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Effectiveness of Copula-Extreme Value Theory in Estimating Value-at-Risk: Empirical Evidence from Asian Emerging Markets

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Abstract

Traditional Monte Carlo simulation using linear correlations induces estimation bias in measuring portfolio value-at-risk (VaR) due to the well-documented existence of fat-tail, skewness, truncations, and non-linear relations of return distributions. In this paper, we evaluate the effectiveness using copula-extreme-value-based semi-parametric approaches in assessing portfolio risks in six Asian markets based on their different return distribution shapes. We incorporate extreme value theory (EVT) to model the tails of the return distributions and various copulas to build the joint distribution of returns. The backtesting analysis of the Monte Carlo VaR simulation suggests that the Clayton copula-EVT has the best performance regardless of the shapes of the return distributions, and in general the copulas with the EVT perform better estimation of VaRs than the traditional copulas. This concludes the economic significance in incorporating the down-side shock in risk management.

JEL classifications: G15, F31, C46

Keywords: Copulas; Dependence; Emerging Markets; EVT; Backtesting

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I. Introduction

Value-at-Risk (VaR) is widely utilized by financial institutions to manage their market risk. To correctly assess the possible losses of a portfolio over a certain horizon, an accurate measurement of correlations between asset returns in the portfolio is essential. Though previous studies have suggested that conditional models, such as copulas, serve as a better estimation for dependence among asset returns, several critical issues emerge: Which copulas can appropriately model the structure of return dependence? Does incorporating extreme value theory (EVT) in copulas enhance the effectiveness in estimating VaRs? Can the models be applied to any returns regardless of their different shapes of distribution? How do the results of different models diverge across countries? How does the selection of models associate with foreign exchange policies and price movement limits? In the last three decades, due to rising economic significance and wealth accumulation, research of selecting risk management tools in Asian developing countries has become an indispensable element in global wealth management. Using daily data, we examine the potential economic values of applying various copulas with the EVT on estimating Monte Carlo VaRs in six Asian countries based on their different return distribution shapes.

The selection of approaches to estimate a VaR of portfolio is the core of risk management. The estimation of the parameters has become critical to finance academia and professionals after the recent financial crisis. In the traditional Monte Carlo VaR approach, correlations are estimated by the Pearson product-moment correlation coefficient or Gaussian-based copulas, under which returns of financial assets are assumed to be normally distributed and the relationships between financial assets are assumed to be linear. However, recent studies (e.g., Ang and Chen, 2002; Boyer, Gibson, and Loretan, 1999; Kolari, Moorman, and Sorescu, 2008;

Longin and Solnik, 2001; and Tastan, 2006) have shown that return correlations among assets are non-linear and time-varying. Specifically, most return distributions show asymmetric downside and upside movements and fat tails. Although recently the Gaussian copula is widely used in financial applications, such as J.P.Morgan's RiskMetrics^{TM} system, it ignores fat tail phenomena when it models correlation with empirical distributions of asset return. Finding an appropriate approach to model dependences between asset returns has become a challenge to risk management.

Using extreme value theory with copulas is one of the solutions to model dependences among multivariate distributions. The structure of dependence can then be used in Monte Carlo simulation to determine the portfolio VaRs. The copula provides a robust and flexible method of consistent estimation for dependence. Recently, the copula theory has been extended to the conditional case, allowing the use of copulas to model dynamic structures. Available conditional copulas are, however, limited to pairwise or low dimensions, as documented by Dias and Embrechts (2004), Patton (2004, 2006a and b), and Jondeau and Rockinger (2006). We extend the framework by incorporating copulas and EVT by adopting a semi-parametric form of the marginal distribution. The copula-EVT semi-parametric approach uses extreme value theory to enhance accuracy of modeling the distribution tails of asset returns and uses copulas to model the dependence among asset return distributions.

This paper synthesizes the *modi operandi* of various copulas and the EVT and aims to examine their practicality in risk management by using data from six Asian emerging markets. Recent empirical studies applying copulas focus on developed countries but pay less attention to emerging markets. Due to financial liberalization since early 1990’s, increasing foreign investments, active government interventions, and policy limitations in Asian developing countries, truncation and tail-dependence should be taken into account modeling distribution. Appropriately modeling dependences between asset markets and foreign exchange markets, especially during periods of excessive volatility, is a major concern to international investors. We apply various copulas and EVT to model dependences among returns and, furthermore, investigate their effectiveness in managing risk.

