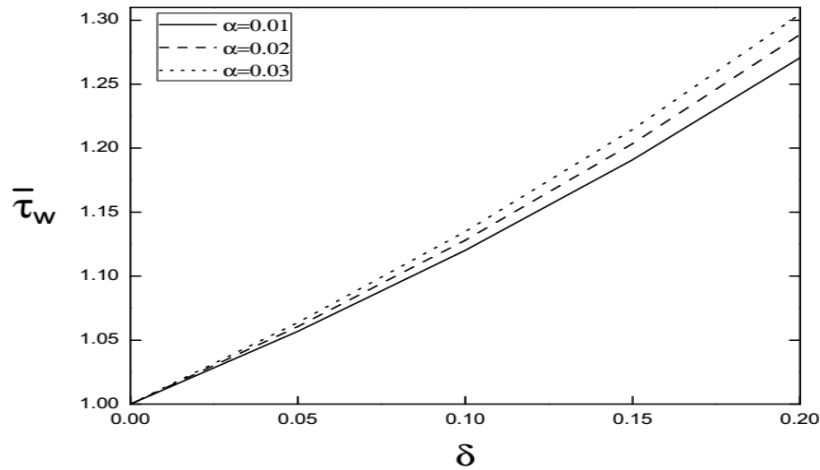**Figure 6.** Effect of M on $\bar{\lambda}$ ($L_0 = 0.2, d_1 = 0.4, \alpha = 0.02, m = 0.2, Da = 0.002$)

Figure 7. Effect of α on $\overline{\tau}_w$ ($L_0 = 0.2, d_1 = 0.4, x = 0.5, Da = 0.002, m = 0.2, M = 5$)
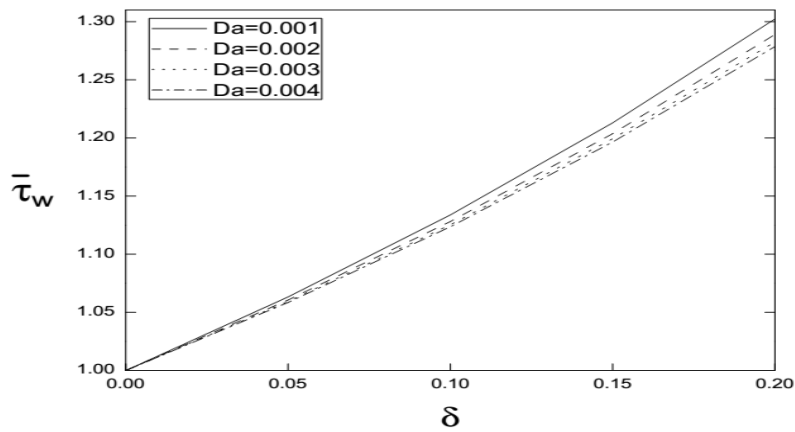

Figure 8. Effect of Da on $\overline{\tau}_w$ ($L_0 = 0.2, d_1 = 0.4, x = 0.5, \alpha = 0.02, m = 0.2, M = 5$)

Table 1: Effect of m on $\overline{\tau_w}$ $(L_0 = 0.2, d_1 = 0.4, x = 0.5, Da = 0.002, \alpha = 0.02, M = 5)$

$\downarrow \delta$	$m \rightarrow$		
	0.1	0.2	0.3
0	1	1	1
0.05	1.06066	1.0606	1.0605
0.1	1.12819	1.12808	1.12789
0.15	1.20382	1.20365	1.20338
0.2	1.28905	1.28883	1.28848
$\overline{\tau_w} \rightarrow$			

Table 2: Effect of M on $\overline{\tau_w}$ $(L_0 = 0.2, d_1 = 0.4, x = 0.5, Da = 0.002, m = 0.2, \alpha = 0.02)$

$\downarrow \delta$	$M \rightarrow$			
	4	5	6	7
0	1	1	1	1
0.05	1.06063	1.0606	1.06057	1.06053
0.1	1.12813	1.12808	1.12802	1.12794
0.15	1.20372	1.20365	1.20356	1.20345
0.2	1.28893	1.28883	1.28871	1.28857
$\overline{\tau_w} \rightarrow$				

5. Conclusion

A mathematical model of the steady flow of an incompressible couple stress fluid in a two dimensional channel with stenosis has been studied. It is considered that a uniform magnetic field is applied to the fluid normal to the wall of the channel. Further, assuming the stenosis to be mild and using slip boundary condition, a closed form solution has been obtained and

expressions for the resistance to flow and the wall shear stress have been derived. It is observed that both the resistance to flow and the wall shear stress increase with the height of the stenosis and the slip parameter but decrease with the couple stress parameter, the Darcy number and the magnetic parameter.

Acknowledgement

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