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EFFECT OF THERMAL RADIATION, CHEMICAL REACTION AND VISCOUS DISSIPATION ON MHD FLOW

B. ZIGTA
University of Debreberhan, School of Natural and Computational Science
Department of Mathematics
P.O. Box 445. ETHIOPIA
E-mail: tzigta@yahoo.com

This study examines the effect of thermal radiation, chemical reaction and viscous dissipation on a magneto-hydro-dynamic flow in between a pair of infinite vertical Couette channel walls. The momentum equation accounts the effects of both the thermal and the concentration buoyancy forces of the flow. The energy equation addresses the effects of the thermal radiation and viscous dissipation of the flow. Also, the concentration equation includes the effects of molecular diffusivity and chemical reaction parameters. The gray colored fluid considered in this study is a non-scattering medium and has the property of absorbing and emitting radiation. The Roseland approximation is used to describe the radiative heat flux in the energy equation. The velocity of flow transforms kinetic energy into heat energy. The increment of the velocity due to internal energy results in heating up of the fluid and consequently it causes increment of the thermal buoyancy force. The Eckert number being the ratio of the kinetic energy of the flow to the temperature difference of the channel walls is directly proportional to the thermal energy dissipation. It can be observed that increasing the Eckert number results in increasing velocity. A uniform magnetic field is applied perpendicular to the channel walls. The temperature of the moving wall is high enough due to the presence of thermal radiation. The solution of the governing equations is obtained using regular perturbation techniques. These techniques help to convert partial differential equations to a set of ordinary differential equations in dimensionless form and thus they are solved analytically. The following results are obtained: from the simulation study it is observed that the flow pattern of the fluid is affected due to the influence of the thermal radiation, the chemical reaction and viscous dissipation. The increment in the Hartmann number results in the increment of the Lorentz force but a decrement in velocity of the flow. An increment in the radiative parameter results in a decrement in temperature. An increment in the Prandtl number results in a decrement in thermal diffusivity. An increment in both the chemical reaction parameter and molecular diffusivity results in a decrement in concentration.

Key words: MHD flow, thermal radiation, chemical reaction, viscous dissipation.

1. Introduction

In fluid dynamics the word dissipation means conversion of energy from one form to other forms. The fluid flow requires kinetic energy and hence its velocity takes energy from the motion and transforms it into internal energy. The kinetic energy is dissipated, i.e., during the flow it is converted into internal energy. The increment of the velocity of the fluid results in heating up of the fluid.

The cited process is irreversible in case of a viscous fluid and is known as viscous dissipation. The characteristics of viscous dissipation effects depend on the non-dimensional quantity Eckert number. Viscous dissipation is applied in geophysical flows and industries. The work done for deforming an elastic material is stored as potential energy which can be recovered mechanically. The dissipation function is always positive when applied to a viscous fluid. But it may be either positive or negative when it is applied to elastic and viscoelastic materials.

Many scholars have studied a natural convection-magneto-hydrodynamic flow of vertical plates. Free convection at a vertical plate with transpiration is studied in [1]. Natural convection adjacent to a surface with three thermal boundary conditions is investigated in [2]. A numerical study for natural convective cooling of a vertical plate is conducted in [3]. Thermal radiation has very wide applications in fluid dynamics, viz. aerospace, chemical, mechanical, industrial and environmental engineering and sciences.

The radiation effects on mixed convection along a vertical plate with a uniform surface temperature using the Rosseland flux model are investigated in [4]. The radiation effects on a magneto-hydrodynamic unsteady free convection flow over a vertical plate with variable temperature are studied in [5].The hydro magnetic flow of a viscous incompressible fluid past an oscillating vertical plate with radiation and variable mass diffusion is considered in [6]. Even though the effects of viscous dissipation are significant and important, they have not been considered in any of the above studies.

The influences of viscous heating dissipation effects in natural convective flows have been investigated in [7]. The thermal radiation effects on a hydro-magnetic-free convection flow past an impulsively started vertical plate with a variable surface temperature and concentration is analyzed in [8]. Recently, the effects of thermal radiation on natural conductive heat and mass transfer of a viscous incompressible grey absorbing emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation are studied in [9]. Very recently the boundary layer steady flow and heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field has been studied in [10].

Magneto-hydrodynamic flow through a vertical Couette plates oscillating in its own plane has many industrial applications. An exact solution of the Navier Stokes equation concerned with flow of a viscous incompressible fluid past a horizontal plate oscillating in its own plate is given in [11]. Natural convection effects on Stokes problem was first studied in [12]. Free convective effect on an impulsively started or oscillating plate has been studied in [13]. An exact solution to the flow of a viscous incompressible unsteady flow past an infinite vertical oscillating plate with a variable temperature and mass diffusion is studied in [14]. Free convection flow of a viscous incompressible fluid past an infinite vertical oscillating plate with a uniform heat flux in the presence of thermal radiation was studied in [15]. The effects of chemical reaction and diffusion in an isothermal laminar flow along a soluble flat plate are studied in [16]. The effects of chemical reaction and mass transfer on flow past an impulsively started infinite vertical plate with a constant heat flux are analyzed in [17]. The effects of chemical reaction on a moving isothermal vertical infinitely long surface with suction are studied in [18]. The effects of homogeneous first order chemical reaction and mass diffusion on unsteady flow past an impulsively started semi-infinite vertical plate with a variable temperature in the presence of thermal radiation have been studied in [19].