Our paper is the first research to estimate portfolio VaRs in emerging markets by applying the copula-EVT semi-parametric method. This method involves two aspects: first, for modeling every return distribution, the approach combines a parametric approach with the generalized Pareto distribution (GPD) to model the tails of the return distribution and a non-parametric approach with the empirical distribution to model the center of the return distribution; second, for correlation estimation, a copula function is applied to model dependences between return distribution pairs. We examine the effectiveness of copula functions of normality (Gaussian), right-tail dependence (Gumbel), and right-tail dependence (Clayton) incorporating extreme value theory. We further perform backtesting analysis to verify whether the copula-EVT approaches provide better performance in measuring VaRs in different distributions.

Our empirical results suggest that these countries have positive but weak correlations between stock returns and change in currency exchange rate. The difference in foreign exchange rate policies and government intervention affects the distribution and tail dependence in each country. This is particularly significant for countries (i.e., Taiwan and Korea) with strictly stock price movement restriction during a period of market downturn. The backtesting analysis of the Monte Carlo simulation suggests that the copulas with the EVT perform better estimations of VaRs than the pure copulas for the return distributions existing fat tails and truncations. Among the copula approaches that are applied in this study, the Clayton copula-EVT approach has the highest significant level regardless of the shapes of return distributions.

This paper is organized as follows: Section 2 reviews literature. Section 3 presents the approaches for copulas and the EVT. Section 4 describes the data. Section 5 discusses the major empirical results. Section 6 reports the results of backtesting analysis. Section 7 presents the conclusions.

II. Literature Review

The precision of computation of portfolio VaRs primarily relies on the modeling of the joint distribution of asset returns. One of the most important elements for implementing the Monte Carlo simulation in computing a portfolio VaR is how to appropriately estimate correlations between asset returns. Though dependences of financial asset returns can be computed by the Pearson product-moment coefficient, it is widely known that asset returns hardly follow normal distribution. In addition, the relationships among financial assets generally are non-linear and time-varying. The price jumps brought by economic shock also make modeling co-movement among asset returns more challenging.

Previous study suggests that correlations are asymmetric across downside and upside market movements, and the tails of the return distributions are fatter than normal distribution. Ang and Chen (2002); Boyer, Gibson, and Mico (1999); Kolari, Moorman, and Sorescu (2008); Longin and Solnik, (2001); and Tastan (2006) suggest that correlation based on the normal distribution may generate misleading results in portfolio risk management. One of the possible solutions is to apply the dynamic conditional correlation (DCC) model proposed by Engle (2002) and Engle and Colacito (2006). The family of the DCC method considers the time-variation issue but does not take in account departure of normality as well as extreme values. Silvennoinen and Terasvirta (2009) and Tsafack (2009) find that the presence of asymmetry in the tail correlation may lead the DCC family models to biased estimation in portfolio management.

Recent studies show that copulas can be an alternative for modeling correlation in the characterization of nonlinearity and asymptotic dependence (e.g., Chen and Fan, 2006; Kole, Koedijk, and Verbeek, 2007; Longin and Solnik, 2001; Patton, 2006a and b; and Rodriguez, 2007). Copulas can model multivariate distribution functions of random variables with flexible one-dimensional marginal distribution functions of each random variable set. Chen and Fan (2006), Okimoto (2008), and Palaro and Hotta (2006) suggest that asymmetric copulas are useful in constructing tail dependence. Compared with traditional methods of VaR estimation, copulas can be a very powerful tool in this regard.

How to deal with fat tail is another crucial issue in risk modeling. Da Silva and Mendes, (2003); Ho, Burridge, Cadle, and Theobald (2000); and Neftci (2000) propose that modeling the tails of a multivariate distribution in extreme value theory (EVT) can provide a better measurement. Bali (2007) and Straetmans, Verschoor, and Wolff (2008) find that the EVT models perform better estimations in an extreme volatile market than the standard approach that assumes the normal distribution.

Previous research of the dependence between equity and currency markets generates mixed conclusions. Solnik (1987) applies ordinary least squares (OLS) to study industrial countries and suggests that the relationship between real stock returns and changes in the real exchange rate is positively weak, especially during the period of October 1979 to December 1983. Ajayi and Mougoue (1996) employ the error correction model (ECM) to examine short term and long term dynamic correlations between stock prices and the exchange rate in developed countries. They conclude that the relationship between domestic stock prices and domestic currency values is negative in the short-run but positive in the long-run. Patro, Wald, and Wu (2002) find that stock returns in countries with high exports show positive exposure to exchange rate depreciation. On the other hand, stock returns in the countries of high import are associated with negative exposure. Moreover, several studies have examined the relationship between stock returns and exchange rates at the firm level. Doidge, Griffin, and Williamson (2006) report that, during periods of large currency depreciations, firms with high international sales generate higher stock returns than firms with no (low) international sales. Kolari, Moorman, and Sorescu (2008) find that firms that are highly sensitive to foreign exchange rate exposure tend to have low stock returns. Using various copulas to revisit this issue in emerging markets will provide insight regarding the relationship between asset return and currency value as well as model selection.

