

Jean-Daniel Boissonnat
Monique Teillaud

Editors

Effective Computational Geometry for Curves and Surfaces

With 120 Figures and 1 Table

 Springer

Contents

1 Arrangements

Efi Fogel, Dan Halperin, Lutz Kettner, Monique Teillaud, Ron Wein, Nicola Wolpert* 1

1.1 Introduction	1
1.2 Chronicles	3
1.3 Exact Construction of Planar Arrangements	5
1.3.1 Construction by Sweeping	7
1.3.2 Incremental Construction	20
1.4 Software for Planar Arrangements	25
1.4.1 The CGAL Arrangements Package	26
1.4.2 Arrangements Traits	33
1.4.3 Traits Classes from EXACUS	36
1.4.4 An Emerging CGAL Curved Kernel	38
1.4.5 How To Speed Up Your Arrangement Computation in CGAL . .	40
1.5 Exact Construction in 3-Space	41
1.5.1 Sweeping Arrangements of Surfaces	41
1.5.2 Arrangements of Quadrics in 3D	45
1.6 Controlled Perturbation: Fixed-Precision Approximation of Arrangements	50
1.7 Applications	53
1.7.1 Boolean Operations on Generalized Polygons	53
1.7.2 Motion Planning for Discs	57
1.7.3 Lower Envelopes for Path Verification in Multi-Axis NC-Machining	59
1.7.4 Maximal Axis-Symmetric Polygon Contained in a Simple Polygon	62
1.7.5 Molecular Surfaces	63
1.7.6 Additional Applications	64
1.8 Further Reading and Open problems	66

2 Curved Voronoi Diagrams

<i>Jean-Daniel Boissonnat*, Camille Wormser, Mariette Yvinec</i>	67
2.1 Introduction	68
2.2 Lower Envelopes and Minimization Diagrams	70
2.3 Affine Voronoi Diagrams	72
2.3.1 Euclidean Voronoi Diagrams of Points	72
2.3.2 Delaunay Triangulation	74
2.3.3 Power Diagrams	78
2.4 Voronoi Diagrams with Algebraic Bisectors	81
2.4.1 Möbius Diagrams	81
2.4.2 Anisotropic Diagrams	86
2.4.3 Apollonius Diagrams	88
2.5 Linearization	92
2.5.1 Abstract Diagrams	92
2.5.2 Inverse Problem	97
2.6 Incremental Voronoi Algorithms	99
2.6.1 Planar Euclidean diagrams	101
2.6.2 Incremental Construction	101
2.6.3 The Voronoi Hierarchy	106
2.7 Medial Axis	109
2.7.1 Medial Axis and Lower Envelope	110
2.7.2 Approximation of the Medial Axis	110
2.8 Voronoi Diagrams in CGAL	114
2.9 Applications	115

3 Algebraic Issues in Computational Geometry

<i>Bernard Mourrain*, Sylvain Pion, Susanne Schmitt, Jean-Pierre Tédourt, Elias Tsigaridas, Nicola Wolpert</i>	117
3.1 Introduction	117
3.2 Computers and Numbers	118
3.2.1 Machine Floating Point Numbers: the IEEE 754 norm	119
3.2.2 Interval Arithmetic	120
3.2.3 Filters	121
3.3 Effective Real Numbers	123
3.3.1 Algebraic Numbers	124
3.3.2 Isolating Interval Representation of Real Algebraic Numbers	125
3.3.3 Symbolic Representation of Real Algebraic Numbers	125
3.4 Computing with Algebraic Numbers	126
3.4.1 Resultant	126
3.4.2 Isolation	131
3.4.3 Algebraic Numbers of Small Degree	136
3.4.4 Comparison	138
3.5 Multivariate Problems	140
3.6 Topology of Planar Implicit Curves	142
3.6.1 The Algorithm from a Geometric Point of View	143

