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Effectiveness of A-ϕ Method in a Parallel Computing With an Iterative Domain Decomposition Method

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Effectiveness of the A-ϕ method in a parallel computing with an iterative domain decomposition method is considered. Convergence of interface problems is faster with the electric scalar potential than without it. A simple model is considered as a numerical example.

***Index Terms*—Domain decomposition method, eddy current problem, finite-element method, parallel processing.**

I. Introduction

THERE ARE many machines or devices where the electromagnetic phenomena are applied such as a motor, a magnetic head, a magnetic shield, and a magnetic sensor, etc. In order to analyze these engineering or physical phenomena, computer simulation is a reliable and yet economical approach.

Moreover, a computational object is made to a large scale and is complicated for numerical analysis recently. In addition, subdivision of the mesh is performed for the improvement of accuracy. Therefore, the requirement of large-scale computations increases greatly in electromagnetic field problems. To meet this requirement, we have already applied the hierarchical domain decomposition method (HDDM) [1], [2] to three-dimensional (3-D) eddy current problems using the A method, where the magnetic vector potential A is used as an unknown function [3].

On the other hand, 3-D eddy current problems using the A-ϕ method that uses furthermore the electric scalar potential ϕ are also considered. Apparently, it might be that the A method is more effective, and that the central processing unit (CPU) time of the A method is less than that of the A-ϕ method, because the number of unknown variables is reduced. However, recent papers insist that convergence of the iterative solver, the conjugate orthogonal conjugate gradient method (COCG) [4], is faster with the electric scalar potential than without it, see [5]–[11], and [12].

In this paper, we introduce HDDM to 3-D eddy current problems using the A-ϕ method. We analyze a simple model to compare CPU time and number of iterations of the A-ϕ method with those of the A method, changing degrees of freedom (DOF) several times up to 5 000 000.

II. Formulation

A. A-ϕ Method

Let Ω be a polyhedral domain with boundary $\partial\Omega$. Assume that the domain Ω consists of two nonoverlapping regions, a conducting part R and a nonconducting one S, with the interface Γ between two regions. In this section, for simplicity, assume that the conducting part R is also a polyhedral region, and that the part R is strictly included in Ω.

We use the magnetic vector potential A [Wb/m] and the electric scalar potential ϕ [V] as unknown complex functions, and neglect the Coulomb gauge condition. Then, the following 3-D eddy current problem is considered:

$$\mathrm{rot}\,(\nu \mathrm{rot} A) - i\omega\sigma A + \sigma \mathrm{grad}\phi = J \quad \text{in } \Omega \tag{1a}$$

$$\mathrm{div}(i\omega\sigma A - \sigma \mathrm{grad}\phi) = 0 \quad \text{in } R \tag{1b}$$

$$A \times n = 0 \quad \text{on } \partial\Omega \tag{1c}$$

$$(i\omega\sigma A - \sigma \mathrm{grad}\phi) \cdot n = 0 \quad \text{on } \Gamma \tag{1d}$$

where J denotes an excitation current density [A/m^2], ν the magnetic reluctivity [m/H], σ the conductivity [S/m], ω the angular frequency [rad/s], and i the imaginary unit. In this section, assume that ν is a piecewise positive constant, that σ is a positive constant in R, and is equal to 0 in S, and that the divergence of J vanishes in Ω

$$\mathrm{div} J = 0 \quad \text{in } \Omega. \tag{2}$$

Now we consider a weak form for (1) and a finite-element method for this form. The domain Ω is decomposed into a union of tetrahedral elements. Let A_h be a magnetic vector potential approximated by the Nedelec elements of simplex type so that $A_h \times n = 0$ on $\partial\Omega$, and let A_h^* be corresponding test functions. Let ϕ_h be an electric scalar potential approximated by the conventional piecewise linear tetrahedral elements, and let ϕ_h^* be corresponding test functions. Then the finite-element approximation is as follows.

Find A_h and ϕ_h such that

$$(\nu \mathrm{rot} A_h, \mathrm{rot} A_h^*) - (i\omega\sigma A_h, A_h^*) + (\sigma \mathrm{grad}\phi_h, A_h^*) = (\tilde{J}_h, A_h^*) \tag{3a}$$

$$(\sigma \mathrm{grad}\phi_h, \mathrm{grad}\phi_h^*) - (i\omega\sigma A_h, \mathrm{grad}\phi_h^*) = 0 \tag{3b}$$

where $(\cdot, \cdot)$ is the complex valued L^2-inner product and $\tilde{J}_h$ is a corrected excitation current density so that (2) holds in a weak sense; for example, see [3].

