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Effect of variable viscosity on thermal boundary layer over a permeable flat plate with radiation and a convective surface boundary condition †

Oluwole Daniel Makinde*

Institute for Advance Research in Mathematical Modelling and Computations, P. O. Box 1906, Bellville 7535, South Africa

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Abstract

In this paper, the combined effects of radiation, temperature dependent viscosity, suction and injection on thermal boundary layer over a permeable flat plate with a convective heat exchange at the surface are investigated. By taking suitable similarity variables, the governing boundary layer equations are transformed into a boundary value problem of coupled nonlinear ordinary differential equations and solved numerically using the shooting technique with sixth-order Runge-Kutta integration scheme. The solutions for the velocity and temperature distributions together with the skin friction coefficient and Nusselt number depend on six parameters; Prandtl number *Pr*, Brinkmann number *Br*, the radiation parameter *Ra*, the viscosity variation parameter *a*, suction/injection parameter f_w and convection Biot number *Bi*. Numerical results are presented both in tabular and graphical forms illustrating the effects of these parameters on thermal boundary layer. The thermal boundary layer thickens with a rise in the local temperature as the viscous dissipation, wall injection, and convective heating each intensifies, but decreases with increasing suction and thermal radiation. For fixed *Pr*, *Ra*, *Br* and *Bi*, both the skin friction coefficient and the Nusselt number increase with a decrease in fluid viscosity and an increase in suction. A comparison with previously published results on special case of the problem shows excellent agreement.

Keywords: Permeable flat plate; Thermal radiation; Variable viscosity; Convective boundary condition; Heat transfer; Similarity solution

1. Introduction

The study of thermal boundary layer over a flat surface is important in many industrial applications such as in the design of cooling systems for electronic devices, in the field of solar energy collection, geothermal reservoirs, heat exchangers, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, etc. In a pioneering work, Blasius [1] obtained a similarity solution for boundary layer flow of a Newtonian fluid over a flat surface. He introduced the concept of similarity solution into fluid mechanics by simplify the governing equations using a coordinate transformation to reduce the number of independent variables. Thereafter, several authors have extended the work in order to explore various aspects of the flow and heat transfer occurring in an infinite domain of the fluid surrounding the flat surface [2-4]. Fang [5] reported a similarity solution of thermal boundary layer for a moving plate. The flow of incompressible viscous fluid past a continuously moving semi-infinite plate with variable viscosity and variable temperature were investigated by Soundalgekar et al. [6]. Ghaly and Seddeek [7] analysed the influence of variable viscosity on heat and mass transfer over a flat plate. Lai and Kulacki [8] examined the effect of variable viscosity on boundary layer flow with convective heat and mass transfer in saturated porous media. Meanwhile, thermal radiation is one of the vital factors controlling the heat transfer in a non-isothermal system, especially when the processes take place at high temperatures [9]. The interaction of radiation with mixed convection flows of variable viscosity fluid permeated by transverse magnetic filed was investigated by Makinde and Ogulu [10]. Hossain and Takhar [11] studied the influence of thermal radiation on mixed convection along a vertical plate with uniform surface temperature. Abo-Eldahab and Elgendy [12] studied the combined effects of radiation and magnetic field on boundary layer flow over a stretching surface with variable viscosity and uniform free stream. Bataller [13] investigated the effect of thermal radiation on boundary layer flow over a flat plate and reported that the Blasius flow provides a thicker thermal boundary layer, but this trend can be reversed at low values of parameters entering the problem. Makinde [14] investigated the unsteady free convection interaction with thermal radiation and mass transfer in a boundary layer flow past a vertical porous plate. Moreover, in most studies on boundary layer flow

*Corresponding author. Tel.: +27 21 9596644, Fax.: +27 21 9596641
E-mail address: makinded@cput.ac.za
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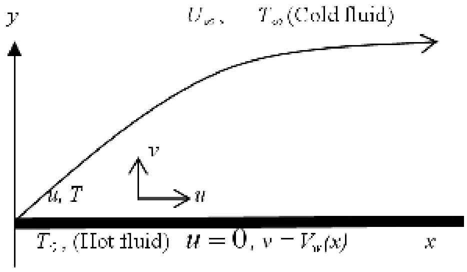

