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Effect of Void Fraction and Two-Phase Dynamic Viscosity Models on Prediction of Hydrostatic and Frictional Pressure Drop in Vertical Upward Gas–Liquid Two-Phase Flow

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In gas–liquid two-phase flow, the prediction of two-phase density and hence the hydrostatic pressure drop relies on the void fraction and is sensitive to the error in prediction of void fraction. The objectives of this study are to analyze dependence of two-phase density on void fraction and to examine slip ratio and drift flux model-based correlations for their performance in prediction of void fraction and two-phase densities for the two extremes of two-phase flow conditions, that is, bubbly and annular flow or, alternatively, the low and high region of the void fraction. It is shown that the drift flux model-based correlations perform better than the slip ratio model-based correlations in prediction of void fraction and hence the two-phase mixture density. Another objective of this study is to verify performance of different two-phase dynamic viscosity models in prediction of two-phase frictional pressure drop. Fourteen two-phase dynamic viscosity models are assessed for their performance against 616 data points consisting of 10 different pipe diameters in annular flow regime. It is found that none of these two-phase dynamic viscosity models are able to predict the frictional pressure drop in annular flow regime for a range of pipe diameters. The correlations that are successful for small pipe diameters fail for large pipe diameters and vice versa.

INTRODUCTION

The gas–liquid two-phase flow finds its practical application in oil–gas, nuclear, refrigeration, and chemical industrial processes. Independent of whether the gas–liquid two-phase flow is boiling or nonboiling in nature, one of the key parameters required in the design of engineering processes involving two-phase flow is the correct estimation of two-phase mixture density and the total two-phase pressure drop. The correct estimation of hydrostatic pressure drop, one of the components of the total two-phase pressure drop, is sensitive to the correct prediction of two-phase mixture density, which in turn is strongly influenced by the accurate prediction of void fraction. This study is focused on two extremes of the two-phase flow conditions based on the low and high values of void fraction or, alternatively, bubbly and annular flow regimes. The two-phase flow phenomenon for small values of the void fraction (bubbly flow) is gravity dominated, while that for large values of void fraction (annular flow) is inertia dominated. The intermediate two-phase flow conditions are a combination of these two mechanisms. Thus, it is of interest to analyze the performance of different two-phase flow models in these extreme two-phase flow scenarios. In the bubbly flow regime (low region of void fraction), the two-phase literature reports the validity of using a homogeneous flow model as an approximate method to predict two-phase mixture density required in calculation of hydrostatic and frictional two-phase pressure drop. However, there is not enough investigation done in the literature on verifying this model by extending its application to the other extreme of two-phase flow, i.e., annular flow. The homogeneous flow model assumes no slip between the two phases ($S = 1$) and hence performs well in the bubbly flow regime due to the fact that the slip ratio (S) between the two phases in this flow regime is close to unity. However, this is not true for the other extreme of two-phase flow, i.e., the annular flow regime. In this flow pattern, there exists a significant

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slippage between the two phases, and the application of a homogeneous flow model may fail drastically in the prediction of different two-phase flow parameters.

Two-phase flow literature reports several models to predict void fraction and verify its accuracy against the measured void fraction spanning over all flow patterns, mostly using percentage error, mean error, and standard deviations. The void fraction as a stand-alone two-phase parameter is of no use for practical applications unless it is used to derive other quantities such as the mixture density, viscosity, and heat transfer coefficient. The void fraction correlation with a good accuracy may fail to predict the two-phase mixture density satisfactorily due to the weighted nature of the two-phase mixture density equation. Thus, in order to address this issue, this study attempts to analyze the performance of different void fraction correlations with the perspective of its influence and accuracy in estimation of the two-phase mixture density. A similar type of study but oriented toward estimation of two-phase mixture density and hence the refrigerant charge inventory for two-phase flow of refrigerants in horizontal tubes was carried out by Farzad and O'Neal [1] and Ma et al. [2]. Farzad and O'Neal [1] analyzed different slip model-based void fraction-based correlations for their impact on the estimation of two-phase density in determination of refrigerant charge that governed the optimum performance of an air conditioner. They concluded that refrigerant two-phase mixture density is sensitive to the void fraction and requires the most accurate correlation to be selected for system optimization. They found that the void fraction correlations of Barcozy [3], Hughmark [4], and Zivi [5] predicted the refrigerant charge closest to the experimental data; however, they did not quantify their performance. Ma et al. [2] recommended that since a typical two-phase flow in an evaporator may go through different flow pattern transitions, a flow-pattern-independent correlation or a combination of different flow-pattern-specific void fraction correlations should be used to determine the two-phase mixture density. They found that correlations of Zivi [5], Smith [6], and Permoli et al. [7] predicted the experimental data with a mean error between 4.3% and 7.8%. This quantitative performance is not conclusive since the mean error numbers are influenced by the number of data points and the flow patterns. In the present study, the top performing void fraction correlations classified as those based on the slip ratio model and drift flux model (DFM) are analyzed for their accuracy in prediction of void fraction and two-phase mixture density, and is illustrated qualitatively using the probability density function (PDF). Additionally, their performance is quantified in terms of the percentage of data points predicted within specific error bands in bubbly and annular flow regimes, respectively.

Another objective of this study is to investigate the performance of different two-phase viscosity models available in the literature in prediction of frictional pressure drop in annular flow regime. The two-phase flow literature reports the existence of 14 two-phase viscosity models to calculate the two-phase mixture Reynolds number and hence the two-phase friction factor. Some of the two-phase dynamic viscosity models such as those by Akers et al. [8], Beattie and Whalley [9], Ciccihitti et al. [10], Davidson et al. [11], Dukler et al. [12], and McAdams et al. [13] were developed between 1940 and 1980, while other models such as those of Fourar and Boris [14], Garcia et al. [15], and Awad and Muzychka [16] were developed more recently (1990–2010). This shows a renewed interest by the research community regarding use of two-phase dynamic viscosity models to predict two-phase frictional pressure drop. Beattie and Whalley [9] extended the theory of two-phase solid–liquid viscosity proposed by Einstein [17] and proposed a gas–liquid two-phase dynamic viscosity correlation for bubbly and annular flow regimes. Lin et al. [18] presented a two-phase viscosity model to predict frictional pressure drop in capillary tubes. The two-phase viscosity model was claimed to work well for $0 < x < 0.25$. Davidson et al. [11] developed a two-phase viscosity model based on the experimental data of highly pressurized steam–water two-phase flow. The main problem with this correlation is that the two-phase viscosity does not approach the single-phase gas viscosity as the flow quality approach unity. Thus, it is likely that this correlation will fail for the annular flow regime. The correlations of McAdams et al. [13], Ciccihitti et al. [10], and Dukler et al. [12] account for a weighted average of the liquid and gas viscosities in terms of the quality but are presented in different forms. Awad and Muzychka [16] proposed four different sets of two-phase viscosity models with reference to the analogy between viscosity in two-phase flow and the thermal conductivity in porous media. The performance of these correlations was verified against the data of refrigerants mostly for the mini- and micro-size pipes and no recommendation was made on the part of selection of the particular equation.

As shown in Figure 1, all of these models show a considerable variation with respect to each other in terms of variation of two-phase dynamic viscosity with change in two-phase flow quality and hence need to be evaluated to identify the best performing correlation. The present study aims to compare the performance of these two-phase viscosity models in prediction of two-phase frictional pressure drop in annular flow regime against the experimental data for a range of pipe diameters.

VOID FRACTION AND TWO-PHASE DYNAMIC VISCOSITY MODELS

The total pressure drop in gas–liquid two-phase flow consists of the hydrostatic, frictional, and accelerational components of the pressure drop, as shown in Eq. (1). The calculation of two-phase hydrostatic components requires the knowledge of the two-phase mixture density, which in turn depends on accurate prediction of the void fraction as expressed by Eqs. (2) and (3), respectively. Another approach reported in the literature to calculate the two-phase mixture density is based on two-phase quality as shown in Eq. (4). This approach of the homogeneous flow model assumes no slip between the two phases and hence a slip ratio of unity. The two-phase density expressed by Eq. (4) is found to be in agreement with the two-phase density defined by Eq. (3) in bubbly flow regime, but these two densities

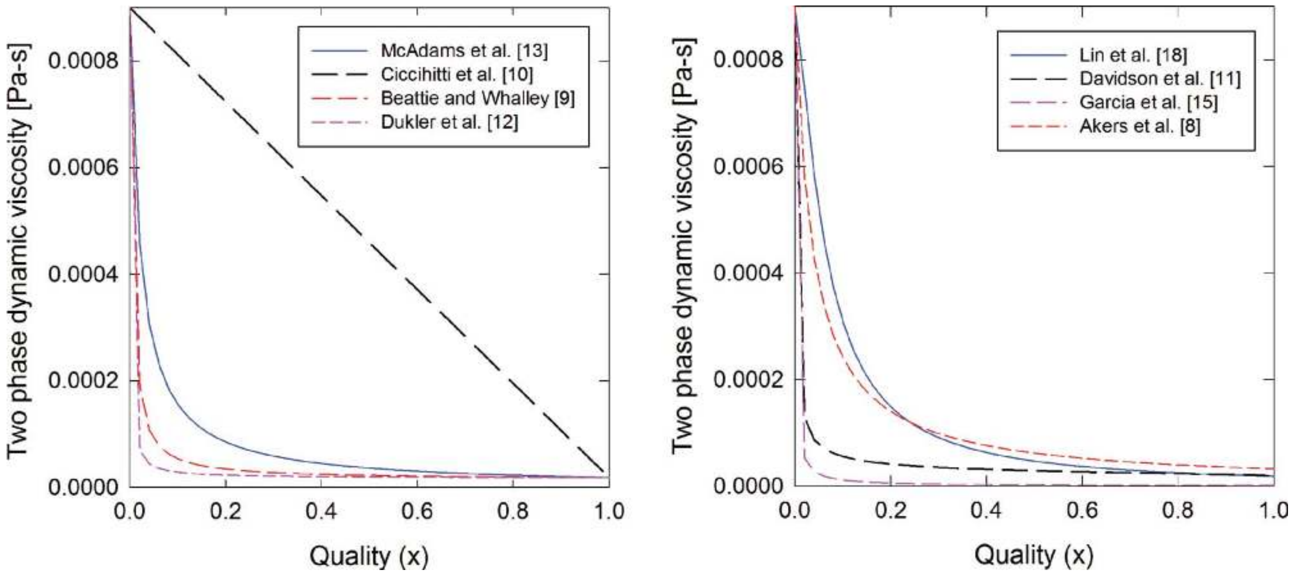


Figure 1 Variation of two-phase dynamic viscosity models with change in two-phase flow quality. (Color figure available online.)

