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Effective Sensor Scheduling Schemes in a Sensor Network by Employing Feedback in the Communication Loop

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Abstract— **In this paper, we consider a state estimation problem over a bandwidth limited network. A sensor network consisting of N sensors is used to observe the states of M plants, but only $p \leq N$ sensors can transmit their measurements to a centralized estimator at each time. Therefore a suitable scheme that schedules the proper sensors to access the network at each time so that the total estimation error is minimized is required. We propose four different sensor scheduling schemes. The static and stochastic schemes assume no feedback from the estimator to the scheduler, while the two dynamic schemes, Maximum Error First (MEF) and Maximum Deduction First (MDF) assume such feedback is available. We compare the four schemes via some examples and show MEF and MDF schemes perform better than the static and stochastic schemes, which demonstrates that feedback can play an important role in this remote state estimation problem. We also show that MDF performs better than MEF as MDF considers the total estimation error while MEF considers the individual estimation error.**

I. Introduction

Advances in fabrication technology and computer architecture have led to the rapid growth of computation capabilities while simultaneously decreasing chip size and power consumption. The latter gave birth to the fast developing field of sensor networks which have gained great attention in recent years [1], [2]. Many control applications now take advantage of sensor networks and the loops are closed via the network [3]. These types of control system are called a networked control systems (NCS). NCS provide many advantages which classical control systems do not have, for example, reducing the system wiring, making the system easy to operate and maintain and increasing system agility. Despite the many advantages NCS has brought, finite bandwidth, network induced delays and possibly data packet drops severely degrade the system performance and may even cause system instability [4].

In the past decade, researchers have studied different networked control problems, mostly analyzing how the network in the closed loop affects the system performance and designing controllers that consider this effect to optimize system performance. For example, in [5], Sinopoli and et al studied how the packet drops in the network affects state estimation and provided upper and lower bounds on the critical packet arrival rate below which the estimation error diverges. In [6], Liu and Goldsmith studied the same estimation problem and gave similar results when only partial observation is received. Nilsson [7] studied how stochastic delays affect the control performance. The effect of finite bandwidth of the network on the control performance was studied by Wong and Brockett in [8], [9] and further pursued by [10], [11] where the authors provided the minimum data rate that the network has to provide in order to have stable state estimate and closed loop stability.

When bandwidth is limited so that a single network is shared by many users, effective access control or network access scheduling schemes are required. Walsh and et al studied in [12] and [13] the problem of when to schedule which plant to access to the network so that all plants remain stable. They proposed a protocol MEF-TOD (Max Error First Try Once Discard) and showed that under certain conditions global exponential stability can be achieved. Tiwari and et.al studied the sensor scheduling problem in [14] where a single sensor has to determine which one of the two plants it needs to observe at each time step so as to minimize the estimation error. In [15], Gupta and et al considered a different scheduling problem where there is only one plant but with multiple sensors. They proposed a stochastic sensor scheduling scheme and provided the optimal probability distribution over the sensors to be selected.

In this paper, we study a networked state estimation problem. A sensor network consisting of N sensors is used to observe the states of M plants, but only $p \leq N$ sensors can transmit their measurements to the estimator at each time. The estimator is a Kalman filter. Since the estimator is usually attached to the controller which has enough power capability, we assume feedback from the estimator to the network scheduler is possible. We attempt to explore how much this feedback can improve the estimator performance. Four sensor scheduling schemes are proposed for this purpose. The static and stochastic schemes utilize no feedback while the two dynamic schemes, Maximum Error First (MEF) and Maximum Deduction First (MDF) assume such feedback is available.

The idea of MEF is similar to the MEF-TOD protocol in [12], however we face a more complicated issue where there are two levels of scheduling. The first is to schedule which p sensors to access the network. The second is to schedule which plants those p sensors observe. We show that MEF and MDF are both better than the static and stochastic schemes, which suggests that feedback can play an important role for this estimation problem. We further show that MDF is better than MEF. It turns out MDF is even better than the locally optimal solution using a combinatorial approach as

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shown in Section VI.