Fig. 1. Flow configuration and coordinate system.

over a flat surface, the surface temperature is either prescribed as constant or a function of an independent variable, but for many practical applications that involve the surface undergoing cooling or heating, the presence of convective heat exchange between the surface and the surrounding fluid cannot be neglected [15, 16]. This invariably causes the surface temperature to vary. Aziz [17] presented a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Ishak [18] presented a local similarity solution for the effects of suction and injection on boundary layer flow of a uniform viscosity fluid with a convective surface boundary condition. Makinde [19] analyzed the effect of a convective surface heat transfer on the hydromagnetic boundary layer flow over a moving vertical plate in a quiescent ambient fluid.

The purpose of the present work is to study the combined effects of radiation, suction and injection, variable viscosity and viscous dissipation on thermal boundary layer over a flat plate with a convective surface boundary condition. In the subsequent sections the classical similarity reductions of the boundary layer equations are derived and the resulting ordinary differential equations are solved numerically using shooting iteration technique together with Runge-Kutta sixth-order integration scheme [20]. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2. Mathematical model

Consider the thermal boundary layer flows over a permeable flat plate in a stream of cold fluid at temperature T_∞ in the presence of thermal radiation. It is assumed that the lower surface of the plate is heated by convection from a hot fluid at temperature T_0 which provides a heat transfer coefficient h_f. The cold fluid on the upper side of the plate is assumed to be Newtonian, heat generating or absorbing and its property variations due to temperature is limited to viscosity as shown in Fig. 1.

Let the x-axis be taken along the direction of plate and y-axis normal to it. If u, v and T are the fluid x-component of velocity, y-component of velocity and the temperature respectively, then for steady incompressible boundary layer flow, the governing equations for conservation of mass, momentum and energy can be written as follows [8, 15-19]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where U_∞ is the cold free stream velocity, c_p is the specific heat at constant pressure, $V_w(x) < 0$ corresponds to suction and $V_w(x) > 0$ corresponds to blowing or injection, α is the thermal diffusivity and ρ is the fluid density. Following Lai and Kulacki [8], the fluid dynamical viscosity μ is assumed to be an inverse linear function of temperature given by:

$$\mu(T) = \frac{\mu_\infty}{1 + \gamma(T - T_\infty)} \tag{4}$$

where μ_∞ is the cold free stream viscosity and γ is a constant. Using the Rosseland approximation [9] for the thermal radiation, the radiative heat flux is simplified as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* is the Stephan-Boltzmann constant and k^* is the mass absorption coefficient. The temperature differences within the flow are assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature T using a truncated Taylor series about the free stream temperature T_∞ i.e.

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{6}$$

The boundary conditions at the plate lower surface and far into the cold fluid above the plate upper surface may be written as [20]:

$$u(x,0) = 0,\; v(x,0) = V_w(x),$$
$$-k\frac{\partial T}{\partial y}(x,0) = h_w[T_0 - T(x,0)], \tag{7a}$$

$$u(x,\infty) = U_\infty, \quad T(x,\infty) = T_\infty \tag{7b}$$

where k is the thermal conductivity coefficient. The variable wall permeability function is given as

$$V_w(x) = -f_w\sqrt{U_\infty \upsilon / x}\Big/2 \tag{7c}$$

where f_w is constant constants with $f_w > 0$ representing the transpiration (suction) rate at the plate surface, $f_w < 0$ corresponds to injection and $f_w = 0$ for an impermeable surface. The stream function ψ, satisfies the continuity Eq. (1) automatically with

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \tag{8}$$

A similarity solution of Eqs. (1)-(3) is obtained by defining an independent variable η and a dependent variable f in terms of the stream function ψ as

$$\eta = y\sqrt{\frac{U_\infty}{\upsilon x}}, \quad \psi = \sqrt{\upsilon x U_\infty} f(\eta), \quad \upsilon = \frac{\mu_\infty}{\rho}, u = U_\infty f'(\eta),$$
$$v = -\frac{1}{2}\sqrt{\frac{U_\infty \upsilon}{x}}\left(f - \eta f'\right), \quad \theta(\eta) = \frac{T - T_\infty}{T_0 - T_\infty}. \tag{9}$$