significantly differ from each other in annular flow regime as shown in Figure 2. The data used in Figure 2 are collected at the Oklahoma State University Two Phase Flow Laboratory and the flow patterns are distinguished based on visual observation. Overall in annular flow regime, Eq. (4) underpredicts the two-phase density and hence the hydrostatic pressure drop.

$$\left(\frac{dP}{dL}\right)_{t,TP} = \left(\frac{dP}{dL}\right)_{h,TP} + \left(\frac{dP}{dL}\right)_{f,TP} + \left(\frac{dP}{dL}\right)_{a,TP} \quad (1)$$

$$\left(\frac{dP}{dL}\right)_{h,TP} = \rho_m g \sin\theta \quad (2)$$

$$\rho_m = \rho_g \alpha + \rho_l (1-\alpha) \quad (3)$$

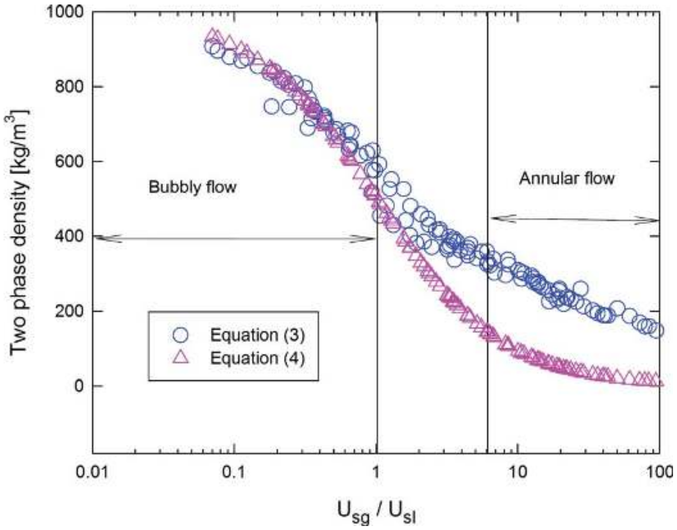


Figure 2 Two-phase density defined by Eqs. (3) and (4). Equation (3) is based on the measured values of void fraction at Oklahoma State University, Two-Phase Flow Laboratory. (Color figure available online.)

$$\rho_m = \left(\frac{x}{\rho_g} + \frac{1-x}{\rho_l}\right)^{-1} \quad (4)$$

The literature reports Eq. (3) as a standard approach to calculate two-phase mixture density. Thus the equation based on void fraction should be used to calculate hydrostatic pressure drop.

For the nonboiling gas–liquid two-phase flow, the contribution of the accelerational component to the total two-phase pressure drop is very small and hence can be neglected. In the case of boiling two-phase flow, the contribution of the accelerational component to the total pressure drop may be noticeable depending upon the change in quality, void fraction, and gas (vapor) density at inlet and outlet of the pipe and is calculated using Eq. (5).

$$\left(\frac{dP}{dL}\right)_{a,TP} = \frac{G^2}{L}\left[\left\{\frac{(1-x)^2}{\rho_l(1-\alpha)} + \frac{x^2}{\rho_g \alpha}\right\}_o - \left\{\frac{(1-x)^2}{\rho_l(1-\alpha)} + \frac{x^2}{\rho_g \alpha}\right\}_i\right] \quad (5)$$

The present study is focused on the nonboiling gas–liquid two-phase flow and hence we ignore the contribution of the accelerational component to the two-phase pressure drop. The frictional pressure drop is due to friction of single-phase liquid at the pipe wall and the friction at the gas–liquid interface.

$$\left(\frac{dP}{dL}\right)_{f,TP} = \frac{f_{TP} G^2}{2D\rho_m} \quad (6)$$

The calculation of the two-phase friction factor in Eq. (6) depends upon the estimation of the two-phase Reynolds number, which in turn depends upon two-phase dynamic viscosity. The two-phase mixture Reynolds number required for calculation of two-phase friction factor is calculated as given by Eq. (7):

$$\mathrm{Re}_m = \frac{GD}{\mu_m} \tag{7}$$

The two-phase dynamic viscosity model of Akers et al. [8] requires formulating the two-phase equivalent mass flux as given by Eq. (8) and is used to calculate two-phase mixture Reynolds number.

$$G_{eq} = G\left((1-x) + x\sqrt{\frac{\rho_l}{\rho_g}}\right) \tag{8}$$

The literature reports the use of the Blasius [19] or Colebrook [20] equation to calculate the two-phase friction factor based on the two-phase Reynolds number. The correlation of Blasius [19] is for smooth pipes and doesn't provide a smooth transition between laminar and turbulent flows, while the equation of Colebrook [20] is for a turbulent flow regime. Hence, the Churchill [21] friction factor correlation is used in this study, as it provides a smooth transition between laminar and turbulent flows and also accounts for the pipe surface roughness effects. The Churchill [21] friction factor equation is given by Eq. (9). The variables A and B are expressed by Eqs. (10) and (11), respectively.

$$f_{TP} = 8\left[\left(\frac{8}{\mathrm{Re}_m}\right)^{12} + \frac{1}{(A+B)^{1.5}}\right]^{\left(\frac{1}{12}\right)} \tag{9}$$

$$A = \left\{2.457\ln\left[\frac{1}{(7/\mathrm{Re}_m)^{0.9} + (0.27\varepsilon/D)}\right]\right\}^{16} \tag{10}$$

$$B = \left(\frac{37530}{\mathrm{Re}_m}\right)^{16} \tag{11}$$

It should be noted that the two-phase dynamic viscosity models recommend use of two-phase density based on the homogeneous flow model defined by Eq. (4) to be used in Eq. (6) to calculate two-phase frictional pressure drop. Although the two-phase mixture density defined by Eq. (3) is more realistic since it accounts for slip between the two phases, it is found in the present study that the two-phase viscosity models are designed in such a way that they give good accuracy only if used with the homogeneous two-phase mixture density to calculate two-phase frictional pressure drop.

Since the hydrostatic pressure drop and frictional pressure drop are governed by two different mechanisms with different contributions to the total two-phase pressure drop at fixed flow conditions, it is desired to calculate these two components separately and then add them together to estimate the total two-phase pressure drop. To continue this discussion, the rest of this article presents discussion on the effect of void fraction on prediction of two-phase mixture viscosity and hence the hydrostatic pressure drop and the effect of two-phase mixture viscosity on prediction of frictional pressure drop.

The literature reports several slip ratio model and drift flux model-based correlations to predict void fraction and hence the mixture density. Most of the void fraction studies done so far recommend the best performing correlations based on their percentage accuracy, mean error, and standard deviation for the entire range of flow patterns. However, the recent studies of Godbole et al. [22], Ghajar and Tang [23], and Bhagwat and Ghajar [24] recommend the top performing models based on the performance of correlations in four different ranges of void fraction that approximate the flow patterns in vertical upward, downward, and horizontal pipe orientations. The scope of this study is to verify performance of void fraction correlations for bubbly and annular flow regimes using a qualitative method of plotting probability density function (PDF) that shows the under- or overprediction tendency of different correlations. The six top performing slip ratio void fraction correlations widely used in the refrigeration industry [5, 6, 25–28] and five drift flux model based correlations generally of interest to the chemical and nuclear industry [29–33] that are considered in this study are documented in Tables 1 and 2, respectively.