We also attempt to include the plug-and-play feature so that those schemes will be suitable for general ad hoc sensor networks where the sensors available to use are in general not fixed. Ad hoc networks have attracted much attention in recent years due to their flexibility and easy operation [16]–[18]. Developing efficient algorithms and methods that are suitable for ad hoc networks is becoming increasingly important. We show that the MEF and MDF schemes are well suitable for such a situation.

The rest of the paper is organized as follows. In Section II, the mathematical model of the problem is given. In Section III through V we propose four schemes for scheduling the access of the sensors and compare their performances in Section VI. Conclusions and future work are given at the end.

II. Problem Set Up

Consider a sensor network consisting of N sensors which can observe the states of M different plants (Figure 1).

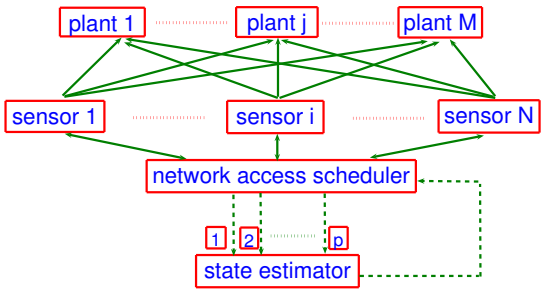

Fig. 1. System block diagram

Plant j has the following dynamics

$$x_{k+1}^j = A^j x_k^j + w_k^j, \tag{1}$$

where $j = 1, \cdots, M$. When sensor i observes plant j, it returns

$$y_k^{ij} = C^{ij} x_k^j + v_k^{ij}, \tag{2}$$

where $i = 1, \cdots, N$. In Eqn (1) and (2), $x_k^j \in \mathbb{R}^{n_j}$ is the state vector, $y_k^{ij} \in \mathbb{R}^{m_{ij}}$ is the observation vector, w_k^j and v_k^{ij} are process and measurement noises which are assumed to be white, Gaussian and zero mean with covariance matrices $Q^j \geq 0$ and $R^{ij} > 0$ respectively. The sensors send their measurements to a state estimator via a bandwidth limited network so that only $p \leq N$ sensors are allowed to access the network at each time step.

From now on denote the sensor space as $\mathbf{S} = \{\mathbf{S}_1, \cdots, \mathbf{S}_N\}$ and the plant space as $\mathbf{P} = \{\mathbf{P}_1, \cdots, \mathbf{P}_M\}$, where $\mathbf{S}_i$ and $\mathbf{P}_j$ are the individual sensors and plants. Further denote $\mathbf{\Sigma}_k^j$ as the set of sensors that all observe $\mathbf{P}_j$ at time k and denote $\mathcal{Y}_k^j$, $\mathcal{C}_k^j$ and $\mathcal{R}_k^j$ as the representation of all sensors in $\mathbf{\Sigma}_k^j$. For example, if $\mathbf{\Sigma}_k^j = \{\mathbf{S}_1, \cdots, \mathbf{S}_p\}$, then $\mathcal{Y}_k^j = [y_k^{1j}; \cdots; y_k^{pj}]$, $\mathcal{C}_k^j = [C^{1j}; \cdots; C^{pj}]$ and $\mathcal{R}_k^j = \text{diag}\,(R^{1j}, \cdots, R^{pj})$.

The state estimator is a Kalman filter. Denote $\hat{x}_k^j$ as the estimated state of $\mathbf{P}_j$ at time k given all previous measurements. Also denote P_k^j as the a priori estimation error covariance (simply write as error covariance later). Then $\hat{x}_k^j$ and P_k^j evolve as

$$\hat{x}_{k+1}^j = A^j \hat{x}_k^j + K_k^j(\mathcal{Y}_k^j - \mathcal{C}_k^j \hat{x}_k^j) \tag{3}$$

$$K_k^j = A^j P_k^j (\mathcal{C}_k^j)^T [\mathcal{C}_k^j P_k^j (\mathcal{C}_k^j)^T + \mathcal{R}_k^j]^{-1} \tag{4}$$

$$\begin{aligned} P_{k+1}^j &= A^j P_k^j (A^j)^T + Q^j \\ &\quad - A^j P_k^j (\mathcal{C}_k^j)^T [\mathcal{C}_k^j P_k^j (\mathcal{C}_k^j)^T + \mathcal{R}_k^j]^{-1} \mathcal{C}_k^j P_k^j (A^j)^T. \end{aligned} \tag{5}$$