After introducing Eq. (9) into Eqs. (1)-(3), we obtain

$$f''' + \frac{1}{2}(1 + a\theta) ff'' - \frac{a\theta' f''}{(1 + a\theta)} = 0, \tag{10}$$

$$\theta'' + \frac{1}{2}\Pr \beta f\theta' + \frac{Br\beta \left(f''\right)^2}{(1 + a\theta)} = 0, \tag{11}$$

$$f(0) = f_w, f'(0) = 0, \quad \theta'(0) = Bi[\theta(0) - 1], \tag{12a}$$

$$f'(\infty) = 1, \quad \theta(\infty) = 0 \tag{12b}$$

where $\beta = 3Ra/(3Ra + 4)$, the prime symbol represents the derivative with respect to η and

$Br = \dfrac{\mu_\infty U_\infty^2}{k(T_0 - T_\infty)}$ (Brinkmann number),

$\Pr = \dfrac{\upsilon}{\alpha}$ (Prandtl number),

$a = \gamma(T_0 - T_\infty)$ (variable viscosity parameter)

$Ra = \dfrac{kk^*}{4\sigma^* T_\infty^3}$ (thermal radiation parameter).

$Bi = \dfrac{h_w}{k}\sqrt{\dfrac{\upsilon x}{U_\infty}}$ (local Biot number).

We remark here the case of $\beta = 1$ corresponds to the absence of thermal radiation influence and $a = 0$ indicates the constant viscosity scenario, the positive value of parameter ($a > 0$) corresponds to liquids while the negative value ($a < 0$) represents gases. It is noteworthy that the local Biot number Bi in Eq. (12a) is functions of x. In order to have a similarity solution, all the parameters must be constant and we can therefore assume [17]

$$h_w = mx^{-\frac{1}{2}} \tag{13}$$

where m is constant. The set of Eqs. (10)-(11) with the boundary conditions (12) have been solved numerically by applying the Nachtsheim and Swigert [19] shooting iteration technique together with Runge-Kutta sixth-order integration scheme. From the process of numerical computation, the plate surface temperature, the local skin-friction coefficient and the local Nusselt number which are respectively proportional to $\theta(0)$, $f''(0)$ and $-\theta'(0)$, are also worked out and their numerical values are presented in a tabular form.

Table 1. Computations showing comparison with Bataller [15] results for $f_w = a = Br = 0$.

Bi	Pr	Ra	$\theta(0)$ Bataller [15]	$\theta(0)$ Present
0.1	5.0	0.7	0.1996265	0.19957406
0.5	5.0	0.7	0.5548979	0.55489763
1.0	5.0	0.7	0.7137422	0.71374169
10.0	5.0	0.7	0.9614407	0.96143981
20.0	5.0	0.7	0.9803475	0.98034087
1.0	0.72	0.7	0.8334487	0.83312107
1.0	1.0	0.7	0.8156143	0.81555469
1.0	5.0	0.7	0.7137422	0.71374169
1.0	10	0.7	0.6630187	0.66301284
1.0	100	0.7	0.4759402	0.47592614
5.0	5.0	0.7	0.9257453	0.92574298
5.0	5.0	5.0	0.9037694	0.90376783
5.0	5.0	10.0	0.9004477	0.90044458
5.0	5.0	100.0	0.8970060	0.89700322

Table 2. Computations showing comparison with Aziz [17] results for $f_w = 0 = a = Br = 0$, $Pr = 0.72$.

Bi	$\theta(0)$ Aziz [17]	$-\theta'(0)$ Aziz [17]	$\theta(0)$ Present	$-\theta'(0)$ **Present**
0.05	0.1447	0.0428	0.14466	**0.04276**
0.60	0.6699	0.1981	0.66991	**0.19805**
1.00	0.7718	0.2282	0.77182	**0.22817**
5.00	0.9441	0.2791	0.94417	**0.27913**
20.00	**0.9854**	**0.2913**	**0.98543**	**0.29132**

