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Effective anisotropic elastic constants for wave propagation through cracked solids

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Summary. Theoretical developments of Hudson demonstrate how to calculate the variations of velocity and attenuation of seismic waves propagating through solids containing aligned cracks. The analysis can handle a wide variety of crack configurations and crack geometries. Hudson associates the velocity variations with effective elastic constants. In this paper we associate the variation of attenuation with the imaginary parts of complex effective elastic constants. These complex elastic constants permit the simulation of wave propagation through two-phase materials by the calculation of wave propagation through homogeneous anisotropic solids.

1 Introduction

Wave propagation in material containing a uniform weak concentration of aligned cracks can be simulated under a wide range of conditions by wave propagation in a purely elastic anisotropic solid which has the same velocity variations with direction as the cracked solid (Crampin 1978). A key step in this procedure is the use of effective elastic constants for the cracked material so that wave propagation can be calculated by the range of computer programs for seismic anisotropy reviewed by Crampin (1981). Crampin (1978) obtained effective elastic constants by modelling the variation of wave velocity through a cracked solid derived in a first-order approximation theory by Garbin & Knopoff (1973, 1975a, b). Hudson has now developed a more general theoretical approach for calculating the elastic constants of cracked solids that includes first-order (Hudson 1981) and second-order (Hudson 1982) interactions between the scattering inclusions. These methods include the results of Garbin & Knopoff and other authors for specific crack geometries.

Hudson's developments, including the correction of a minor copying error in Garbin & Knopoff (1975a), allow a more systematic treatment for calculating elastic constants than was available to Crampin (1978). In particular, Hudson (1981) develops techniques for modelling attenuation in cracked solids. The most convenient formulation for anisotropic attenuation is to represent the attenuation parameter $1/Q$ as the eigenvalue of a matrix of the imaginary parts of complex elastic constants (Crampin 1981), where the real parts model the purely elastic behaviour. This then allows attenuative wave-propagation to be calculated by the same range of anisotropy programs as the purely elastic behaviour (Crampin 1981).

In this paper, we outline the procedure and calculate some complex elastic constants for two representative systems of cracks.

2 Theoretical formulations for velocity variations

We consider a weak distribution of parallel penny-shaped cracks, normal to the x_1-direction, with crack density $\epsilon = Na^3/v$ ($\epsilon \ll 1$), where N is the number of cracks of radius a in volume v in an isotropic solid with Lame constants λ and μ. Hudson (1981, 1982) showed that the general expression for effective elastic constants c_{jkmn} applicable to the propagation of long-wavelength seismic waves through a cracked solid is:

$$c_{jkmn} = c^0_{jkmn} + c^1_{jkmn} + c^2_{jkmn}; \tag{1}$$

where c^1_{jkmn} is the first-order and c^2_{jkmn} the second-order perturbation of the isotropic (or anisotropic) elastic constants c^0_{jkmn} of the uncracked solid. The effective first-order perturbation for cracks normal to the x_1-axis in an isotropic solid can be written (Hudson 1981):

$$\{c^1_{jkmn}\} = -\frac{\epsilon}{\mu}\begin{bmatrix} (\lambda+2\mu)^2 & \lambda(\lambda+2\mu) & \lambda(\lambda+2\mu) & 0 & 0 & 0 \\ \lambda(\lambda+2\mu) & \lambda^2 & \lambda^2 & 0 & 0 & 0 \\ \lambda(\lambda+2\mu) & \lambda^2 & \lambda^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu^2 \end{bmatrix} D; \tag{2}$$

and the second-order perturbation (Hudson 1982):

$$\{c^2_{jkmn}\} = \frac{\epsilon^2}{15}\begin{bmatrix} (\lambda+2\mu)q & \lambda q & \lambda q & 0 & 0 & 0 \\ \lambda q & \lambda^2 q/(\lambda+2\mu) & \lambda^2 q/(\lambda+2\mu) & 0 & 0 & 0 \\ \lambda q & \lambda^2 q/(\lambda+2\mu) & \lambda^2 q/(\lambda+2\mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X & 0 \\ 0 & 0 & 0 & 0 & 0 & X \end{bmatrix} D^2; \tag{3}$$

where $q = 15(\lambda/\mu)^2 + 28(\lambda/\mu) + 28$; $X = 2\mu(3\lambda + 8\mu)/(\lambda + 2\mu)$; and D is the diagonal matrix with trace (U_{11}, U_{11}, U_{11}, 0, U_{33}, U_{33}). The quantities U_{km} depend on the conditions at the crack face and are integrands over the face of μ/a times the kth components of the discontinuity in displacement across the crack due to the imposed tractions in the x_m-direction (Hudson 1981).