Another objective of this study is to examine performance of two-phase dynamic viscosity models for annular flow regimes. Fourteen two-phase dynamic viscosity models analyzed in this study are reported in Table 3 [8–16, 18, 34]. The two-phase frictional pressure drop calculated using two-phase dynamic viscosity models is compared against the measured pressure drop data in annular flow regime for different pipe diameters. The experimental database used for this comparison consists of 616 data points including data for 10 different pipe diameters as tabulated in Table 4 [35–44]. The predictions of two-phase frictional pressure drop correlations based on dynamic viscosity models are compared only against the data in the annular flow

Table 1 Void fraction correlations based on the concept of slip model

Author	Void fraction correlation
Chisholm [25]	$\alpha = (1 + \sqrt{1 - x(1 - \frac{\rho_l}{\rho_g}})(\frac{1-x}{x})(\frac{\rho_g}{\rho_l}))^{-1}$
Lockhart and Martinelli [26]	$\alpha = (1 + 0.28(\frac{1-x}{x})^{0.64}(\frac{\rho_g}{\rho_l})^{0.36}(\frac{\mu_l}{\mu_g})^{0.07})^{-1}$
Spedding and Chen [27]	$\alpha = (1 + 2.22(\frac{1-x}{x})^{0.65}(\frac{\rho_g}{\rho_l})^{0.65})^{-1}$
Smith [6]	$\alpha = (1 + (0.4 + 0.6\sqrt{[\frac{\rho_l}{\rho_g} + 0.4(\frac{1-x}{x})]/[1 + 0.4(\frac{1-x}{x})]}(\frac{1-x}{x})\frac{\rho_g}{\rho_l}))^{-1}$
Thom [28]	$\alpha = (1 + (\frac{1-x}{x})(\frac{\rho_g}{\rho_l})^{0.89}(\frac{\mu_l}{\mu_g})^{0.18})^{-1}$
Zivi [5]	$\alpha = (1 + (\frac{1-x}{x})(\frac{\rho_g}{\rho_l})^{0.67})^{-1}$

Table 2 Void fraction correlations based on the concept of drift flux model

Author	Distribution parameter (C_o)	Drift velocity (U_{gm})
Bhagwat and Ghajar [29]	$[\frac{1}{[1+\cos\theta]^{1.25}}]^{(1-\alpha)^{0.5}} + 0.18(\frac{U_{sl}}{U_{sl}+U_{sg}})^{0.1}$	$R(0.35\sin\theta + 0.54\cos\theta)\sqrt{(\frac{gD(\rho_l-\rho_g)}{\rho_l})}(1-\alpha)^{-0.5\sin\theta}$ Where, $R = (\frac{\mu_l}{\mu_w})^{-0.25}$
Gomez et al. [30]	1.15	$1.53(g\sigma(\frac{\rho_l-\rho_g}{\rho_l^2}))^{0.25}(1-\alpha)^{0.5}\sin\theta$
Hibiki and Ishii [31]	$1.2 - 0.2\sqrt{\frac{\rho_g}{\rho_l}}$	$1.41(\frac{g\sigma(\rho_l-\rho_g)}{\rho_l^2})^{0.25}(1-\alpha)^{1.75}$
Rouhani and Axelsson [32]	$1 + 0.2(1-x)(\frac{gD\rho_l^2}{G^2})^{0.25}$ $1 + 0.2(1-x)$	$1.18(g\sigma(\frac{\rho_l-\rho_g}{\rho_l^2}))^{0.25}$
Woldesemayat and Ghajar [33][a]	$\frac{U_{sg}}{U_{sl}+U_{sg}}[1 + (\frac{U_{sl}}{U_{sg}})^{(\rho_g/\rho_l)^{0.1}}]$	$2.9(\frac{gD\sigma(1+\cos\theta)(\rho_l-\rho_g)}{\rho_l^2})^{0.25}(1.22 + 1.22\sin\theta)^{\frac{P_{atm}}{P_{sys}}}$

[a] The leading constant 2.9 in the drift velocity equation carries a unit of $[m^{0.25}]$.

regime since all of these correlations predict data correctly in bubbly flow regime.

RESULTS AND DISCUSSION

Effect of Void Fraction on Prediction of Hydrostatic Pressure Drop

As mentioned earlier, the two-phase hydrostatic pressure drop depends upon the calculation of the two-phase mixture density, which in turn depends on the accurate prediction of void fraction. The two-phase mixture density is found to be sensitive to the error in prediction of void fraction. Moreover, this sensitivity of two-phase density and hence the hydrostatic pressure drop to the error in prediction of void fraction is different for different values of void fraction, and in comparison to other pipe orientations it is observed to magnify for vertical upward pipe orientation and large values of void fraction as shown in Figure 3. In vertical upward flow, for the small range of void fraction (bubbly flow), a 10% error in prediction of void fraction virtually does not affect the two-phase mixture density, whereas a 10% error in the high range of void fraction significantly alters the calculated two-phase mixture density and hence the hydrostatic pressure drop. The influence of error in void fraction on hydrostatic pressure drop may be significant for large-diameter pipes where the contribution of hydrostatic pressure drop to the total pressure drop is significant compared to small-diameter pipes.

The effect of void fraction on two-phase mixture density and hence the hydrostatic pressure drop in vertical upward bubbly flow regime for two different pipe diameters is shown in Figure 4. In Figures 4a and b, the error bands on hydrostatic pressure drop component are due to ±10% error in prediction of void fraction in bubbly flow for D = 12.5 mm and D = 44.5 mm,

Table 3 Two-phase mixture dynamic viscosity models

Number	Author	Expression for two-phase dynamic viscosity
1	Akers et al. [8]	$\mu_m = \frac{\mu_l}{(1-x)+x\sqrt{\rho_l/\rho_g}}$
2	Beattie and Whalley [9]	$\mu_m = \mu_l(1-\beta)(1+2.5\beta) + \mu_g\beta$
3	Ciccihitti et al. [10]	$\mu_m = x\mu_g + (1-x)\mu_1$
4	Davidson et al. [11]	$\mu_m = \mu_l(1 + x(\frac{\rho_l}{\rho_g} - 1))$
5	Dukler et al. [12]	$\mu_m = \rho_m(x(\frac{\mu_g}{\rho_g}) + (1-x)(\frac{\mu_l}{\rho_l}))$
6	Fourar and Boris [14]	$\mu_m = (1-\beta)\mu_l + \beta\mu_g + 2\sqrt{\beta(1-\beta)\mu_l\mu_g}$
7	Garcia et al. [15]	$\mu_m = \frac{\mu_l\rho_g}{x\rho_l+(1-x)\rho_g}$
8	Lin et al. [18]	$\mu_m = \frac{\mu_l\mu_g}{\mu_g+x^{1.4}(\mu_l-\mu_g)}$
9	McAdams et al. [13]	$\mu_m = (\frac{x}{\mu_g} + \frac{1-x}{\mu_l})^{-1}$
10	Oliemans [34]	$\mu_m = \frac{\mu_l(1-\beta)+\mu_g\alpha}{(1-\beta+\alpha)}$
11	Awad and Muzychka [16] model 1	$\mu_m = \mu_l\frac{2\mu_l+\mu_g-2(\mu_l-\mu_g)x}{2\mu_l+\mu_g+(\mu_l-\mu_g)x}$
12	Awad and Muzychka [16] model 2	$\mu_m = \mu_g\frac{2\mu_g+\mu_l-2(\mu_g-\mu_l)(1-x)}{2\mu_g+\mu_l+(\mu_g-\mu_l)(1-x)}$
13	Awad and Muzychka [16] model 3	Arithmetic mean of model 1 and model 2
14	Awad and Muzychka [16] model 4	$\mu_m = \frac{1}{4}\left(\begin{array}{c}(3x-1)\mu_g + [3(1-x)-1]\mu_l+\\ \sqrt{[(3x-1)\mu_g + (3\{1-x\}-1)\mu_l]^2 + 8\mu_g\mu_l}\end{array}\right)$

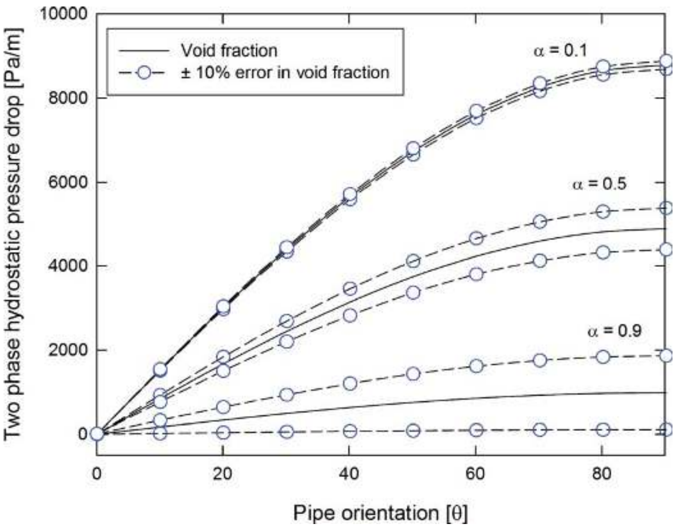


Figure 3 Effect of error in the prediction of void fraction on the estimation of two-phase hydrostatic pressure drop. (Color figure available online.)

respectively. However, in Figures 5a and b, the error bands on hydrostatic pressure drop component are for ±10% error in prediction of the void fraction in the annular flow regime for the aforementioned pipe diameters. The error bars represent actual deviation and not the percentage error between the measured and predicted values of hydrostatic pressure drop. It is evident that the error in void fraction virtually does not affect the hydrostatic pressure drop in bubbly flow regime, whereas the error in void fraction has a significant impact on hydrostatic pressure drop and is pronounced for large-diameter pipes in an annular flow regime. This implies that for a fixed pipe diameter and void fraction, a 10% error in prediction of void fraction in bubbly region gives a small deviation in measured and predicted values of hydrostatic pressure drop compared to that in the annular flow regime. Thus, while analyzing the void fraction correlations for their accuracy, relaxed performance criteria may be considered acceptable for bubbly flow (low region of the void fraction) while stringent criteria may be imposed for annular flow regime (high region of the void fraction).