Notice that if there is no sensor observing $\mathbf{P}_j$ at time k, $\mathbf{\Sigma}_k^j = \emptyset$, hence $\mathcal{C}_k^j = 0$ and $\mathcal{R}_k^j = 0$. Therefore we simply have

$$\hat{x}_{k+1}^j = A^j \hat{x}_k^j, \tag{6}$$

$$P_{k+1}^j = A^j P_k^j (A^j)^T + Q^j. \tag{7}$$

We are interested in solving the following problem.

Problem 1: Design a sensor scheduling scheme to minimize

$$\sum_{j=1}^{M} \text{Tr}(P_k^j)$$

at each time k.

The reason we choose to minimize the cost function at each time step rather than the steady state estimation error as in most previous other works is that in general ad hoc networks N is a varying number, as existing sensors can quit due to the power drainage or malfunctioning and new sensors can join. Consequently the sensors available at one time may be quite different than at another time step. It therefore does not make sense to consider the long term behavior of the estimation error. This can be thought as a best effort minimization problem.

In the following sections, we present four scheduling schemes and compare their performances through some examples.

III. Static Scheduling

Static sensor scheduling schemes are the simplest among all schemes which only require little computation and are very easy to implement, for example, in a network using a token ring or polling. There are many static schemes and we present one possible scheme in Algorithm I.

Algorithm I is periodic and it takes $\text{LCM}(M, N)$ cycles to repeat, where $\text{LCM}(M, N)$ denotes the least common multiple of M and N. Apparently, Algorithm I guarantees fair use of the sensors for each plant and hence avoids overusing some particular sensors and extends the whole network life.

Given the explicit parameters of the plants and sensors, we can provide conditions for the convergence of the upper

TABLE I

ALGORITHM I: STATIC SCHEME

1) $k = 1, i = 1$.

2) $\mathbf{S}_{i \mod N}, \cdots, \mathbf{S}_{i+p-1 \mod N}$ are selected to access the network.

3) For $j = 0, \cdots, p-1$, $\mathbf{S}_{i+j \mod N}$ observes $\mathbf{P}_{i+j \mod M}$.

4) $k = k+1, i = i+p$. Go to Step 2.

TABLE II

ALGORITHM II: STOCHASTIC SCHEME

1) $k = 1$.

2) Select p sensors out of N sensors according to some distribution π.

3) If $\mathbf{S}_i$ is selected, it observes $\mathbf{P}_j$ with probability χ_{ij}.

4) $k = k+1$. Go to Step 2.

bound of $\sum_{j=1}^{M} \mathrm{Tr}(P_k^j)$ when using Algorithm I. Different static schemes will have different convergence conditions of the upper bound. For example, with $M = 2, N = 3$ and $p = 1$, we have the following convergence conditions. Define the following functions for any positive semi-definite $X \geq 0$.

$$\begin{aligned} g_{ij}(X) &= A^j X (A^j)^T + Q^j \\ &\quad - A^j X (C^{ij})^T [C^{ij} X (C^{ij})^T + R^{ij}]^{-1} C^{ij} X (A^j)^T \\ h_j(X) &= A^j X (A^j)^T + Q^j, \end{aligned} \tag{8}$$

where $i = 1, 2, 3$ and $j = 1, 2$. Then for $k = 6z, z = 0, 1, \cdots$, the error covariances for the two plants evolve as

$$\begin{aligned} P_{k+6}^1 &= h_1 g_{21} h_1 g_{31} h_1 g_{11}(P_k^1), \\ P_{k+6}^2 &= g_{32} h_2 g_{12} h_2 g_{22} h_2(P_k^1). \end{aligned}$$

