where $\Gamma = -b\omega \sin\phi / c \cos\theta$, so that I_4 exhibits gain for positive Γ. We note that the total energy is conserved,

$$\frac{dI_0}{dz} = 0, \tag{5}$$

so that I_0 is constant. The solution of Eqs. (3) and (4) is

$$I_1(z) = \frac{I_0}{1 + [I_4(0)/I_1(0)]e^{\Gamma z}},$$
$$I_4(z) = \frac{I_0}{1 + [I_1(0)/I_4(0)]e^{-\Gamma z}}. \tag{6}$$

When we apply boundary condition appropriate to a ring oscillator,

$$I_4(0) = (1 - L)I_4(l), \tag{7}$$

where L represents the cavity's fractional linear intensity loss per round trip we find that the ratio of oscillating energy to pumping energy is

$$\frac{I_4(0)}{I_1(0)} = \frac{1 - e^{-\Gamma l}}{L} - 1. \tag{8}$$

We note that $I_4(0)/I_1(0)$ is infinite for $L = 0$ and becomes zero, i.e., no oscillation at $L = 1 - e^{-\Gamma l}$.

In conclusion, we have demonstrated a number of new oscillators involving four-wave mixing in $BaTiO_3$. Oscillation is achieved with a low-power He-Ne laser at 632.8 nm supplying the pump beam(s).

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Effect of superluminescence on the modulation response of semiconductor lasers

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The small-signal modulation response of semiconductor lasers with a very small mirror reflectivity is analyzed. Superluminescent effects inside the laser cavity provide yet another mechanism for damping relaxation oscillation resonance. These results can serve as useful guides in designing high frequency semiconductor lasers.

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Semiconductor lasers are potentially important as light source for optical communication systems. Consequently, methods for improving the modulation response of these devices are topics of great interest. The small-signal modulation behavior of the laser is well established experimentally and theoretically.[1] The absence of a resonance peak in the modulation response of some lasers has been attributed to a variety of causes including the high spontaneous emission factor and/or lateral carrier diffusion. In this letter we point out yet another mechanism that can lead to a flat frequency response. This mechanism involves the nonuniform distribution of photons and carriers along the length of a laser cavity with a small mirror reflectivity. This effect is particularly important when one attempts to increase the modulation speed by reducing the end-facet reflectivities in order to reduce the lifetime of the cavity.[2]

Previous considerations of nonuniform distribution of photons and carriers occurred in the context of superluminescent light emitting diodes (LED's)—diodes with totally suppressed mirror feedback to prevent lasing.[3–7] Total suppression of mirror feedback is unattainable in practice. A very low reflectivity at one mirror can be obtained by leaving a long section of the laser unpumped[3]; nevertheless, scattering from material defects and waveguide irregularities make it very difficult if not impossible to suppress the reflectivity to much below 10^{-4}. Even with such a small feedback from one facet, the device will lase and will not operate as a LED unless the pump current is kept at moderate levels. Above the conventional lasing threshold, such low-feedback lasers behave quite differently from a common laser due to superluminescent effects resulting from a large single-pass amplification inside the cavity.

The spatially uniform rate equations commonly employed are not valid and one must use the local photon and injected carrier conservation equations[8]

$$\frac{\partial X^+}{\partial t} + c\frac{\partial X^+}{\partial z} = c\kappa(N - N_{om})X^+ + \beta\frac{N}{2\tau_s} - cfX^+, \tag{1a}$$

$$\frac{\partial X^-}{\partial t} - c\frac{\partial X^-}{\partial z} = c\kappa(N - N_{om})X^- + \beta\frac{N}{2\tau_s} - cfX^-, \tag{1b}$$

$$\frac{dN}{dt} = \frac{J}{ed} - \frac{N}{\tau_s} - \kappa c(N - N_{om})(X^+ + X^-), \tag{1c}$$

where X^+ and X^- are the forward and backward propagating photon densities (which are proportional to the light intensities), N is the carrier density, N_{om} is the carrier density for transparancy, c is the group velocity of the waveguide mode, κ is the gain constant in cm^{-1}/(unit inversion), β is the fraction of spontaneous emission entering the lasing mode, τ_s is the spontaneous recombination lifetime of the carriers, f is the distributed internal loss, z is the distance along the active medium with $z = 0$ at the center of the laser, J is the pump current density, e is the electronic charge, and d is the thickness of the active region. Neglected in Eq. (1) is the fact that the carrier density $N(z)$, in a semiconductor is directly related to the quasi-Fermi level (which in turn is related to the local potential) and therefore a nonuniform carrier distribution in the longitudinal direction leads to local currents and variation in injection rates in that direction, which tend to wash out the nonuniformity. Equation (1) can thus be applied only to diode lasers where the active and the cap layers have relatively high sheet resistances—simple estimation indicates that a value $> 50\,\Omega$/square suffices. Equation (1) is solved subjected to the boundary conditions

$$X^-(L/2) = R_2X^+(L/2);\; X^+(-L/2) = R_1X^-(-L/2), \tag{2}$$

where L is the length of the laser, R_1 and R_2 are the reflectivities of the end mirrors. Equation (1) can be shown to reduce to the common rate equations when the mirror reflectivities[9] are high ($\geqslant 0.3$) in which case the carrier distribution is approximately uniform along the length of the cavity.