Performance of Void Fraction Correlations in Prediction of Two-Phase Mixture Density

Once the dependency and sensitivity of two-phase mixture density and hence the hydrostatic pressure drop on void fraction are established, it is important to analyze performance of different void fraction correlations available in the literature and their accuracy in prediction of two-phase density. The experimental database of void fraction used in this study to compare different void fraction correlations is reported in Godbole et al. [22]. The qualitative and quantitative performance of different slip ratio models and DFM-based correlations is presented in the following paragraphs. The qualitative performance analysis is carried out in terms of PDF (probability density function) to get an idea of over- or underprediction tendency of these correlations. The quantitative performance analysis of slip ratio models and DFM-based correlations in prediction of void fraction and hence the two-phase mixture density for bubbly and annular flow regimes is documented in Tables 5 and 6, respectively. The quantitative performance analysis presented in the form of percentage error is based upon direct comparison of the predicted values with the measured data (296 for bubbly flow and 476 for annular flow) reported by Godbole et al. [22]. Figures 6 and 7 show the probability density functions of different slip ratio model and DFM-based void fraction correlations for bubbly flow, respectively. It is evident that the DFM correlations perform better than slip ratio model-based correlations in prediction of void fraction. All of the slip ratio model-based correlations with the exception of Thom [28] tend to significantly overpredict the void fraction in bubbly flow regime. Among DFM correlations, Bhagwat and Ghajar [29], Gomez et al. [30], Hibiki and Ishii [31], and Rouhani and Axelsson [32] give comparable performance, while that of Woldesemayat and Ghajar [33] tends to overpredict the data. The effect of prediction of slip ratio model- and DFM-based void fraction correlations on the two-phase mixture density in bubbly flow regime is shown in Figures 8 and 9, respectively. The performance of Zivi [5] in prediction of low values of void fraction (bubbly flow) is observed to be

Table 4 Experimental data used to analyze two-phase dynamic viscosity models

Source	D (mm)	Number of data points^{a}	Void fraction range	L/D	Fluid combination
MacGillivray [35]	9.5	113 (R)	0.87–0.94	76	Air–water
Aggour [36]	11.7	22 (R)	0.77–0.93	130	Helium–water
Vijay [37]	11.7	20 (R)	0.76–0.89	130	Air–water
Sujumnong [38]	11.7	45 (R)	0.79–0.93	130	Air–water Air–glycerol + water
Tang et al. [44]	12.52	32 (S), 34 (R)	0.76–0.98	100	Air–water
Chiang [39]	12.7	94 (S)	0.77–0.89	205	Air–water
	15.7	32 (S)		184	
Asali [40]	22.9	30 (S)	0.92–0.98	—	Air–water
	42	47 (S)			Air–glycerol + water
Oshinowo [41]	25.4	26 (S)	0.82–0.96	100	Air–water
Nguyen [42]	44.5	81 (S)	0.81–0.98	40	Air–water
Belt et al. [43]	50	40 (S)	0.92 –0.98	170	Air–water

[a] (S) and (R) represent smooth and rough pipes, respectively.

Table 5 Quantitative performance of slip model based correlations in prediction of void fraction and mixture density

	Void fraction				Two-phase density			
	±10% Error bands		±20% Error bands		±20% Error bands		±30% Error bands	
Correlation	Bubbly	Annular	Bubbly	Annular	Bubbly	Annular	Bubbly	Annular
Chisholm [25]	22.6	86.3	39.8	98.5	86.8	31.4	93	45.6
Lockhart and Martinelli [26]	29.72	85.2	56	97.4	92.5	36.8	98	53.5
Spedding and Chen [27]	34.5	81.6	58.1	97.4	93.9	36.8	97.3	52.9
Smith [6]	28.7	82.4	48.3	97.8	89.2	29.1	93.9	39.6
Thom [28]	3.7	63.3	6.75	95.07	93.9	10.5	98.3	17.8
Zivi [5]	0	58.2	0	95.5	70.6	8.9	86.4	15.2

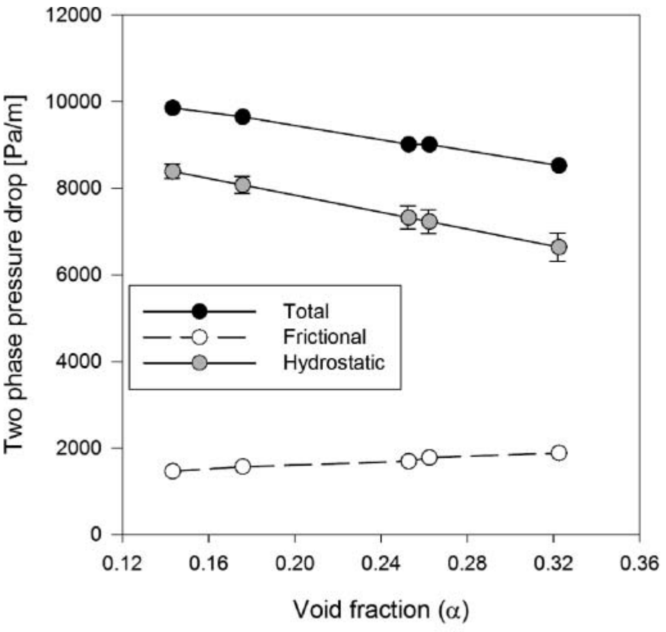
drastically poor and hence its PDF is not included in Figures 6 and 8.

Further analysis of performance of void fraction correlations in the annular flow regime using PDFs is as shown in Figures 10 and 11. As shown in Figure 10, the slip-ratio-based correlations of Lockhart and Martinelli [26], Smith [6], and Chisholm [25] predict majority of the data within ±20% error bands, while the correlation of Zivi [5] tends to overpredict the data. The performance of DFM correlations illustrated in Figure 11 shows that the correlations by Woldesemayat and Ghajar [33] and Bhagwat and Ghajar [29] give comparable performance, while the least accuracy is given by the Rouhani and Axelsson [32] correlation. In fact, the correlations of Woldesemayat and Ghajar [33] and Bhagwat and Ghajar [29] predict more than 90% of data points within ±10% error bands. The effect of accuracy of these slip ratio model- and DFM-based correlations on two-phase mixture density in annular flow regime is illustrated in Figures 12 and 13, respectively. Based on the PDF values and the shape and distribution of the curves, it is clear that the DFM-based void fraction correlations perform better than the slip ratio model-based correlations in prediction of two-phase mixture density for annular flow regime. In this flow regime, the correlations of Woldesemayat and Ghajar [33] and Bhagwat and Ghajar [29] give performance superior to all other DFM-based correlations considered in this study. It should be noted that the performance of different correlations in terms of their PDF distribution is a statistical tool inclined more toward a qualitative comparison and hence gives a qualitative guess of comparison among different void fraction correlations.

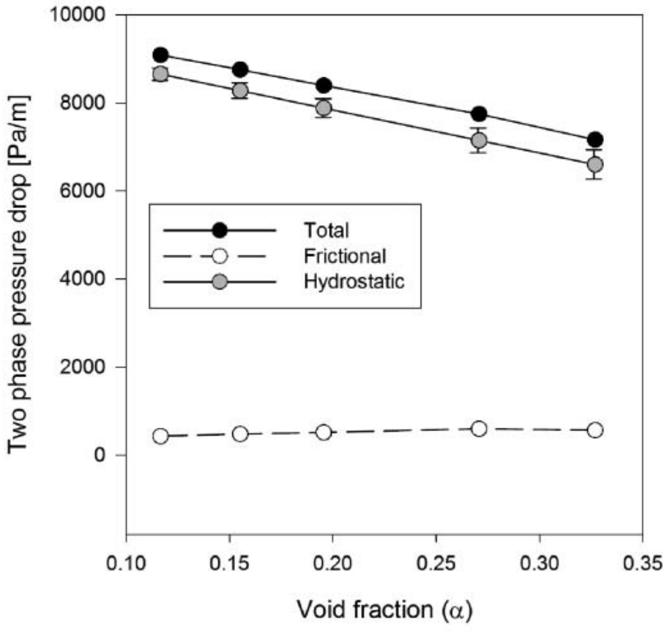
Overall, in comparison to DFM correlations the slip ratio model-based correlations are found to perform poorly in prediction of void fraction in bubbly flow regime. The maximum accuracy is given by Lockhart and Martinelli [26] and Spedding and Chen [27] and predicts only more than 55% of data points within ±20% error bands whereas the top performing DFM correlations such as by Bhagwat and Ghajar [29] and Rouhani and Axelsson [32] predict more than 80% of data points within this performance criterion. However, it should be noted that in spite of poor performance of the slip ratio model-based void fraction correlations in bubbly flow regime, the quantitative performance of both slip ratio model and DFM void fraction correlations is comparable in prediction of two-phase mixture density. This justifies that the two-phase mixture density is not a strong function of void fraction in bubbly flow regime. Although these correlations predict more than 90% of two-phase mixture density data within ±30% error bands, for the small region of the void fraction (bubbly flow) ±30% error in mixture density translates to ±30% error in hydrostatic pressure drop. As mentioned earlier, the total two-phase pressure drop for nonboiling two-phase flow is composed of hydrostatic and frictional pressure drops and hence is biased to inaccuracies induced in both of these pressure drop components. Thus, for accurate estimation of total two-phase pressure drop it is always desired to keep the error in two-phase mixture density and hence hydrostatic pressure drop and frictional pressure drop to a minimum. From this accuracy standpoint, further performance analysis of drift flux model-based correlations for bubbly flow regimes shows that the correlation of Woldesemayat and Ghajar [33] can predict only

Table 6 Quantitative performance of DFM correlations in prediction of void fraction and mixture density

