

EFFECT OF STRESS ON ENERGY FLUX DEVIATION OF ULTRASONIC WAVES IN  
ULTRASONIC WAVES IN GR/EP COMPOSITES

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## ABSTRACT

Ultrasonic waves suffer energy flux deviation in graphite/epoxy because of the large anisotropy. The angle of deviation is a function of the elastic coefficients. For nonlinear solids, these coefficients and thus the angle of deviation is a function of stress. Acoustoelastic theory was used to model the effect of stress on flux deviation for unidirectional T300/5208 using previously measured elastic coefficients. Computations were made for uniaxial stress along the  $x_3$  axis (fiber axis) and the  $x_1$  axis for waves propagating in the  $x_1x_3$  plane. These results predict a shift as large as three degrees for the quasi-transverse wave. The shift in energy flux offers a new nondestructive technique of evaluating stress in composites.

## INTRODUCTION

In anisotropic media, the energy flux associated with an elastic wave does not propagate along the same direction as the normal to the wavefront except along symmetry directions. This effect is known as energy flux deviation. Both Musgrave [1] and Love [2] derived the relationship between the energy flux and the elastic coefficients of the material.

In composite materials which are highly anisotropic, the amount of energy flux deviation can be quite large. Kriz and Ledbetter [3] calculated the flux deviation angle as a function of fiber orientation for unidirectional composites. They also showed the influence of fiber content

on the flux deviation. Experimental measurement of the flux deviation has been made by Kriz and Stinchcomb [4] and Prosser [5] among others while measuring the elastic coefficients by ultrasonic methods. Kriz [6] further showed that since the flux shift was a function of the elastic coefficients, it could be used to monitor changes in the composite elasticity. Thus moisture absorption by the matrix and fiber degradation which altered the elastic coefficients could be determined by measurements of the energy flux shift.

In this research the effect of stress on energy flux deviation was modeled. Due to nonlinear elastic effects, the energy flux deviation was shown to be a function of stress. Second and third order elastic coefficients for unidirectional T300/5208 gr/ep, previously measured by Prosser [5] and Wu and Prosser [7], were used in these model calculations. The effect of uniaxial stress along both the fiber direction ( $x_3$ ) and perpendicular to the fibers along the laminate stacking direction ( $x_1$ ) was calculated for waves propagating at various angles with respect to the fiber direction in the  $x_1x_3$  plane.

## THEORY

Assuming linear elasticity the energy flux vector ( $E_j$ ) can be expressed in terms of the linear elastic stiffness coefficients ( $C_{ijkl}$ ) as

$$E_j = -C_{ijkl} \frac{\partial u_k}{\partial x_l} \frac{\partial u_i}{\partial t} \quad (1)$$

where  $u_i$  is the displacement vector [1,

2]. The angle between the energy flux vector and the normal to the plane wave front is the energy flux deviation angle.

However, if nonlinear elastic effects are included, the effective elastic stiffnesses become functions of the applied stress. An effective stiffness tensor ( $C_{nlij}^*$ ) can be derived from acoustoelastic theory and was presented by Barnett [8]. It is given by

$$C_{nlij}^* = k_{nlij} + \sigma_{nj}\delta_{li} \quad (2)$$

where  $\sigma_{nj}$  is the applied stress, assumed to be within the linear elastic regime,  $\delta_{li}$  is the Kronecker delta, and  $k_{nlij}$  is a combination of second and third order coefficients given by

$$\begin{aligned} K_{nlij} = & C_{nlij} + C_{rlij}\epsilon_{nr} + C_{nsij}\epsilon_{ls} \\ & + C_{nlpj}\epsilon_{ip} + C_{nliq}\epsilon_{jq} + C_{nlijuv}\epsilon_{uv} \\ & + C_{rlijuv}\epsilon_{uv}\epsilon_{nr} + C_{nsijuv}\epsilon_{uv}\epsilon_{ls} \\ & + C_{nlpjuv}\epsilon_{uv}\epsilon_{ip} + C_{nliquv}\epsilon_{uv}\epsilon_{jq} \end{aligned} \quad (3)$$

In this expression,  $c_{ijkluv}$  are the third order stiffness coefficients and  $\epsilon_{uv}$  are the applied strains which are related to the applied stresses by

$$\epsilon_{ij} = s_{ijkl}\sigma_{kl} \quad (4)$$

where  $s_{ijkl}$  are the linear elastic compliances.