The effects of magnetic field on the flow of a fluid between two vertical parallel Couette plates have been studied. The fluid considered here has the following properties viz., unsteady, viscous, incompressible, electrically conducting, Newtonian, thermal radiating and chemical reacting. Free stream velocity of the fluid is assumed to be fluctuating. Furthermore, it is assumed that temperature and concentration of the fluid also fluctuate with time. Although the effects of thermal radiation, chemical reaction and viscous dissipation are very important and significant, they have not been considered [20].

To fill some of the gaps of the fore-cited works the present study is taken up. The main objective of this paper is to study the effect of the thermal radiation, the chemical reaction and viscous dissipation on the flow pattern.

2. Theoretical experiment, mathematical model and analysis

Consider a flow is identical in parallel Couette channel walls. Here the fluid flow is bounded by two infinite vertical channel walls separated by a distance h. The fluid considered is an: unsteady, incompressible, viscous, electrical conducting, Newtonian, chemical reacting and radiating fluid.
The geometrical representation of the model and the coordinate system are shown in Fig.1. The x' axis is taken along the infinite channel walls and the y' axis is taken normal to the channel walls. The vertical moving channel wall is located at $y'=0$ along x' where the temperature is T_w' and the concentration is C_w'. The other but stationary channel wall is located at $y'=h$ where the temperature is T_h' and the

concentration is C_h'. It is assumed that the radiation heat flux in the x' direction is negligible as compared to that in the y' direction.

Initially, at $t' = 0$, the stationary channel wall and the fluid are at the same temperature T_h'; and concentration level of the fluid C_h' is the same at all points. At a later time, $t' > 0$, the temperature of the moving wall and concentration of the fluid do raise to T_w' and C_w', respectively.

The magnetic field of uniform strength is applied perpendicular to the channel walls and the induced magnetic field of the fluid is negligible. Hence the magnetic Reynolds number is small and this is a valid assumption on laboratory scale under the assumption of a small magnetic Reynolds number [21].The influence of density variation with both temperature and concentration in the body force term or Boussinesq's approximation is not constant. This approximation involved neglecting all variable fluid properties for density variation with approximation of $\rho g \beta \left(T' - T_h'\right)$ and $\rho g \beta_c \left(C' - C_h'\right)$. Furthermore, it is considered that the fluid is gray in color, has a radiation absorbing nature, emits radiation, and is a non-scattering medium. The Roseland approximation is used to describe the radiative heat flux in the energy equation.

In nature, there is no pure air and water due to the presence of foreign masses. The foreign masses may exist either naturally or mixed with air or water. The foreign masses in air or water cause some kinds of chemical reaction and hence heat is generated. It is important to study such kinds of chemical reactions to improve a number of technologies such as food preservation, polymer production, manufacturing of ceramics and glassware.

The free stream velocity has the form

$$U'\left(t'\right) = U_o \left(1 + \varepsilon e^{i\omega' t'}\right). \tag{2.1}$$

In Eq.(2.1), U_0 is the mean constant free stream velocity; ω' is the frequency; and t' is the time.

Based on the following model assumptions, the governing equations of the model are derived:

(1) All fluid properties except density in the thermal and concentration buoyancy force term are constant.
(2) The influence of the density variations in other terms of the momentum, energy and concentration equations are negligible.
(3) The Eckert number Ec and the magnetic Reynolds number are small so that the induced magnetic field can be neglected.
(4) The external electric field is supposed to be zero.
(5) All the physical variables are independent of x'.

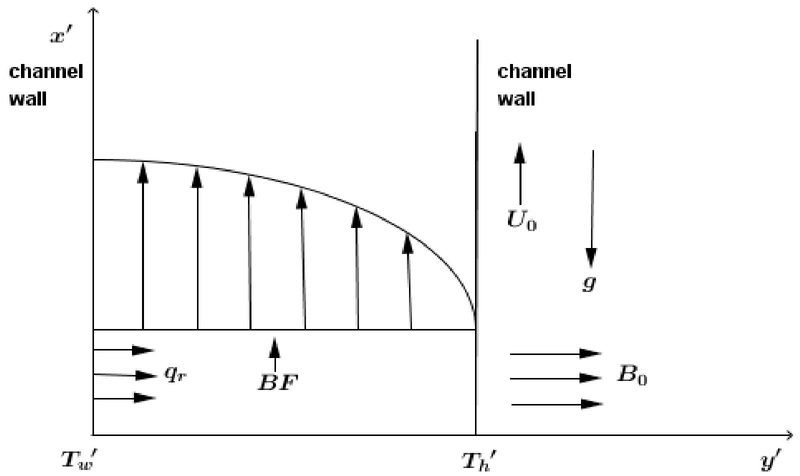

Fig.1. Flow configuration and coordinate representation of the model.

The problem of the model, based on the model assumptions, can be governed by the following set of equations. The momentum equation of the problem is given by

$$\frac{\partial u'}{\partial t'}=\frac{\partial U'}{\partial t'}+\nu\left(\frac{\partial^2 u'}{\partial y'^2}\right)+g\beta\left(T'-T_h'\right)+g\beta_c\left(C'-C_h'\right)-\left(\frac{J\,X\,B}{\rho}\right). \tag{2.2}$$

In Eq.(2.2), the vector cross product $\left(JxB\right)$ represents the Lorentz force. This term is a body force corresponding to the magneto hydrodynamic flow. The total magnetic field is represented by B. The density of the current is represented by J. Using Ohm's law, the expression for the density of current can be constructed as $J=\sigma\left(E+v'xB\right)$. Upon substituting $E=0$, since the electric field is assumed to be negligible, the expression for J reduces to $J=\sigma\,(v'xB)$. Also, the expression for the Lorentz force reduces and takes the form as $JxB=-\sigma B^2 u'$. In view of these result, Eq.(2.2) reduces to

$$\frac{\partial u'}{\partial t'}=\frac{\partial U'}{\partial t'}+\nu\frac{\partial^2 u'}{\partial y'^2}+g\beta\left(T'-T_h'\right)+g\beta_c\left(C'-C_h'\right)-\frac{\sigma B^2(u'-U')}{\rho}. \tag{2.3}$$

The third and fourth terms on the right hand side of the momentum Eq.(2.3) denote thermal and concentration buoyancy effects, respectively; and the fifth term is the magneto hydrodynamic effect due to the Lorentz force.