	Void fraction				Two-phase density			
	±10% Error bands		±20% Error bands		±20% Error bands		±30% Error bands	
Correlation	Bubbly	Annular	Bubbly	Annular	Bubbly	Annular	Bubbly	Annular
Bhagwat and Ghajar [29]	49.6	95.2	80	99.5	99.3	41.5	99.6	55.8
Gomez et al. [30]	46.2	85.2	74.3	100	98.6	38.7	99.3	54.6
Hibiki and Ishii [31]	37.8	74.9	66.5	100	95.2	38.8	97.9	49.8
Rouhani and Axelsson [32]	53.3	44.1	81	86.7	99.3	19.9	99.6	25.4
Woldesemayat and Ghajar [33]	19.2	93.14	33.1	99.8	98.3	40.8	99.6	64.2

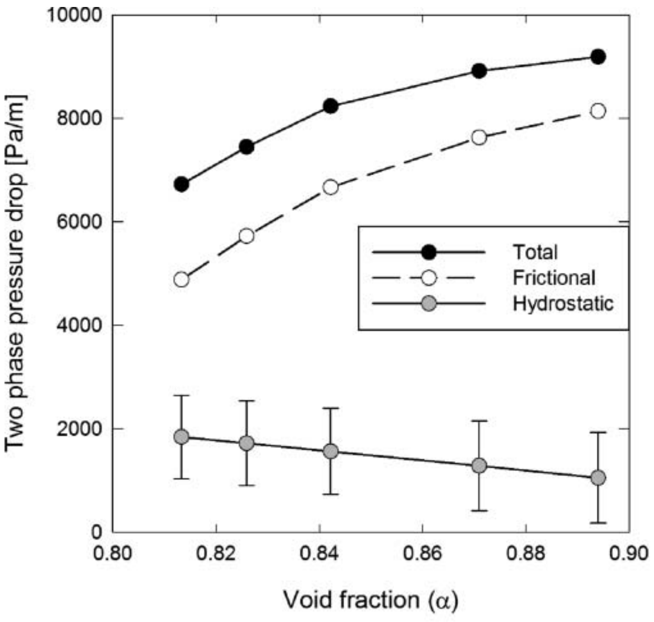


(a) Bubbly flow (D = 12.5 mm)

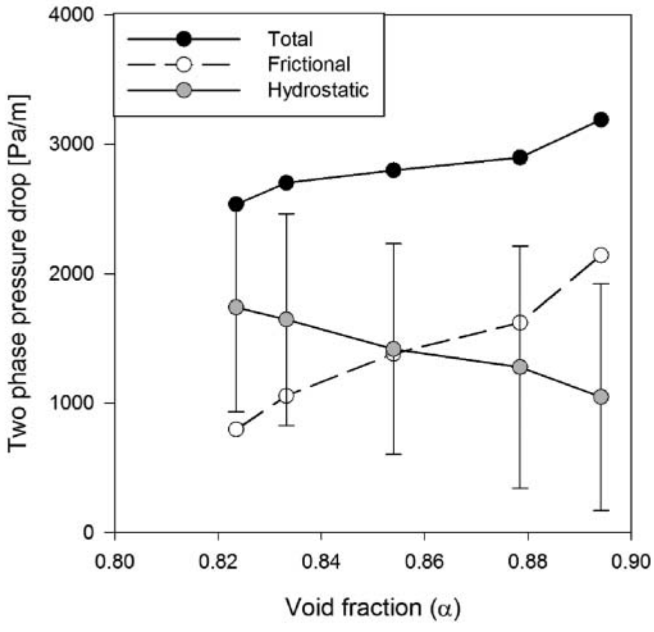


(b) Bubbly flow (D = 44.5 mm)

Figure 4 Contribution of hydrostatic and frictional component of pressure drop to the total two-phase pressure drop for low region of void fraction.



(a) Annular flow (D = 12.5 mm)



(b) Annular flow (D = 44.5 mm)

Figure 5 Contribution of hydrostatic and frictional component of pressure drop to the total two-phase pressure drop for high region of void fraction.

43.6% and 9.8% of data points of two-phase mixture density within ±10% and ±5% error bands, respectively. However, the correlation of Bhagwat and Ghajar [29] can predict 98.6% and 90.5% of two-phase mixture density data points within ±10% and ±5% error bands, respectively.

For the annular flow regime, although the majority of correlations predict more than 95% of void fraction data within ±20% error bands, these correlations perform poorly in prediction of two-phase mixture density. The best accuracy is displayed by Woldesemayat and Ghajar [33] by predicting 64.2% of two-phase mixture density data points within ±30% error bands criterion.

Overall, due to superior performance of the DFM void fraction correlations it is recommended to use the correlation of Bhagwat and Ghajar [29] for the low region of void fraction (bubbly flow), while for the high region of void fraction

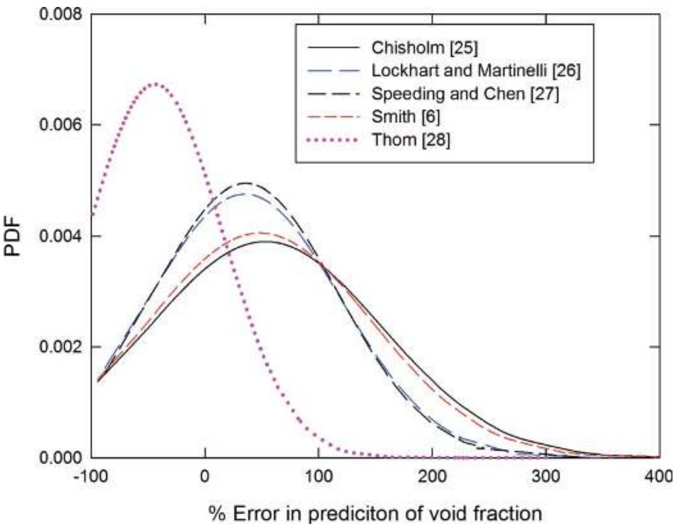


Figure 6 PDF of slip ratio model-based correlations in prediction of void fraction in bubbly flow regime. (Color figure available online.)

(annular flow), the correlation of Woldesemayat and Ghajar [33] may be preferred.

Effect of Two-Phase Dynamic Viscosity Models on Prediction of Two-Phase Frictional Pressure Drop

The two-phase frictional pressure drop correlations based on two-phase dynamic viscosity models predict the measured frictional pressure drop correctly for bubbly flow regime and hence their performance for bubbly flow is not reported in this study. However, the performance of these models in predicting frictional pressure drop data in annular flow regime needs to be investigated. The following section deals with the performance analysis of two-phase dynamic viscosity models in prediction of two-phase frictional pressure drop. Based on the overall performance of two-phase dynamic viscosity models it is decided to analyze their accuracy based on the percentage of data predicted within ±30% error bands as shown in Table 7. A similar type of performance criteria had been used by Awad and Muzychka [16] and Dalkilic et al. [45]. Dalkilic et al. [45] carried out a similar type of study to investigate the different two-phase viscosity models available in the literature and compared their performance against the experimental data of R134 refrigerant undergoing condensation in vertical downward pipe. They found that the correlations of Fourar and Boris [14] and Davidson et al. [11] showed consistent discrepancy with the measured data, whereas the correlations of Dukler et al. [12], Lin et al. [18], Garcia et al. [15], and McAdams et al. [13] predicted the measured two-phase friction factor within ±30% error bands. They used the friction factor equation of

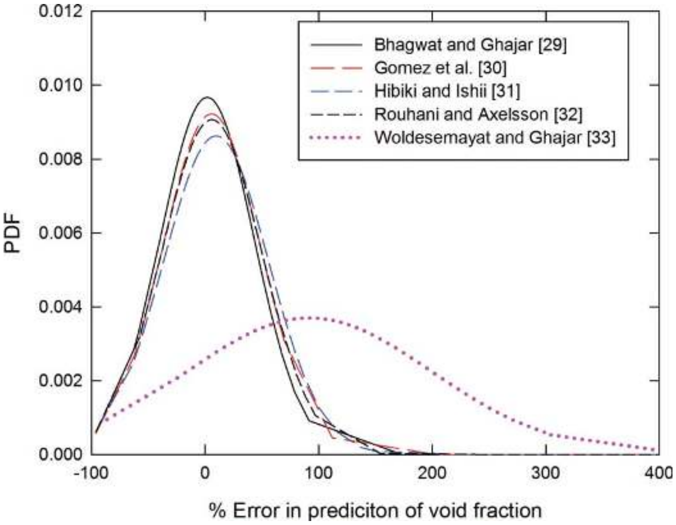


Figure 7 PDF of DFM-based correlations in prediction of void fraction in bubbly flow regime. (Color figure available online.)

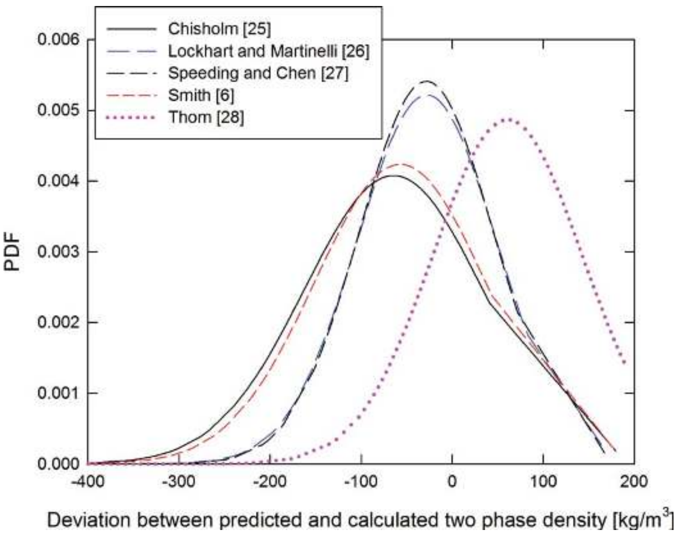


Figure 8 PDF of slip ratio model-based void fraction correlations in prediction of two-phase mixture density for bubbly flow (low region of void fraction). (Color figure available online.)

