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Effective field theory for fluids: Hall viscosity from a Wess-Zumino-Witten term

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Abstract: We propose an effective action that describes a relativistic fluid with Hall viscosity. The construction involves a Wess-Zumino-Witten term that exists only in (2+1) spacetime dimensions. We note that this formalism can accommodate only a Hall viscosity which is a homogeneous function of the entropy and particle number densities of degree one.

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Contents

1 Introduction

Hydrodynamics provides a universal description of the long wavelength dynamics of finite-temperature interacting systems. It can be considered an effective theory organized in a derivative expansion. To leading order one has a dissipationless theory (ideal hydrodynamics), and effects of dissipation enter at the next-to-leading order (first-order hydrodynamics). The dissipative effects are parameterized by kinetic coefficients (viscosities, thermal conductivity etc.)

Recently, it has been found that new, dissipationless terms are possible in first-order hydrodynamics. In 3+1 dimensions, these effects are related to the anomalies of the underlying quantum field theory. In 2+1 dimensions, one such possible term involves the Hall viscosity [1]. In a gapped quantum Hall fluid the Hall viscosity has a topological nature, it is related to the shift and the angular momentum density. In a gapless fluid such a connection is not expected to hold. Holographic models typically lead to a Hall viscosity which is not related to the angular momentum density, although there are indications of a relationship in holographic $p_x + ip_y$ superfluids near the phase transition [2].

Since hydrodynamics, including Hall viscosity, is a dissipationless theory, it is natural to ask whether a description based on an action is possible. This question has been addressed before, but no term has been identified with the Hall viscosity. The goal of this paper is to show that such a term exists. However, the Hall viscosity in this description cannot be an arbitrary function of entropy density s and the particle number density n, but must have the form of $sf(n/s)$. In particular, for an uncharged plasma, the Lagrangian description corresponds to a constant ratio of the Hall viscosity and the entropy density.

2 Review of effective field theory formulation of fluid dynamics

We begin with a brief review of an effective field theory (EFT) approach to fluid dynamics (for details see refs. [3, 4]). We will at first concentrate on the simplest case of a fluid without conserved charges, where the fundamental degree of freedom in the EFT description is a

JHEP11(2014)004

set of scalars $\phi^I(x,t)$, $I=1,\ldots d$ mapping spatial slices to a d dimensional internal space $\mathcal{F}$. We will set $d=2$ for the entirety of this paper. One can interpret ϕ^I as Lagrangian coordinates, labeling which fluid particle occupies the point x at the time t (such an interpretation, however, should not be taken literally.) We seek to write down an action functional of ϕ^I and its derivatives, invariant under shifts $\phi^I \to \phi^I + a^I$ and area preserving diffeomorphisms,

$$\phi^I \to \tilde{\phi}^I(\phi), \qquad \det(\partial_I \tilde{\phi}^J) = 1. \tag{2.1}$$

Invariance with respect to area preserving diffeomorphisms in ϕ^I space is what tells us we're working with a fluid, not a solid.

We define, following ref. [3], a scalar

$$s = \sqrt{\det \partial_\mu \phi^I \partial^\mu \phi^J} = \frac{1}{\sqrt{-g}} \det \partial_i \phi^I (1-\dot{x}^2)^{1/2} \tag{2.2}$$

and a unit timelike vector

$$u^\mu = \frac{1}{2s} \varepsilon^{\mu\alpha\beta} \epsilon_{IJ} \partial_\alpha \phi^I \partial_\beta \phi^J, \tag{2.3}$$

where $\varepsilon^{\mu\nu\lambda}$ is the completely antisymmetric tensor $\varepsilon^{012} = 1/\sqrt{-g}$. The vector u^μ is tangent to curves of constant ϕ^I:

$$u^\mu \partial_\mu \phi^I = 0. \tag{2.4}$$

The current

$$s^\mu = s u^\mu = \frac{1}{2} \varepsilon^{\mu\alpha\beta} \epsilon_{IJ} \partial_\alpha \phi^I \partial_\beta \phi^J \tag{2.5}$$

is then identically conserved $\nabla_\mu s^\mu = 0$, independent of equations of motion and is identified with the entropy current and s with the entropy density [3].