3.6.2 Algebraic Ingredients	144
3.6.3 How to Avoid Genericity Conditions	145
3.7 Topology of 3d Implicit Curves	146
3.7.1 Critical Points and Generic Position	147
3.7.2 The Projected Curves	148
3.7.3 Lifting a Point of the Projected Curve	149
3.7.4 Computing Points of the Curve above Critical Values	151
3.7.5 Connecting the Branches	152
3.7.6 The Algorithm	153
3.8 Software	154
4 Differential Geometry on Discrete Surfaces	
<i>David Cohen-Steiner, Jean-Marie Morvan*</i>	157
4.1 Geometric Properties of Subsets of Points	157
4.2 Length and Curvature of a Curve	158
4.2.1 The Length of Curves	158
4.2.2 The Curvature of Curves	159
4.3 The Area of a Surface	161
4.3.1 Definition of the Area	161
4.3.2 An Approximation Theorem	162
4.4 Curvatures of Surfaces	164
4.4.1 The Smooth Case	164
4.4.2 Pointwise Approximation of the Gaussian Curvature	165
4.4.3 From Pointwise to Local	167
4.4.4 Anisotropic Curvature Measures	174
4.4.5 ϵ -samples on a Surface	178
4.4.6 Application	179
5 Meshing of Surfaces	
<i>Jean-Daniel Boissonnat, David Cohen-Steiner, Bernard Mourrain,</i> <i>Günter Rote*, Gert Vegter</i>	181
5.1 Introduction: What is Meshing?	181
5.1.1 Overview	187
5.2 Marching Cubes and Cube-Based Algorithms	188
5.2.1 Criteria for a Correct Mesh Inside a Cube	190
5.2.2 Interval Arithmetic for Estimating the Range of a Function	190
5.2.3 Global Parameterizability: Snyder's Algorithm	191
5.2.4 Small Normal Variation	196
5.3 Delaunay Refinement Algorithms	201
5.3.1 Using the Local Feature Size	202
5.3.2 Using Critical Points	209
5.4 A Sweep Algorithm	213
5.4.1 Meshing a Curve	215
5.4.2 Meshing a Surface	216
5.5 Obtaining a Correct Mesh by Morse Theory	223
5.5.1 Sweeping through Parameter Space	223

5.5.2 Piecewise-Linear Interpolation of the Defining Function	224
5.6 Research Problems	227
6 Delaunay Triangulation Based Surface Reconstruction	
<i>Frédéric Cazals, Joachim Giesen</i>	231
6.1 Introduction	231
6.1.1 Surface Reconstruction	231
6.1.2 Applications	231
6.1.3 Reconstruction Using the Delaunay Triangulation	232
6.1.4 A Classification of Delaunay Based Surface Reconstruction Methods	233
6.1.5 Organization of the Chapter	234
6.2 Prerequisites	234
6.2.1 Delaunay Triangulations, Voronoi Diagrams and Related Concepts	234
6.2.2 Medial Axis and Derived Concepts	244
6.2.3 Topological and Geometric Equivalences	249
6.2.4 Exercises	252
6.3 Overview of the Algorithms	253
6.3.1 Tangent Plane Based Methods	253
6.3.2 Restricted Delaunay Based Methods	257
6.3.3 Inside / Outside Labeling	261
6.3.4 Empty Balls Methods	268
6.4 Evaluating Surface Reconstruction Algorithms	271
6.5 Software	272
6.6 Research Problems	273
7 Computational Topology: An Introduction	
<i>Günter Rote, Gert Vegter*</i>	277
7.1 Introduction	277
7.2 Simplicial complexes	278
7.3 Simplicial homology	282
7.4 Morse Theory	295
7.4.1 Smooth functions and manifolds	295
7.4.2 Basic Results from Morse Theory	300
7.5 Exercises	310
8 Appendix - Generic Programming and The CGAL Library	
<i>Efi Fogel, Monique Teillaud</i>	313
8.1 The CGAL Open Source Project	313
8.2 Generic Programming	314
8.3 Geometric Programming and CGAL	316
8.4 CGAL Contents	318
References	321
Index	341