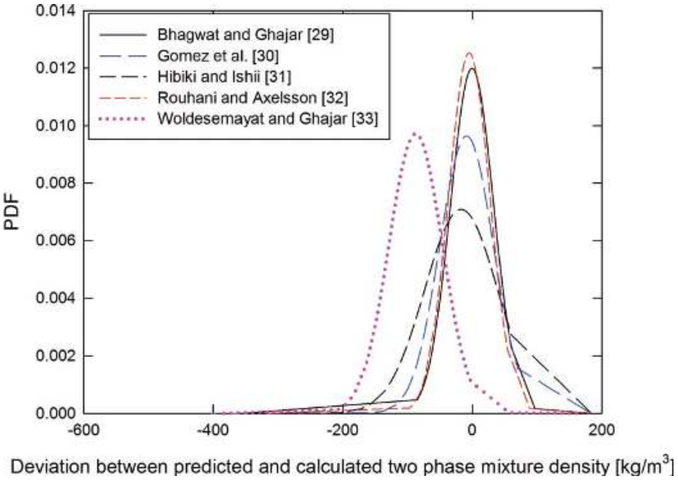


Figure 9 PDF of DFM-based void fraction correlations in prediction of two-phase mixture density for bubbly flow (low region of void fraction). (Color figure available online.)

Table 7 Quantitative performance of different two-phase dynamic viscosity models in prediction of two-phase frictional pressure drop

Data set, pipe diameter (S = smooth pipe, R = rough pipe)	Two-phase dynamic viscosity models[a]													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Percentage of data points within ±30% error bands													
MacGillivray [35], D = 9.5 mm (R)	93.8	99.1	81.4	62.8	91.2	100.0	58.4	100.0	100.0	91.2	85.0	100.0	40.7	85.8
Aggour [36] (R), Vijay [37] (R) and Sujumnong [38], D = 11.7 mm (R)	66.7	76.7	53.3	55.6	56.7	73.3	44.4	76.7	75.6	60.0	55.6	76.7	68.9	57.8
Tang et al. [44], D = 12.5 mm (S)	59.4	96.9	3.1	0.0	96.9	96.9	71.9	15.6	50.0	96.9	3.1	15.6	0.0	3.1
OSU, D = 12.5 mm[b] (R)	94.1	61.8	100	88.2	61.8	82.4	61.8	55.9	82.4	88.2	82.4	94.1	100	82.4
Chiang [39], D = 12.7 mm (S)	4.2	5.3	67.4	62.1	2.1	11.6	0.0	53.7	29.5	2.1	73.7	80.0	84.2	76.8
Chiang [39], D = 15.8 mm (S)	0.0	0.0	87.5	78.1	0.0	0.0	0.0	28.1	9.4	0.0	90.6	40.6	90.6	87.5
Asali [40], D = 22.9 mm (S)	1.4	12.5	9.7	6.9	8.3	20.8	0.0	34.7	27.8	8.3	18.1	41.7	33.3	30.6
Oshinowo [41], D = 25.4 mm (S)	80.8	88.5	0.0	3.8	96.2	84.6	65.4	38.5	69.2	96.2	0.0	26.9	0.0	0.0
Asali [40], D = 42 mm (S)	0.0	0.0	91.5	59.6	0.0	0.0	0.0	2.1	2.1	0.0	89.4	6.4	70.2	44.7
Nguyen [42], D = 44.5 mm (S)	43.2	34.6	55.6	44.4	22.2	44.4	9.9	69.1	63.0	23.5	58.0	72.8	65.4	58.0
Belt et al. [43], D = 50 mm (S)	0.0	0.0	100.0	67.5	0.0	0.0	0.0	0.0	0.0	0.0	97.5	0.0	75.0	55.0
Entire data	40.7	44.6	57.6	47.5	38.8	47.2	27.0	53.9	51.2	39.3	60.4	59.3	56.9	57.3

[a] Two-phase dynamic viscosity models are in the same order as numbered in Table 3.

[b] Pressure drop data measured at Oklahoma State University Two-Phase Flow Laboratory in 12.5-mm schedule 10 S stainless-steel pipe.

Blasius [19] to calculate the two-phase friction factor for the experimental data measured in a smooth copper pipe.

As shown in Table 7, for the two-phase frictional pressure drop data of MacGillivray [35] measured in a 9.5-mm-diameter pipe, the correlations of Awad and Muzychka [16] model 2, Fourar and Boris [14], Lin et al. [18], and McAdams et al. [13] predict 100% of data points within ±20% error bands. In particular, Lin et al. [18] and McAdams et al. [13] predict more than 80% and 90% of data points for more restricted error bands of ±15% and ±20%, respectively. The best performance for the pressure drop data of Aggour [36], Vijay [37], and Sujumnong [38] (D = 11.7 mm) is shown by Awad and Muzychka [16] model 2, Beattie and Whalley [9], Lin et al. [18], and McAdams et al. [13]. All these correlations are found to predict more than 75% of data points within ±30% error bands. In case of pressure drop data in smooth and rough pipes with a slightly bigger pipe diameter of 12.5 mm, Beattie and Whalley [9], Dukler et al. [12], Fourar and Boris [14], and Oliemans [34] predict more than 90% and 80% of data points within ±30% error bands for smooth and rough pipes, respectively. In comparison to the performance for smooth pipes, the accuracy of Awad and Muzychka [16] model 2 and McAdams et al. [13] is observed to improve drastically for the flow through rough pipes. Both of these correlations are able to predict 100% of data points within ±30% error bands.

For the pressure drop data of Chiang [39] measured in 12.7-mm pipe diameter, Awad and Muzychka [16] model 2 and model 3 give best performance by predicting more than 80% of data points within ±30% error bands, whereas for flow through

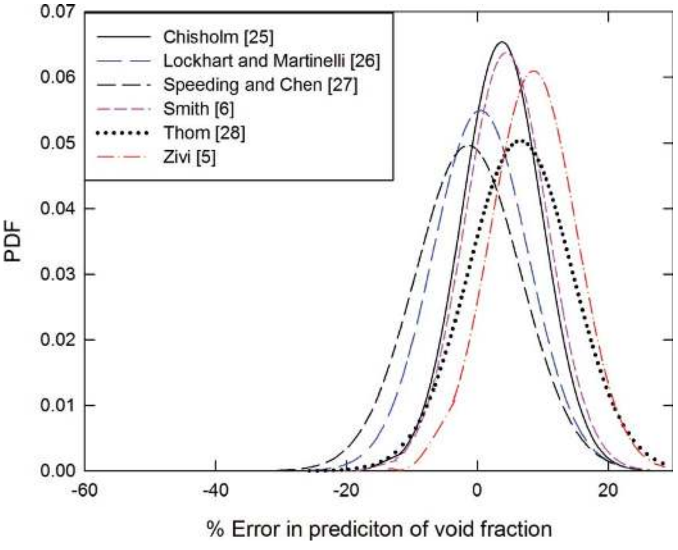


Figure 10 PDF of slip ratio model-based correlations in prediction of void fraction in annular flow regime. (Color figure available online.)

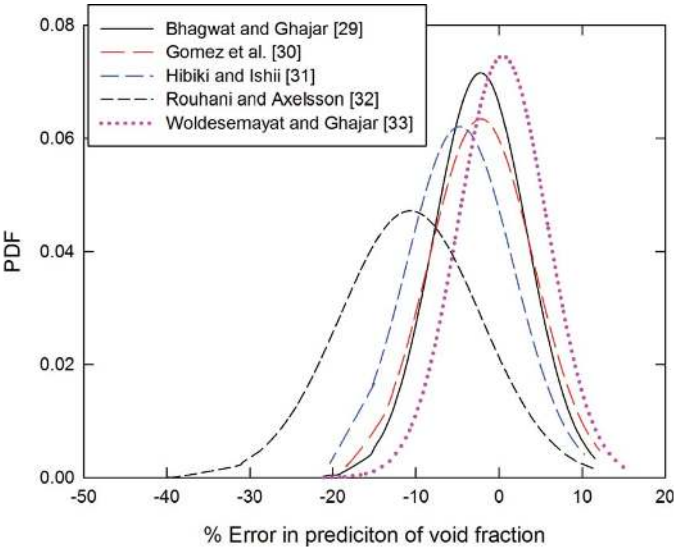


Figure 11 PDF of DFM model-based correlations in prediction of void fraction in annular flow regime. (Color figure available online.)

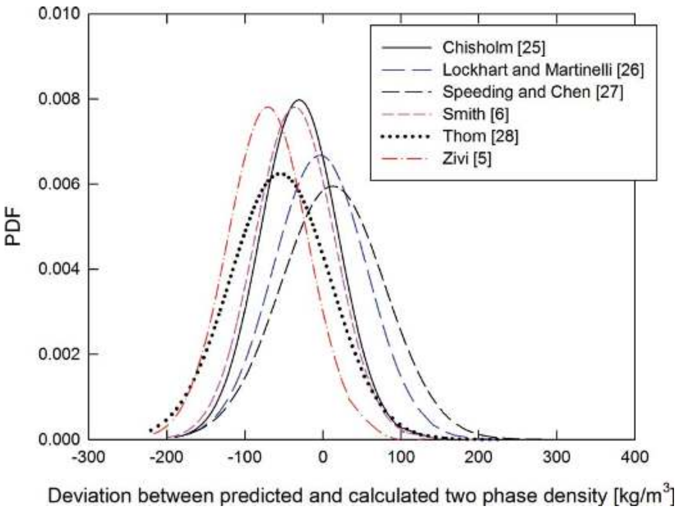


Figure 12 PDF of slip ratio model-based void fraction correlations in prediction of two-phase mixture density for annular flow (high region of void fraction). (Color figure available online.)