Due to the shift symmetry the fields ϕ^I always appear in the Lagrangian with at least one derivative so in the construction of the effective action we use a power-counting scheme in which $\partial_\mu \phi^I$ is counted as having no spatial derivatives, $\partial_\mu \partial_\nu \phi^I$ as having one derivative, etc. To leading order the only possible term in the action is

$$S = -\int d^3x \sqrt{-g}\, \epsilon(s), \tag{2.6}$$

where s is an arbitrary function of one variable. Under metric variations, $\delta s = \frac{1}{2} s P_{\mu\nu} \delta g^{\mu\nu}$, and the stress-energy tensor computed from (2.6) is

$$T_{\mu\nu} = \epsilon u_\mu u_\nu + P_{\mu\nu} P, \tag{2.7}$$

where $P = s\partial_s \epsilon - \epsilon$ and $P_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ is the spatial projector. This is exactly the stress-energy tensor of a perfect fluid if one identifies $\epsilon(s)$ as the energy density as a function of the entropy density and its Legendre transform P as the pressure.

3 Wess-Zumino-Witten term

Now we turn our attention to first-order corrections to hydrodynamics. Previous attempts [5, 6] to include parity-breaking terms have not produced nontrivial transport coefficients: the stress contribution of all local, first order terms that may be added to the Lagrangian may be removed by some suitable field redefinition. We show that previous analyses miss one term which corresponds precisely to the Hall viscosity. This term has the form of a Wess-Zumino-Witten term, and our construction is very similar to that of ref. [7].

To begin, we define an induced metric on the space of comoving fluid particles ϕ^I,

$$G^{IJ} = s^{-1}\partial_\mu \phi^I \partial^\mu \phi^J. \tag{3.1}$$

The inclusion of the factor s^{-1} implies that $\det G = 1$. Note that G^{IJ} transforms as a tensor under area-preserving diffeomorphism. We denote the inverse of G^{IJ} by G_{IJ} and the volume form by ϵ_{IJ}. Since the metric is unimodular, $\epsilon_{12} = 1$. In what follows, we will often use matrix notation in which we denote G_{IJ} as G, G^{IJ} as G^{-1}, and both ϵ_{IJ} and ϵ^{IJ} by ϵ (context should make clear whether we are working with raised or lowered indices).

We follow the general philosophy behind the construction of Wess-Zumino-Witten (WZW) terms [8]. We first concentrate on one particular fluid element, parameterized by some values of the comoving coordinates ϕ^I. Points on the world line of this fluid element can parameterized further by a parameter τ. For example, τ can be (but does not have to be) chosen as the proper time. At the chosen values of ϕ^I, the field $G_{IJ}(\tau)$ defines a map from the world line to the space of unimodular matrices, which we call $\mathbb{H}^2$.

There is a two-form on $\mathbb{H}^2$ which is invariant under area preserving diffeomorphisms,

$$\omega = \frac{1}{2}\mathrm{Tr}\big(\epsilon\, dG\, G^{-1} \wedge dG\big). \tag{3.2}$$

This form is clearly closed, as it is a two-form on a two-dimensional space. Since $\mathbb{H}^2$ is contractible, the form is also exact so there exists a one-form α so that $\omega = d\alpha$. We define the WZW action for each fluid volume element to be the integral of α. Note that α is defined only up to a differential of a zero-form, but a replacement $\alpha \to \alpha + d\beta$ merely changes the Lagrangian by a total derivative. For the whole fluid, we sum up contributions from each of the fluid elements,

$$S_{\mathrm{WZW}} = f \int d^2\phi \int \alpha, \tag{3.3}$$

where f is some constant. The second integral is taken along the worldline of each fluid element.

