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Effective Proton-Neutron Interaction and Spectroscopy of the Nuclei with $N=29$

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The nuclei with $N=29$ and $Z=20\sim28$ are described by a shell model, assuming $(1f_{7/2})^{Z-20}$ configuration for protons and $(2p_{3/2}, 2p_{1/2}, 1f_{5/2})^1$ for one neutron outside the ^{48}Ca core. Effective two-body matrix elements between the $1f_{7/2}$-proton and $(2p_{3/2}, 2p_{1/2}, 1f_{5/2})$-neutron are determined by a least-square fitting to the observed level energies of the $N=29$ isotones. These best-fit matrix elements reproduce the observed properties of the low-lying levels of these nuclei and also the experimental spectroscopic factors for the single-neutron transfer reactions. The observed $M1$-moments are adequately reproduced by using the resulting wave function and an effective g-factor for the neutron.

§ 1. Introduction

In the shell-model calculations, an inert core is assumed and nucleons outside the core are treated as active nucleons in order to describe the low-lying levels of the nucleus. Usually a closed-shell nucleus can be adopted as such an inert core. In the calculation of the $1f$-$2p$ shell nuclei, ^{40}Ca, ^{48}Ca and ^{56}Ni can be assumed as such closed-shell nuclei. The role of ^{48}Ca core is slightly different from that of the other two, because the number of protons and neutrons in ^{48}Ca differs from each other and, therefore, the protons and the neutrons outside the ^{48}Ca core occupy different orbits. Thus, the problem of a nucleus, for which ^{48}Ca can be regarded as an inert core, will be a good example to investigate the correlation between the protons and the neutrons in different orbits.

The rigidity of the ^{48}Ca core can be investigated by the one-nucleon transfer reaction on the ^{48}Ca target. A recent experiment of the (t, α) reactions on Ca-isotopes[1)] shows that the component of the excitation of the proton from the $2s$-$1d$ shell is negligible in the ground state of ^{48}Ca, whereas it is appreciable for ^{40}Ca. The complete closure of the $1f_{7/2}$-neutron shell of ^{48}Ca, however, is shown less clearly than for the system of protons in ^{48}Ca. The amount of the component of the one-neutron excitation from the $1f_{7/2}$ shell to upper shells can be studied by the one-neutron pickup reactions on a ^{48}Ca target. The ^{48}Ca$(p, d)^{47}$Ca reaction[2)] shows only small value of the spectroscopic factor for the excitation to the first $J=3/2^-$ state (2.02 MeV) of ^{47}Ca which is similar to the single-particle state of $2p_{3/2}$ neutron. But such an excitation has not been observed in

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the ^{48}Ca(^{3}He, α)^{47}Ca and the ^{48}Ca(d, t)^{47}Ca reactions.[3] These experimental facts suggest that the $1f_{7/2}$-neutron shell in the ground state of ^{48}Ca is almost completely filled.

Investigating the nuclei with $N=29$ and $20<Z<28$ enables us to determine the effective interactions between the proton and the neutron outside the ^{48}Ca core. The odd nuclei ^{50}Sc and ^{56}Co have been the subject of several investigations,[4),5)] because they can be described in the shell model as the one-neutron and one-proton (particle or hole) system, when protons are restricted to the $1f_{7/2}$ shell.[6] The central forces have usually been assumed as interactions between the proton and the neutron, and the spins of the ground states have been discussed in connection with the Nordheim rule.

Several models are proposed for odd-A nuclei, ^{51}Ti, ^{53}Cr and ^{55}Fe. Ramavataram[7] applied the unified model in which the interactions between the phonons and the odd neutron are treated as an adjustable parameter. Maxwell and Parkinson[8] calculated these three odd-A nuclei using a shell model but considering only the central forces between protons and neutrons. They reproduced the excitation energies of the low-lying states in these nuclei, but it is impossible to understand the single-neutron spectrum in ^{57}Ni by using the central forces. Ohnuma[9] has calculated the spectra of five $N=29$ nuclei with $22\leqq Z\leqq 26$, using the central interactions between the protons and the neutron and by changing the single-neutron energies of the $2p_{1/2}$ and $1f_{5/2}$ orbits relative to that of the $2p_{3/2}$ orbit as free parameters for each nucleus.

The dependence of the single-neutron (or proton) energy on the number of protons (or neutrons) is one of the well-known phenomena which can be explained in terms of the interaction between the proton and the neutron. Such phenomena have been investigated in various mass regions.[10),11)] In the case of nuclei with $N=29$ where the $2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ levels are available, the neutron single-particle spectra difference between ^{49}Ca and ^{57}Ni must be explained by the ieteraction between the $1f_{7/2}$ protons and one neutron outside the ^{48}Ca core.

In this paper we shall explain this phenomenon in the single-particle energies and for the properties of the low-lying states of all nuclei with $N=29$ and $21\leqq Z\leqq 27$. The matrix elements of the effective interactions between the $1f_{7/2}$ proton and the $2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ neutrons are determined in § 2 by a least-square fitting to observed energy levels. Such a method has been applied already to these nuclei by Vervier,[12] but in his investigation the neutron is restricted to the $2p_{3/2}$ orbit. Level structures of individual nuclei are discussed in § 3. The spectroscopic factors of the single-neutron transfer reactions both for the stripping and the pickup processes are predicted and compared with experimental results in § 4. In § 5, the magnetic dipole moments are calculated, and the discrepancies from the experimental values are discussed. We summarize our main conclusions in § 6.

§ 2. Determination of the proton-neutron interaction

A) *Single-particle spectra in* ^{49}Ca *and* ^{57}Ni

The average interaction energy between a particle in j orbit and the one in j' orbit ($j \neq j'$) is expressed in terms of the antisymmetrized two-body matrix elements as

$$\overline{V}(j,j')_T = \sum_J (2J+1)\langle jj'|V|jj'\rangle_{TJ}/(2j+1)(2j'+1),$$

in the respective isospin T states. The interaction energy between the particles in the closed j orbit and one particle in the j' orbit is given in terms of the average energies:

$$E_j(j')_{T=1} = (2j+1)\overline{V}(j,j')_{T=1},$$

if the particles in both orbits are of the same kind of nucleons, and

$$E_j(j')_{pn} = (2j+1)\{\overline{V}(j,j')_{T=0} + \overline{V}(j,j')_{T=1}\}/2,$$

if the particles in both orbits are of different kinds.

The values of $E_j(j')$ can be extracted from the experimental single-particle spectra. The observed single-neutron energies of the $2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ levels in ^{41}Ca, ^{49}Ca and ^{57}Ni are shown in Fig. 1. The spectra of ^{41}Ca and ^{49}Ca are taken from the ^{40}Ca$(d,p)^{41}$Ca and the ^{48}Ca$(d,p)^{49}$Ca reactions,[13),14)] respectively, and the single-neutron energies of these nuclei are estimated by averaging the level energies of the same spin weighted by the spectroscopic factors. By taking three single-particle levels of ^{57}Ni from the ^{58}Ni$(^3$He$,\alpha)^{57}$Ni reaction,[15)] we can determine the values of $E_{f_{7/2}}(j_n)$, $j_n = 2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ for both the the neutron-neutron and the proton-neutron interactions by assuming that the eight neutrons (protons) added in going from ^{40}Ca to ^{48}Ca (from ^{48}Ca to ^{56}Ni) occupy the $1f_{7/2}$ orbit. These values are shown in Table I. The binding energies which are necessary for extracting these values are taken from the 1964 Mass Table.[16)] Table I shows that the interaction between the $1f_{7/2}$ neutron and the $2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ neutrons are all repulsive and rather weak

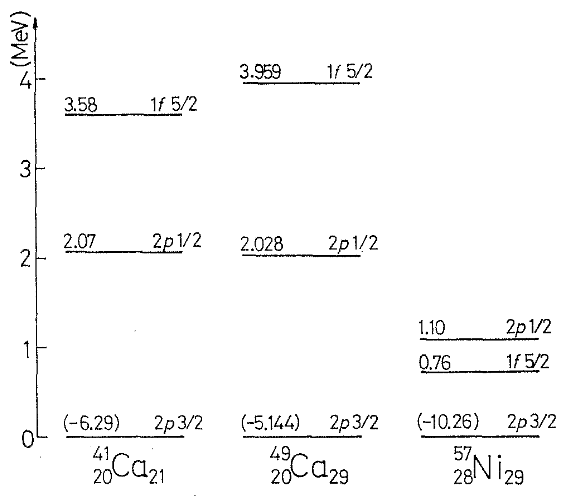

Fig. 1. Single-particle energies of the $2p_{3/2}$-, $2p_{1/2}$- and $1f_{5/2}$-neutron states in the f-p shell region. The single-neutron energies in ^{41}Ca and ^{49}Ca are determined by the averaging over the level energies of the same spin weighed by the spectroscopic factors of the ^{40}Ca$(d,p)^{41}$Ca and the ^{48}Ca$(d,p)^{49}$Ca reactions,[13),14)] respectively. The ^{58}Ni$(^3$He$,\alpha)^{57}$Ni reaction[15)] is used for ^{57}Ni. Energies relative to the ground state of the $N=28$ nucleus with the same Z are given for the $2p_{3/2}$ level in each nucleus. Excitation energies are given for the $2p_{1/2}$ and $1f_{5/2}$ levels.