The steady-state solutions of Eq. (1) are obtained numerically employing the shooting method. The calculated static photon and carrier distributions are shown in Figs. 1(a) and 1(b), where case (a) is that of a common laser (with equal reflectivities of 0.3 at both facets) and that in (b) the reflectivity of one facet is reduced to 10^{-4}. The highly nonuniform carrier distribution is evident in (b). The length of the device in (b) is longer that that in (a) so that the photon lifetimes in both cases are identical. The distributed internal loss (f) is taken to be $60\ \text{cm}^{-1}$. The small-signal dynamic solutions to

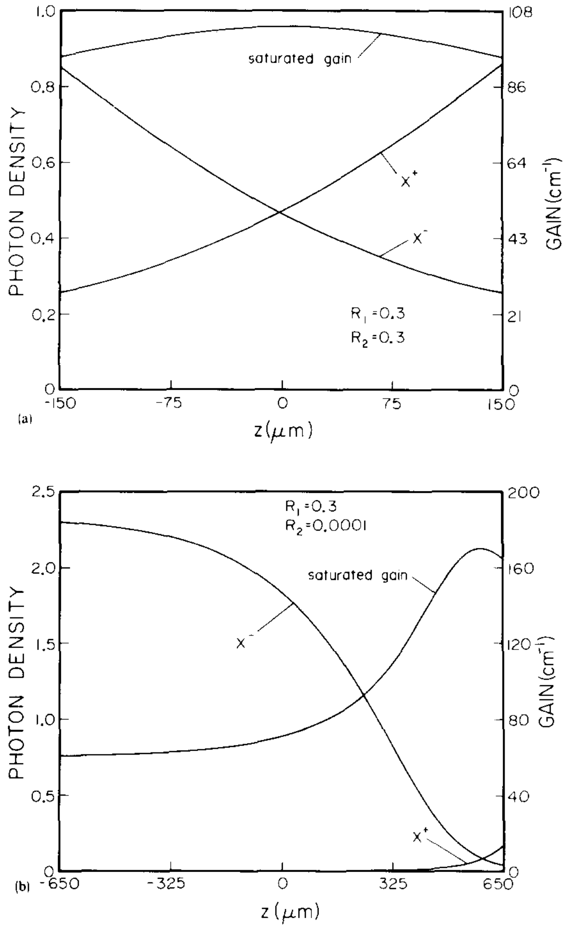

FIG. 1. (a) Steady-state normalized forward and backward photon density flux and local gain distributions in a common semiconductor laser with mirror reflectivities of 0.3 (b) Steady-state distributions in a laser with mirror reflectivities of 0.3 and 0.0001. The distributed loss and photon lifetime are respectively $60\ \text{cm}^{-1}$ and 1 ps in both the above cases.

Eq. (1) have been considered for the case of perfect mirrorless devices[10] where lasing cannot occur. We shall consider the more realistic case of finite mirror reflectivities. We use the perturbation expansion:

$$X^{\pm}(z,t) = X_0^{\pm}(z) + x^{\pm}(z)e^{i\omega t}, \tag{3a}$$

$$N(z,t) = N_0(z) + n(z)e^{i\omega t}, \tag{3b}$$

where $x^{\pm}$ and n are "small" variations about the steady states. This assumes that the electron and photon densities throughout the length of the diode vary in unison, which is true when the modulation frequency is small compared to the inverse of the photon transit time. This corresponds to above 15 GHz for diode lengths shorter than 1 mm.

Substituting (3) into Eq. (1) and neglecting the nonlinear product terms, we obtain the following small-signal equations:

$$\frac{dx^+}{dz} = Ax^+ + Bx^- + C, \tag{4a}$$

$$\frac{dx^-}{dz} = Dx^+ + Ex^- + F, \tag{4b}$$

where A,B,C,D,E,F are given by the following:

$$A = g_0 - \frac{i\omega}{c\tau_s} - \frac{(X_0^+ + \beta)g_0}{1 + i\omega + (X_0^+ + X_0^-)} - f, \tag{5a}$$

$$B = \frac{-(X_0^+ + \beta)g_0}{1 + i\omega + (X_0^+ + X_0^-)}, \tag{5b}$$

$$C = \frac{g_m(X_0^+ + \beta)}{1 + i\omega(X_0^+ + X_0^-)}, \tag{5c}$$

$$D = \frac{(X_0^- + \beta)g_0}{1 + i\omega + (X_0^+ + X_0^-)}, \tag{5d}$$

$$E = -\left(g_0 - \frac{i\omega}{c\tau_s} - \frac{(X_0^- + \beta)g_0}{1 + i\omega + (X_0^+ + X_0^-)}\right) + f \tag{5e}$$

$$F = \frac{-g_m(X_0^- + \beta)}{1 + i\omega(X_0^+ + X_0^-)}, \tag{5f}$$

where $X_0^\pm$ are the static photon density distributions, $g_0(z) = \alpha(N_0(z) - N_{om})$ = steady-state gain distribution, $g_m = \alpha j\tau_s/(ed)$ = small-signal gain due to rf drive, and ω has been normalized by the inverse of spontaneous lifetime.