The form ω can be interpreted as the volume form of a natural geometry on $\mathbb{H}^2$. To make our discussion more concrete, we follow ref. [7] and parameterize $\mathbb{H}^2$ via

$$G_{IJ} = \begin{pmatrix} T+X & Y \\ Y & T-X \end{pmatrix}. \tag{3.4}$$

The condition $\det G = 1$ implies $T^2 - X^2 - Y^2 = 1$, and the positive definiteness of G implies $T > 0$. Thus the space of all G_{IJ} is the two-dimensional hyperbolic plane which caries a natural metric induced by the Minkowski metric of the (T, X, Y) space. To see that ω is the volume form, we can choose to use the coordinates (Q, φ)

$$X = \sinh Q \cos\varphi, \qquad Y = \sinh Q \sin\varphi, \qquad T = \cosh Q, \tag{3.5}$$

with $0 \leq Q \leq \infty$ and $0 \leq \varphi < 2\pi$, and find by explicit calculation from eq. (3.2) that $-\frac{1}{2}\omega = \sinh Q\, dQ \wedge d\varphi$, which is the volume form on the hyperbolic plane.

One convenient way to write the WZW action is by extending the metric G_{IJ} to an extra dimension u, $G_{IJ} = G_{IJ}(\phi, \tau, u)$, so that it interpolates between a reference metric, say δ_{IJ}, at $u = 0$ to the physical metric at $u = 1$,

$$G_{IJ}(\phi, \tau, 0) = \delta_{IJ}, \qquad G_{IJ}(\phi, \tau, 1) = G_{IJ}(\phi, \tau). \tag{3.6}$$

The WZW term then can be written as

$$S_{\text{WZW}} = f \int d^2\phi\, d\tau\, du\, \text{Tr}\big(\epsilon \partial_u G G^{-1} \partial_\tau G\big). \tag{3.7}$$

Instead of writing the action as an integral over the fluid (Lagrangian) coordinates (τ, ϕ^I), one can also write it in the physical (Eulerian) coordinates x^μ,

$$S_{\text{WZW}} = f \int d^3x\, du \sqrt{-g}\, s\, \text{Tr}\big(\epsilon \partial_u G G^{-1} u^\mu \partial_\mu G\big) \tag{3.8}$$

where $\partial_\tau = u^\mu \partial_\mu$ and $G = G(\phi, \tau, u)$. This makes explicit that S_{WZW} is first order in derivatives, and should be included in our expansion. Though not manifest in this form, it is insensitive to the details of the interpolation and the choice of reference metric since in the end it is just a rewriting of eq. (3.3).

Let's check this explicitly and vary the interpolation δG. Since the interpolation must begin at a fixed point, and end at the physical metric, we have $\delta G|_{u=1} = 0$ and $\partial_\tau \delta G|_{u=0} = 0$. Under such a change

$$\delta S_{\text{WZW}} = f \int d^2\phi\, d\tau\, du\, \text{Tr}(\epsilon \partial_u \delta G G^{-1} \partial_\tau G + \epsilon \partial_u G \delta G^{-1} \partial_\tau G + \epsilon \partial_u G G^{-1} \partial_\tau \delta G). \tag{3.9}$$

The second term vanishes and using integration by parts on τ, the first and third can be combined as a total u-derivative.

$$\delta S_{\text{WZW}} = f \int d^2\phi\, d\tau\, du\, \partial_u \big(\text{Tr}(\epsilon \delta G G^{-1} \partial_\tau G)\big) = 0 \tag{3.10}$$