The energy equation of the model can be expressed as

$$\frac{\partial T'}{\partial t'}=\alpha\left(\frac{\partial^2 T'}{\partial y'^2}\right)-\left(\frac{1}{\rho C_p}\right)\left(\frac{\partial q_r}{\partial y'}\right)+\left(\frac{\mu}{\rho C_p}\right)\left(\frac{\partial u'}{\partial y'}\right)^2 . \tag{2.4}$$

The second term on the right hand side of the energy Eq.(2.4) denotes radiation and the third term is viscous dissipation and is always positive when applied to a viscous fluid.

The concentration equation of the model can be expressed as

$$\frac{\partial C'}{\partial t'}=D\left(\frac{\partial^2 C'}{\partial y'^2}\right)-k_r(C'-C_h'). \tag{2.5}$$

The first term on the right hand side of the concentration Eq.(2.5) is molecular diffusivity and second term on the right hand side is generative chemical reaction.

Based on Fig.1 the boundary conditions for Eqs (2.3) – (2.5) can be chosen as follows

$$y'=0,\quad u'=U_o\left(1+\varepsilon e^{i\omega' t'}\right), \tag{2.6}$$

$$T'=T_w'+\varepsilon(T_w'-T_h')e^{i\omega' t'}, \tag{2.7}$$

$$C'=C_w'+\varepsilon(C_w'-C_h')e^{i\omega' t'}, \tag{2.8}$$

$$y'=b,\quad u'=0,\quad T'=T_h',\quad C'=C_h' . \tag{2.9}$$

It is assumed that the radiation heat flux is to be presented in the form of an unidirectional flux in the y' direction. Using the Roseland approximation for radiative heat transfer and the Roseland approximation for diffusion and also following other works [22], the expression for the radiative heat flux q_r can be given as

$$q_r = \left(\frac{-4\sigma}{3k_s}\right)\left(\frac{\partial T'^4}{\partial y'}\right). \tag{2.10}$$

Here in Eq.(2.10), the parameters σ and k_s represent the Stefan Boltzmann constant and the Roseland mean absorption coefficient, respectively.

Now on assuming that the temperature differences within the fluid flow are sufficiently small, T'^4 in Eq.(2.10) can be expressed as a linear function of T_h' using the Taylor series expansion. The Taylor series expansion of T'^4 about T_h', after neglecting the higher order terms, takes the form

$$T'^4 \cong 4T_h'^3 T' - 3T_h'^4 . \tag{2.11}$$

Using Eqs (2.10) and (2.11) in Eq.(2.4), it can be obtained that

$$\frac{\partial T'}{\partial t'} = \alpha\left(\frac{\partial^2 T'}{\partial y'^2}\right) + \left(\frac{1}{\rho C_p}\right)\left(\frac{16\sigma T_h'^3}{3k_s}\right)\left(\frac{\partial^2 T'}{\partial y'^2}\right) + \left(\frac{Q_0}{\rho C_p}\right)\left(T' - T_h'\right) + \left(\frac{\mu}{\rho C_p}\right)\left(\frac{\partial u'}{\partial y'}\right)^2 . \tag{2.12}$$

In Eq.(2.12), the third term denotes heat absorption of the fluid.

3. Non-dimensionalization of the model

In order to solve Eqs (2.3) – (2.5) of the model it is convenient to deal with its dimensionless form. Hence, the dimensionless form of the model is found by introducing the following non-dimensional quantities

$$y = y'/h;\quad R = 4\sigma T_h'^3/\kappa k_s;\quad u = u'/U_0;\quad U = U'/U_0;\quad t = \omega' t';\quad \omega = \omega' h^2/\nu;$$

$$Gc = \left[\left[g\beta_c h^2\left(C'_w - C'_h\right)\right]\Big/\nu U_0\right];\quad \mathrm{Gr} = \left[g\beta h^2\left(T'_w - T'_h\right)\Big/\nu U_0\right];\quad \mathrm{Sc} = \nu/D;\quad \mathrm{Pr} = \nu/\alpha;$$

$$M = \sqrt{\left(\sigma B^2 h^2/\rho\nu\right)};\qquad C = \left[\left(C' - C'_h\right)/\left(C'_w - C'_h\right)\right];$$

$$\mathrm{Ec} = U_o^2/\mathrm{C_p}\Delta T;\qquad \Delta T = T'_w - T'_h;\qquad \theta = \left[\left(T' - T'_h\right)/(T'_w - T'_h)\right];$$

After substituting the above non-dimensional quantities in Eqs (2.3) – (2.5) and after simple algebraic manipulations, the non-dimensional form of the model takes the following form

$$\omega\frac{\partial u}{\partial t} = \omega\frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} + \mathrm{Gr}\,\theta + GcC + M^2(u - U), \tag{3.1}$$

$$\omega\frac{\partial\theta}{\partial t}=\frac{1}{pr}\left(1+\frac{4R}{3}\right)\frac{\partial^2\theta}{\partial y^2}+\frac{Q_o}{\rho C_p \upsilon}\theta h^2+\mathrm{Ec}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3.2}$$

$$\omega\mathrm{Sc}\frac{\partial C}{\partial t}=\frac{\partial^2 C}{\partial y^2}-\frac{K_r C h^2}{D}. \tag{3.3}$$

The corresponding boundary conditions (2.6) – (2.9) take the form

$$y=0,\quad u=1+\varepsilon e^{it},\quad \theta=1+\varepsilon e^{it},\quad C=1+\varepsilon e^{it}, \tag{3.4}$$

$$y=1,\quad u=0,\quad \theta=0,\quad C=0. \tag{3.5}$$

The system (3.1) – (3.3) together with the boundary conditions (3.4) – (3.5) forms the non-dimensional form of the present model.