Lemma 2: Let P_∞^1 and P_∞^2 satisfy

$$P_\infty^1 = h_1 g_{21} h_1 g_{31} h_1 g_{11}(P_\infty^1), \tag{9}$$

$$P_\infty^2 = g_{32} h_2 g_{12} h_2 g_{22} h_2(P_\infty^2). \tag{10}$$

Then $\sum_{j=1}^{2} \mathrm{Tr}(P_k^j)$ is bounded at each time step if and only if Eqn (9) and (10) have bounded solutions.

Proof: We omit the proof as it is straightforward to show. ■

In many situations, the plants may have different dynamics and some are even unstable. It is then natural to schedule more sensors to observe the more unstable plants and hence help to control the unstable process. Most good scheduling schemes are very unfair, *i.e.*, allocating more resources (sensors in this case) to the plant who has the highest priority according to some cost functions. The dynamic scheduling schemes in Section V provide such examples.

IV. STOCHASTIC SCHEDULING

Like the tatic scheme, the stochastic scheduling scheme also utilizes no feedback from the estimator to the scheduler. The details of the stochastic scheme are presented Algorithm II.

In Algorithm II, π is the distribution over the subsets of $\mathbf{S}$ which consists of p sensors exactly. $\sum_{j=1}^{M} \chi_{ij} = 1$ for each $i = 1, \cdots, N$. We are able to give conditions on the convergence of the upper bound of $E[\sum_{j=1}^{M} \mathrm{Tr}(P_k^j)]$ which is similar to that in [15] and we refer reader to [15] for details. The advantage of this algorithm is that by properly choosing the distributions π and χ according to the plant and sensor parameters, the upper bound of $E[\sum_{j=1}^{M} \mathrm{Tr}(P_k^j)]$ can be minimized. The disadvantage is that it assumes no feedback from the estimator to the scheduler as in the static scheme. As we show in the next two sections, by allowing such feedback the estimator performance is greatly enhanced.

V. DYNAMIC SCHEDULING

In this section, we first present a *locally optimal* solution using a combinatorial approach and show that this locally optimal solution is intractable due to its high computational complexity. [1] We then propose two dynamic sensor scheduling schemes which use feedback from the estimator to the scheduler and show they are computationally tractable.

A. Locally Optimal Solution

There are $\mathbf{C}_N^p = \frac{N!}{p!(N-p)!}$ ways to select p out of N sensors, and each selected sensor can observe any one of the M plants. As a result, in total there are $\mathbf{C}_N^p M^p$ ways to determine which p sensors access the network and their associated plants to observe. Denoting ω as one such way and Ω as the space consisting of all such ways, then Ω has cardinality $\mathbf{C}_N^p M^p$. Problem 1 can be cast as

$$\min_{\omega \in \Omega} \sum_{i=1}^{M} \mathrm{Tr}(P_k^i).$$

Clearly as N becomes large, the above minimization problem is intractable as it takes $O(\mathbf{C}_N^p M^{p+1})$ times to obtain. Therefore we seek tractable solutions but possibly at the price of losing local optimality. We propose two locally suboptimal solutions to Problem 1 in the next two sections. The advantages of these solutions are that they only take polynomial time in M, N, p and are suitable for general ad hoc networks where N might be varying.

B. Dynamic Scheduling: Max Error First (MEF)

We present a dynamic scheduling scheme and call it with Maximum Error First (MEF). The idea is that the plant having the largest open loop error has the highest priority to use the sensors. Once a sensor is selected by this plant, it has access to the network. Recall that we have defined $\mathbf{\Sigma}_k^j$ as the set of sensors observing $\mathbf{P}_j$ at time k. Let $\mathbf{S}_i$ be one

[1] By *locally optimal*, we mean the minimization is taken at each time step rather than over a time horizon.

TABLE III

ALGORITHM III: DYNAMIC SCHEDULING - MAX ERROR FIRST

1) $k = 1$.