4 Stress-energy tensor

We now vary this with respect to the metric to obtain its stress contribution $\Delta T_{\mu\nu} = -2\delta S_{\text{WZW}}/\delta g^{\mu\nu}$. The variation is the total u-derivative (3.10) which integrates to

$$\delta S = f \int d^2\phi\, d\tau\ \text{Tr}(\epsilon \delta G^{-1} G \partial_\tau G^{-1}), \tag{4.1}$$

where G is the physical ($u=1$) metric. Using the following identities

$$\epsilon_{IJ} = -su^\mu \varepsilon_{\mu\nu\lambda}\partial_I x^\nu \partial_J x^\lambda, \qquad G_{IJ} = s\partial_I x^\mu \partial_J x_\mu, \qquad \partial_\mu \phi^I \partial_I x^\nu = P_\mu{}^\nu,$$
$$\delta G^{IJ} = \left(s^{-1}\partial_\mu\phi^I\partial_\nu\phi^J - \frac{1}{2}G^{IJ}P_{\mu\nu}\right)\delta g^{\mu\nu}, \qquad u^\nu\nabla_\nu\nabla_\mu\phi = -\nabla_\mu u^\nu \nabla_\nu\phi, \tag{4.2}$$

we find the stress takes precisely the form of the Hall viscosity

$$\Delta T_{\mu\nu} = -\eta_H u^\alpha \varepsilon_{\alpha\beta(\mu}\sigma_{\nu)}{}^\beta, \tag{4.3}$$

where $\sigma_{\mu\nu} = P_\mu{}^\alpha P_\nu{}^\beta(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{1}{2}\nabla_\gamma u^\gamma P_{\alpha\beta})$ is the shear tensor and the Hall viscosity is $\eta_H = 2sf$. One may check that this gives a nontrivial contribution to the tensor frame-invariant

$$C_T^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\Delta T_{\alpha\beta} - \frac{1}{2}P^{\mu\nu}P_{\alpha\beta}\Delta T^{\alpha\beta} = -\eta_H u_\alpha \varepsilon^{\alpha\beta(\mu}\sigma^{\nu)}{}_\beta \tag{4.4}$$

and so cannot be removed by any field redefinition [5, 6].

Moreover, η_H represents a true response to geometric perturbations. Consider transverse, traceless perturbations about flat space with zero wavevector and frequency ω,

$$\delta g_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \delta g_{ij} \end{pmatrix}, \qquad \delta g_{ij} = h_{ij}e^{-i\omega_+ x^0}, \qquad h^i{}_i = 0, \tag{4.5}$$

where $\omega_+ = \omega + i\epsilon$ and $\epsilon \to 0^+$. These perturbations do not disturb the static solution $s = \text{const}, u^\mu = (1, 0, 0)$ to first order and the stress-tensor two point function is easily evaluated. We obtain the traceless part of the long-wavelength Hall response

$$\begin{aligned}\eta^{\mu\nu\lambda\rho}(\omega) &= \int d^3x\, e^{i\omega_+ x^0}\left(\frac{1}{i\omega_+}\left\langle\frac{\delta T_{\mu\nu}(x)}{\delta g_{\lambda\rho}(0)}\right\rangle + \frac{1}{2\omega_+}\Theta(x^0)\left\langle[T^{\mu\nu}(x), T^{\lambda\rho}(0)]\right\rangle\right)\\ &= \int d^3x\,\frac{e^{i\omega_+ x^0}}{i\omega_+}\frac{\delta\langle T^{\mu\nu}(x)\rangle}{\delta g_{\lambda\rho}(0)} = \eta_H u_\alpha \varepsilon^{\alpha\beta(\mu}P^{\nu)(\lambda}P_\beta{}^{\rho)}\end{aligned} \tag{4.6}$$

up to a contribution proportional to $g^{\mu\nu}g^{\lambda\rho}$ which is left undefined in this calculation.

5 Number current and review of Friedman's formulation of fluid dynamics

We now turn our attention to the case when the fluid has a conserved charge. Thus, we will expand our formalism to include other independent currents. One possible generalization involves introducing a phase ψ with a so-called chemical shift symmetry $\psi \to \psi + c$ [4]. The resulting Noether current $n^\mu = nu^\mu$ is then interpreted as a conserved particle density. However, for our purposes this formalism is not entirely suitable, as it breaks the symmetry between the conserved entropy and particle number currents, leading to a less general form of the Hall viscosity coefficient. Instead, we will employ the formalism of Friedman [10], used in his study of the stability of relativistic stars, though in this discussion we principally follow the treatment of Green, Schiffrin and Wald [11].