3. Results and discussion

Computations have been carried out for various values of Pr ($0.72 \le Pr \le 100$), ($0.05 \le Bi_x \le 20$), Br ($0.1 \le Br \le 10$), a($-0.5 \le a \le 1$), f_w ($-0.5 \le f_w \le 1$), and Ra ($0.7 \le Ra \le 100$). The edge of the boundary layer (η_∞) has been taken between 6 and 10 depending on the values of the parameters. In order to validate the accuracy of our numerical procedure, we have compared our results with those of Bataller [15] (i.e. $a = 0, f_w = 0$) and Aziz [17] (i.e. $a = 0$, $f_w = 0$, $Ra = 0$). The results are found in excellent agreement and some of the comparisons are shown in Tables 1 and 2. In Table 3 the computational results showing the effects of various embedded parameters on the local skin-friction coefficient $f''(0)$, the local Nusselt $-\theta'(0)$ and the plate surface temperature $\theta(0)$ are displayed. It is noteworthy that $f''(0)$ increases with increasing values of Bi_x, $a > 0$, Br, $f_w > 0$ and decreases with increasing values of Pr,

Table 3. Computation showing $f''(0)$, $\theta(0)$, and $\theta'(0)$ for various values of key parameters.

Bi	a	Br	Ra	Pr	f_w	$f''(0)$	$-\theta'(0)$	$\theta(0)$
0.1	0.1	0.1	0.7	0.72	0.1	0.37739	0.06554	0.34450
1.0	0.1	0.1	0.7	0.72	0.1	0.38955	0.16938	0.83061
10	0.1	0.1	0.7	0.72	0.1	0.39323	0.20146	0.97985
0.1	0.5	0.1	0.7	0.72	0.1	0.41035	0.06593	0.34063
0.1	1.0	0.1	0.7	0.72	0.1	0.44792	0.06634	0.33652
0.1	-0.1	0.1	0.7	0.72	0.1	0.35972	0.06533	0.34669
0.1	-0.5	0.1	0.7	0.72	0.1	0.32143	0.06482	0.35170
0.1	0.1	1.0	0.7	0.72	0.1	0.38195	0.04457	0.55421
0.1	0.1	10	0.7	0.72	0.1	0.42238	0.15046	2.50461
0.1	0.1	0.1	5.0	0.72	0.1	0.37654	0.07074	0.29259
0.1	0.1	0.1	10	0.72	0.1	0.37645	0.07130	0.28690
0.1	0.1	0.1	0.7	3.00	0.1	0.37509	0.07760	0.22392
0.1	0.1	0.1	0.7	7.10	0.1	0.37379	0.08342	0.16570
0.1	0.1	0.1	0.7	0.72	0.5	0.53555	0.06888	0.31119
0.1	0.1	0.1	0.7	0.72	1.0	0.74624	0.07199	0.28007
0.1	0.1	0.1	0.7	0.72	-0.1	0.30299	0.06345	0.36545
0.1	0.1	0.1	0.7	0.72	-0.5	0.16691	0.05784	0.42157

Ra, $a < 0$, $f_w < 0$. This implies that the skin friction on the upper surface of the plate is enhanced by combined effects of increasing convective heat transfer, decreasing in fluid viscosity, viscous dissipation increase and suction. The skin friction decreases with an increase in the intensity of radiation, Prandtl number, fluid viscosity increase and injection. On the other hand, the local Nusselt increases with increasing values of Bi_x, $a > 0$, Br, $f_w > 0$, Ra, Pr and decreases with increasing values of $a < 0$, $f_w < 0$. This can be attributed to the fact that as Biot number increases, the rate of heat transfer from the hot fluid on the lower side of the plate to the cold variable viscosity fluid on the upper side of the plate increases.

3.1 Effects of parameter variation on velocity profiles

In Figs. 2-4 the effects of various thermophysical parameters on the fluid velocity profiles are demonstrated. Generally, the fluid velocity is lowest at the place surface due to no-slip condition and increases to the free stream value satisfying the far field boundary condition. From Fig. 2, we observe that a decrease in the fluid viscosity ($a > 0$) due to an increase in the fluid temperature may lead to an overshoot of velocity profile towards the plate surface while the trend is opposite when ($a < 0$) with an increase in the fluid velocity. Fig. 3 shows the velocity profiles for different values of suction/injection parameter f_w when Pr = 0.72 (such as air). We observed that in case of injection ($f_w < 0$), the fluid is carried away from the plate surface as compared to the case when the plate surface is impermeable ($f_w = 0$), causing a reduction in velocity gradient as it tries to maintain the same velocity over a small region near the surface. In the case of suction ($f_w > 0$), the fluid is pushed towards the plate surface leading to an increase in the velocity gradient and the surface shear stress as compared to the case when the plate surface is impermeable ($f_w = 0$). We also note in Fig. 4, that an increase in Br in the presence of suction ($f_w > 0$) produced a further increase in the fluid velocity within the boundary layer region. This can be attributed to the fact that the fluid becomes lighter and flows faster with an increase in viscous heating as Br increases.