The variables and parameters used in this study and their physical meanings are given in the Appendix.

4. Analytical solution of the model problem

If the amplitude of oscillations ($\varepsilon \ll 1$) is very small then the solutions of flow velocity u, temperature field θ and concentration C near to the moving plate can be assumed as the sum of steady and small oscillating components. Thus, the following

$$u(y,t)=u_0(y)+\varepsilon u_1(y)e^{it}, \tag{4.1}$$

$$\theta(y,t)=\theta_0(y)+\varepsilon\theta_1(y)e^{it}, \tag{4.2}$$

$$C(y,t)=C_0(y)+\varepsilon C_1(y)e^{it}. \tag{4.3}$$

Here in Eqs (4.1) – (4.3) u_0, θ_0, and C_0 represent mean velocity, mean temperature and mean concentration, respectively.

Also, the free stream velocity takes the form

$$U=1+\varepsilon e^{it}. \tag{4.4}$$

Substituting Eqs (4.1) - (4.4) into Eqs (3.1) - (3.3); equating harmonic and non-harmonic terms and neglecting higher orders of ε the following system of equations is obtained

$$u_o''-M^2u_0=-\mathrm{Gr}\,\theta_0-GcC_0-M^2, \tag{4.5}$$

$$u_1''-\left(i\omega+M^2\right)u_1=-\mathrm{Gr}\,\theta_1-GcC_1-\left(i\omega+M^2\right), \tag{4.6}$$

$$C_0''-\frac{K_r C_o h}{D}=0, \tag{4.7}$$

$$C_1'' - C_1\left(\frac{K_r h^2}{D} + i\omega\mathrm{Sc}\right) = 0, \tag{4.8}$$

$$\frac{1}{\mathrm{Pr}}\left(1 + \frac{4R}{3}\right)\theta_0'' + \frac{Q_0\theta_0 h^2}{\rho C_p \nu} = -\mathrm{Ec}\,u_0'^2, \tag{4.9}$$

$$\frac{1}{\mathrm{Pr}}\left(1 + \frac{4R}{3}\right)\theta_1'' + \left(\frac{Q_0 h^2}{\rho C_p \nu} - i\omega\right)\theta_1 = -2\mathrm{Ec}\,u'_0\,u'_1. \tag{4.10}$$

Further, the new boundary conditions corresponding to Eqs (3.4) – (3.5) are obtained as

$$y = 0, \quad u_0 = 1, \quad u_1 = 1, \quad \theta_0 = 1, \quad \theta_1 = 1, \quad C_0 = 1, \quad C_1 = 1, \tag{4.11}$$

$$y = 1, \quad u_0 = 0, \quad u_1 = 0, \quad \theta_0 = 0, \quad \theta_1 = 0, \quad C_0 = 0, \quad C_1 = 0. \tag{4.12}$$

The variables $u_0, u_1, \theta_0, \theta_1, C_0$ and C_1 are still coupled in Eqs (4.5) – (4.10). To solve (4.5) – (4.10) we assume the Eckert number Ec to be very small for incompressible fluid and assume that

$$F(y) = F_0(y) + \mathrm{Ec}\,F_1(y) + o\left(Ec^2\right).$$

Here F stands for any variable $u_0, u_1, \theta_0, \theta_1, C_0$ and C_1. These variables can be expanded in powers of Ec as follows

$$u_0(y) = u_{00}(y) + Ecu_{01}(y), \tag{4.13}$$

$$u_1(y) = u_{10}(y) + \mathrm{Ec}u_{11}(y), \tag{4.14}$$

$$\theta_0(y) = \theta_{00}(y) + \mathrm{Ec}\,\theta_{01}(y), \tag{4.15}$$

$$\theta_1(y) = \theta_{10}(y) + \mathrm{Ec}\,\theta_{11}(y), \tag{4.16}$$

$$C_0(y) = C_{00}(y) + \mathrm{Ec}\,C_{01}(y), \tag{4.17}$$

$$C_1(y) = C_{10}(y) + \mathrm{Ec}\,C_{11}(y). \tag{4.18}$$

Upon substituting Eqs (4.13) – (4.18) in to Eqs (4.5) – (4.10) and equating terms free from Ec and with coefficients Ec and neglecting higher orders of Ec the following equations are obtained

$$u''_{00} - M^2 u_{00} = -\mathrm{Gr}\,\theta_{00} - GcC_{00} - M^2, \tag{4.19}$$

$$u''_{01} - M^2 u_{01} = -\mathrm{Gr}\,\theta_{01} - GcC_{01}, \tag{4.20}$$

$$u''_{10} - u_{10}\left(i\omega + M^2\right) = -\mathrm{Gr}\,\theta_{10} - GcC_{10}\,, \tag{4.21}$$

$$u''_{11} - u_{11}\left(i\omega + M^2\right) = -\mathrm{Gr}\,\theta_{11} - GcC_{11}\,, \tag{4.22}$$