The boundary conditions for solving (4) are the same as those employed in solving the steady-state case, Eq. (2). Equation (4) is solved numerically for each value of ω. Figures 2(a) and 2(b) show the amplitude and phase response of five diodes. One diode corresponds to a common laser with a facet feedback of 0.3 from both facets and a length of 300 μm. In each of the other diodes, one of the facets has a reflectivity of 0.3 whereas the other facet takes on reflectivities of 0.1, 0.01, 0.005, and 0.0001. In each of these four cases, the diode length is adjusted so that they possess a photon lifetime identical to that of the first laser diode. The values of f, β, and τ_s are taken to be 60 cm^{-1}, 10^{-4}, and 3 ns, respectively. These devices possess the same threshold gain of 100 cm^{-1}. The devices are pumped to an identical level corresponding to an unsaturated gain of 200 cm^{-1}, which translates into a J/J_{th} of 1.25 (when κN_{om} assumes the measured value of $\approx$300 cm^{-1}). The commonly employed rate equations analysis predicts that these five diodes have identical output powers and frequency responses, in obvious disagreement with that shown in Fig. 2. Notice that for the two cases where $R_1 = 0.3$ and 0.1, both the amplitude and phase curves are almost identical to that calculated from the simple rate equations, displaying a resonance at 2.9 GHz. The resonance is suppressed in devices with a smaller feedback from a mirror and at a value of R_1 between 0.001 and 0.005, the response is maximally flat. Further decrease in R_1 reduces the modulation bandwidth. These data show that the simple rate equations fail to give the correct frequency response when the reflectivity of one of the mirrors is reduced to below 0.1. The simple approach does, however, give the correct corner frequency in the small-signal response even when the reflectivity of one of the mirrors is as low as 10^{-3}, as evident from Fig. 2. These conclusions are not restricted to the particular case above but are in general true over a wide range of pump currents and photon lifetimes.

The above results have important implications in the design of high frequency semiconductor lasers. First, the prescription described in Ref. 2 derived from the simple rate equations remains valid over a wide range of mirror reflectivities down to 10^{-3}. Secondly, given the choices of reducing the photon lifetime by suppressing feedback from one facet

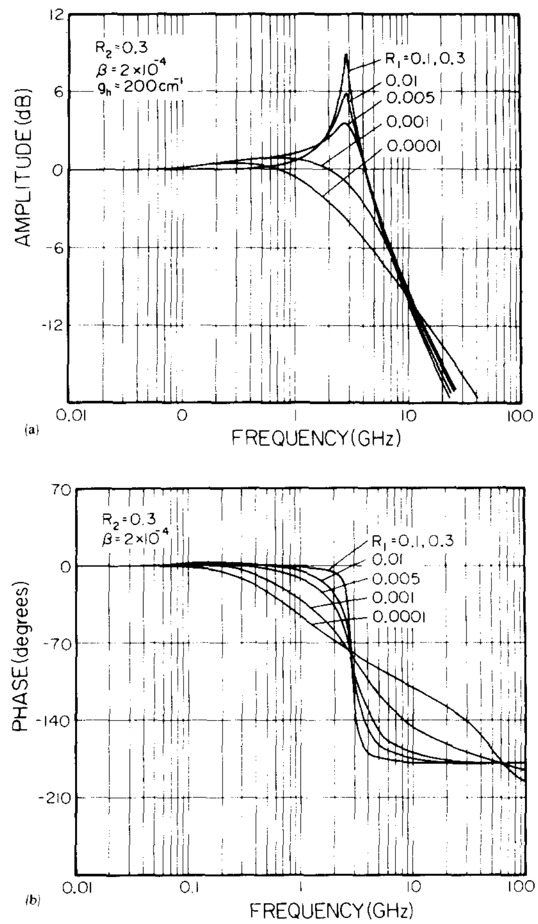

FIG. 2. (a) Amplitude and (b) phase responses of diodes with different mirror reflectivities but different lengths (so that they all possess the same photon lifetimes of 1 ps). They are pumped to an unsaturated gain of 200 cm^{-1} which is above the classical threshold of all of these lasers.

or by shortening the laser cavity, the former has the advantage of suppressing relaxation oscillation resonance. And finally, the amount of suppression of feedback can be controlled via a double-section structure (similar to that of a bistable laser recently reported,[12]) so that one section constitutes the laser cavity while the other section controls the amount of feedback at one end of the laser. Experiments are in progress to evaluate these design schemes.

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