Table I. Average interaction energies $\overline{V}(j, j')_T$ for $j'=2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$, j being $f_{7/2}$. The values are extracted from the observed single-neutron spectra of ^{41}Ca, ^{49}Ca and ^{57}Ni which are shown in Fig. 1.

j	j'	$\overline{V}(j\,j')_{T=1}$	$\overline{V}(j\,j')_{pn}$
$f_{7/2}$	$p_{3/2}$	0.14	−0.64
$f_{7/2}$	$p_{1/2}$	0.14	−0.76
$f_{7/2}$	$f_{5/2}$	0.21	−1.04

compared with the values of the corresponding proton-neutron interaction. This property of the average interaction energies between the like nucleons has been indicated generally by Talmi[6] in connection with the nuclear binding energies. The large negative values of the interaction energies between the proton and the neutron show that these interactions are strongly attractive.

The differences in the single-neutron spectra between ^{49}Ca and ^{57}Ni are summarized as follows:

1) the large depression of the $1f_{5/2}$-neutron level when the $1f_{7/2}$-proton orbit is completely filled, and

2) the change of the spin-orbit splitting between the $2p_{3/2}-2p_{1/2}$ levels.

If we calculate the radial integral $I(n_1l_1, n_2l_2)$ for the δ-function interactions

$$I(n_1l_1, n_2l_2)=\frac{1}{2}\int R^2_{n_1l_1}(r)\,R^2_{n_2l_2}(r)\,r^2dr\,,$$

using harmonic oscillator wave functions as the radial functions $R_{nl}(r)$, the value of $I(1f, 1f)$ is about twice that of $I(1f, 2p)$.[17] Thus the lowering of the $1f_{5/2}$ level relative to the $2p_{3/2}$ level in going from ^{49}Ca to ^{57}Ni, namely $|E_{f_{7/2}}(f_{5/2})_{pn}|\gg|E_{f_{7/2}}(p_{3/2})_{pn}|$ can easily be explained by the attraction of the two-body proton-neutron interaction. On the other hand, the shift of the spin-orbit splitting, namely $|E_{f_{7/2}}(p_{3/2})_{pn} - E_{f_{7/2}}(p_{1/2})_{pn}|=0.95$ MeV, is difficult to understand by a simple force, such as the delta force, between the proton and the neutron.[10][11] Even if we consider the contribution of the central interaction with any range and reasonable mixture parameters, the spin-orbit splitting remains substantially unaffected. Only the charge-exchange force can slightly change such a splitting, but the amount is very small.

These facts indicate that the effects of proton-neutron forces other than the central forces must be considered, if we are to explain the change of single-particle spectra in the $N=29$ nuclei. The effects might be caused by non-central forces, such as tensor or two-body spin-orbit forces, or by radial wave functions more realistic than the harmonic oscillator wave function. We have therefore adopted the effective interaction method introduced by Talmi,[18] in which we treat the two-body matrix elements of the effective proton-neutron interaction as free parameters and determine them by a least-square fitting to the observed spectra of the nuclei with $N=29$ and $Z=21\sim28$.

B) *Least-square fitting of the proton-neutron interaction*

The assumptions made in describing the low-lying state of the nuclei with $N=29$ (^{49}Ca, ^{50}Sc, ^{51}Ti, ^{52}V, ^{53}Cr, ^{54}Mn, ^{55}Fe, ^{56}Co and ^{57}Ni) are as follows:

1) ^{48}Ca is assumed to be an inert core.
2) $Z-20$ protons are restricted to the $1f_{7/2}$ shell and one active neutron is restricted to the $2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ shells.

We neglect the components of the excitation of the protons in the *s-d* shell and of the neutron in the $1f_{7/2}$ shell. It seems reasonable to neglect the proton excitation from the *s-d* shell to the $1f_{7/2}$ shell, but the neutron excitation from the $1f_{7/2}$ shell to the upper *f-p* shell ($2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ shells) might be enhanced by the interaction with one neutron outside the ^{48}Ca core. It is thus possible that some states in $N=29$ nuclei might include such a state as a main component. In the present investigation, however, we exclude the effects of such excited states on the states assumed here because the coupling between them would not be strong.

The present restriction on the proton configuration can be supported by the systematics of the binding energies and the excitation energies in the nuclei with $N=28$.[6] The $J=3/2^-$ state, whose wave function contains mainly the $2p_{3/2}$ single-proton component, lies at about 3 MeV in ^{49}Sc and at 2.17 MeV in ^{55}Co. Thus some excited states above 2 MeV would be affected by this proton excitation, although its omission seems permissible for almost all low-lying states. We discuss this point in more detail in later sections.

The wave function for each state in the above-assumed configurations can be obtained by the diagonalization of the Hamiltonian H:

$$H=H_{\text{s.p.}}+V_{pp}+V_{pn}.$$

Table II. The excitation energies (in MeV) of the proton states, $|f_{7/2}^{Z-20}v_pJ_p\rangle$, used in the present calculation, where v_p and J_p represent seniority and total spin of the $Z-20$ protons in the $1f_{7/2}$ shall. These values are taken from the observed spectra of the nuclei with $N=28$. Experimental data are from Ref. 9) for ^{50}Ti, Ref. 20) for ^{52}Cr, Ref. 9) for ^{54}Fe, Ref. 9), 20) for ^{51}V and Ref. 22) for ^{53}Mn.

$(v_p, J_p)=$	(0, 0)	(2, 2)	(2, 4)	(2, 6)	(4, 2)	(4, 4)	(4, 5)	(4, 8)
^{50}Ti	0.0	1.570	2.690	3.215				
^{52}Cr	0.0	1.434	2.766	3.112	2.965	2.370	3.614	5.20
^{54}Fe	0.0	1.408	2.540	2.96				
^{56}Ni	0.0							

$(v_p, J_p)=$	(1, 7/2)	(3, 3/2)	(3, 5/2)	(3, 9/2)	(3, 11/2)	(3, 15/2)
^{49}Sc	0.0					
^{51}V	0.0	0.93	0.32	1.81	1.61	2.70
^{53}Mn	0.0	1.29	0.38	1.89	1.52	2.91
^{55}Co	0.0					

Here, $H_{s.p.}$ is the Hamiltonian of the single-particle and we use the value of the single-particle energies,

$$\varepsilon_n(2p_{3/2}) = -5.144 \text{ MeV},$$
$$\varepsilon_n(2p_{1/2}) = -3.116 \text{ MeV},$$
$$\varepsilon_n(1f_{5/2}) = -1.186 \text{ MeV},$$

for the neutron states which are taken form the neutron separation energies of ^{49}Ca, and $\varepsilon_p(1f_{7/2}) = 0.0$ for the proton. V_{pp} represents the interactions among $1f_{7/2}$ protons, while the matrix elements of these interactions for the states with spin J_p and seniority v_p can be obtained from the experimental spectra of the $N=28$ nuclei, and the values adopted in this paper are shown in Table II. V_{pn} represents the effective proton-neutron interactions. Its matrix elements between the $Z-20$ protons and one neutron can be expanded in terms of the two-body matrix elements

$$\langle j_p j_n | V_{pn} | j_p j_n' \rangle_J = \tfrac{1}{2}\{\langle j_p j_n | V | j_p j_n' \rangle_{J,T=0} + \langle j_p j_n | V | j_p j_n' \rangle_{J,T=1}\},$$

where $j_p = 1f_{7/2}$ and j_n, $j_n' = 2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$.

There are 20 two-body matrix elements of proton-neutron interactions. They are treated as free parameters and their values are determined by a least-square fitting to reproduce accurately the 38 observed energies of the low-lying states of the $N=29$ nuclei with $21 \leqq Z \leqq 28$. Eight of these are binding energies of the ground states relative to that of the $N=28$ nucleus with the same Z (the neutron separation energies) while the other 30 are the excitation energies of the excited states. It is mainly in this respect that the present calculations differ from a previous one[23] in which the calculated energies of the excited states also are fitted to the separation energies of the neutron.

The best-fit values of the twelve diagonal and eight nondiagonal elements of the effective proton-neutron interaction are searched for in equal weight. Suitable sets of values are assumed initially and the root-mean-square deviation (rms) is minimized. The rms is defined as

Table III. The two-body matrix elements of the effective proton-neutron interaction $\langle j_p j_n | V_{pn} | j_p j_n' \rangle_J$, $j_p = 1f_{7/2}$. These best-fit values are determined in § 2(B) by the least-square procedure.

j_n	j_n'	$J=1$	$J=2$	$J=3$	$J=4$	$J=5$	$J=6$
$p_{1/2}$	$p_{1/2}$			−0.760	−0.695		
$p_{1/2}$	$p_{3/2}$			−0.536	0.202		
$p_{1/2}$	$f_{5/2}$			−0.866	−0.602		
$p_{3/2}$	$p_{3/2}$		−0.787	−0.444	−0.141	−1.026	
$p_{3/2}$	$f_{5/2}$		0.294	0.306	0.034	−0.074	
$f_{5/2}$	$f_{5/2}$	−2.544	−1.430	−0.592	−1.336	−0.005	−1.345

Table IV. Experimental and calculated eneriges (in MeV) of the levels adopted in the least-square fitting method. For ground states (gnd), the value of the binding energy relative to the N=28 nuclei with the same Z is tabulated; for all other states, the excitation energy is given.