Recent empirical studies applying copulas, e.g., Chen and Fan (2006); Kole, Koedijk, and Verbeek (2007); Okimoto (2008); Patton (2006a and b); and Rodriguez (2007), focus on developed countries but pay less attention to emerging markets, in particular the dependence between stock markets and currency exchange markets. Due to financial liberalization since the early 1990's, active government interventions, and stock price movement limitations in these countries, a study using models that take into account distribution truncation and tail-dependence is needed. To our best knowledge, this article is the first in the literature to focus on the correlation between stock return and change in foreign exchange.

III. Methodology

The purpose of this paper is to investigate the effectiveness of various models in estimating portfolio risk. The first step is to fashion the dynamics between stock return and change in currency value. The continuous return of an international portfolio investment can be described as follows:

$$\mathrm{r_{p,t}} = \ln\left(\frac{i_t \times e_t}{i_{t-1} \times e_{t-1}}\right) = \ln\left(\frac{i_t}{i_{t-1}}\right) + \ln\left(\frac{e_t}{e_{t-1}}\right) = \mathrm{r_{i,t}} + \mathrm{r_{e,t}} \tag{1}$$

where $r_{p,t}$ is the continuous rate of return of the portfolio at time t, i is the stock index quoting in the local currency, and e represents the American terms foreign currency exchange rate, which is defined as the price of one unit of foreign currency in U.S. dollars, $r_{i,t}$ is the stock index return at time t, and $r_{e,t}$ is the change in exchange rate at time t. Value-at-Risk (VaR) represents the possible loss of a portfolio that is held over a target horizon under a specific confidence level. Mathematically, VaR can be defined as:

$$\mathrm{VaR}_{\alpha}^{h} = \inf\left\{r \in R \middle| P\left(L \geq V\hat{a}R\right) = \alpha\right\} \tag{2}$$

where L is a sequence number, $-r_{p,t}$, $-r_{p,t-1}$, $-r_{p,t-2}$, $-r_{p,t-3}$, …$-r_{p,t-h}$, denoting the portfolio negative return at times t, t-1, t-2, ..., t-h, respectively; α is a small percentage close to 0, and h is the target horizon (the differential period between time t and time $t+h$). In this study, we use the Monte Carlo approach to measure the value of VaR, in which the VaR is estimated with return data simulated repeatedly from a random process based on an appropriate joint distribution that describes bilateral correlations between stock index returns and changes in exchange rate. Thus, the Monte Carlo process is defined as follows

$$\begin{bmatrix} \dfrac{dR_i}{r_i} \\ \dfrac{dR_e}{r_e} \end{bmatrix} = \begin{bmatrix} \mu_i \\ \mu_e \end{bmatrix} dt + \Phi \begin{bmatrix} \sqrt{dt_i} \\ \sqrt{dt_e} \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \sigma_i^2 & \sigma_{i,e} \\ \sigma_{i,e} & \sigma_e^2 \end{bmatrix}, \sigma_{i,e} \equiv \rho_{i,e} \times \sigma_i \times \sigma_e$$

$\rho_{,i,e}$ is the correlation coefficient between r_{i} and r_{e}. Thus, how to construct the comovement between r_i and r_e is important in measuring portfolio Monte Carlo VaR. Traditionally, a conventional Pearson's product-moment coefficient is widely used to estimate the correlation. However, this method is challenged by some frequently observed features in return data, such as departure from normality and nonlinearity.

3.1. Copulas

The copula methodology has become a significant technique for handling the correlation between markets and risks with larger flexibility in recent years (Cherubini, Luciano, and Vecchiato, 2004). Suppose U_1, U_2, …, U_n, are n random variable sets with a joint distribution function:

$$F_{u_1,u_2,\ldots,u_n}(u_1,u_2,\ldots,u_n) = p(U_1 \le u_1, U_2 \le u_2, \ldots, U_n \le u_n)\, for (u_1,u_2,\ldots,u_n) \in \Re^n \quad (3)$$

The marginal distribution functions of U_1, U_2 …, U_{n} are

$$\begin{aligned} F_{u_1}(u_1) &= p(U_1 \le u_1) \\ F_{u_2}(u_2) &= p(U_2 \le u_2) \\ &\vdots \\ F_{u_n}(u_n) &= p(U_n \le u_n). \end{aligned} \quad (4)$$