2) $t = 1$. $\mathbf{S}$ has $N - t + 1$ sensors. $\Sigma_k^j = \emptyset, j = 1, \cdots, M$.

3) • For each $\mathbf{P}_j, j = 1, \cdots, M$
- Compute the open loop errors according to Eqn (7).
• Store the trace of those errors in a buffer $\mathcal{B}$.
• Sort $\mathcal{B}$ in descending order.

4) • Let $\mathbf{P}_t$ be the plant having the first value in $\mathcal{B}$.
• For each $\mathbf{S}_i$ in $\mathbf{S}$
- Compute E_{it} according to Eqn (11) or (12).
• Let $\mathbf{S}_{q(t)}$ be the one that minimizes the trace of E_{it}
• Replace the first value in $\mathcal{B}$ by the trace of $E_{q(t)t}$.

5) • Sort $\mathcal{B}$.
• Record $C^{q(t)t}$ into array $\mathcal{A}$.
• Move $\mathbf{S}_{q(t)}$ from $\mathbf{S}$ to Σ_k^j.

6) $t = t + 1$. If $t \neq p + 1$, goto Step 4.

7) $k = k + 1$. Clear array $\mathcal{A}$ and goto Step 2.

TABLE IV

ALGORITHM IV: DYNAMIC SCHEDULING - MAX DEDUCTION FIRST

1) $k = 1$.

2) $t = 1$. $\mathbf{S}$ has $N - t + 1$ sensors. $\Sigma_k^j = \emptyset, j = 1, \cdots, M$.

3) For each $\mathbf{P}_j, j = 1, \cdots, M$
- Compute Δ_k^{ij} according to Eqn (13) or (14) for each $\mathbf{S}_i$ from $\mathbf{S}$

4) For each $\mathbf{P}_j, j = 1, \cdots, M$
- Store the trace of Δ_k^{ij} in a buffer $\mathcal{B}_j$, $i = 1, \cdots, N - t + 1$.
- Sort $\mathcal{B}_j$ in descending order.
- Store $\mathcal{B}_j[1]$ in $\mathcal{D}$.

5) • Sort $\mathcal{D}$. Let $\mathbf{P}_t$ have the first value in $\mathcal{D}$.
• Let $\mathbf{S}_{q(t)}$ returns the first value in $\mathcal{B}_t$.
• Record $C^{q(t)t}$ into an array $\mathcal{A}$.
• Move $\mathbf{S}_{q(t)}$ from $\mathbf{S}$ to Σ_k^j.

6) $t = t + 1$. If $t \neq p + 1$, goto Step 3.

7) $k = k + 1$. Clear array $\mathcal{A}, \mathcal{D}$ and goto Step 2.

sensor from $\mathbf{S}$ which consists of all the available sensors that $\mathbf{P}_j$ can use. If $\mathbf{\Sigma}_k^j$ is empty, define

$$E_{ij} = A^j P_k^j (A^j)^T + Q^j - A^j P_k^j (C^{ij})^T [C^{ij} P_k^j (C^{ij})^T + R^{ij}]^{-1} C^{ij} P_k^j (A^j)^T \quad (11)$$

where C^{ij}, R^{ij} are parameters for $\mathbf{S}_i$. Otherwise add $\mathbf{S}_i$ to $\mathbf{\Sigma}_k^j$ and define

$$E_{ij} = A^j P_k^j (A^j)^T + Q^j - A^j P_k^j (\mathcal{C}_k^j)^T [\mathcal{C}_k^j P_k^j (\mathcal{C}_k^j)^T + \mathcal{R}_k^j]^{-1} \mathcal{C}_k^j P_k^j (A^j)^T \quad (12)$$

where $\mathcal{C}_k^j$ and $\mathcal{R}_k^j$ are defined in Section II. With these notations the MEF scheme is presented in Algorithm III.