In Friedman's formalism, the d-dimensional space $\mathcal{F}$ is replaced by a $(d+1)$-dimensional fiducial space $\mathcal{M}'$ diffeomorphic to the spacetime $\mathcal{M}$. The fluid configuration throughout time is described by a diffeomorphism $\chi : \mathcal{M} \to \mathcal{M}'$. We will specialize to the case $d = 2$. $\mathcal{M}'$ comes furnished with a fixed closed two-form S_{AB} and a scalar function n' so that $N_{AB} = n' S_{AB}$ is also closed

$$dS = dN = 0. \tag{5.1}$$

We denote $\mathcal{M}'$ indices with uppercase Latin letters near the beginning of the alphabet A, B, C etc. The physical number and entropy currents are then the pullbacks $S_{\mu\nu} = \chi^*(S_{AB})$ and $N_{\mu\nu} = \chi^*(N_{AB}) = n' S_{\mu\nu}$, respectively. The entropy and particle number currents $s^\mu = \frac{1}{2}\varepsilon^{\mu\nu\lambda}S_{\nu\lambda}$ and $n^\mu = n's^\mu$ are identically conserved and any given fluid configuration satisfying the two conservation laws may be described by an appropriate choice of S_{AB} and n'. We have thus restricted our description to include only those fluid configurations that do not change the particle and entropy content of a given fluid element

$$\nabla_\mu s^\mu = 0, \qquad \nabla_\mu n^\mu = 0, \qquad u^\mu \nabla_\mu n' = 0 \qquad \text{for all configurations.} \tag{5.2}$$

Other relevant fluid variables are

$$s = (-s^\mu s_\mu)^{1/2}, \qquad u^\mu = \frac{1}{s}s^\mu, \qquad n = n's, \tag{5.3}$$

and their transformations with respect to the metric are precisely as before

$$\delta s = -\frac{1}{2}sP^{\mu\nu}\delta g_{\mu\nu}, \qquad \delta n' = 0, \qquad \delta u^\mu = \frac{1}{2}u^\mu u^\nu u^\lambda \delta g_{\nu\lambda}. \tag{5.4}$$

If we set

$$S = -\int d^3x \sqrt{-g}\, \epsilon(s, n'), \tag{5.5}$$

we again obtain the perfect fluid stress tensor

$$T = \epsilon u^\mu u^\nu + PP^{\mu\nu}, \tag{5.6}$$

where $P = s\partial_s \epsilon - \epsilon$.

To construct the WZW term above, consider the pushed forward metric $g_{AB} = \chi_*(g_{\mu\nu})$ (we will always denote the pushforward of a spacetime tensor by the same symbol but with Greek indices replaced by upper case Latin indices). The form

$$\omega = -f(n')S_{AB}dg^{BC}g_{CD} \wedge dg^{DA} \tag{5.7}$$

is then a closed two form on the space of Lorentzian metrics for any function f of the particle number-entropy ratio. Define h to be the orthogonal projector

$$h_{AB} = g_{AB} + \frac{s_A s_B}{s^2}, \tag{5.8}$$

then we also have

$$\omega = \frac{1}{2}f(n')s^E \varepsilon_{EAB}dh^{BC}h_{CD} \wedge dh^{DA}. \tag{5.9}$$

We may then recover eq. (3.2) by introducing coordinates ϕ^I, $I = 1, 2$ on a spatial slice. Transport them along u^A, and take ϕ^0 to be the affine parameter to obtain a coordinate patch on $\mathcal{M}'$. We call such a system "comoving coordinates" on $\mathcal{M}'$. ω then becomes

$$\omega = \frac{1}{2} f(n') s \epsilon_{IJ} dH^{JK} H_{KL} \wedge dH^{LI}, \tag{5.10}$$

where H_{IK} is h_{IK} normalized to be unimodular, and $\epsilon_{12} = 1$, as in the previous section.