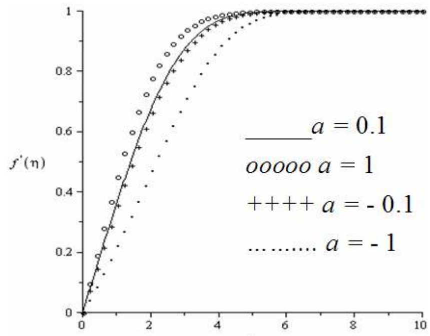

Fig. 2. Velocity profiles for various values of a with Pr = 0.72, Ra = 0.7, Br = 0.1, f_w = 0.1, Bi = 0.1.

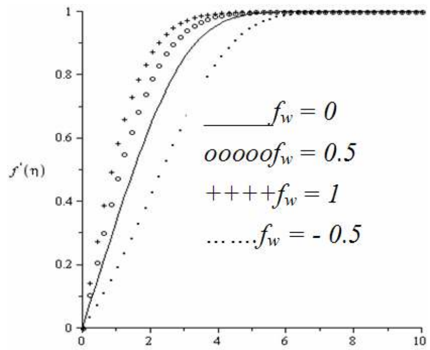

Fig. 3. Velocity profiles for various values of f_w with Pr = 0.72, Ra = 0.7, Br = 0.1, a = 0.1, Bi = 0.1.

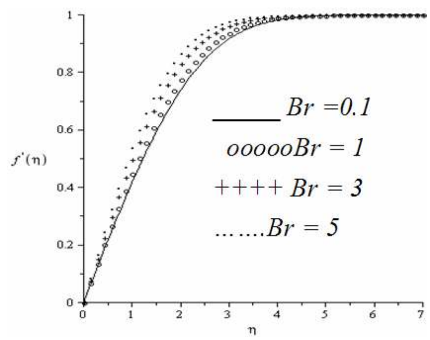

Fig. 4. Velocity profiles for various values of Br with Pr = 0.72, Ra = 0.7, f_w = 0.1, a = 1, Bi = 0.1.

3.2 Effects of parameter variation on temperature profiles

The effects of variation in the thermophysical parameters on the fluid temperatures profiles are illustrated in Figs. (5)-(10).

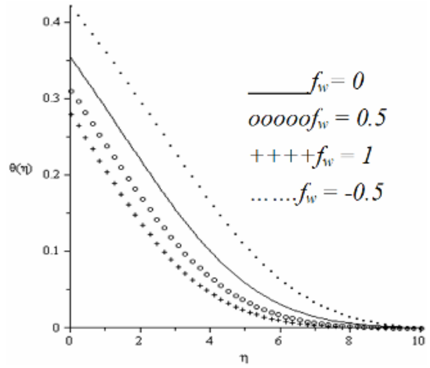

Fig. 5. Temperature profiles for various values of f_w with $Pr = 0.72$, $Ra = 0.7$, $Br = 0.1$, $a = 0.1$, $Bi = 0.1$.

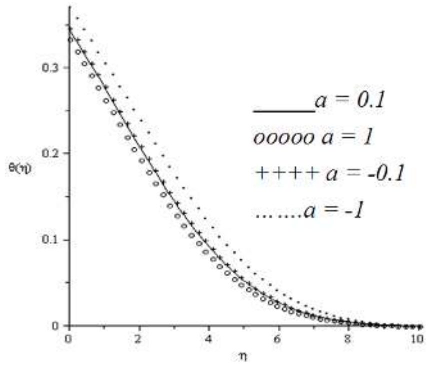

Fig. 6. Temperature profiles for various values of a with $Pr = 0.72$, $Ra = 0.7$, $Br = 0.1$, $f_w = 0.1$, $Bi = 0.1$.