B. A Method

For the A method, we can consider similarly. We use only the magnetic vector potential A[Wb/m] as an unknown com-

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plex function. Then the following 3-D eddy current problem is considered.

$$\mathrm{rot}(\nu \mathrm{rot} A) - i\omega\sigma A = J \quad \text{in } \Omega \tag{4a}$$

$$\mathrm{div} A = 0 \quad \text{in } S \tag{4b}$$

$$A \times n = 0 \quad \text{on } \partial\Omega \tag{4c}$$

$$\int_{\Gamma} A \cdot n \, ds = 0. \tag{4d}$$

Neglecting again the Coulomb gauge condition, the finite-element approximation for (4) is as follows.

Find A_h such that

$$(\nu \mathrm{rot} A_h, \mathrm{rot} A_h^*) - (i\omega\sigma A_h, A_h^*) = (\tilde{J}_h, A_h^*). \tag{5}$$

For details and a practical example, see [3].

III. Iterative Domain Decomposition Method

We introduce an iterative domain decomposition method to eddy current problems using the A-ϕ method and the A method. Let us denote the finite-element equations of these methods by the matrix form as follows:

$$Ku = f \tag{6}$$

where K denotes the coefficient matrix, u the unknown vector, and f the known vector.

The polyhedral domain Ω is partitioned into the nonoverlapping subdomains. Then the linear system (6) is rewritten as follows:

$$\begin{pmatrix} K_{II} & K_{IB} & K_{IE} \\ K_{BI} & K_{BB} & K_{BE} \\ K_{EI} & K_{EB} & K_{EE} \end{pmatrix} \begin{pmatrix} u_I \\ u_B \\ u_E \end{pmatrix} = \begin{pmatrix} f_I \\ f_B \\ f_E \end{pmatrix} \tag{7}$$

where the subscripts I, B, E correspond to the nodal points in the interior of subdomains, on the interface boundary, and on the essential boundary. (7) leads to linear systems as follows:

$$K_{II} u_I = (f_I - K_{IB} u_B - K_{IE} u_E) \tag{8}$$

$$(K_{BB} - K_{BI} K_{II}^{\dagger} K_{IB}) u_B = f_B - K_{BI} K_{II}^{\dagger} f_I - (K_{BE} - K_{BI} K_{II}^{\dagger} K_{IE}) u_E \tag{9}$$

where $K_{II}^{\dagger}$ is a generalized inverse of K_{II}.

At first, the unknown vector u_B is obtained from the application of the following algorithm based on the COCG method to (9):

Choose u_B^0;
Compute u_I^0 by $(K_{II}\ K_{IB}\ K_{IE})(u_I^0\ u_B^0\ u_E)^T = f_I$;
$p^0 = r^0 = (K_{BI}\ K_{BB}\ K_{BE})(u_I^0\ u_B^0\ u_E)^T - f_B$;
for $n = 0, 1, \ldots$;
  Compute p_I^n by $(K_{II}\ K_{IB}\ K_{IE})(p_I^n\ p^n\ 0)^T = 0$;
  $q^n = (K_{BI}\ K_{BB}\ K_{BE})(p_I^n\ p^n\ 0)^T$;
  $\alpha^n = r^n \cdot r^n / p^n \cdot q^n$;
  $u_B^{n+1} = u_B^n - \alpha^n p^n$;
  $r^{n+1} = r^n - \alpha^n q^n$;
  If $\|r^{n+1}\| \langle \delta \| r^0 \|$, break;
  $\beta^n = r^{n+1} \cdot r^{n+1} / r^n \cdot r^n$;
  $p^{n+1} = r^{n+1} + \beta^n p^n$;
end;

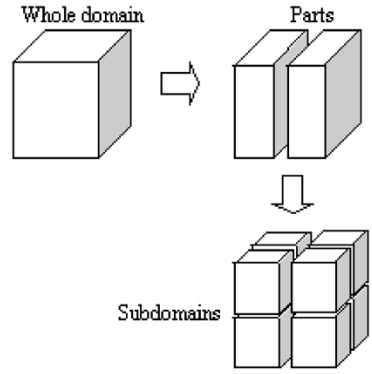

Fig. 1. Hierarchical domain decomposition.

where $\| \cdot \|$ is the Euclidean norm and δ is a positive constant. Because the matrix K_{II} is block diagonal corresponding to each subdomain, the vectors u_I^0 and p_I^n can be solved independently in each subdomain. After solving u_B, the unknown u_I is obtained from (8). The vector u_I is solved by the COCG method with a preconditioner, and can also be solved independently in each subdomain. Hence, we can get the unknown u in the whole domain.