Thus, if the linear elastic stiffnesses and compliances, the third order stiffnesses, and the applied stress are known, an effective stiffness tensor can be calculated. These can be then used to predict the energy flux deviation as a function of applied stress.

#### MODEL CALCULATIONS

The material modeled in this study was unidirectional T300/5208 graphite/epoxy which was assumed to be transversely isotropic. The fiber direction was taken to be the  $x_3$  axis and the laminate stacking direction was chosen to be the  $x_1$  direction. The five independent linear elastic

stiffness coefficients are listed in Table 1 along with the linear elastic compliance coefficients. Listed in Table 2 are the nine independent third order elastic stiffness coefficients.

	GPa		GPa <sup>-1</sup>
$C_{11}$	14.26	$S_{11}$	0.092
$C_{12}$	6.78	$S_{12}$	-0.042
$C_{13}$	6.5	$S_{13}$	-0.003
$C_{33}$	108.4	$S_{33}$	0.0096
$C_{44}$	5.27	$S_{44}$	0.190

Table 1. Linear elastic stiffness and compliance coefficients.

	GPa		GPa
$C_{111}$	-196	$C_{155}$	-49.1
$C_{112}$	-89	$C_{344}$	-47
$C_{113}$	-4	$C_{133}$	-236
$C_{123}$	65	$C_{333}$	-829
$C_{144}$	-33.4		

Table 2. Third order elastic stiffness coefficients.

Propagation of acoustic waves was considered for waves propagating in the  $x_1x_3$  plane. In this plane, three modes of propagation are predicted. The first is a pure mode transverse wave (PT) at all angles. Even though it is a pure mode, it suffers energy flux deviation except for propagation along either of the axes. The second mode is a quasi-transverse mode (QT) and the third is a quasi-longitudinal mode (QL). Both of these modes also suffer energy flux deviation in this plane.

For fiber orientations of less than 60 degrees, the QL mode propagates with a faster velocity and deviates toward the fiber direction while the QT mode deviates in the opposite direction. Between 60 and 90 degrees, these modes transition with the QL becoming the QT and vice-versa. Because of the complexity in this cross over region, it was excluded from this study.

Two cases of uniaxial stress were considered. The first was that of loading along the  $x_3$  axis. To estimate the maximum effect, the stress level was taken to be 1.0 GPa which is approximately 70 percent of the ultimate tensile strength of this material. The flux deviation angles were calculated as a function of fiber orientation at this stress level. The flux deviation at zero stress was then subtracted from these values to yield the energy flux deviation shift for the three modes at this load.

The second loading case was that of uniaxial stress along the  $x_1$  direction which is perpendicular to the fibers. In this case the maximum stress was taken to be 0.1 GPa and again the change in flux deviation from the zero stress state was calculated.

## RESULTS AND DISCUSSIONS

The flux deviation angles of the three modes as a function of fiber orientation at zero stress are plotted in Figure 1. The fiber orientation angle is the angle that the fibers make with respect to the direction of propagation. Thus, a negative flux deviation shift indicates a shift back toward the fiber axis while a positive shift deviates away from the fibers. Over the 0 to 60 degree range the QL and PT waves deviate toward the fiber direction while the QT deviates toward the  $x_1$  or the laminate stacking direction. The magnitude of the shift of the QL and QT waves is quite large while the PT suffers much smaller deviation.

In Figure 2, the energy flux deviation shift is shown for the three modes for the case of 1 GPa. stress along the fiber axis. The QT mode suffers the maximum flux shift reaching 3 degrees at 20 degrees fiber orientation. The PT wave suffers a somewhat smaller shift while the QL mode suffers almost no flux deviation.

A larger shift in the QT mode as compared to the QL mode might be expected if the ratio of the nonlinear coefficients to the linear elastic coefficients is considered. The magnitudes of  $c_{111}$  and  $c_{112}$  are over an

order of magnitude larger than  $c_{11}$  and  $c_{12}$ . These coefficients dominate the propagation of the QT wave. The magnitude of the third order coefficients along the  $x_3$  direction is much smaller relative to the linear coefficients in the same direction even though they are larger than the nonlinear coefficients along  $x_1$ . Thus the relative effect of nonlinearity along the fibers which controls the flux shift of the QL wave is smaller. The work by Kriz [6] on the effect of matrix degradation on flux deviation shift also showed that the QT wave mode was most affected with the QL showing almost no shift.