Nucleus	J	$E_{\rm exp}$	$E_{\rm cal}$
^{51}Ti	$(3/2^-)_{\rm gnd}$	−6.379	−6.664
	$(1/2^-)_1$	1.160	1.119
	$(5/2^-)_2$	2.136	2.290
	$(3/2^-)_2$	2.189	2.069
^{53}Cr	$(3/2^-)_{\rm gnd}$	−7.941	−8.053
	$(1/2^-)_1$	0.567	0.569
	$(5/2^-)_1$	1.007	1.211
	$(7/2^-)_1$	1.286	1.332
	$(3/2^-)_2$	2.319	2.234
	$(5/2^-)_3$	2.646	2.528
^{55}Fe	$(3/2^-)_{\rm gnd}$	−9.299	−9.239
	$(1/2^-)_1$	0.417	0.348
	$(5/2^-)_1$	0.935	0.906
	$(7/2^-)_1$	1.327	1.359
	$(1/2^-)_2$	1.926	1.851
	$(3/2^-)_2$	2.061	1.962
	$(5/2^-)_2$	2.159	2.119
	$(3/2^-)_3$	2.490	2.432
^{57}Ni	$(3/2^-)_{\rm gnd}$	−10.26	−10.04
	$(5/2^-)_1$	0.76	0.78
	$(1/2^-)_1$	1.10	1.14
^{50}Sc	$5^+_{\rm gnd}$	−6.060	−6.171
	2_1^+	0.259	0.214
^{52}V	$3^+_{\rm gnd}$	−7.309	−7.470
	2_1^+	0.017	0.000
	5_1^+	0.023	−0.006
	1_1^+	0.142	0.241
	4_1^+	0.148	0.018
	2_2^+	0.437	0.510
	3_2^+	0.794	0.773
	4_2^+	0.846	0.721
	3_3^+	1.418	1.385
	4_3^+	1.559	1.369
^{54}Mn	$3^+_{\rm gnd}$	−8.941	−9.005
^{56}Co	$4^+_{\rm gnd}$	−10.088	−9.931
	3_1^+	0.164	0.267
	5_1^+	0.560	0.511
	2_1^+	0.975	1.057

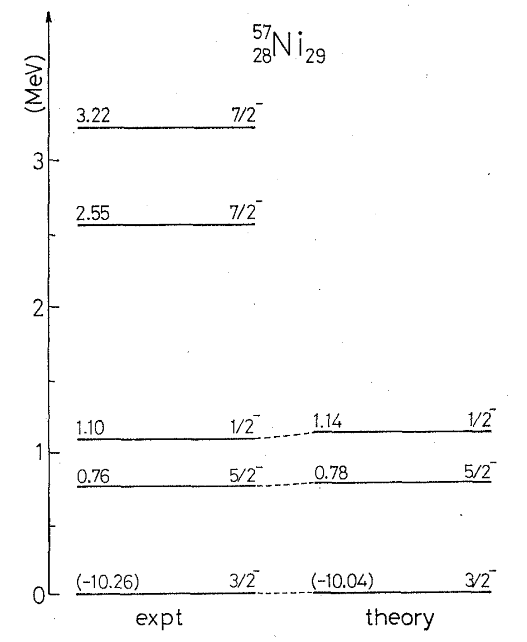

Fig. 2. Experimental[15] and calculated energy levels of ^{57}Ni. The dotted lines connect the experimental levels adopted for the least-square fitting with the corresponding calculated values. The ground-state energies are the values relative to the experimental binding energy of ^{56}Ni. Excitation energies (in MeV) and J^{π} are shown for each level.

$$\mathrm{rms}=\sqrt{\frac{1}{N-k}\sum_{i=1}^{N}(E_{\mathrm{cal}}^{(i)}-E_{\mathrm{exp}}^{(i)})^2},$$

where N is the number of the levels adopted and k is the number of free parameters.

The resulting two-body matrix elements of the effective proton-neutron interactions are given in Table III. The rms is 0.15 MeV for this set of parameters, and agreement between the calculated and the experimental energies is satisfactory (Table IV).

The calculated and observed spectra of ^{57}Ni are shown in Fig. 2. In our model the calculated spectra of ^{57}Ni depend only on the values of the average energies $\overline{V}(f_{7/2}, j_n)_{pn}$ obtained from Table III. They are -0.62, -0.73 and -1.01 for $j_n=p_{3/2}$, $p_{1/2}$ and $f_{5/2}$, respectively. These values are close enough to the experimental values shown in Table I and can satisfactorily reproduce the spectra of ^{57}Ni, i.e., the lowering of the $1f_{5/2}$-neutron orbit and the isospin dependence of the spin-orbit splitting of the $2p$-neutron orbits.

§ 3. Properties of the individual nucleus

In this section we describe calculated results of all the spin states which can be constructed by using our configuration.

A) *Even-Z nuclei* (^{51}Ti, ^{53}Cr *and* ^{55}Fe)

Experimentally, the ground states of ^{51}Ti, ^{53}Cr and ^{55}Fe all have the spin $J=3/2^-$ and these nuclei also have $J=1/2^-$, $5/2^-$ and $7/2^-$ excited states just above the ground states. In our calculations these three excited states are obtained in the observed energy region based upon the assumed configurations.

We suspect, however, that there are effects of other configurations due to the excitations of $f_{7/2}$-neutrons to the upper levels. Estimation of the energies of the neutron states $|f_{7/2}^{-1}\times(p_{3/2}, p_{1/2}, f_{5/2})_0^2; J=7/2^-\rangle$ in these even-Z nuclei by making use of the binding energies of nuclei with $N=30$ and $N=27$, gives 3.1, 2.2 and 2.3 MeV in ^{51}Ti, ^{53}Cr and ^{55}Fe, respectively. Thus, at least one of the low-lying $7/2^-$ levels of these nuclei are to be understood in terms of the states of ^{48}Ca core excitation.

^{51}Ti

The calculated levels with spin $J=1/2^-$, $3/2^-$, $\cdots$, $17/2^-$ are compared with the experimental spectrum[24] in Fig. 3. The agreement between theory and experiment is satisfactory for excited levels up to 2.5 MeV.

The $J=3/2^-$ ground state and the $J=1/2^-$ level at 1.16 MeV are reproduced as states of almost single-particle nature. The two levels at 1.43 MeV and 1.56 MeV which are weakly excited by the (d, p) reaction[34] might be assigned to $J=5/2^-$ or $7/2^-$, according to the weak coupling between the two protons and the 29th neutron. We can calculate the two levels in this energy region, i.e., the $J=(5/2^-)_1$ level*) at 1.37 MeV and the $J=(7/2^-)_1$ level at 1.57 MeV. The calculated excitation energies and the wave functions of these two levels are only weakly affected by the details of the effective proton-neutron interactions. These wave functions have a large component of $|J_p=2^+\times p_{3/2};\ J\rangle$, $J=5/2^-$ and $7/2^-$, where J_p is the total spin of the proton system. The wave function of the $J=(3/2^-)_2$ state at 2.07 MeV is similar to these two levels, i.e., the $|J_p=2^+\times p_{3/2};\ J=3/2^-\rangle$ component is predominant, while the $J=(5/2^-)_2$ state at 2.29 MeV is reproduced as a single-particle-like state, i.e., the $|J_p=0^+\times f_{5/2};\ J=5/2\rangle$ state is predominant.

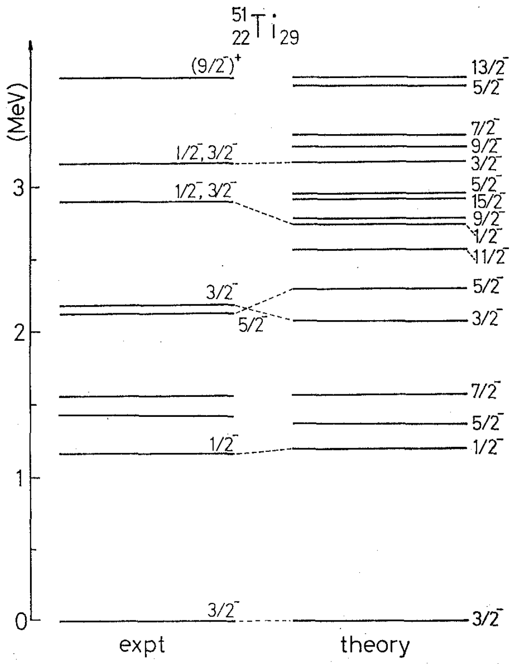

Fig. 3. Experimental[24] and calculated energy levels of ^{51}Ti. Levels with the spin $J=1/2^-$, $3/2^-$,$\cdots$, $17/2^-$ are calculated and the levels below 3.8 MeV in excitation energy are shown. The dotted lines connect the experimental levels adopted for the least-square fitting with the corresponding calculated levels.