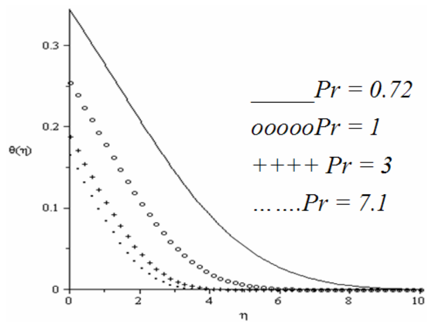

Fig. 7. Temperature profiles for various values of Pr with $Ra = 0.7$, $a = 0.1$, $Br = 0.1$, $f_w = 0.1$, $Bi = 0.1$.

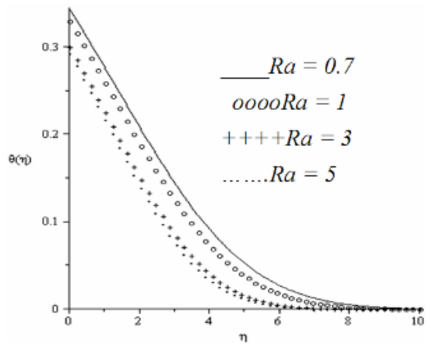

Fig. 8. Temperature profiles for various values of Ra with $Pr = 0.72$, $a = 0.1$, $Br = 0.1$, $f_w = 0.1$, $Bi = 0.1$.

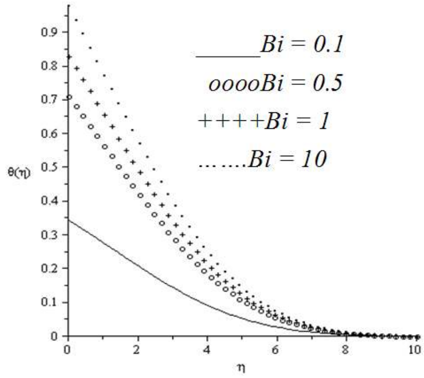

Fig. 9. Temperature profile for various values of Bi with $Pr = 0.72$, $a = 0.1$, $Br = 0.1$, $f_w = 0.1$, $Ra = 0.7$.

The fluid temperature is highest at the plate surface and decreases exponentially to the free stream zero value away from the plate satisfying the boundary condition. Fig. 5 shows the numerical solutions for different values of f_w when $Pr = 0.72$ (such as air) and $a = 0.1$ It is seen from Fig. 5 that the surface temperature $\theta(0)$ decreases for suction ($f_w > 0$) and increases for injection ($f_w < 0$). This can be attributed to the fact that the as suction increases, the convection resistance of the hot fluid on the lower side of the plate increases and consequently, the surface temperature $\theta(0)$ decreases. In Fig. 6, we observe that the thermal boundary layer thickness decreases with a decrease in the fluid viscosity ($a > 0$) and increases with an increase in the fluid viscosity ($a < 0$). Fig. 7 shows the temperature profiles for various values of Pr (i.e. from Pr = 0.72 Air to $Pr = 7.1$ Water) when the other parameters are fixed. It is evident from this figure that the temperature gradient at the plate surface increases as Pr increases, which implies an increase in the heat transfer rate at the surface. This is because a higher Prandtl number fluid has a relatively low thermal diffusivity, which reduces conduction, and thereby the thermal boundary layer thickness, and as a consequence increases the heat transfer rate at the surface. The effects of thermal radiation parameter Ra on temperature profiles are shown in Fig. 8. An increase in Ra causes a decrease in the plate surface temperature leading to a reduction in the thermal boundary layer thicknesses. This agreed perfectly with the observation of Bataller [13]. Since the flow system is operating at a high dimensionless temperature value ($0.3 < \theta < 0.5$) due to combined effects of viscous heating and Newtonian heating of the plate from the hot fluid at the lower surface, an increase in the radiative heat loss to the surrounding environment causes a general decrease in the fluid temperature. In Fig. 9, the plate surface temperature increases with an increase in the local Biot number (Bi) due to convective heat exchange between the hot fluid on the lower side of the plate to the cold fluid on its upper surface, consequently, the thickness of the thermal