Clearly at each time step the array $\mathcal{A}$ tells which p sensors will access the network and which plants they observe. Step 3 in Algorithm III takes $O(M \log M)$ times to sort M elements. Step 4 takes $O(N - t + 1)$ times. Before step 4, $\mathcal{B}$ is already sorted and step 4 only changes the first value in $\mathcal{B}$, hence it takes only $O(\log M)$ times to resort $\mathcal{B}$ in step 5. Since $t = 1, \cdots, p$, in total it takes $O(M \log M + pN + p \log M)$ times to execute the algorithm for each k.

With this algorithm, it is possible to schedule multiple sensors to observe the same plant which may be very unstable compared with other plants.

C. Dynamic Scheduling: Max Deduction First (MDF)

We present another scheme and call it Dynamic Scheduling with Maximum Deduction First (MDF). The difference between this and MEF scheme is the different priorities of plants using the sensors. If $\mathbf{\Sigma}_k^j$ is empty, define

$$\Delta_k^{ij} = A^j P_k^j (C^{ij})^T [C^{ij} P_k^j (C^{ij})^T + R^{ij}]^{-1} C^{ij} P_k^j (A^j)^T. \quad (13)$$

Otherwise add $\mathbf{S}_i$ to $\mathbf{\Sigma}_k^j$ and call this new set $\bar{\mathbf{\Sigma}}_k^j$ with new parameters $\bar{\mathcal{C}}_k^j = [\mathcal{C}_k^j; C^{ij}]$ and $\bar{\mathcal{R}}_k^j = \text{diag}\,(\mathcal{R}_k^j, R^{ij})$. Define

$$\begin{aligned} \Delta_{k-}^{ij} &= A^j P_k^j (\mathcal{C}_k^j)^T [\mathcal{C}_k^j P_k^j (\mathcal{C}_k^j)^T + \mathcal{R}_k^j]^{-1} \mathcal{C}_k^j P_k^j (A^j)^T. \\ \Delta_{k+}^{ij} &= A^j P_k^j (\bar{\mathcal{C}}_k^j)^T [\bar{\mathcal{C}}_k^j P_k^j (\bar{\mathcal{C}}_k^j)^T + \bar{\mathcal{R}}_k^j]^{-1} \bar{\mathcal{C}}_k^j P_k^j (A^j)^T. \\ \Delta_k^{ij} &= \Delta_{k+}^{ij} - \Delta_{k-}^{ij}. \end{aligned} \quad (14)$$

With these notations, the MDF scheme is presented in Algorithm IV.

Similar to Algorithm III, array $\mathcal{A}$ gives a solution to Problem 1. It is also easy to show that step 3 in Algorithm IV takes $O(MN)$ times. Step 4 takes $O(MN \log N)$ times because it needs to sort N elements M times. Step 5 takes $O(M \log M)$ times to sort M elements. All the other steps take constant time. Since $t = 1, \cdots, p$, in total it takes $O(pMN + pMN \log N + pM \log M)$ times to execute the algorithm for each k. This algorithm is also able to schedule multiple sensors for the same plant.

Compared with the $O(\mathbf{C}_N^p M^{p+1})$ time complexity, these two schemes provide unbeatable computational advantage, though MDF is slightly worse than MEF. As we see from the examples in the next section, MDF gives better performance than MEF. The reason is that MDF gives the highest priority to the plant who reduces most of the total estimation error while MEF gives the highest priority to the plant having the largest open loop error, *i.e.*, it only considers the individual estimation error.

These two schemes are also compatible with varying N. However, unlike the static or stochastic scheduling schemes in the previous sections, it is quite difficult to give conditions for convergence for these two schemes. The reason is that the sensors to be chosen and the plants to be observed are dynamically changing at all times. There are no clear static or statistical patterns of the selected sensors and their associated