Many calculated levels in the region just above 2.5 MeV can be predicted, but only the two levels at 2.90 MeV and 3.16 MeV can be clearly observed by the (d, p) reaction with $l_n=1$. We can obtain two $l_n=1$ levels, i.e., the $J=(1/2^-)_2$ level at 2.63 MeV and the $J=(3/2^-)_3$ level at 3.19 MeV, which might correspond to the observed two levels previously mentioned. Other predicted levels are not observed by the (d, p) reaction. This is because the wave function of these levels mainly contain the excited states of the protons, such as the $|J_p=4^+\times p_{3/2};\ J=11/2^-\rangle$ component for the $J=(11/2^-)_1$ state at 2.56 MeV and the $|J_p=6^+\times p_{3/2};\ J=15/2^-\rangle$ for the $J=(15/2^-)_1$ state at 2.93 MeV.

^{53}Cr

The energy levels with spins $J=1/2^-$, $3/2^-$, $\cdots$, $21/2^-$ which are allowed by the $f^4_{7/2}(v_pJ_p)\times(p_{3/2}, p_{1/2}, f_{5/2})^1$ configuration, are calcurated using the effective proton-neutron interactions obtained in § 2. Comparison between the experimental[25] and the calculated spectra for levels below 3.0 MeV is shown in Fig. 4.

The lowest three levels, the $J=3/2^-$ ground state, the $J=1/2^-$ state at 0.57 MeV and the $J=5/2^-$ state at 1.01 MeV are well reproduced and the agreement, shown in § 4, between the experimental and the theoretical spectroscopic factors of the (d, p) reaction is satisfactory. Just above these three levels, two levels at 1.29

*) $(J^\pi)_n$ represents the n-th calculated state with spin J and parity π, e.g., $J=(5/2^-)_1$ represents the first calculated $J=5/2^-$ level.

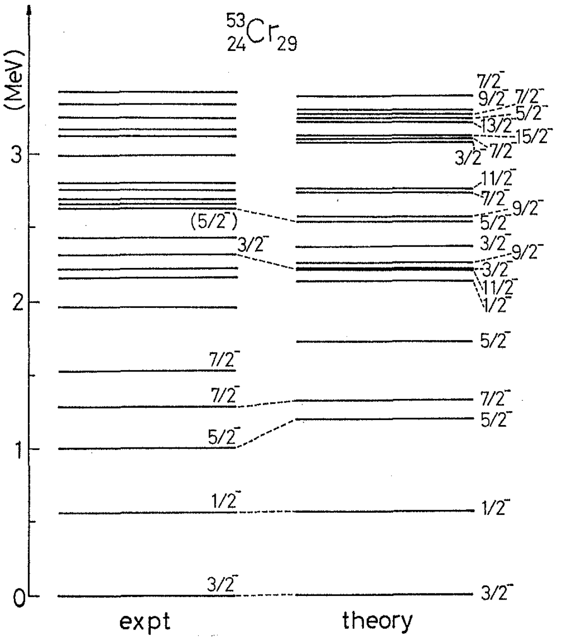

Fig. 4. Experimental[25] and calculated energy levels of ^{53}Cr. Levels with the spin $J=1/2^-$, $3/2^-,\cdots$, $21/2^-$ are calculated and the levels below 3.4 MeV in excitation energy are shown. See the caption of Fig. 3.

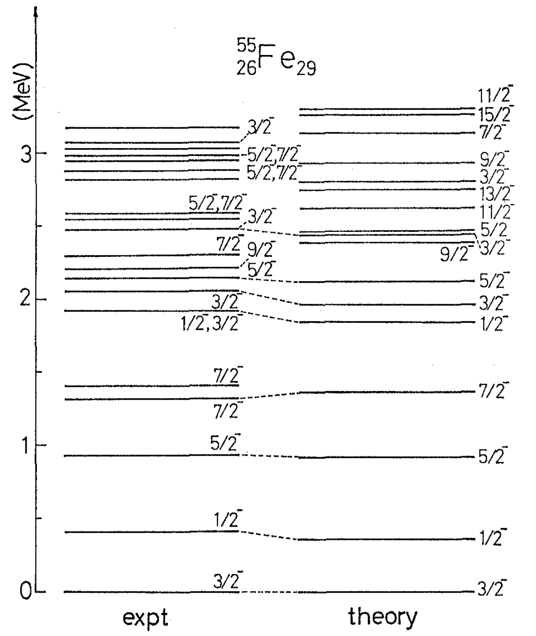

Fig. 5. Experimental[8] and calculated energy levels of ^{55}Fe. Levels with the spin $J=1/2^-$, $3/2^-,\cdots 17/2^-$ are calculated and the levels below 3.3 MeV in excitation energy are also shown. See the caption of Fig. 3.

MeV and 1.59 MeV are observed by the (d, p) reaction.[25~27] Rollefson et al. assigned the spins of both of these states as $J=7/2^-$ [53] by the Cr$(p, p\gamma)^{53}$Cr reaction,[28] but it is almost impossible to obtain two $J=7/2^-$ states in this energy region using the assumed configurations. The recent experiment of the ^{54}Cr(p, d) ^{53}Cr reaction by Whitten[29] shows that although the level at 1.29 MeV is weakly excited, the level at 1.54 MeV is strongly excited. The large $l_n=3$ spectroscopic factor for the 1.54 MeV state in the neutron pickup reaction suggests that this state is achieved by picking out the $1f_{7/2}$ neutron and that the main component is the two-particle one-hole state of the neutron system. Only the level at 1.29 MeV is adopted as the $J=(7/2^-)_1$ level for the present least-square fitting but we can also obtain the $J=(7/2^-)_1$ state at 1.33 MeV. The calculated energy of the $J=(7/2^-)_2$ state is 2.75 MeV.

The level at 1.97 MeV is weakly excited by the (p, d) and the (d, p) reactions. In this energy region there is one calculated level at 1.72 MeV with $J=(5/2^-)_2$ which has a small spectroscopic factor for the one-neutron stripping reaction. We may predict that this level decays mainly to the ground state by the $E2$ transition, which is consistent with the experimental fact that this state decays only to the ground state.[28]

^{55}Fe

The comparison between the experimental[8] and the theoretical energy levels of ^{55}Fe below 3.2 MeV is shown in Fig. 5.

The present calculation clearly reproduces the observed tendency for the excitation energies of the $J=(1/2^-)_1$ and the $J=(5/2^-)_1$ levels to decrease as the number of proton pairs in the $f_{7/2}$ shell increases. Agreement between the calculated and the observed spectroscopic factors is also satisfactary for these two states and for the $J=3/2^-$ ground state. The $J=(5/2^-)_1$ state decays to the $J=3/2^-$ ground state by the $M1$ and $E2$ transitions. The measured value of the mixing ratio of these two transitions, $\delta(E2/M1)$, is 0.36.[30] The calculated value of δ is 1.9 when we introduce the effective charge which reproduces the $E2$ transition from the $J=2_1^+$ state to the $J=0^+$ ground state of ^{54}Fe. The discrepancy between the calculated and measured δ values would decrease if the effect of the small admixture of the other configuration is introduced to this l-forbidden $M1$ transition.

The levels at 1.32 MeV and 1.41 MeV are weakly excited by the (d, p) reaction,[8] whereas they are strongly excited by the (p, d) reaction[31] which can be assigned to the states with the spin $J=7/2^-$ from the J-dependence in the angular distribution of this pickup reaction. It is difficult to explain the two $J=7/2^-$ levels in this energy region for the same reason as mentioned in the case of ^{53}Cr. The ratio of the (p, d) cross sections for the 1.32 MeV and 1.41 MeV states is 2.4 : 8.0. The 1.43 MeV state seems to be the two-particle one-hole state for the neutrons, so we have adopted the level at 1.32 MeV as the $J=(7/2^-)_1$ state in the least-square fitting. The calculated $J=(7/2^-)_1$ state is at 1.36 MeV whose main component is the $|J_p=2^+\times p_{3/2};\ J=7/2^-\rangle$ state while the $J=(7/2^-)_2$ state is calculated to be at 3.13 MeV.

The level at 1.92 MeV, which is excited by the (d, p) reaction with the $l_n=1$ neutron angular distribution, is assigned as the $J=(1/2^-)_2$ state, because we can always obtain the $J=(1/2^-)_2$ and $J=(3/2^-)_2$ states in this energy region and the $(n, \gamma\gamma)$ reaction[32] shows that the level at 2.05 MeV is the $J=3/2^-$ state. The $J=9/2^-$ level calculated at 2.38 MeV is in good agreement with the level at 2.21 MeV which is observed by the β-decay of the ^{55}Co.[33] The wave function obtained shows that this state is composed of the components $|J_p=2^+\times f_{5/2};\ J=9/2^-\rangle$, $|J_p=4^+\times p_{3/2};\ J=9/2^-\rangle$ and so on.