$$C''_{00} - \frac{k_r C_{00} h^2}{D} = 0\,, \tag{4.23}$$

$$C''_{01} - \frac{k_r C_{01} h^2}{D} = 0\,, \tag{4.24}$$

$$C''_{10} - C_{10}\left[\frac{k_r h^2}{D} + i\omega\mathrm{Sc}\right] = 0\,, \tag{4.25}$$

$$C''_{11} - C_{11}\left[\frac{k_r h^2}{D} + i\omega\,\mathrm{Sc}\right] = 0\,, \tag{4.26}$$

$$\frac{1}{\mathrm{Pr}}\left(1 + \frac{4R}{3}\right)\theta''_{00} + \frac{Q_0 \theta_{00} h^2}{\rho C_p \nu} = 0\,, \tag{4.27}$$

$$\left[\frac{1}{\mathrm{Pr}}\left(1 + \frac{4R}{3}\right) + \frac{Q_0 h^2}{\rho C_p \nu}\right]\theta_{01} = u'^{\,2}_{00}\,, \tag{4.28}$$

$$\frac{1}{\mathrm{Pr}}\left(1 + \frac{4R}{3}\right)\theta''_{10} + \left[\frac{Q_0 h^2}{\rho C_p \nu} - i\omega\right]\theta_{10} = 0\,, \tag{4.29}$$

$$\frac{1}{\mathrm{Pr}}\left(1 + \frac{4R}{3}\right)\theta''_{11} + \left[\frac{Q_0 h^2}{\rho C_p \nu} - i\omega\right]\theta_{11} = -2u'_{00}\,u'_{10}\,. \tag{4.30}$$

Equations (4.19) - (4.30) are subjected to the new boundary conditions as given below

$$y = 0, \quad u_{00} = u_{10} = \theta_{00} = \theta_{10} = C_{00} = C_{10} = 1, \quad u_{01} = u_{11} = \theta_{01} = \theta_{11} = C_{01} = C_{11} = 0\,, \tag{4.31}$$

$$y = 1, \quad u_{00} = u_{01} = u_{10} = u_{11} = \theta_{00} = \theta_{01} = \theta_{10} = \theta_{11} = C_{00} = C_{01} = C_{10} = C_{11} = 0\,. \tag{4.32}$$

Solving Eqs (4.19) - (4.30) together with the boundary conditions (4.31) - (4.32), the analytical solutions are obtained

$$u_{00} = Ae^{My} + Be^{-My} + \frac{\mathrm{Gr}\,\theta_{00} + GcC_{00} + M^2}{M^2}\,, \tag{4.33}$$

$$u_{01} = A^{'}e^{My} + B^{'}e^{-My} + \frac{\mathrm{Gr}\,\theta_{01} + GcC_{01}}{M^2} = 0\,, \tag{4.34}$$

$$u_{10} = ce^{\beta y} + de^{-\beta y} + \frac{\mathrm{Gr}\,\theta_{10} + GcC_{10}}{M^2}\,, \tag{4.35}$$

$$u_{11} = c^{'}e^{\beta y} + d^{'}e^{-\beta y} + \frac{\mathrm{Gr}\,\theta_{11} + GcC_{11}}{M^2} = 0\,, \tag{4.36}$$

$$\theta_{00} = c_1 e^{\lambda y} + c_2 e^{-\lambda y}\,, \tag{4.37}$$

$$\theta_{01} = c_1^{'} e^{\lambda y} + c_2^{'} e^{-\lambda y} = 0\,, \tag{4.38}$$

$$\theta_{10} = k_1 e^{\mu y} + k_2 e^{-\mu y}\,, \tag{4.39}$$

$$\theta_{11} = k_1^{'} e^{\mu y} + k_2^{'} e^{-\mu y} = 0\,, \tag{4.40}$$

$$C_{00} = d_1 e^{my} + d_2 e^{-my}\,, \tag{4.41}$$

$$C_{01} = f_1 e^{my} + f_2 e^{-my} = 0\,, \tag{4.42}$$

$$C_{10} = h_1 e^{\gamma y} + h_2 e^{-\gamma y}\,, \tag{4.43}$$

$$C_{11} = p_1 e^{\gamma y} + p_2 e^{-\gamma y} = 0\,. \tag{4.44}$$

Also $u(y,t), \theta(y,t)$ and $C(y,t)$ are given by

$$u(y,t) = Ae^{My} + Be^{-My} + \frac{\mathrm{Gr}\,\theta_{00} + GcC_{00} + M^2}{M^2} + \varepsilon\left(ce^{\beta y} + de^{-\beta y} + \frac{\mathrm{Gr}\,\theta_{10} + GcC_{10}}{M^2}\right)e^{it}\,, \tag{4.45}$$

$$\theta(y,t) = c_1 e^{\lambda y} + c_2 e^{-\lambda y} + \varepsilon\left(k_1 e^{\mu y} + k_2 e^{-\mu y}\right)e^{it}\,, \tag{4.46}$$

$$C(y,t) = d_1 e^{my} + d_2 e^{-my} + \varepsilon\left(h_1 e^{\gamma y} + h_2 e^{-\gamma y}\right)e^{it}\,. \tag{4.47}$$

Various symbols used here and above are given in the APPENDIX.

5. Simulation study

In this paper the flow pattern of the fluid which is affected due to the influence of the thermal radiation, the chemical reaction and viscous dissipation on a magneto-hydro-dynamic flow in infinite vertical Couette channel walls have been analyzed. Perturbation technique is used to solve the governing equations of the flow model. The effects of physical parameters like the Grashof number based on temperature, the modified Grashof number based on concentration difference, the Schmidt number, the thermal radiation parameter, the chemical reaction parameter, the Prandtl number, Hartmann number, Eckert number and molecular diffusivity on the solution of the model have been analyzed.