The elastic constants are fully specified for systems of parallel cracks if U_{11} and U_{33} can be calculated for the particular crack configuration. The constants for mixtures of cracks with different parameters and different orientations can be obtained, by adding the perturbations ($c^1_{jkmn} + c^2_{jkmn}$) calculated separately for each parallel element of the mixture with appropriate values of ϵ, U_{11}, and U_{33} and appropriate orientations.

Hudson (1981) determines expressions for U_{11} and U_{33} in the long-wavelength limit for ellipsoidal inclusions with semi-axes a, a and c, small aspect ratio $d = c/a$ ($c \ll a$), and filled with weak isotropic material having Lamé constants λ' and μ'. Hudson obtains:

$$U_{11} = (4/3)\,[(\lambda + 2\mu)/(\lambda + \mu)]/(1 + K); \tag{4}$$

and

$$U_{33} = (16/3)\,[(\lambda + 2\mu)/(3\lambda + 4\mu)]/(1 + M);$$

where

$$K = [(\kappa' + (4/3)\mu')/(\pi\, d\,\mu)]\,[(\lambda + 2\mu)/(\lambda + \mu)];$$

$$M = [4\mu'/(\pi d \mu)]\,[(\lambda + 2\mu)/(3\lambda + 4\mu)];$$

and $\kappa' = \lambda' + (2/3)\,\mu'$ is the bulk modulus of the weak material.

Equations (4) model saturated water-filled cracks with aspect ratio d by putting $\mu' = 0$ and $\kappa' = \lambda' \approx 2.25 \times 10^9\,\mathrm{N\,m^{-2}}$. The condition for thin cracks $d \to 0$ then gives $U_{11} = 0$ and $U_{33} = (16/3)\,(\lambda + 2\mu)/(3\lambda + 4\mu)$. This leads to the same expressions for the velocity variation with direction as those in Garbin & Knopoff (1975b) for weak anisotropy.

Equations (4) model dry cracks with aspect ratio d by putting $\lambda' = 0$ and $\mu' = 0$, giving $U_{11} = (4/3)\,(\lambda + 2\mu)/(\lambda + \mu)$ and $U_{33} = (16/3)\,(\lambda + 2\mu)/(3\lambda + 4\mu)$. These lead to expressions demonstrating the physically plausible result that the scattering due to cracks is independent of the aspect ratio for small values of the ratio.

These expressions have the same velocity variations as the corrected expressions of Garbin & Knopoff (1973, 1975a) for weak anisotropy. The correction (Hudson finds a factor of 2 missing in the last term in equation (60) of Garbin & Knopoff 1975a) results in the contributions with 4θ terms in the squares of the velocity variations of the qP-wave and the shear wave with coplanar polarization (qSP) being equal and opposite in sign in the symmetry plane through the crack normal. This equality is a necessary feature of propagation in anisotropic symmetry planes (Crampin 1981), and the previous inequality caused ambiguity in Crampin's (1978) modelling of the Garbin & Knopoff expressions for dry cracks.

Note that Hudson (1981, 1982) writes the elastic tensor as a 6×6 matrix $\{c_{ij}\}$ for $i, j = 1, 2, \ldots 6$ with the notation that $c_{ij} = c_{klmn}$ for $i, j < 4$ (which is standard) and $c_{ij} = 2c_{klmn}$ for $4 \leqslant i, j$ (non-standard) where k, l, m, n take appropriate value 1, 2, 3 given by equation (48) of Hudson (1982).