B) *Odd-Z nuclei* (^{50}Sc, ^{52}V, ^{54}Mn *and* ^{56}Co)

^{50}Sc

The structure of ^{50}Sc is the simplest that can be described within the 1 proton-1 neutron configuration. Since the lowest neutron level in ^{49}Ca is $2p_{3/2}$ and the next level, $2p_{1/2}$ is about 2 MeV higher, the four lowest calculated levels of ^{50}Sc are almost pure $|(f_{7/2})_p(p_{3/2})_n;\ J\rangle$ states with $J=2^+, \cdots, 5^+$. Experimentally the four levels are observed below 1 MeV and the spins of the ground state and of the first excited state can be assigned as $J=5^+$ and $J=2^+$, respectively, which agree with the present calculation (Fig. 6). The spins of the other two levels which are observed at 0.331 MeV and 0.761 MeV are not yet assigned, but in

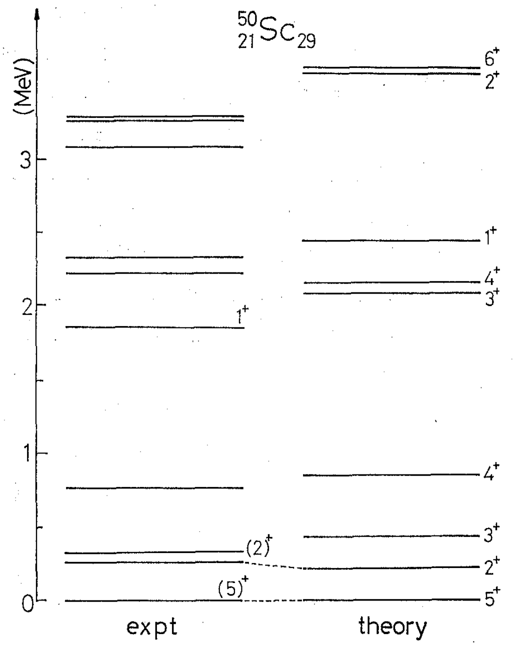

Fig. 6. Experimental[36] and calculated energy levels of ^{50}Sc. Levels with the spin $J=1^+$, 2^+, $\cdots$, 6^+ are calculated and the levels below 3.7 Mev in excitation energy are also shown. See the caption of Fig. 3.

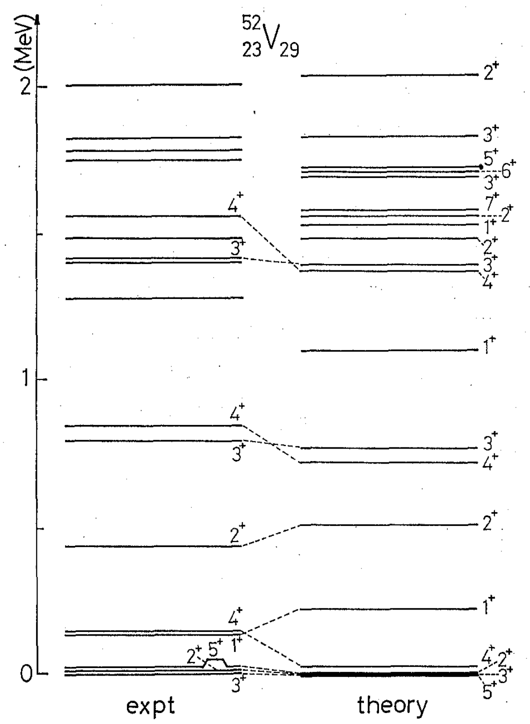

Fig. 7. Experimental[38] and calculated energy levels of ^{52}V. Levels with the spin $J=0^+$, 1^+, $\cdots$, 10^+ are calculated and the levels below 2.1 MeV in excitation energy are also shown. See the caption of Fig. 3.

the present model the $J=3^+$ and $J=4^+$ states are calculated at 0.43 MeV and 0.85 MeV regions, respectively.

The $J=1^+$ state is observed at 1.84 MeV by the β-decay of ^{50}Ca[34),35)] and the ^{48}Ca(^{3}He, p)^{50}Sc reaction,[36)] but in our model the $J=1^+$ state can be formed only from the $(f_{7/2})_p{}^1(f_{5/2})_n{}^1$ configuration. The measured log ft value, 4.2 and the enhanced cross section of the ^{48}Ca(^{3}He, p)^{50}Sc reaction[36)] to this $J=1^+$ state suggest extensive mixing of the $p_{3/2}$-proton component. It would be reasonable for the main component of this $J=1^+$ state to be the $|(p_{3/2})_p(p_{3/2})_n; J=1^+\rangle$ state. The $|(f_{7/2})_p(f_{5/2})_n; J=1^+\rangle$ state is calculated at 2.44 MeV. Hughes and Soga[37)] have recently calculated this $J=1^+$ state by including the components of the $f_{7/2}$ proton excitations to the upper shells. They predicted the $J=1^+$ level at 1.76 MeV which agrees well with the experimental result. The prediction of the $J=1^+$ level of ^{50}Sc is one of the weakest points in our model due to the restriction of the proton configuration. We shall mention a similar situation in ^{56}Co.

^{52}V

Precise measurement of the low-lying states of ^{52}V was carried out by the ^{51}V(n, γ)^{52}V reaction.[38)] The low-lying spectra of ^{52}V is characterized by the existence of two doublets above the $J=3^+$ ground state. The energy levels with the spin $J=0^+$, 1^+, $\cdots$, 10^+ are calculated and compared with the observed levels below 2.0 MeV excitation energy (Fig. 7).

The $J=3^+$ ground state and the first doublet (the $J=2_1^+$ at 17 keV and the $J=5_1^+$ at 21 keV) are reproduced well. The mixing of the $p_{1/2}$- and $f_{5/2}$-neutron states at these levels is small, but the component of the $f_{7/2}^3(v_p=3, J_p)$ proton state cannot be ignored. The second doublet of the $J=1_1^+$ level at 142 keV and the $J=4_1^+$ level at 148keV have not been reproduced so satisfactorily as the first, i.e., the calculated excitation energy of the $J=4_1^+$ level is too low. The calculated energy of the $J=1_1^+$ state, which contains the $|f_{7/2}^3(v_p=3, J_p=5/2)p_{3/2}; J=1^+\rangle$ state as its main component (about 90%), is 0.24 MeV.

^{54}Mn

The levels with the spin $J=0^+, 1^+, \cdots, 10^+$ are calculated within the present assupmptions on the configuration. The observed spectrum of ^{54}Mn obtained by the ^{56}Fe$(d, \alpha)^{54}$Mn reaction[27] is shown with the theoretical spectrum in Fig. 8. Only $J=3^+$ ground state is adopted for the present least-square fitting. Agreement between theoretical and experimental ground state energy values and the level spacings at low energy is satisfactory.

^{56}Co

The low-lying levels of ^{56}Co can be described as the one-particle one-hole states relative to a doubly magic nucleus ^{56}Ni. Comparison of the calculated

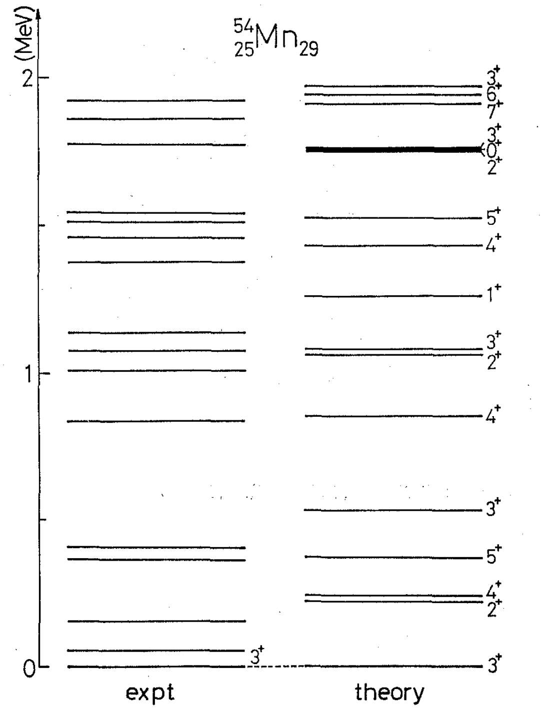

Fig. 8. Experimental[27] and calculated energy levels of ^{54}Mn. $0^+, 1^+, \cdots, 10^+$ are calculated and the levels below 2.1 MeV in excitation energy are also shown. See the caption of Fig. 3.