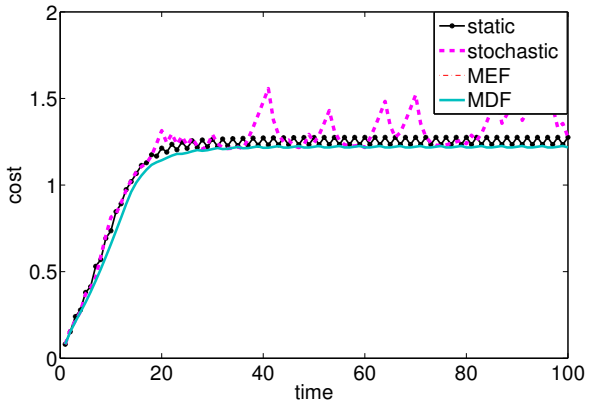

Fig. 2. Comparison of different scheduling schemes

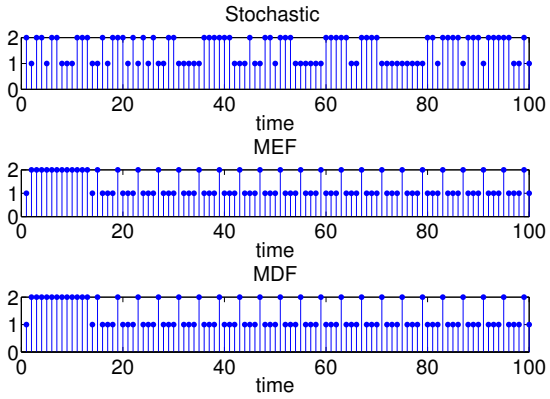

Fig. 3. Sensor selection based on stochastic or dynamic schemes

plants. We leave this part as future work to be pursued. Intuitively, the two schemes make the best effort in minimizing the cost function and their actual performances are better than the static or stochastic schemes as demonstrated through the examples in the next section.

VI. Examples

We compare the different schemes through some examples in this section.

Example 3: Here we consider a simple example which is taken from [15] with slight modification of the measurement noise covariances. In this case, $M = 1, N = 2, p = 1$. The plant and sensors parameters are given as follows.

$$
\begin{aligned}
A &= \begin{bmatrix} 1 & 0 & 0.2 & 0 \\ 0 & 1 & 0 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
Q &= \begin{bmatrix} 0.0004 & 0.0001 & 0.0040 & 0.0010 \\ 0.0001 & 0.0004 & 0.0010 & 0.0040 \\ 0.0040 & 0.0010 & 0.0400 & 0.0100 \\ 0.0010 & 0.0040 & 0.0100 & 0.0400 \end{bmatrix} \\
C^{11} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}, C^{21} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
R^{11} &= \begin{bmatrix} 2.4 & 0 \\ 0 & 1.0 \end{bmatrix}, R^{21} = \begin{bmatrix} 1.8 & 0 \\ 0 & 0.1 \end{bmatrix}
\end{aligned}
$$

We compare the four different schemes here and the results are shown in Figure 2. We also plot the selected sensors for the stochastic and dynamic scheduling schemes in Figure 3.

The two schemes from dynamic scheduling turn out to be the same for this case. This is because $M = 1$ and there is no other plant competing to use the sensors. It is also easy to see the local optimal solution using the combinatorial approach also overlaps with these two schemes and therefore the result is not shown in the figures. For this example, the static scheme is better than the stochastic scheme. In general this may not be true as we choose an arbitrary distribution π and χ for this particular example. As shown in the next example where we optimize the distribution, *i.e.*, put more weight for those sensors having smaller noise covariances, the static scheme is worse than the stochastic scheme.

Example 4: We consider a more interesting example where six sensors are available to observe three plants, but only two sensors are allowed to access the network each time, that is $M = 3, N = 6$ and $p = 2$. To save space, we only list the parameters for plant two and sensor two respectively.

$$
\begin{aligned}
A^2 &= \begin{bmatrix} 1 & 0 & 0.15 & 0 \\ 0 & 1 & 0 & 0.15 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
Q^2 &= \begin{bmatrix} 0.0003 & 0.0000 & 0.0034 & 0.0006 \\ 0.0000 & 0.0003 & 0.0006 & 0.0034 \\ 0.0034 & 0.0006 & 0.0450 & 0.0079 \\ 0.0006 & 0.0034 & 0.0079 & 0.0450 \end{bmatrix} \\
C^{21} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, C^{22} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \\
C^{23} &= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \\
R^{21} &= \begin{bmatrix} 1.5 & 0 \\ 0 & 0.1 \end{bmatrix}, R^{22} = \begin{bmatrix} 2.0 & 0 \\ 0 & 0.8 \end{bmatrix} \\
R^{23} &= \begin{bmatrix} 3.0 & 0 \\ 0 & 0.1 \end{bmatrix}.
\end{aligned}
$$

We compare the four schemes and plot the results in Figure 4. Figure 5 is the zoomed in version for close comparison.