3 Theoretical formulations for attenuation variations

Hudson (1981) also determined the angular variations of the dissipation coefficient $1/Q$ for parallel cracks. Hudson obtained the expressions:

$$\left.\begin{aligned} &1/Q_{qP}(\theta) = [\alpha\,\epsilon/(\beta\,15\,\pi)]\,(\omega a/\alpha)^3\,[\![X \sin^2 2\theta + Y[(\alpha/\beta)^2 - 2\sin^2\theta]^2]\!];\\ &1/Q_{qSP}(\theta) = [\epsilon/(15\pi)]\,(\omega a/\beta)^3\,(X\cos^2 2\theta + Y\sin^2 2\theta);\\ &\text{and}\\ &1/Q_{qSR}(\theta) = [\epsilon/(15\pi)]\,(\omega a/\beta)^3\,X\cos^2\theta; \end{aligned}\right\} \tag{5}$$

where

$$X = [3/2 + (\beta/\alpha)^5]\,U_{33}^2;$$

$$Y = [2 + (15/4)/(\beta/\alpha) - 10(\beta/\alpha)^3 + 8(\beta/\alpha)^5]\,U_{11}^2;$$

θ is the angle from the normal to the crack; ω is the angular frequency; α and β are the compressional and shear-wave velocities in the uncracked solid; qSP, and qSR are the quasi shear-waves polarized parallel, and at right angles, respectively, to the symmetry plane through the crack normals.

Crampin (1981) demonstrated that anisotropic dissipation coefficients may be modelled as the imaginary parts c^I_{jkmn} of complex elastic constants $c_{jkmn} = c^R_{jkmn} + \mathrm{i}\, c^I_{jkmn}$, where the real parts are the conventional elastic constants (we may drop the suffix R on the real part without causing ambiguity). These imaginary parts form a fourth-order tensor, and the variation of the dissipation in symmetry planes has similar approximate equations to those of the velocity variations (Crampin 1981).

The real parts of the imaginary elastic constants equivalent to (5) are obtained by solving the approximate equations for specific values of the dissipation coefficient. We have:

$$\{c^I_{jkmn}\} = \begin{bmatrix} c_{1111}/C & A & A & 0 & 0 & 0 \\ A & c_{2222}/D & B & 0 & 0 & 0 \\ A & B & c_{2222}/D & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{2323}/E & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{3131}/F & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{3131}/F \end{bmatrix}; \quad (6)$$

where

$$A = [(c_{1111} + c_{2222})/2 + c_{1122} + 2c_{1212}]\, Q_{qP}(45^\circ) - (c^I_{1111} + c^I_{2222})/2 - 2c^I_{1212};$$

$$B = c^I_{2222} - 2\,c^I_{2323};\, C = Q_{qP}(0^\circ);\, D = Q_{qP}(90^\circ);\, E = Q_{qSR}(90^\circ);\; F = Q_{qSR}(0^\circ);$$

and the constants c_{jkmn} without superscripts are defined in equation (1).

4 Specimen cracked solids: elastic propagation

Specimen elastic constants for dry and water-saturated parallel cracks in an isotropic solid, with crack densities $\epsilon = 0.1$, are listed for specified parameter in Table 1. The velocity variations are compared with the equivalent Garbin & Knopoff expressions in Fig. 1.

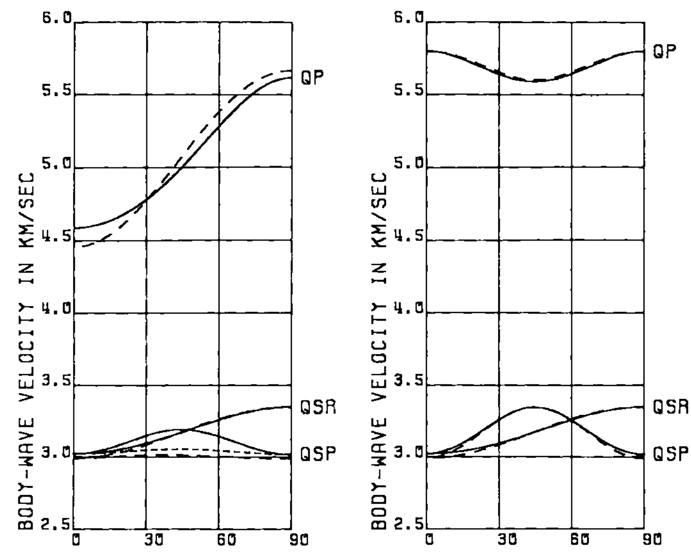
Fig. 1(a) compares the second-order Hudson velocity variations in a solid with dry cracks with the first-order variations of the equivalent Garbin & Knopoff model. The uncorrected Garbin & Knopoff model GKFF1 (Crampin 1978) is also shown. The Hudson velocities and corrected Garbin & Knopoff velocities are in good agreement. Note that, unusually for hexagonal symmetry systems, the two shear-wave velocity surfaces do not touch except in the direction of the symmetry axis normal to the plane of the cracks. This means that the delay between the split shear-waves, which is zero in the direction of the normal, increases smoothly with increasing angle from the normal.