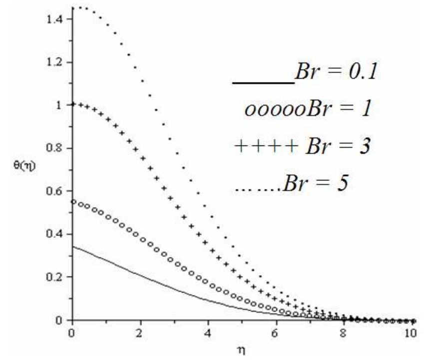

Fig. 10. Temperature profiles for various values of Br with Pr = 0.72, a = 0.1, Bi = 0.1, f_w = 0.1, Ra = 0.7.

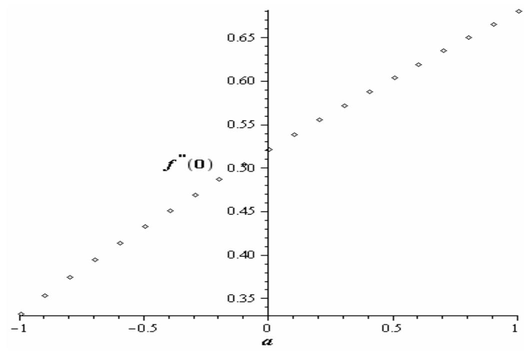

Fig. 11. Effects of viscosity variation on the skin friction for Pr = 0.72, Br = 0.1, Bi = 0.1, f_w = 0.5, Ra = 0.1.

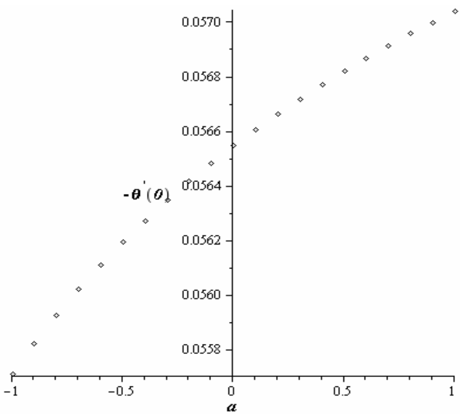

Fig. 12. Effects of viscosity variation on the plate surface heat transfer rate for Pr = 0.72, Br = 0.1, Bi = 0.1, f_w = 0.5, Ra = 0.1.

boundary layer is enhanced. Similar trend is observed in Fig. 10 with an increase in the viscous dissipation (Br). This can be attributed to the fact that the presence of viscous dissipation serves as an additional heat source in the flow field leading to elevation in the fluid temperature as expected.

3.3 Skin friction and local Nusselt number

Figs. 11-14 depict the effects of variable viscosity and suction/injection on the plate surface heat transfer rate and the

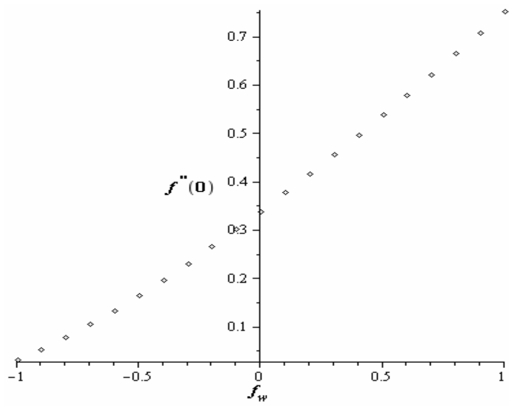

Fig. 13. Effects of suction/injection on the skin friction for Pr = 0.72, Br = 0.1, Bi = 0.1, a= 0.1, Ra = 0.1.