Clearly the two dynamic schemes are much better than the static and stochastic schemes. The MDF and MEF schemes are about 4 times better than the static scheme and 2 times better than the optimized stochastic scheme.

The MDF scheme is also observed to be better than the MEF scheme as we have discussed before. Since the number of M, N, p are not too big, we can compute the locally optimal solution in the figure for comparison purpose. With little surprise, MDF is even better than the locally optimal solution for most of the time. Again as we have discussed

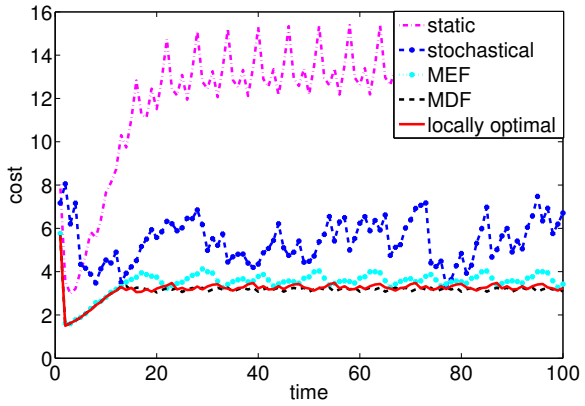

Fig. 4. Comparison of Different Scheduling Schemes

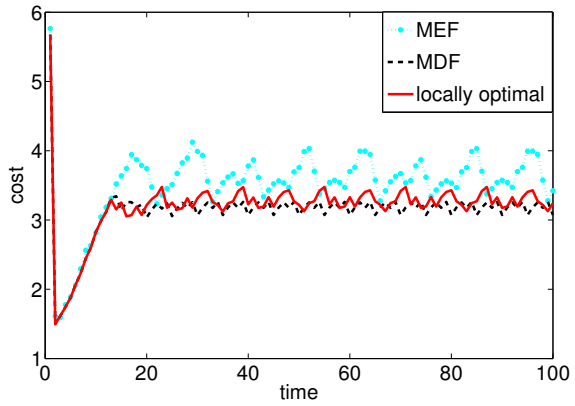

Fig. 5. Close Comparison

before, local optimality does not necessarily imply global optimality. Therefore other local schemes, even though not locally optimal, might have better performances.

VII. Conclusions and Future Work

We have considered a remote centralized state estimation problem with multiple sensors able to observe multiple plants in this paper. Only a subset of the available sensors are able to send their measurement to an estimator due to the finite bandwidth of the network and each sensor can only measure one of the plants. In order to minimize the total estimation error at each time step, we have proposed four different sensor scheduling schemes and showed the dynamic schemes with feedback from the estimator to the scheduler outperform those without feedback.

The analysis of the dynamic schemes is quite difficult as there are no static or stochastic patterns of the selected sensors and their associated plants. Hence it is not straightforward to show the convergence of the total estimation error as we have done for the static and stochastic schemes. In the future work, we will look at this issue and pursue the convergence conditions for the two dynamic schemes.

Another interesting problem is to determine the minimum number p such that the total estimation error is bounded. It is clear that if p is sufficiently small and M, N are sufficiently large, the total estimation error quickly diverges. It will be of great interest to obtain a closed form relationship between these numbers and the plants and the sensors parameters, which shows the fundamental tradeoff between the accuracy of the estimation and the resources available.

Finally, it would be interesting and natural to close the loop via the network which is the dual of the estimation problem we have considered.

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