Fig. 1(b) shows that the second-order Hudson velocity variations for saturated cracks are essentially identical with those of the equivalent first-order Garbin & Knopoff model GKLF1. In this case the two shear-wave sheets do intersect and there are significant shear-wave delays for all directions except for very close to the normal, and for directions approximately 60° to the normal.

Table 1. Effective real and imaginary elastic constants for wave propagation through parallel cracks (normal to the x_1-axis) in an isotropic solid with density $\rho = 2.6\,\text{g cm}^{-3}$ and velocities $\alpha = 5.8$ and $\beta = 3.349\,\text{km s}^{-1}$. The real elastic constants are derived from equations (1), (2) and (3), and the imaginary constants from equation (6). All constants are in units $10^9\,\text{N m}^{-2}$. The cracks have radius 5 m, aspect ratio 0.0001, crack density $\epsilon = 0.1$, and are penetrated by waves of frequency 40 Hz.

		Perturbations of the elastic constants (HCD1) Dry cracks			(HCS1) Saturated cracks		
jkmn	c^0_{jkmn}	c^1_{jkmn}	c^2_{jkmn}	c^I_{jkmn}	c^1_{jkmn}	c^2_{jkmn}	c^I_{jkmn}
1111	87.464	−52.473	16.555	0.1910	−0.142	0.000	0.0000
2222, 3333	87.464	−5.825	1.838	0.0343	−0.016	0.000	0.0000
2233	29.142′	−5.825	1.838	0.0343	−0.016	0.000	0.0000
1122, 1133	29.142	−17.483	5.516	0.1008	−0.047	0.000	0.0072
2323	29.161	0.000	0.000	0.0000	0.000	0.000	0.0000
1212, 1313	29.161	−6.666	0.745	0.0213	−6.666	0.745	0.0213

These differences between the shear-wave delays in dry and saturated cracks could be a valuable criterion to assess the degree of water saturation in system of parallel cracks. The differences are more clearly shown in the projections in Fig. 2. These are equal-area projections of the delays for diagonal paths through spheres with diagonals of 10 km for three orientations of parallel cracks. The delays are plotted for group-velocity (ray-path) propagation, such as would be observed in a horizontal plane over a source of seismic waves. The deviations in the direction between the group (energy) and phase propagation are comparatively small at these crack densities (see Crampin & McGonigle 1981), and replacing



(a) (b)

Figure 1. Variations in the phase velocities of the three body-waves for angles of incidence between 0° (normal) and 90° (tangential) to parallel cracks. The *qP*- and *qSP*-waves are polarized parallel, and the *qSR*-wave perpendicular to, the incident plane of symmetry. (a) Dry cracks: solid line – GKFF1 (Crampin 1978); short dash – GKFF1 with Hudson's (1981) correction to Garbin & Knopoff (1975b); and long dash – HCD1 specified in Table 1. (b) Saturated cracks: solid line – GKLF1 (Crampin 1978); and long dash – HCS1 specified in Table 1.