15.8-mm-diameter pipe, Awad and Muzychka [16] model 1 and model 3 predict more than 90% of data points within ±30% error bands. For the pressure drop data of Oshinowo [41] through a D = 25.4 mm smooth pipe, the best performance is given by Dukler et al. [12] and Oliemans [34] by predicting more than 95% of data points within ±30% error bands. This is followed by the performances of Beattie and Whalley [9] and Fourar and Boris [14] that predict more than 85% and 80% of data points for error criteria similar to that already mentioned. The pressure drop data of Asali [40], Nguyen [42], and Belt et al. [43] is for large pipe diameters compared to other experimental data used in this study. The correlations of Ciccihitti et al. [10] and Awad and Muzychka [16] model 1 predict 91% and 100% of data and 89.4% and 97.5% of data within ±30% error bands for D = 42 mm and D = 50 mm, respectively. Although, the data of Nguyen [42] are for a pipe diameter similar to that of Asali [40], the correlations of Ciccihitti et al. [10] and Awad and Muzychka [16] model 1 can predict only 56% and 58% of data points within ±30% error bands. A plausible reason for the inability of these correlations to predict the data of Nguyen [42] is that this experimental frictional pressure drop data is collected at a pipe axial location of L/D = 40. Literature review shows that pressure drop data collected for small L/D ratios may not represent the actual pressure gradient, since for this L/D = 40 pipe axial distance, the two phases may not be completely aligned with each other and their distribution with respect to each other and hence the pressure gradient may change at downstream of inlet. More details about the change in pressure gradient with respect to the nondimensional pipe axial length (L/D) are reported by Wolf et al. [46].

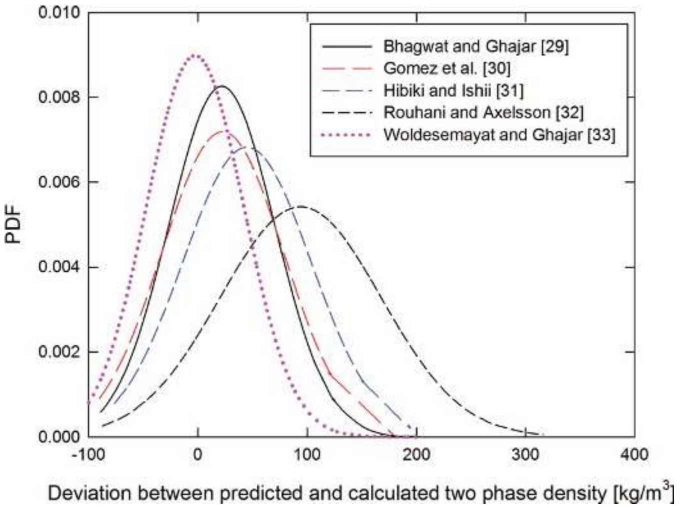


Figure 13 PDF of DFM based void fraction correlations in prediction of two-phase mixture density for annular flow (high region of void fraction). (Color figure available online.)

Overall, performance analysis of two-phase dynamic viscosity models shows that they underpredict the pressure drop data for large pipe diameters of Asali [40] and Belt et al. [43]. The probable reason for the correlations to underpredict the large pipe diameters is that, in addition to friction at the pipe wall, friction at the gas–liquid interface contributes significantly to the total frictional pressure drop and the two-phase viscosity models do not account for the increase in interfacial friction with increase in the pipe diameter. In addition to the correlation of Awad and Muzychka [16] model 1, the correlations of Ciccihitti et al. [10] and Davidson et al. [11] that predict the frictional pressure drop for large diameter with good accuracy essentially tend to overpredict the frictional pressure drop for comparatively smaller pipe diameters at given flow conditions and hence probably account for the added effect of interfacial friction in large diameter pipes. The overprediction tendency of Ciccihitti et al. [10] and comparative underprediction trend of Fourar and Boris [14] and McAdams et al. [13] are clear from the general shift in the predicted data shown in Figures 14–16, respectively.

It should be noted that in comparison to the flow through smooth pipes, the majority of the correlations perform better

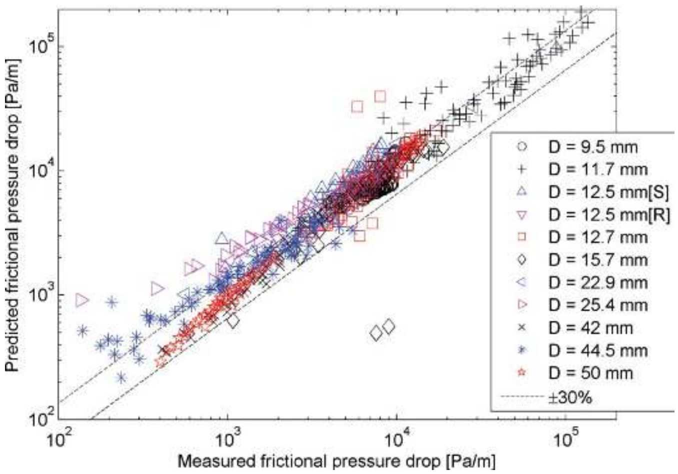


Figure 14 Performance of Ciccihitti et al. [10] in prediction of two-phase frictional pressure drop for data reported in Table 4 ([S] = smooth, [R] = rough). (Color figure available online.)

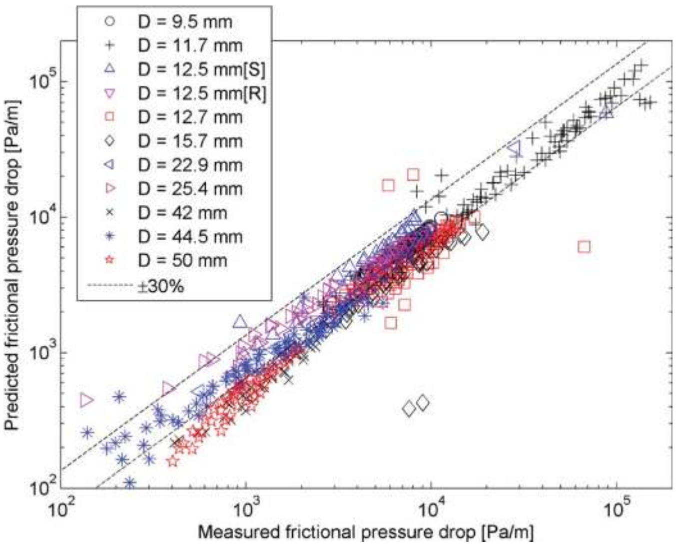


Figure 15 Performance of Fourar and Boris [14] in prediction of two-phase frictional pressure drop for data reported in Table 4 ([S] = smooth, [R] = rough). (Color figure available online.)

against the data for rough pipes. This is possibly due to the fact that these correlations tend to overpredict the data for smooth pipes and for similar mass flow rates; as reported by Shannak [47], the two-phase frictional pressure drop in rough pipes for annular flow regime is greater than that in smooth pipes and consequently these correlations perform well for two-phase flow through rough pipes.

It is also of interest to see how different two-phase viscosity models proposed by Awad and Muzychka [16] perform with respect to each other. As tabulated in Table 3, model 1 and model 2 are based on the liquid and gas viscosity definitions, respectively while model 3 is the arithmetic mean of first two

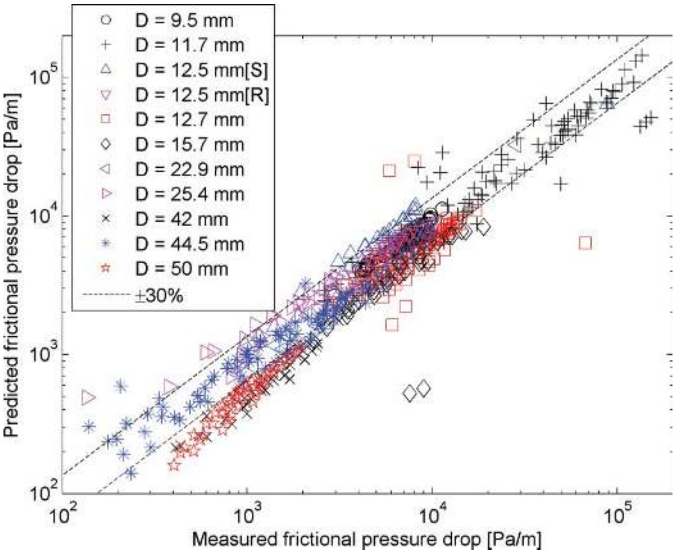


Figure 16 Performance of McAdams et al. [13] in prediction of two-phase frictional pressure drop for data reported in Table 4 ([S] = smooth, [R] = rough). (Color figure available online.)