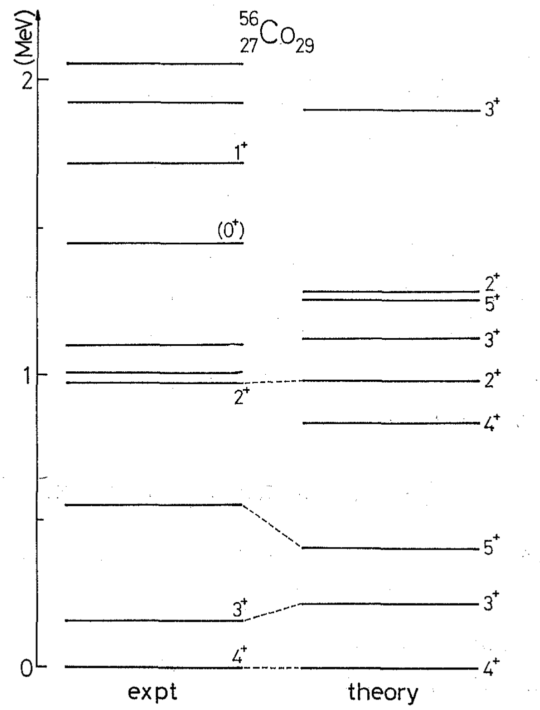

Fig. 9. Experimental[41] and calculated energy levels of ^{56}Co. Levels with the spin $J=1^+, 2^+, \cdots, 6^+$ are calculated and the levels below 2.1 MeV in excitation energy are shown. See the caption of Fig. 3.

spectrum with experimental values[40),41)] is shown in Fig. 9.

The $J=4^+$ ground state and the $J=3^+$ state at 0.16 MeV are reproduced satisfactorily as almost pure $|(f_{7/2}^{-1})_p(p_{3/2})_n; J\rangle$ states. It is natural to consider that the $J=5^+$ state would appear in low-excitation-energy region, because it can be formed by the coupling of the $f_{7/2}$ proton to the $p_{3/2}$ neutron. There is an observed level at 0.56 MeV which cannot be reached by the γ-decay of the upper states with low spin,[40),41)] but this level is excited by the reactions of ^{56}Fe$(p, n)^{56}$Co[42)] and ^{58}Ni$(d, \alpha)^{56}$Co.[27)] In employing the least-square fitting we have assumed that this observed level is the $J=5_1^+$ state, but the calculated excitation energy of the $J=5_1^+$ state is 0.41 MeV. This discrepancy is small.

The $J=4_2^+$ state calculated at 0.84 MeV has strongly mixed configurations. In this energy region one level is observed at 0.83 MeV, but its spin is not assigned. The calculated wave function of the $J=2_1^+$ state at 0.98 MeV which corresponds with the level observed at 0.98 MeV, has the large component of $|(f_{7/2}^{-1})_p(f_{5/2})_n; J=2^+\rangle$.

Although we can get good agreements for the levels below 1 MeV, we cannot explain the $J=1^+$ state at 1.72 MeV observed by β-decay of ^{56}Ni.[41),42)] The $J=1^+$ state can be produced only by $|(f_{7/2}^{-1})_p(f_{5/2})_n; J=1^+\rangle$ in the present model in which the calculated excitation energy, due to the repulsion of the particle-hole matrix element $\langle f_{7/2}^{-1}f_{5/2}|V_{pn}|f_{7/2}^{-1}f_{5/2}\rangle_{J=1}$, is 3.50 MeV. The calculated $\log ft$ value is 2.6 but the observed value is 4.5. This disagreement would be disappear if we take account of the $p_{3/2}$-proton state. Armstrong et al. observed the almost $p_{3/2}$-single proton state at 2.19 MeV in ^{55}Co by the ^{54}Fe$(^3$He$, d)^{55}$Co reaction,[43)] which suggests that the proton excitation from the $f_{7/2}$ shell has important effects at about 2 MeV region in the excitation of ^{56}Co. Recently, Belote et al.[44)] investigated the low-lying states of ^{56}Co with ^{54}Fe$(^3$He$, p)^{56}$Co reaction. The large cross sections to the 1.43 MeV and 1.72 MeV levels indicate that these states are formed by adding the proton-neutron pair in the $p_{3/2}$ shell to the ^{54}Fe target. They assigned the spin of these states as $J=0^+$ and $J=1^+$, respectively. More precise theoretical calculation including the excitations from the $f_{7/2}$ shell would be required for these levels to be explained.

§ 4. Spectroscopic factors for one-nucleon transfer reactions

A) *One-neutron stripping reactions*

We give here a comparison between the observed and calculated spectroscopic factors for one-neutron transfer reactions which will be a good test of the model and especially of the neutron configuration assumed. Except for the ^{49}Sc, ^{53}Mn and ^{55}Co nuclei, the experimental data derived from the (d, p) reaction on the $N=28$ nuclei are available. In this case, the spectroscopic factor S_{lj} is defined as follows:

$$S_{lj}=\langle\psi_{N=28}(J_{gnd})\times\phi_{lj}|\psi_{N=29}(J)\rangle^2,$$

where ϕ_{lj} is a single neutron wave function of the lj-orbit, and $\psi_{N=28}(J_{gnd})$ and $\psi_{N=29}(J)$ are wave functions of the target and of the final states, respectively.

In Tables V$\sim$VIII the calculated values of $(2J+1)S_{lj}$ are compared with those obtained by experiments. Agreements between the theoretical and experimental values are generally satisfactory, although there are discrepancies for some levels. In the ^{52}Cr$(d, p)^{53}$Cr reaction, the transition to the $J=7/2^-$ level at 1.29 MeV which is forbidden within the DWBA theory, shows a small but observable value of the spectroscopic foctors. The component of the neutron excitation from the $f_{7/2}$ shell may be included in this lower $J=7/2^-$ state. There is also a large discrepancy for the $J=3/2^-$ state observed at 2.32 MeV. The calculated spectroscopic factors of the $J=3/2^-$ levels at 2.21 MeV and 2.30 MeV are small, $(2J+1)S=0.34$ and 0.33, respectively, but the experimental spectroscopic factor of this level is 1.26, suggesting that more than 30% admixture of the single $p_{3/2}$ neutron states occurs.

The calculated spectroscopic factors for one-neutron stripping reaction to ^{55}Fe are compared with the results of the ^{54}Fe$(d, p)^{55}$Fe reaction.[8] Agreement between them is fairly good for the levels which are adopted for the least-square fitting procedure. We cannot explain the transition to the two $J=7/2^-$ states near 1.4 MeV in which the two-neutron excitation from the $f_{7/2}$ shell must be considered in the ground state of ^{54}Fe. The calculated spectroscopic factor for the $J=(5/2^-)_3$ level at 2.45 MeV is fairly large $((2J+1)S=1.26)$; but it is not certain to which observed level this corresponds.

^{51}V is the only stable nucleus with $N=28$ and odd-Z. The ^{51}V$(d, p)^{52}$V reaction has been carried out by Catala et at.[44] The calculated and experimental spectroscopic factors shown in Table VIII are relative values which are normalized to a value of 100 for the $l_n=1$ transition to the $J=3^+_{gnd}$ of ^{52}V. The mixing

Table V. Experimental[24] and calculated spectroscopic factors S for the ^{50}Ti (d, p) ^{51}Ti reaction. The target nucleus is assumed as a neutron-closed nucleus. Angular momentum of the transferred neutron, l_n, is 1 for the excitation to the $J=3/2^-$ and $1/2^-$ states, and $l_n=3$ for the $J=5/2^-$.

Experiment			Calculation		
E	J^π	$(2J+1)S$	E	J^π	$(2J+1)S$
g.s.	$3/2^-$	3.65	g.s.	$(3/2^-)_1$	3.60
1.160	$1/2^-$	1.71	1.247	$(1/2^-)_1$	1.41
1.429			1.365	$(5/2^-)_1$	0.06
1.559			1.570	$(7/2^-)_1$	
2.136	$5/2^-$	2.36	2.154	$(5/2^-)_2$	3.02
2.189	$3/2^-$	0.34	2.027	$(3/2^-)_2$	0.22
2.986	$1/2^-$, $3/2^-$	0.74	2.672	$(1/2^-)_2$	0.26
3.164			2.894	$(5/2^-)_3$	1.14
3.759	$1/2^-$, $3/2^-$	0.41	2.961	$(3/2^-)_3$	0.15

of the $f_{5/2}$ neutron in the low-lying levels of ^{52}V is small and these levels are excited mainly by capturing the $l_n=1$ neutron. This tendency is also observed by the experiment.

B) *One-neutron pickup reactions*

In addition to the one-neutron stripping reactions, the one-neutron pickup reactions on the $N=30$ nuclei can be used to check the neutron configuration. In Tables IX, X and XI, we show some results of the calculations for the one-

Table VI. Experimental[25] and calculated spectroscopic factors S for the ^{52}Cr$(d, p)^{53}$Cr reaction. See the caption of Table V.