For simplicity, only graphical representations of temperature profile for different values of the radiation parameter, temperature profile for different values of the Prandtl number, velocity profile for different values of the Hartmann number and also of concentration profile for different values of the chemical reaction parameters and molecular diffusivity have been presented and discussed.

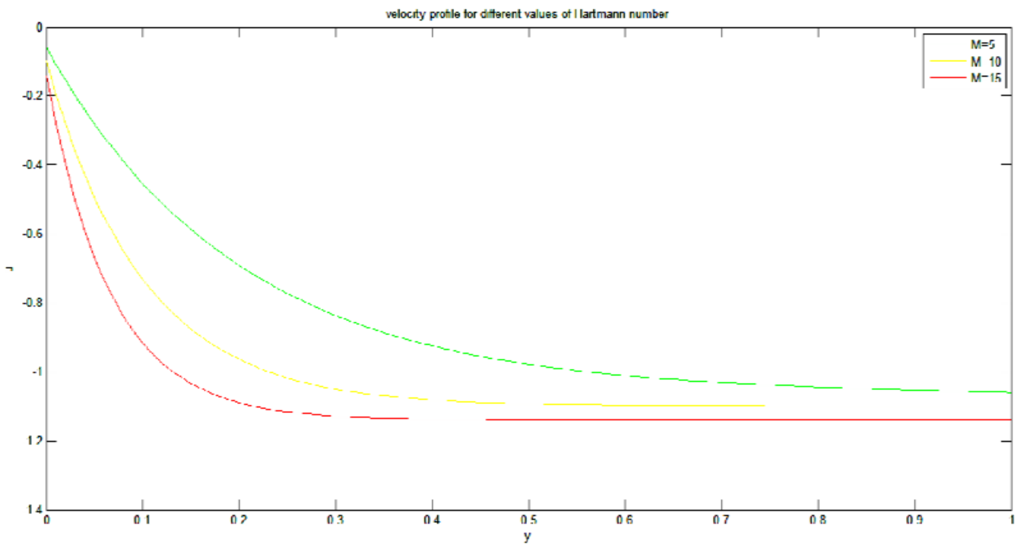

Fig.2. Velocity profile of the model for different values of the Hartmann number M.

In Fig.2, the simulated results of the influence of the Hartmann number on velocity are presented. The graph is drawn in the yu plane representing, respectively, the distance between the channel walls and velocity. Other parameters are held constant. From the figure it is observed that if the value of the Hartmann number M is increased, then the Lorentz force of the fluid flow will increase and as a result the velocity also will decrease.

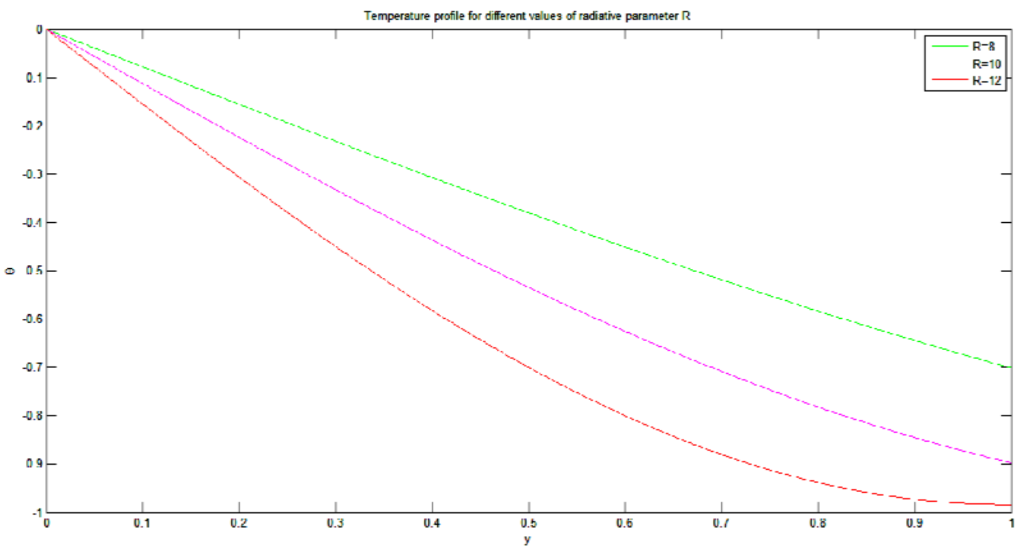

Fig.3. Temperature profile of the model for different values of the radiative parameter R .

In Fig.3, the simulated result of the influence of the radiation parameter R on the transfer of temperature is presented. The graph is drawn in the $y\theta$ plane representing, respectively, the distance between the channel walls and temperature. Other parameters are held constant. The result shows that for a fixed value of the thermal radiation parameter R, the temperature θ starts from a constant value at the moving channel wall. Also as y increases the temperature decreases till it reaches a minimum value. However, thereafter the temperature decreases with the increase of y and ultimately the temperature reaches an upper constant value. Nevertheless, the measure of temperature at the moving channel wall is always at a higher value than that at the stationary channel wall.