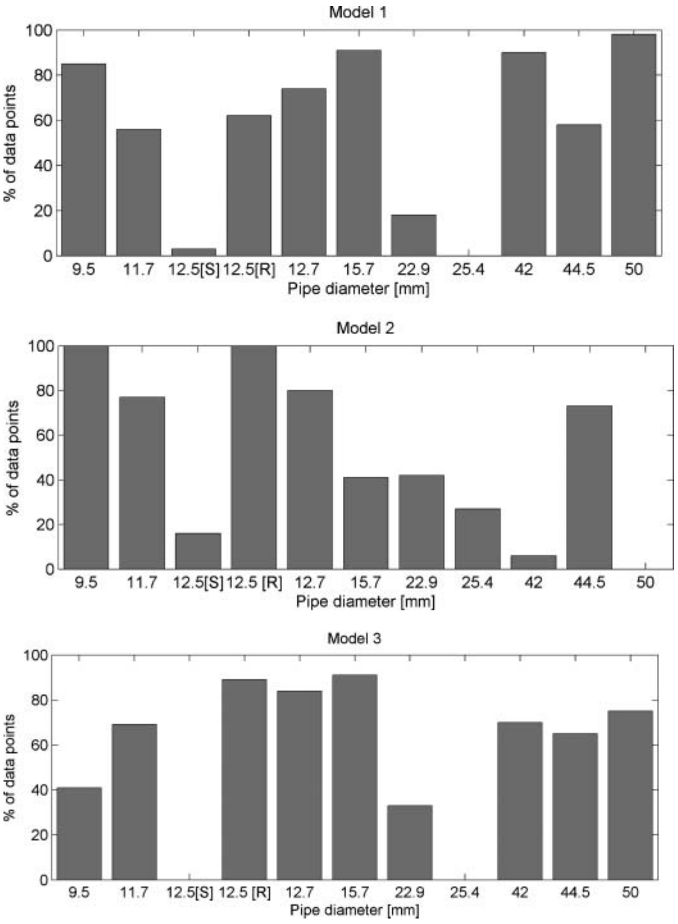


Figure 17 Performance of Awad and Muzychka [16] models 1, 2, and 3 in prediction of two-phase frictional pressure drop for data reported in Table 4 ([S] = smooth, [R] = rough).

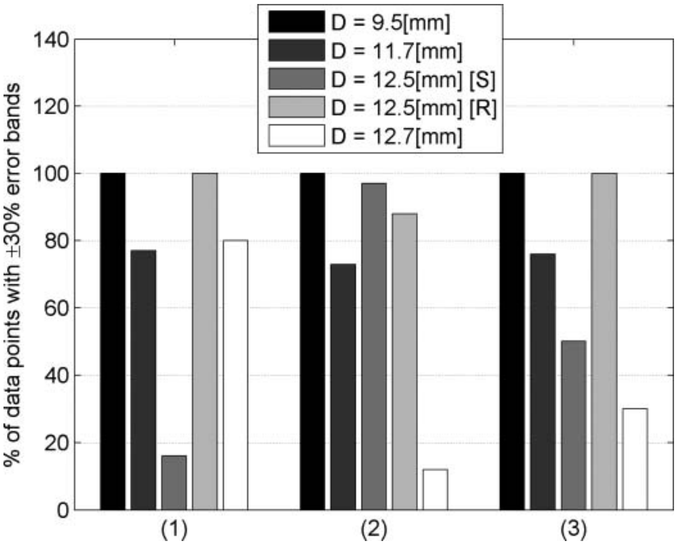


Figure 18 Top performing two-phase dynamic viscosity models for (D ≤ 12.7 mm); [S] = smooth, [R] = rough. (1) Awad and Muzychka [16] model 2, (2) Fourar and Boris [14], and (3) McAdams et al. [13].

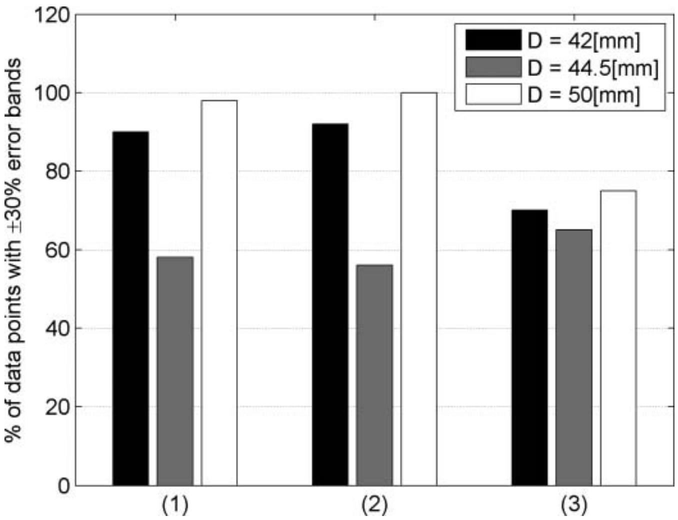


Figure 19 Top performing two-phase dynamic viscosity models for ($D > 40$ mm). (1) Awad and Muzychka [16] model 1, (2) Ciccihitti et al. [10], and (3) Davidson et al. [11].

models. The comparison between these three models is shown in Figure 17. For the overall experimental data used in this study, it is also observed that Awad and Muzychka [16] model 1 based on liquid-phase viscosity performs well for small pipe diameters up to $D = 12.7$ mm, whereas for large pipe diameters, $D > 40$ mm, the Awad and Muzychka [16] model 2 based on the gas-phase viscosity gives good accuracy. In addition to these correlations, Beattie and Whalley [9], Dukler et al. [12], Fourar and Boris [14], and McAdams et al. [13] are among the top performing correlations for pipe diameters less than 15.7 mm. The performance of the top three performing correlations (1) Awad and Muzychka [16] model 2, (2) Fourar and Boris [14], and (3) McAdams et al. [13], for small pipe diameters in the range already mentioned is shown in Figure 18. The top three performing correlations for large diameter pipes are (1) Awad and Muzychka [16] model 1, (2) Ciccihitti et al. [10], and (3) Davidson et al. [11], and their performance is presented qualitatively in Figure 19. Thus, it may be concluded that the correlations successful for small pipe diameters may fail for large pipe diameters. Moreover, the correlations may perform differently for smooth and rough pipe conditions. It is also expected that these two-phase dynamic viscosity models may fail for inclined two-phase flow where both phases may be in contact with pipe wall, contributing to two-phase frictional pressure drop. Hence it is recommended that before adopting these models to predict two-phase pressure drop, their accuracy against a data set consisting of a range of pipe diameters, surface roughness, fluid combinations, and pipe orientations should be verified.

CONCLUSIONS

The present study is divided into two sections on the effect of void fraction in prediction of two-phase mixture density and hence the hydrostatic pressure drop and the effect of two-phase dynamic viscosity models on prediction of two-phase frictional pressure drop. The conclusions and recommendations of this study are outlined next.

It is shown in this study that the two-phase mixture density and hence the two-phase hydrostatic pressure drop component are sensitive to accurate prediction of the void fraction. It is observed that a ±10% error in prediction of void fraction may have a significant effect in prediction of hydrostatic pressure drop and hence calculation of total two-phase pressure drop for large pipe diameters in annular flow regime. Further, the quantitative and qualitative analysis of top performing correlations based on a slip and drift flux model show that the drift flux model-based correlations have better accuracy in prediction of void fraction and hence two-phase mixture density in both extremes of two-phase flow conditions, namely, bubbly and annular flow regimes. Based on the accuracy of these correlations to predict void fraction and most importantly the two-phase mixture density, it is recommended to use correlations of Bhagwat and Ghajar [29] and Woldesemayat and Ghajar [33] for low and high values of void fraction (bubbly and annular flow regimes), respectively.

The performance analysis of two-phase dynamic viscosity models in prediction of two-phase frictional pressure drop shows that none of these models can predict two-phase frictional pressure drop accurately for a range of pipe diameters. However, it is found that the correlation of Awad and Muzychka [16] model 2 gives the best performance for small pipe diameters, typically $D < 15$ mm, while for large pipe diameters ($D > 40$ mm), Awad and Muzychka [16] model 1 gives the best performance among all the two-phase dynamic viscosity models analyzed in this study. Based on the formulation of two-phase dynamic viscosity models and their overall performance for different data sets, it is conjectured that the frictional pressure drop correlations based on the concept of the two-phase dynamic viscosity model may fail to predict the frictional pressure drop in the case where two-phase flow is in large-diameter pipes and interfacial friction is a major contributor to frictional pressure drop, and in another case of two-phase flow through inclined systems where the frictional pressure drop may be due to simultaneous flow and hence friction of both gas and liquid phase at the pipe wall. The general structure of two-phase dynamic viscosity models doesn't show any evidence for their ability to account for interfacial friction in large-diameter pipes and simultaneous contact of two phases with pipe wall in inclined systems. Thus, it is necessary to verify performance of these models in prediction of frictional pressure drop in annular two-phase flow scenarios involving large pipe diameter and inclined systems.

NOMENCLATURE

A	variable in Churchill [21] equation, Eq. (9)
B	variable in Churchill [21] equation, Eq. (9)
C_o	distribution parameter
D	pipe diameter, m

DFM	drift flux model
f	friction factor
g	acceleration due to gravity, m/s^2
G	two-phase mixture mass flux, $(G = G_l + G_g)$, kg/m^2-s
L	pipe length, m
P	pressure, Pa
PDF	probability density function
Re	Reynolds number
S	slip ratio
U	phase velocity, m/s
U_{gm}	drift velocity, m/s
x	quality

Greek Symbols

α	void fraction
β	gas volumetric flow fraction
ρ	phase density, kg/m^3
μ	phase dynamic viscosity, Pa-s
σ	surface tension, N/m
ε	surface roughness, m
θ	pipe orientation (inclination angle), degrees

Subscripts

a	acceleration
atm	atmospheric
eq	equivalent
f	friction
g	gas phase
h	hydrostatic
i	pipe inlet
l	liquid phase
m	mixture
o	pipe outlet
s	superficial
sys	system
t	total
TP	two phase
w	water

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