Experiment			Calculation		
E	J^π	$(2J+1)S$	E	J^π	$(2J+1)S$
g.s.	$3/2^-$	3.06	g.s.	$(3/2^-)_1$	3.27
0.567	$1/2^-$	0.95	0.605	$(1/2^-)_1$	1.09
1.007	$5/2^-$	2.43	1.115	$(5/2^-)_1$	2.16
1.286	$7/2^-$		1.313	$(7/2^-)_1$	
1.531	$7/2^-$				
1.963			1.603	$(5/2^-)_2$	0.52
2.158			2.124	$(1/2^-)_2$	0.26
2.221					
2.319	$3/2^-$	1.26	2.208	$(3/2^-)_2$	0.34
2.439			2.304	$(3/2^-)_3$	0.33
2.646	$5/2^-$	0.8	2.496	$(5/2^-)_3$	1.12
2.658	$1/2^-$, $3/2^-$	0.13			
2.699	$1/2^-$, $3/2^-$	0.049			

Table VII. Experimental[8] and calculated spectroscopic factors S for the ^{54}Fe$(d,p)^{55}$Fe reaction. See the caption of Table V.

Experiment			Calculation		
E	J^π	$(2J+1)S$	E	J^π	$(2J+1)S$
g.s.	$3/2^-$	3.25	g.s.	$(3/2^-)_1$	3.26
0.417	$1/2^-$	1.20	0.392	$(1/2^-)_1$	1.00
0.935	$5/2^-$	3.60	0.859	$(5/2^-)_1$	3.13
1.327	$7/2^-$	0.33	1.343	$(7/2^-)_1$	
1.413	$7/2^-$	0.14			
1.926	$1/2^-$, $3/2^-$	0.26	1.845	$(1/2^-)_2$	0.34
2.061	$3/2^-$	0.38	1.867	$(3/2^-)_2$	0.28
2.159	$5/2^-$	0.89	2.041	$(5/2^-)_2$	0.84
2.218			2.447	$(5/2^-)_3$	1.26
2.30	$7/2^-$			$(7/2^-)_2$	
2.490	$3/2^-$	0.77	2.405	$(3/2^-)_3$	0.43

Table VIII. Experimental[44] and calculated spectroscopic factors S_{rel} for the $^{51}V(d,p)^{52}V$ reaction. $(2J+1)S_{rel}$ are normalized to a value of 100 for the transition to the ground state with $l_n=1$. The calculated S_{rel} value of the $l_n=1$ transition are obtained by summing over the spectroscopic factors for the $2p_{3/2}$- and $2p_{1/2}$-neutron stripping processes and the S_{rel} value of the $l_n=3$ transition corresponding to the $1f_{5/2}$-neutron stripping process.

Experiment				Calculation			
E	J^π	$(2J+1)S_{rel}$		E	J^π	$(2J+1)S_{rel}$	
		$l_n=1$	$l_n=3$			$l_n=1$	$l_n=3$
g.s.	(3^+)	100		g.s.	3_1^+	100	0.6
0.020	$(2^+, 5^+)$	325		0.000	2_1^+	59.5	1.4
				−0.006	5_1^+	205.6	0.0
0.145	$(1^+, 4^+)$	90		0.018	4_1^+	69.3	0.7
				0.242	1_1^+		2.3
0.431	(2^+)	13		0.510	2_2^+	15.1	4.7
0.787	(3^+)	60		0.773	3_2^+	48.3	13.2
0.838	(4^+)	99		0.721	4_2^+	102.1	4.0
				1.098	1_2^+		13.1
1.471	(3^+)	21		1.385	3_3^+	20.2	0.4
1.492		0.5	14.3	1.480	2_3^+	4.9	9.5
1.557	(4^+)	100		1.369	4_3^+	117.2	2.0
1.559		10		1.532	1_3^+		13.1
				1.566	2_4^+	1.7	40.7
				1.583	7_1^+		
1.756		2.5	17.5	1.695	3_4^+	27.1	29.9
1.792		14		1.724	5_2^+	1.4	0.0
1.843		0	14.5	1.711	6_1^+		6.0
2.097		34		1.829	3_5^+	41.9	3.2
				2.119	0_1^+		
				2.140	5_3^+	0.8	86.2
2.143		0	77	2.182	6_2^+		72.1
2.166		22		2.040	2_5^+	18.6	2.0

neutron pickup reactions on the $N=30$ nuclei which are stable. Wave functions of the nuclei with $N=30$ and $20\leqq Z\leqq 28$ have been calculated[46] assuming the $(1f_{7/2}{}^{Z-20})_p(2p_{3/2}, 2p_{1/2}, 1f_{5/2})_n{}^2$ configuration and using the effective proton-neutron interactions determined in § 2 and the effective neutron-neutron interactions fitted to the observed levels of Ni-isotopes.[47] We have used these wave functions as a ground state of the $N=30$ target nuclei.

The calculated S_{lj} for the pickup of the lj-neutron is defined similarly to the S_{lj} in the stripping process, i.e., $S_{lj}=\langle\psi_{N=29}(J)\times\phi_{lj}|\psi_{N=30}(J_{gnd})\rangle$. The spectroscopic factors measured by the $^{54}Cr(p,d)^{53}Cr$ reaction[29] are compared with the calculated values in Table IX. In the present calculations, only the $J=3/2^-$, $1/2^-$ and $5/2^-$ levels can be excited by the pickup of the $p_{3/2}$, $p_{1/2}$ and $f_{5/2}$ neutron in the $J=0$ ground state of the target nucleus ^{54}Cr. The observed excitations of the $J=1/2^-$ and $J=5/2^-$ states as well as the $J=3/2^-$ states suggest mixing of the $(p_{1/2}^2)_n$ and $(f_{5/2}^2)_n$ components in the ground state of ^{54}Cr. These pxperimental facts can be reproduced in our calculation.

In the case of the (p, d) reaction on the ^{56}Fe target, the situation is similar to the ^{54}Cr$(p, d)^{53}$Cr reaction. The calculated spectroscopic factors are compared with the ^{56}Fe$(p, d)^{55}$Fe reaction[47] in Table X. The large $S_{\rm exp}$ with $l_n=3$ for the $J=7/2^-$ level at 1.42 MeV suggests that this level is excited mainly by the pickup of the $1f_{7/2}$ neutron.

The spectroscopy of ^{54}Mn can be investigated by the reaction of the one-neutron pickup from ^{55}Mn. The well-known anomalous spin $J=5/2^-$ of the ground state of ^{55}Mn is well reproduced with the shell model.[46] The pickup of

Table IX. Experimental[29] and calculated spectroscopic factors S, for the ^{54}Cr$(p, d)^{53}$Cr reaction. The shell-model wave function from Ref. 45) is used as a target nucleus. The angular momentum of the transferred neutron l_n is 1 for excitation of the $J=3/2^-$ and $1/2^-$ states and $l_n=3$ for the $J=5/2^-$ state.

Experiment				Calculation		
E	J^π	l_n	C^2S	E	J^π	S
g.s.	$3/2^-$	1	0.83	g.s.	$(3/2^-)_1$	1.41
0.563	$1/2^-$	1	0.31	0.605	$(1/2^-)_1$	0.16
1.001	$5/2^-$	3	0.51	1.115	$(5/2^-)_1$	0.23
1.284	$7/2^-$	3	0.70	1.313	$(7/2^-)_1$	
1.532	$7/2^-$	3	3.2			
1.967		(3)		1.603	$(5/2^-)_2$	0.02
				2.124	$(1/2^-)_2$	0.04
2.319	$3/2^-$	(1)		2.208	$(3/2^-)_2$	0.00
				2.304	$(3/2^-)_3$	0.01
2.654	$5/2^-$	(3)		2.496	$(5/2^-)_3$	0.00

Table X. Experimental[46] and calculated spectroscopic factors S for the ^{56}Fe$(p, d)^{55}$Fe reaction. See the caption of Table IX.

Experiment				Calculation		
E	J^π	l_n	C^2S	E	J^π	S
g.s.	$3/2^-$	1	1.0	g.s.	$(3/2^-)_1$	1.27
0.39		1	0.42	0.392	$(3/2^-)_1$	0.17
0.95	$5/2^-$	3	0.87	0.859	$(5/2^-)_1$	0.34
				1.343	$(7/2^-)_1$	
1.42	$7/2^-$	3	6.1			
				1.845	$(1/2^-)_2$	0.03
				1.867	$(3/2^-)_2$	0.01
				2.041	$(5/2^-)_2$	0.01

Table XI. Experimental[46] and calculated spectroscopic factors S for the $^{55}Mn(p, d)^{54}Mn$ reaction. The calculated S value of the $l_n=1$ transition is obtained by summing the spectroscopic factors for the $2p_{3/2}$-and $2p_{1/2}$-neutron pickup processes and the calculated S value of the $l_n=3$ transition corresponding to the $1f_{5/2}$-neutron pickup process.