IV. Hierarchical Domain Decomposition Method

The original domain is hierarchically divided into parts, which are further decomposed into smaller domains called subdomains (Fig. 1). This is called the hierarchical domain decomposition method (HDDM). HDDM is one of techniques for parallel computing. HDDM has some modes depending on roles of processors.

A. Hierarchical Processor Mode

Hierarchical processor mode (H-mode) [2] classifies processors into three groups, "Grand Parent," "Parent," and "Child." One of the processors is assigned as Grand Parent, a few as Parent, and others as Child. The number of processors assigned as Parent is the same as that of parts. The number of Child processors can be varied; and it affects the parallel performance.

The role of Grand Parent is to organize all processor communications (i.e., message passing) which occur between all processors. Parents prepare mesh data, manage finite-element analysis (FEA) results, and coordinate the COCG iteration, including convergence decision for the COCG iteration. Parents send data to Child processors, where FEA is performed in parallel. After the FEA, Child processors send the results to Parents. This computation will be repeated until the COCG iteration is convergent (Fig. 2).

B. Parallel Processor Mode

The traditional HDDM [2] was introduced in the previous section. However, because most computation is performed in Child processors and the most communication time is taken between Parent processors and Child processors, the communication speed becomes important. Although the communication performance has also been improved in network technology in

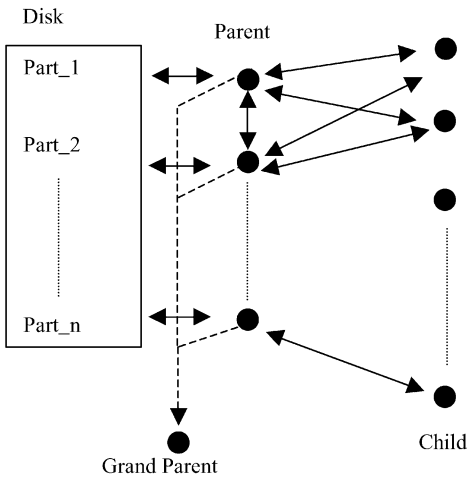

Fig. 2. Hierarchical processor mode.

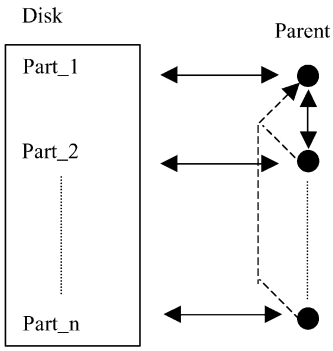

Fig. 3. Parallel processor mode.

recent years, the high-speed network is still expensive. On the other hand, for the PC cluster generally used, the network speed becomes a bottleneck to the processing performance of the CPU. Moreover, when parallel processing performance is considered, it is important to reduce the amount of communications as much as possible. Therefore, the Parent-only type (Parallel processor mode: P-mode) [13] is useful than H-mode.

In the P-mode, Parent processors perform the FEA by themselves, which is computed by Child processors in the H-mode (Fig. 3). Although Parent processors store some of subdomain analysis data and coordinate the COCG iteration as the main work, the idling time of CPU increases in the H-mode, because of less computation in Parent processors. On the other hand, since all processors perform the FEA and CPU can be used without idleness in an environment with 10–20 CPUs, the P-mode is considered to be superior to the H-mode. In the P-mode also, the number of Parent processors should be equal to the number of parts.

V. Numerical Examples

In this section, the P-mode is used for computing.

We consider a simple model which is a model for the accuracy verification of the eddy current analysis and uses the solenoidal coil with unlimited length, see Figs. 4 and 5. The radius of the conductor is 0.1 m, and the height of z-axis is 0.1 m. The magnetic reluctivity is $1/(4\pi) \times 10^7$ [m/H]. The conductivity in the

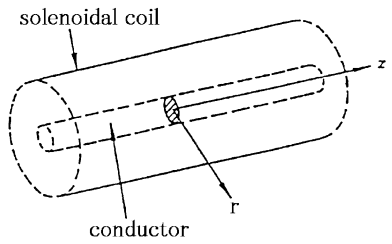

Fig. 4. Solenoidal coil with unlimited length.