Thus the two-form on $\mathbb{H}^2$ considered earlier may be equivalently thought of as a (more obviously slicing independent) two-form on the 6-dimensional space of Lorentzian metrics. Working in Friedman's formalism also buys us the added freedom to add an arbitrary function of the number-entropy ratio since n' is identically conserved along worldlines. This is the reason we stick to this formalism. In the treatment common to the condensed matter literature involving chemical shifts [4], only the entropy current is identically conserved, whereas n^μ is merely conserved on shell. In the off shell formulation, n' is not then merely a function of the fluid coordinates ϕ^I and promoting f to $f(n')$ would be nonsensical in (3.8).

Writing down the WZW term in coordinates

$$S_{\rm WZW} = \frac{1}{2} \int d^3x \, du \sqrt{-g} f(n') S^\mu \, {\rm Tr}(\epsilon \partial_u H^{-1} H \partial_\mu H^{-1}), \tag{5.11}$$

one may use this to explicitly check interpolation independence as in eq. (3.10). Under an infinitesimal change of interpolation, we find δS is a total u derivative plus a term proportional to $\nabla_\mu (f(n') s^\mu) = 0$.

To complete the comparison with our original treatment, note that in the adapted choice of coordinates given above

$$u^\mu \partial_\mu \phi^I = 0, \qquad s^\mu = \frac{1}{2} \varepsilon^{\mu\nu\lambda} S_{IJ} \partial_\nu \phi^I \partial_\lambda \phi^J, \qquad n^\mu = \frac{1}{2} \varepsilon^{\mu\nu\lambda} n' S_{IJ} \partial_\nu \phi^I \partial_\lambda \phi^J. \tag{5.12}$$

Through a ϕ^0 independent redefinition of the spatial coordinates ϕ^I we may further bring N^μ into the form

$$s^\mu = \frac{1}{2} \varepsilon^{\mu\nu\lambda} \epsilon_{IJ} \partial_\nu \phi^I \partial_\lambda \phi^J. \tag{5.13}$$

If we wish to retain this formula, we may only perform area preserving diffeomorphisms of the coordinates ϕ^I. We thus see our original formalism for a single current arises in this setting via a suitable choice of coordinates on the fluid manifold. Specifically, we choose coordinates on $\mathcal{M}'$ that "follow" the paths of fluid particles, which was our original interpretation of the scalars ϕ^I, plus an arbitrary normalization convention that restricts us to area-preserving diffeomorphisms.

The variation of the action proceeds as before, but is much more easily done in the form (5.7) since we simply have $\delta g^{AB} = \partial_\mu \phi^A \partial_\nu \phi^B \delta g^{\mu\nu}$. We again obtain

$$\Delta T_{\mu\nu} = \eta_H u^\alpha \varepsilon_{\alpha\beta(\mu} \sigma_{\nu)}{}^\beta, \tag{5.14}$$

where the Hall viscosity now has the more general form $\eta_H = s f(n')$.

6 Conclusions

We have shown that the Hall viscosity can be incorporated into the Lagrangian description of ideal fluids through a WZW term. This term gives rise to a nontrivial Hall viscosity, which has to have the form $\eta_H = sf(n/s)$.

However, from standard hydrodynamic arguments based, in particular, on the second law of thermodynamics, the Hall viscosity η_H may be an arbitrary function of s and n [6]. The difficulty may be traced to the stringent restrictions that the topological nature of a WZW term places on its form, only allowing us prefactors that are functions of comoving thermodynamic variables. Only one such quantity exists in general: the specific entropy. Adopting Friedman's formalism, which restricts to fluid configurations with comoving specific entropy, we can thus enhance our description to include Hall viscosities not directly tied to the entropy density, but can go no further. Whether it is possible to obtain a completely arbitrary η_H via an action principle remains an open problem. We note that the action of anomalous hydrodynamics in (3+1) dimensions needs to be formulated on a Schwinger-Keldysh contour [12]. It is possible that the most general Hall viscosity requires the theory to be formulated on such a contour.

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JHEP11(2014)004

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JHEP11(2014)004