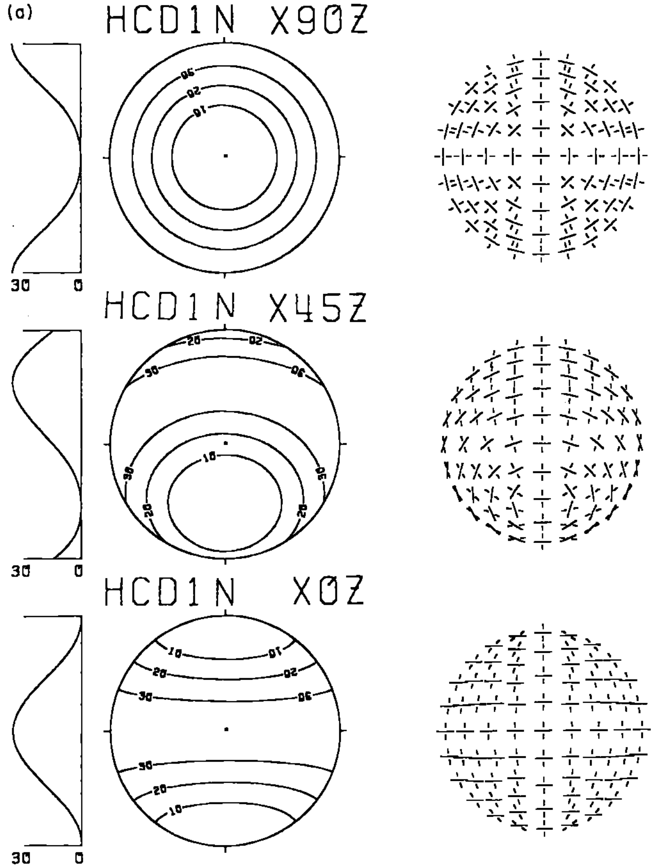


Figure 2. Equal-area projections of the shear-wave delays and shear-wave polarizations for group-velocity (ray-path) propagation through a 10 km radius upper-hemisphere containing parallel cracks. On the left of each delay projection is a north–south section of the delays. Both contours and sections are labelled in hundredths of a second. The solid bar in the polarization stereograms is the projection on a horizontal plane (not a free surface) of the polarization of the faster shear-wave, and the broken line is the projection of the polarization of the slower shear-wave. The orientations of the parallel cracks are, from the bottom: (i) vertical, (ii) dipping 45° to the south and (iii) horizontal. The projections of the delays and polarizations are shown for: (a) dry cracks, HCD1 and (b) saturated cracks, HCS1.

phase by group arrivals does not alter the essential features. Note that the effects of a free surface on the behaviour of shear-waves have not been considered here.

Cracks in the Earth are probably most commonly oriented vertically perpendicular to the minimum compressive stress which is usually horizontal (Hubbert & Willis 1957; Zoback & Zoback 1980). Fig. 2 indicates that the relative delays for dry vertical cracks (bottom set of diagrams in Fig. 2a) are large in a broad stripe of directions nearly parallel to the cracks smoothly decreasing to zero perpendicular to the cracks. The delays for saturated vertical cracks show similar values as those for saturated cracks in the broad stripe parallel to the cracks, but the stripe is now bordered by two bands of small delays before increasing again before returning to zero for the direction normal to the cracks.

Fig. 2 also shows equal-area projections of the shear-wave polarizations as would be observed on horizontal instruments in a horizontal plane over the source; again surface effects have not been considered. The polarizations of the faster shear-waves through dry cracks are approximately parallel to the plane of the cracks over the whole of the projection.