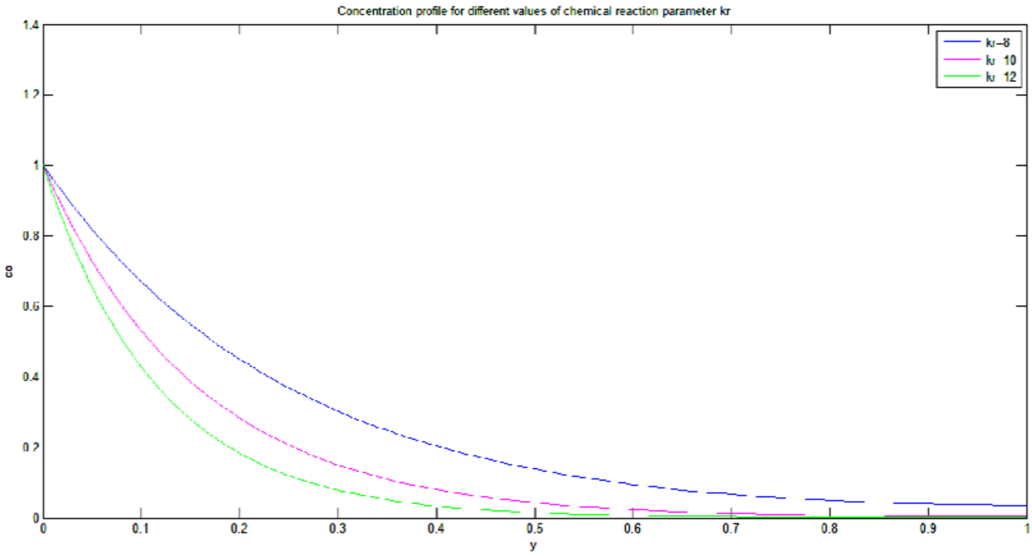

Fig.4. Concentration profile of the model for different values of the chemical reaction parameter kr .

In Fig.4, the simulated result of the influence of the chemical reaction parameter Kr on concentration is presented. The graph is drawn in the yc_0 plane representing, respectively, the distance between the channel walls and concentration. Other parameters are held constant. The result shows that for a fixed value of the chemical reaction parameter kr , the value of fluid concentration denoted by c_0 starts from a maximum constant value at the moving channel wall and as y increases the concentration c_0 decreases and reaches a zero constant value at the stationary channel wall. Nevertheless, the measure of fluid concentration at the moving channel wall is always at a higher value than that at the stationary channel wall.

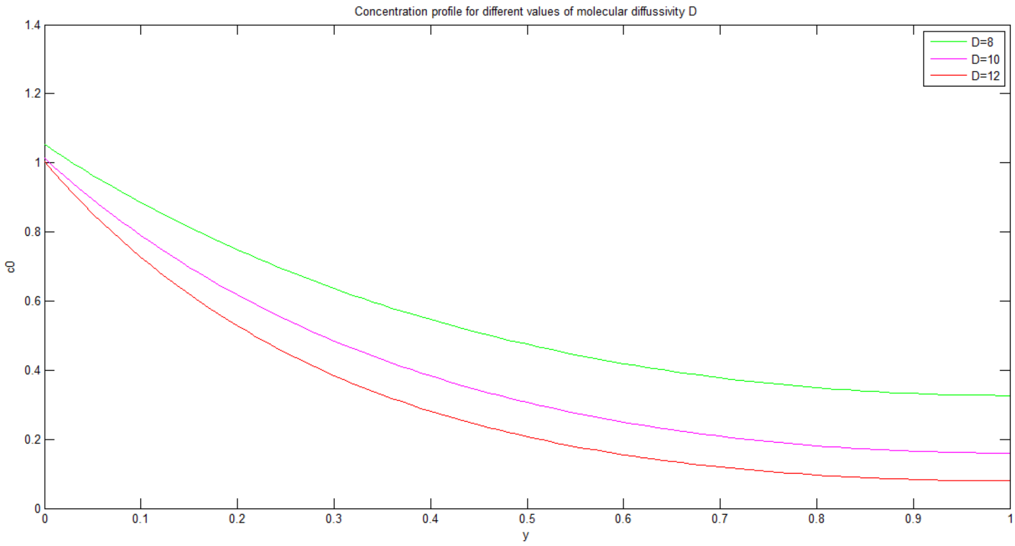

Fig.5. Concentration profile of the model for different values of molecular diffusivity D.

In Fig.5, the result of the simulated graph of the influence of molecular diffusivity on concentration is presented. The graph is drawn in the yc_0 plane where y is the distance between the channel walls and c_0 is the concentration. From the simulated graph it can be concluded that as the molecular diffusivity D increases the concentration of the fluid c_0 decreases. This is due to viscous dissipation. The effect of viscous dissipation of the fluid flow is to increase energy by heating up the fluid yielding greater temperature consequently this results in increasing the buoyancy force. The increase in temperature causes an increase in velocity of the flow. Therefore, the concentration decreases as molecular diffusivity increases.

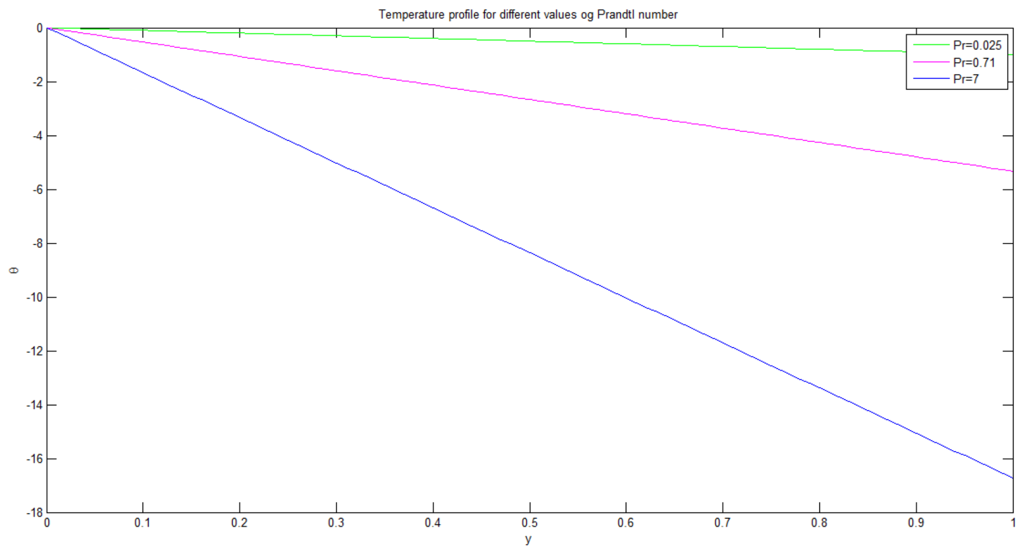

Fig.6. Temperature profile of the model for different values of the Prandtl number Pr.