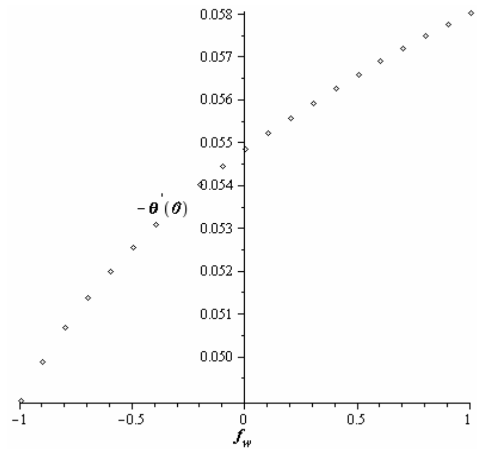

Fig. 14. Effects of suction/injection on the plate surface heat transfer rate for Pr = 0.72, Br = 0.1, Bi = 0.1, a= 0.1, Ra = 0.1.

skin friction. From the figures, it is interesting to note that both the skin friction and plate surface heat transfer rate increase with increasing suction and decrease with increasing injection. This is in agreement with the numerical results displayed in Table 3. A decrease in the fluid viscosity (i.e. increasing values of $a > 0$) causes an increase in both the plate surface heat transfer rate and the skin friction while the trend is opposite with an increase in the fluid viscosity (i.e. increasing values of $a < 0$).

4. Conclusions

The combined effects of temperature dependent viscosity, fluid suction and injection on laminar boundary layer flow and heat transfer over a horizontal flat-plate and heat transfer in the presence of thermal radiation under a convective surface boundary condition is investigated. Using similarity approach, the nonlinear partial differential equations governing the problem together with the boundary conditions are transformed into a set of coupled ordinary differential equations and solved numerically by a shooting technique with sixth-order Runge-Kutta integration scheme. It was found that

(1) an increase in Prandtl number and the radiation parameter may cause a decrease in the fluid temperature and consequently reducing the thermal boundary layer thickness along the plate.

(2) the surface shear stress increases with a decrease in fluid viscosity, a decrease in thermal radiation, an increase suction, an increase local Biot number and an increase viscous dissipation.

(3) the local Nusselt number increases with a decrease in fluid viscosity, an increase suction, an increase local Biot number, an increase viscous dissipation, an increase in the thermal radiation and Prandtl number.

This study has potential applications in understanding the complex dynamics of many engineering and industrial flow systems such as in the cooling systems for electronic devices, cooling of nuclear reactors, packed-bed catalytic reactors, enhanced oil recovery, etc. The interdisciplinary nature of boundary layer flow research presents a great opportunity for exploration and discovery at the frontiers of science and technology. It is hoped that the present work will serve as a stimulus for needed experimental work on this problem.

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Nomenclature

a : Variable viscosity parameter
Bi : Local Biot number
Br : Brinkmann number
f : Dimensionless stream function
f_w : Suction/injection parameter
h_w : Heat transfer coefficient
k : Thermal conductivity coefficient
k^* : Mass absorption coefficient
Pr : Prandtl number
Ra : Radiation parameter
T : Temperature parameter
T_∞ : Free stream temperature
T_0 : Hot fluid temperature
U_∞ : Free stream velocity
(u, v) : Velocity components
V_w : Wall suction/injection velocity
(x, y) : Coordinates

Greek symbols

ψ : Stream function
μ_∞ : Cold fluid dynamic viscosity
μ : Dynamic viscosity
υ : Cold fluid kinematic viscosity
γ : Viscosity variation parameter
ρ : Fluid density
α : Thermal diffusivity
η : Similarity variable
σ^* : Stephan-Boltzmann constant
θ : Dimensionless temperature

Subscripts

∞ : Free stream variables (cold fluid)
0 : Hot fluid variables
w : Variable at the plate surface

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Oluwole Daniel Makinde is currently a Senior Professor and Director of Postgraduate Studies at Cape-Peninsula University of Technology (CPUT) in South Africa. He is also the founder and Director of Institute for Advance Research in Mathematical Modelling and Computations at CPUT. He received his B.Sc. (Hons) First Class and M.Sc. from Obafemi Awolowo University, Ile-Ife, Nigeria and Ph.D at University Bristol, England all in Computational and Applied Mathematics. Professor Makinde has taught and supervised graduate students and served as an external examiner at many universities within and outside African continent. He has co-authored two applied mathematics textbooks, two Open University monographs on Particle Mechanics and Incompressible Flow Theory and published numerous research articles in several reputable international journals. He is presently the Secretary General of African Mathematical Union, Editor and editorial board member of several international journals. Professor Makinde received several research excellence awards, among them are: South African NSTF/National Research Foundation award and African Union Kwame Nkrumah Continental Scientific Award for outstanding contribution to Science, Engineering, Technology and Innovation.