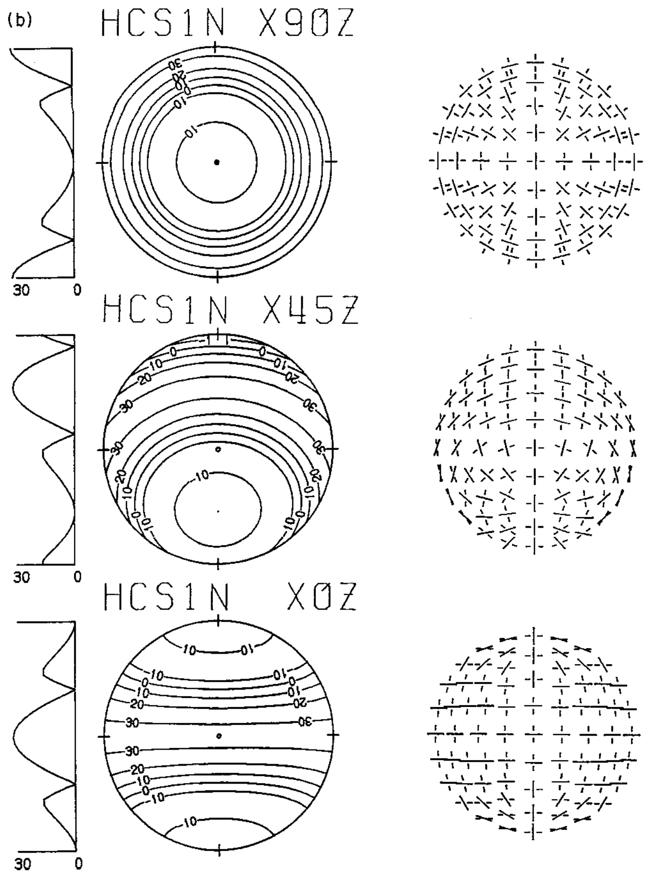


Figure 2 – *continued*

The polarizations of the faster shear-wave through saturated cracks, although parallel to the cracks in a broad stripe across the centre of the projection, change abruptly to approximately normal to the cracks for directions more steeply inclined to the plane of the cracks.

Although the projections show marked differences, both in delays and polarizations, between dry and saturated cracks, it must be remembered that the effect of the free surface is not shown in the projections of Fig. 2. Shear waves at the free surface will only clearly indicate the polarization of the incident waves for angles of incidence less than about 35° (Booth & Crampin 1984), and within this circle the projections for dry and saturated cracks are very similar.

Observations of shear-wave delays and shear-wave polarizations, if they can be obtained, are measurements of two almost independent phenomena determined by the crack configuration. Any valid interpretation or inversion must satisfy both sets of observations, and is a valuable check on the solution.

5 Comparison of the Hudson and Garbin & Knopoff derivations

The first-order expressions of Hudson (1981, 1982) for the velocity variations in a weak distribution of cracks in an isotropic solid may be written:

$$V = V_0\,[1 - \epsilon f(\theta)]^{1/2}; \qquad (7)$$

where V_0 is the appropriate *P*- or *S*-wave velocity in the uncracked isotropic solid; ϵ is the crack density; and $f(\theta)$, a function of the angle to the crack normal, is dependent on the particular wave-type *qP*, *qSP* or *qSR*.

The equivalent expressions of Garbin & Knopoff (1973, 1975a, b) are of the form:

$$V = V_0/[1 + \epsilon f(\theta)]^{1/2}; \tag{8}$$

where $f(\theta)$ is the same function for appropriate wave types in (7) and (8).

Thus the first-order expressions of Hudson and Garbin & Knopoff formally agree to the first order in ϵ ($\epsilon \ll 1$), but they differ significantly for velocity anisotropy greater than 5 per cent, say. However, the Hudson perturbations, correct to the second order, are similar to the first-order Garbin & Knopoff expressions for dry cracks in Fig. 1(a), and are nearly identical for saturated cracks in Fig. 1(b). This agreement is remarkable considering the different approach used in the derivations.

Hudson (private communication) writes that the range of validity of such truncated series depends on how it is expressed: as a straightforward polynomial; as a polynomial raised to some positive or negative power; or as a quotient of two polynomials (such as Padé approximants, see Cabannes 1976). All such representations may be the same when expanded as a straightforward series in ϵ up to some given power, but their ranges of validity will be different. It follows that, to remain valid over the same range of values of ϵ, one representation will need to be taken to a higher power of ϵ than another.

6 Specimen cracked solids: anelastic propagation

The attenuation of wave propagation in cracked solids can be simulated by the imaginary elastic constants in equation (6). The imaginary parts of the elastic constants for specified parameters appropriate to HCD1 and HCS1 are listed in Table 1. The angular variations derived from the imaginary elastic constants in (6) are compared in Fig. 3 with Hudson's first-order expressions (5). The overall pattern of variations compares very well. However,