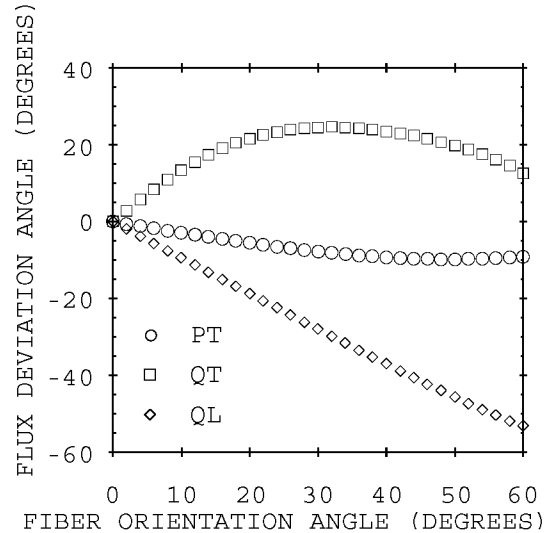


Figure 1. Flux deviation as a function of fiber orientation under condition of no applied stress.

The flux shifts due to stress along  $x_1$  are shown in Figure 3. Again the QT mode exhibits the largest shift. However, the magnitude of the shift is somewhat smaller as the stress level is smaller. It is interesting to note that the direction of shift is in the opposite direction from the case of stress along the fibers.

These calculations demonstrate the effect of stress on the energy flux deviation of ultrasonic waves due to nonlinear elastic effects in composite materials. The models indicate the angles of fiber orientation and wave modes that suffer the maximum shift

which will enable future experimental confirmation of this effect.

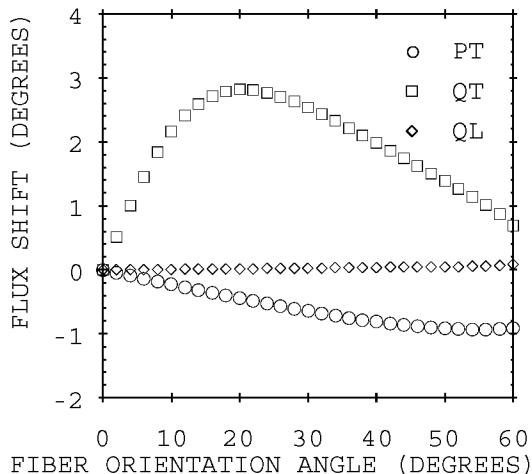


Figure 2. Flux deviation shift due to 1 GPa stress along fiber direction.

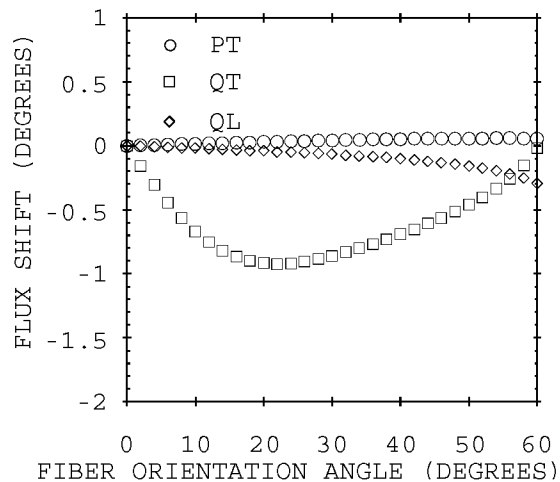


Figure 3. Flux deviation shift due to 0.1 GPa stress perpendicular to fibers.

The shift, particularly in the QT mode, should be measurable with a linear array transducer if a large enough distance of propagation is used. Although the models presented were for bulk waves propagating through a thick composite material, the same effect is expected for plane plate waves propagating in thin plates. The longer

propagation paths across plates would make the effect more measurable and thus could improve the stress resolution possible. This effect could be used to develop a new nondestructive method for monitoring stress in composites.

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