In Fig.6, the result of the simulated graph of the Prandtl number on temperature profile is presented. The graph is drawn in the $y\theta$ plane where y is the distance between the channel walls and θ is temperature. From the simulated graph it can be concluded that as the Prandtl number increases the temperature decreases. The Prandtl number Pr is considered to be directly proportional to the viscosity ν but inversely proportional to the thermal diffusivity α . That is $\Pr = \nu / \alpha$. This theoretical fact has been verified successfully through the simulation study.

Whenever the value of Pr increased, the simulation study results also in an increase of the viscosity and as a result a decrease in the velocity. However, whenever the value of Pr is decreased, it also results in a decrease of viscosity and as a result an increase of the velocity and the same can be observed in the simulated graphs. Similarly, whenever the value of Pr is increased, the simulation study results in a decrease of the thermal diffusivity and as a result a decrease of the thermal boundary layer. However, whenever the value of Pr is decreased, it also results in an increase of the thermal boundary layer and the same can be observed in the simulated graphs. The Eckert number is the ratio of the kinetic energy of the fluid flow to the difference of the channel walls temperature. The Eckert number and temperature are directly proportional.

6. Conclusions

In this paper the effects of the thermal radiation, the chemical reaction and viscous dissipation on MHD flow in an infinite vertical Couette channel walls are analyzed. The flow pattern of the fluid is affected due to the influence of these physical parameters. The effects of physical parameters viz. The Hartmann number, chemical reaction parameter, thermal radiation parameter, Prandtl number, Eckert number and molecular diffusivity on flow variables viz. velocity, temperature and concentration are discussed. The solution of the governing equations is obtained using perturbation techniques. Mat lab code is used to solve the ordinary differential equations and to simulate the graphs. The following results are obtained:

(i) An increment in the value of the Hartmann number M is due to the Lorentz force of the fluid flow but as a result the velocity of the flow will decrease.
(ii) An increment in the radiative parameter results in a decrease in temperature.
(iii) An increment in the Prandtl number results in decreasing thermal diffusivity.
(iv) An increment in both the chemical reaction parameter and molecular diffusivity results in a decrease in concentration. The latter decrement in concentration is due to viscous dissipation.

Nomenclature

C – dimension less concentration
C_p – specific heat at constant pressure
C'_h – concentration at channel wall at $y = h$
C'_w – concentration at channel wall at $y = 0$
D – mass diffusivity
Ec – Eckert number
Gc – modified Grashof number
Gr – thermal Grashof number
g – acceleration due to gravity
J – electric current density
k_r – chemical reaction parameter
M – Hartmann number
Pr – Prandtl number
R – radiation parameter
Sc – Schmidt number
T' – temperature of the fluid in the boundary layer
T'_w – temperature of the moving channel wall
T'_h – temperature of the stationary channel wall
t – time

U – dimensionless free stream velocity
U' – free stream velocity
u' – velocity component in x' direction
q_r – radiative heat flux
v' – velocity component in y' direction
x , y – dimension less Cartesian coordinates
x', y' – Cartesian coordinates
α – thermal diffusivity
β – thermal expansion coefficient
β_c – concentration expansion coefficient
ε – amplitude of free stream velocity
θ – dimensionless temperature
κ – thermal conductivity
μ – dynamic viscosity
ν – kinematic viscosity
σ – electric conductivity
ω – frequency of oscillation

Appendix

$$m=\pm\sqrt{\frac{k_r h^2}{D}},\qquad \gamma=\pm\sqrt{\frac{k_r h^2}{D}+i\omega\mathrm{Sc}},\qquad \lambda=\pm i\sqrt{3Q_0h^2\mathrm{Pr}/\rho C_p\nu(4R+3)}\,,$$

$$c=\frac{-1}{e^{2\beta}-1},\qquad d=\frac{e^{2\beta}}{e^{2\beta}-1},\qquad A=\frac{-1}{e^{2M}-1},\qquad B=\frac{e^{2M}}{e^{2M}-1}\,,$$

$$\beta=\pm\sqrt{i\omega+M^2}\,,\qquad \mu=\pm i\sqrt{3\mathrm{Pr}(Q_0h^2-i\omega\rho C_p\nu)/\rho C_p\nu(4R+3)}\,,$$

$$c_1=\frac{1}{e^{2\lambda}-1},\qquad c_2=\frac{e^{2\lambda}}{e^{2\lambda}-1},\qquad d_1=\frac{-1}{e^{2m}-1}\,,$$

$$d_2=\frac{e^{2m}}{e^{2m}-1},\quad h_1=\frac{2-e^{2\gamma}}{1-e^{2\gamma}},\quad h_2=\frac{-1}{1-e^{2\gamma}}\,,\quad k_1=\frac{-1}{e^{2\mu}-1},\quad k_2=\frac{e^{2\mu}}{e^{2\mu}-1}\,.$$

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