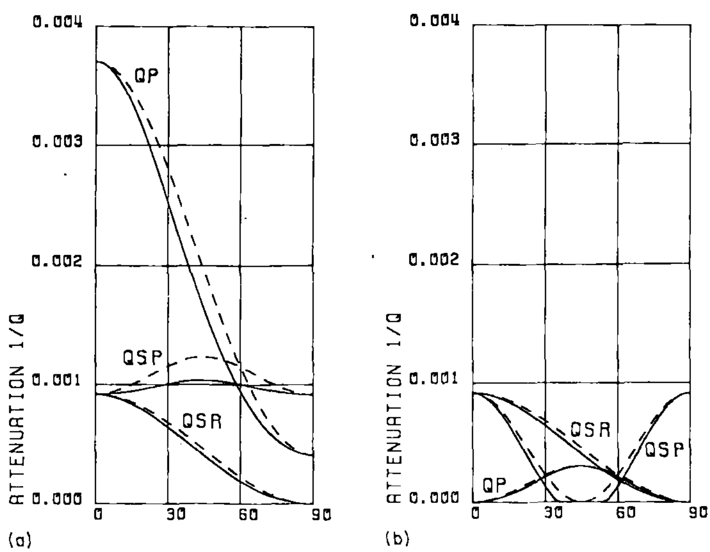


Figure 3. Variations in attenuation of waves of 40 Hz frequency for propagation over the same angles as in Fig. 1 for: (a) dry cracks, HCD1 and (b) saturated cracks, HCS1. Dashed lines are calculated from the expressions of Hudson (1981), and solid lines are from the elastic constants in equation (6).

there is the same problem of the difference of the 4θ variations of qP and qSP, that was present with Crampin's (1978) fit of elastic constants to the (uncorrected) Garbin & Knopoff variations, but the agreement between the two derivations could be improved by empirical fitting as in Crampin (1978). As expected, the variation of attenuation bears an inverse relationship to the velocity variations, in the sense that in directions where the wave type is particularly affected by the presence of cracks the attenuation is high and the velocity is low, whereas in directions less affected by cracks the attenuation is low and the velocity approaches the higher value in the uncracked solid. The attenuation of the qSR-wave is the same for dry and saturated conditions, whereas the attenuation of qP and qSP decreases as the cracks become saturated.

It is worth noting a computational detail. The velocities (Fig. 1) and the attenuation (Fig. 3) are obtained from the complex eigenvalue of a matrix of complex elastic constants. The real and imaginary parts of the eigenvalues (the square of the velocities, and the attenuation $1/Q$, respectively) may be found independently by solving the real and imaginary parts of the matrix separately. This procedure gives the same numerical values, but it is then difficult to order the imaginary eigenvalues correctly. Solving the complex eigenproblem ties the real and imaginary eigenvalues together, and the velocities are naturally ordered by the polarizations of their eigenvectors.

7 Discussion

Equations (2) and (3) show that the perturbations to the effective (real) elastic constants controlling the seismic velocities in aligned cracks are linearly dependent on the crack density ϵ. In the long-wavelength limit, the velocities are insensitive to the crack radius (at constant crack density), to the aspect ratio, and the particular material filling the crack apart from the major effect of the presence or absence of liquid in the cracks. The