Experiment				Calculat.on			
E	J^π	S		E	J^π	S	
		$l_n=1$	$l_n=3$			$l_n=1$	$l_n=3$
g.s.		0.83		g.s.	3_1^+	0.52	0.04
				0.223	2_1^+	0.42	0.03
				0.235	4_1^+	0.22	0.01
				0.370	5_1^+		0.01
0.36		0.27		0.534	3_2^+	0.23	0.06
				0.854	4_2^+	0.01	0.10
1.12		0.12	1.2	1.064	2_2^+	0.03	0.00
				1.074	3_3^+	0.02	0.01
				1.254	1_1^+	0.00	0.00
1.46		0.13	0.67	1.434	4_3^+	0.01	0.01
				1.525	5_2^+		0.02

the $p_{3/2}$, $p_{1/2}$ and $f_{5/2}$ neutron from $^{55}Mn(J=5/2^-)$ excites the level with the spin $J=0^+$, 1^+, $\cdots$, 5^+. In Table XI we show the calculated spectroscopic factors with the results of the experiment on $^{55}Mn(p, d)^{54}Mn$ reaction.[47] The wave function of the $J=5/2^-$ ground state of ^{55}Mn, which has strongly mixed components, i.e., $|J_p=5/2^-\times p_{3/2}^2(0); J=5/2^-\rangle$, $|J_p=7/2^-\times p_{3/2}^2(2); J=5/2^-\rangle$ and so on, yields small excitation on the level of ^{54}Mn.

§ 5. Magnetic dipole moments

The magnetic moments of the ground states of the $N=29$ nuclei are calculated by making use of the wave functions obtained in § 2. The operator of the magnetic moment is given by $\boldsymbol{\mu}=\sum_i \boldsymbol{\mu}_i$, where $\boldsymbol{\mu}_i=g_s\boldsymbol{s}_i+g_l\boldsymbol{l}_i$. The gyromagnetic ratios g_s and g_l are usually assumed as $g_s=5.58$ and -3.82 n.m. and $g_l=1$ and 0 for protons and neutrons, respectively. Incidentally, our phase convention for the wave function is given by the coupling order $\boldsymbol{j}=\boldsymbol{s}+\boldsymbol{l}$.

The calculated values $\mu_{\rm cal}$ are given in Table XII together with experimental values.[48] The calculated values for the $3/2^-$ ground states of ^{51}Ti, ^{53}Cr and ^{55}Fe deviate from the Schmidt value (-1.91 n.m.) of the $p_{3/2}$ neutron. The deviation is mainly caused by the admixture of the $|J_p=2^+\times(p_{3/2})_n; J=3/2^-\rangle$ component to the ground states by the proton-neutron interaction. There is, however, still a large discrepancy between the calculated and the experimental values for ^{53}Cr which cannot be reduced appreciably by changing the proton-neutron interactions. It is likely, therefore, that the small admixtures of the omitted configurations in

Table XII. Magnetic dipole moments of the ground states of the nuclei with N=29. Experimental values μ_{exp} (in n.m.) are compared with the calculated values (μ_{cal} and μ_{cal}'). μ_{cal} and μ_{cal}' are calculated with the bare value of the neutron g-factor ($g_{s,n}=-3.82$ n.m.), and with the effective g-factor for neutron ($g'_{s,n}=0.5g_{s,n}$), respectively. In both cases, $g_{s,n}$=5.58 n.m., $g_{l,p}$=1.0 n.m. and $g_{l,n}$=0.0n.m. are used.

Nuclei	J^{π}	μ_{exp}	μ_{cal}	μ_{cal}'
^{51}Ti	3/2$^-$		−1.65	−0.72
^{53}Cr	3/2$^-$	−0.47	−1.43	−0.51
^{55}Fe	3/2$^-$		−1.44	−0.51
^{57}Ni	3/2$^-$		−1.91	−0.96
^{50}Sc	5$^+$		3.88	4.84
^{52}V	3$^+$		3.30	3.93
^{54}Mn	3$^+$	3.30	3.03	3.63
^{56}Co	4$^+$	3.86	3.64	4.48

the present calculation give rise to rather large deviations of the magnetic moments.[17] If the effect of the configuration mixing can be taken into account by the renormalization of the gyromagnetic ratio, we can replace the neutron g_s from the bare value $g_{s,n}=-3.82$ n.m. by the effective g-factor $g'_{s,n}$. In order to obtain agreement in ^{53}Cr, the effective g-factor must become $g'_{s,n}\cong 0.5g_{s,n}$. The calculated values μ'_{cal} of the magnetic moments of the $N=29$ nuclei with this effective g-factor $g'_{s,n}$ instead of $g_{s,n}$ are also show in Table XII. The calculated values μ_{cal} of the odd nuclei ^{54}Mn and ^{56}Co show fairly good agreement with the experimental values μ_{exp} and the μ'_{cal} also give good agreement, although there are some ambiguities in the effective g-factors for protons in the case of the odd nuclei. Furthermore, by adopting the above effective g-factor $g'_{s,n}$, we can obtain better agreement with experimental values of the magnetic moments for the Ni-isotopes which have been calculated by Cohen et al.[46] In order to understand the $g'_{s,n}$, it is important to include the effect of the excitations of the $f_{7/2}$ neutrons to the upper orbits, especially to the $f_{5/2}$ orbit.

§ 6. Summary and conclusion

The properties of the low-lying states of the $N=29$ nuclei have been investigated in terms of the shell model based upon the assumption that there are $Z-20$ protons in the $f_{7/2}$ orbit and one neutron in the $p_{3/2}$, $p_{1/2}$ and $f_{5/2}$ orbits outside the ^{48}Ca core. The matrix elements of the proton-proton interaction have been taken from the low-lying energy levels of the $N=28$ nuclei, while those of the proton-neutron interaction have been determined by a least-square fitting to the observed energy spectra of the $N=29$ nuclei. The small resulting rms deviation indicates that the present shell-model calculations are appropriate for the states adopted for the least-square fitting mentioned in § 2. The matrix elements derived from the calculations give reasonable results, not only for the energy spectra but also for the spectroscopic factors of the neutron transfer reactions.

The change of the central field of the shell model from ^{48}Ca to ^{56}Ni has been explained in terms of the interactions between the eight additional protons in the $f_{7/2}$ orbit in ^{56}Ni and one neutron in the $p_{3/2}$, $p_{1/2}$ and $f_{5/2}$ orbits. The importance of the non-central forces between the protons and the neutron outside the ^{48}Ca core has been shown experimentally from the change of the $p_{3/2}$-$p_{1/2}$ splitting in the single-particle spectra of ^{49}Ca and ^{57}Ni. This important effects have been included in our calculations of the matrix elements derived from the least-square fitting.

The wave functions of ^{51}Ti, ^{53}Cr and ^{55}Fe calculated by making use of the matrix elements show strong correlation between the even protons and the single neutron. If the proton-neutron interactions were weak, the $J=3/2^-$ ground states of these three nuclei would appear as almost pure $p_{3/2}$ single-neutron states. There would also be degenerate quartets of $J=1/2^-$, $3/2^-$, $5/2^-$ and $7/2^-$ at about 1.5 MeV excitation energy corresponding to the position of the 2^+ state in the adjoining nuclei with $N=28$. However, those four excited states appear as being strongly coupled with other states, especially with $p_{3/2}$, $p_{1/2}$ and $f_{5/2}$ single-neutron states. This may be seen experimentally from the (d, p) reactions and indeed the present calculations can show this property. The ground states contain the $|J_p=2^+\times(p_{3/2})_n;\ J=3/2^-\rangle$ component remarkably which reveals the breakdown of the seniority scheme in the proton configurations. In the odd nuclei, ^{52}V and ^{54}Mn, it is also shown that the ground states contain remarkably the $v_p=3$ proton components due to the proton-neutron interactions.

Some excited levels have not been explained by the present calculations, but the properties of most of the low-lying levels are now understood in terms of the $(f_{7/2})_p{}^{Z-20}\times(p_{3/2}, p_{1/2}, f_{5/2})_n{}^1$ configurations. In ^{53}Cr and ^{55}Fe, there are two $J=7/2^-$ states with almost equal excitation energies in each nucleus. One of them can be understood as a state which is mainly $|J_p=2^+\times(p_{3/2})_n;\ J=7/2^-\rangle$, while the other may be the two-particle one-hole state in the neutron configuration. It may perhaps be inferred that the coupling between two different configurations is weak for the two closely lying $J=7/2^-$ levels. The existence of low-lying 1^+ states in ^{50}Sc and ^{56}Co suggests that the correlation in the $T=0$, $J=1^+$ state in the $(p_{3/2}, p_{1/2}, f_{5/2})$-orbit is important. Therefore, the excitations of $f_{7/2}$ particles to higher orbits are necessary for better understanding of the low-lying levels of the $N=29$ nuclei.

It seems very important to investigate the derivation of the two body matrix elements from the realistic nuclear forces. In this direction, Kuo and Brown[49)] made a calculation by the G-matrix theory and obtained numerical results which are close to our phenomenological values.

It is also interesting to consider the validity of the matrix elements obtained in the present calculations in relation to nuclei other than the $N=29$ nuclei but also in $1f-2p$ shell. The two-body matrix elements applied to the low-lying levels of $N=30$ and 31 nuclei show good agreement with experimental results.[45)]

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