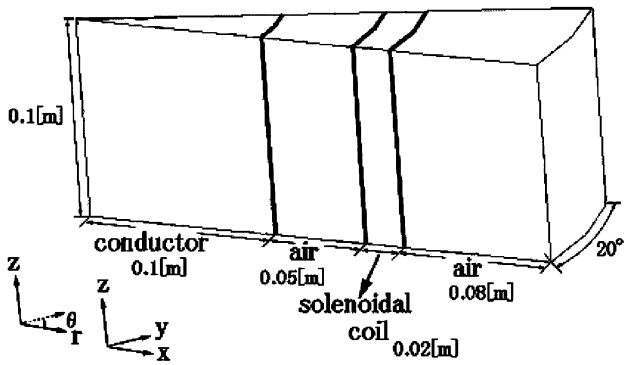

Fig. 5. Geometry of a simple model.

TABLE I
Numbers of DOF, Elements, and Subdomains

Mesh	DOF		elements	subdomains
	A-ϕ method	A method		
(1)	1,107,349	1,029,816	858,505	8 x 1,000
(2)	3,301,101	3,073,873	2,589,930	8 x 3,200
(3)	5,551,903	5,172,591	4,374,369	8 x 5,400

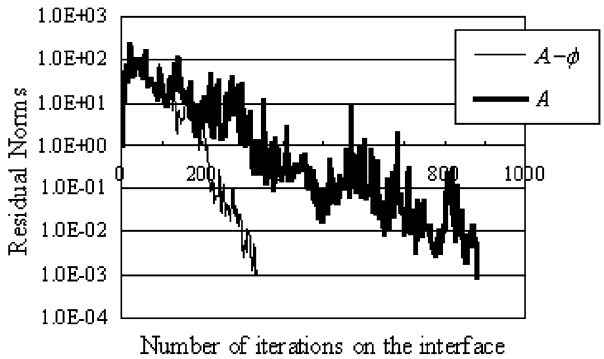

Fig. 6. History of residual norms (Mesh(1)).

conductor is 7.7×10^6 [S/m]. The angular frequency is $2\pi \times 60$ [rad/s]. The absolute value of real (or imaginary) part of the excitation current density $|J_r|$ (or $|J_i|$) in the coil is 50 (or 0) [A/m^2]. Dirichlet boundary conditions of $A \times n = 0$ and $\phi = 0$ are given on the surfaces of $\theta = 0°$ and $\theta = 20°$. Table I shows numbers of DOF, elements, and subdomains.

A simplified block diagonal scaling is used as the preconditioner in the COCG procedure on the interface. Each process is stopped when the residual norm $||r^n||/||r^0||$ becomes less than 10^{-3}. In each subdomain, the COCG method is used as the solver for the complex symmetric (not Hermitian) system

TABLE II
CPU TIME, NUMBER OF ITERATIONS, AND AMOUNT OF MEMORY

Mesh	Formulation	CPU time [h]	# of iterations	Memory per CPU [MB]
(1)	A-ϕ method	1.09	336	88.1
	A method	3.71	881	69.3
(2)	A-ϕ method	4.51	485	267
	A method	17.7	1,484	210
(3)	A-ϕ method	10.7	687	451
	A method	37.0	1,817	354

arising in approximations. A shifted incomplete Cholesky factorization is used as the preconditioner with the accelerative parameter 1.2. The COCG method in each subdomain is stopped when the preconditioned residual norm becomes less than 10^{-10}.

Computation was performed by eight CPUs using Pentium 4 2.0-GHz processors. Fig. 6 shows history of residual norms for Mesh(1), and Table II shows CPU time, number of iterations to solve interface problems, and amount of memory per CPU versus formulation cases. In Mesh(1)-(3), the A-ϕ method needs much more memory than the A method by about 27%, but the CPU time and number of iterations of the A-ϕ method are much less than those of the A method.

VI. CONCLUSION

We have introduced HDDM to 3-D eddy current problems using the A-ϕ method and have shown the possibility of large-scale analysis in eddy current problems. Moreover, we have confirmed the effectiveness of the A-ϕ method in CPU time and number of iterations.

In future research, it is very important for us to reduce number of iterations and computational time.

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