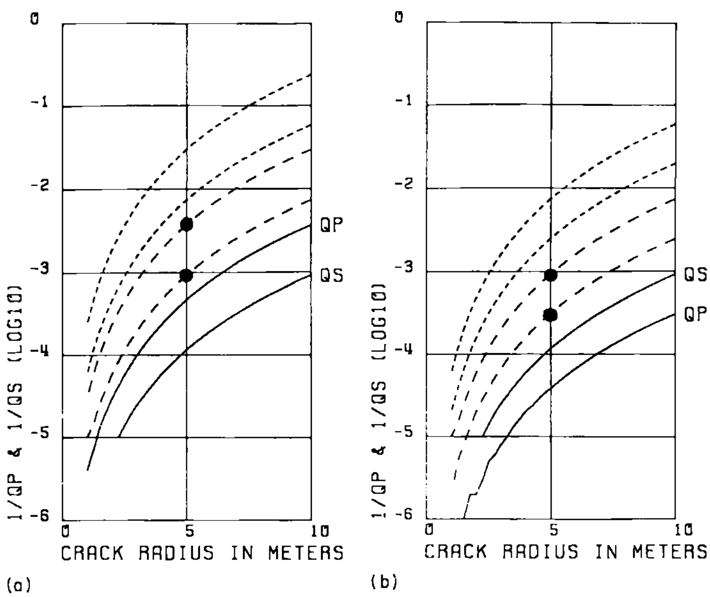


Figure 4. Variations of the maximum values of qP- and qS-wave attenuation for crack radii between 1 and 10 m at a constant crack density of $\epsilon = 0.1$. Solid lines are the attenuation of 20 Hz waves, short dashes are 40 Hz waves; and long dashes are 80 Hz waves in: (a) dry cracks and (b) saturated cracks. The positions of the maxima of the attenuation shown in Fig. 3 are marked by solid circles.

attenuation, however, derived from equations (5) and (6) varies rapidly with crack radius and with the frequency of the wave penetrating the cracked solid. Fig. 4 shows the variation of the maximum *qP*- and *qS*-wave attenuation of waves of 20, 40 and 80 Hz through a cracked solid with crack density $\epsilon = 0.1$ and a range of crack radii from 1 to 10 m.

The maximum of the *qS*-wave attenuation is independent of the degree of saturation of the cracks, but falls off rapidly with decreasing crack radius, and increases with increasing frequency. Note that the maximum attenuation of *qP*-waves is approximately 3 times greater than the maximum *qS*-wave attenuation in dry cracks, and 3 times less than the maximum of the *qS* attenuation in saturated cracks. The presence of cracks, which will tend to be aligned by stress (Crampin, Evans & Atkinson 1984) and hence effectively anisotropic (Crampin 1978), can also cause the rapid fluctuations that characterize observations of shear waves in the Earth. Shear waves propagating through parallel cracks show large varying delays between the split shear-waves with significant differences between dry and saturated cracks (Figs 1 and 2). The split shear-waves also have very different attenuations (Fig. 3). This means that small changes in the direction of propagation, small changes in polarization, or small changes in the degree of water saturation, may have large effects on the appearance of the shear wavetrain particularly on subsurface recoordings and demonstrates the sensitivity of shear-waves to the presence of cracks.

8 Conclusions

The examples of cracked solids we have shown have relatively large crack densities and are approximate solutions. However, the three-dimensional pattern of the velocity variations and shear-wave delays are changed very little by the crack density; although the absolute values may vary widely. In many circumstances, and this is the justification for presenting them, the three-dimensional pattern is more informative than the absolute values. This is supported by Crampin, McGonigle & Bamford (1980) who obtained similar parameters from the inversions of rather different velocity variations observed in cracked limestone at three neighbouring quarries. The differences were attributed to different degrees of saturation by water, with the interpretation that despite the wide variation in the dilatation of the very large cracks, the overall patterns of the variations were similar to those of much smaller crack densities.

Hudson's formulations for calculating real and imaginary effective elastic constants as outlined in this paper are very flexible and will accommodate a wide range of crack geometries by the technique of summing the individual perturbations. It is clear that shear wavetrains contain much more information about the internal structure of the material through which they propagate than *P* wavetrains. If the various problems associated with the interpretation of shear waves can be overcome, particularly the interaction with the free surface at over-critical angles (Booth & Crampin 1984), the study of shear waves may reveal much new information about the interior of the Earth.

The overall patterns of the velocity variations for the parallel cracks of Garbin & Knopoff of both the uncorrected, but particularly the corrected version, are similar to the Hudson variations in Fig. 1. The technique for calculating the effects of crack systems with mixed orientations suggested by Crampin (1978) (by multiplying the relative reductions in velocity for each parallel system) is in agreement with the technique of Hudson (1981, 1982) of adding the perturbations of the elastic contents. This means that the figures and conclusions of Crampin (1978) are largely unaltered by adopting the preferred analysis of cracked solids developed by Hudson, although the detailed behaviour of the shear-waves in dry cracks changes because of the correction to the Garbin & Knopoff expressions. The main conclusion of this paper is that the Hudson (1981) formulations provide a very flexible technique for calculating real and imaginary effective elastic constants of cracked solids, which then provide the essential model parameters for calculating seismic-wave propagation through anisotropic solids by the techniques reviewed by Crampin (1981).

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