According to Sklar's theorem, if $F_{u_1}, F_{u_2}, \ldots, F_{u_n}$ are continuous functions, then there exists a unique copula such that

$$F_{u_1,u_2,\ldots,u_n}(u_1,u_2,\ldots,u_n) = c(F_{U_1}(u_1), F_{U_2}(u_2), \ldots, F_{U_n}(u_n)) \quad (5)$$

For bivariate variables such as the stock index returns r_i and change in exchange rates r_e, the above equation can be rewritten as

$$c(x, y) = p(R_i \leq r_i, R_e \leq r_e), for(x, y) \in [0,1]^2 \tag{6}$$

where, R_i and R_e are the marginal distribution functions of the return variable sets, r_i and r_e respectively, x and y are uniform distribution of stock index returns and changes in exchange rates, and $C(x, y)$ is the copula function. Following previous studies, we apply Gaussian, Gumbel, and Clayton copulas to feature tail dependence and examine their effectiveness in modeling risk. The Gaussian copula describes normally distributed data; the Gumbel copula includes a heavy tail on the right-hand side; and the Clayton copula exhibits a fat tail on the left.

The Gaussian copula is described as follows:

$$c(x, y) = \int_{-\infty}^{\Phi^{-1}(x)} dx \int_{-\infty}^{\Phi^{-1}(y)} dy \frac{1}{2\pi\sqrt{1-\delta^2}} \exp\left\{-\frac{x^2 - 2\delta xy + y^2}{2(1-\delta^2)}\right\} = \Phi_\delta\left(\Phi^{-1}(x), \Phi^{-1}(y)\right) \tag{7}$$

where Φ denotes a univariate standard normal distribution and Φ_δ is a bivariate standard normal distribution with the correlation coefficient $-1 \leq \delta \leq 1$. The benefit of the Gaussian copula is easy to apply and is widely used in financial applications, such as J.P.Morgan's RiskMetrics™ system (Zivot and Wang, 2006). However, its bivariate standard normality assumption is too strong for most financial data, and ignores tail dependence.

Tail dependence is frequently found in financial data. When extreme events occur, financial markets response the events asymmetrically. The upper (lower) tail dependence represents the probability that one variable sequence exceeds (drops below) its q-th quantile given that the other variable sequence also exceeds (drops below) its own q-th quantile, in the limit q tends to infinity.

To deal with tail dependence issues, Embrechts, Frey, and McNeil (2005) suggest that the Gumbel copula can be applied in the case of upper tail dependence, while the Clayton copula can be applied in the case of lower tail dependence[2]. The Gumbel copula can be written as:

$$c(x, y) = \exp\left\{-\left[\left(-\ln(x)^{\delta}\right)+\left(-\ln(y)^{\delta}\right)\right]^{\frac{1}{\delta}}\right\}, \delta \geq 1 \tag{8}$$

δ represents the extent of dependence between x and y. When δ reaches its minimum, δ=1, no upper tail dependence exists between x and y. As δ increases, the upper tail dependence between x and y increases until δ approaches its maximum, which implies x and y have a perfect upper tail dependence. δ can be normalized by the following equation $\lambda_u = 2 - 2^{\frac{1}{\delta}}$, where λ_u is the normalized coefficient of correlation of x and y.

The form of Clayton copula is:

$$c(x, y) = \max\left[\left(x^{-\delta} + y^{-\delta} - 1\right)^{-\frac{1}{\delta}}, 0 \leq \delta \leq \infty\right]. \tag{9}$$

Thus, when δ= 0, no lower tail dependence exists between x and y. As δ increases, the lower tail dependence between the two variables increases until δ approaches its maximum ∞, which implies x and y have a perfect low tail dependence. The dependence parameter δ can also be normalized by $\lambda_l = 2^{\frac{1}{\delta}}$, where λ_l is the normalized coefficient of correlation of x and y.

3.2. Extreme Value Theory

Although copulas model the dependence between two variable sets by the empirical distribution of each variable set, it does not catch jumps of price caused by incidents. We further

[2] Detailed derivation is presented in Appendix I

incorporate the Extreme Value Theory (EVT) when the empirical distribution of the VaR is formed. The extreme values can be modeled by the block maxima or the peaks-over-threshold.[3] There are two reasons that support applying the second method in the current paper. First, the block maxima method defines extreme events as the maximum (minimum) value in each sub-period. Therefore, it tends to abandon a great deal of data. Second, the peaks-over-threshold considers the clustering phenomenon, which is frequently found in financial data.

The generalized Pareto distribution (GPD) is used to model tails of distribution. Consider a sequence of n independent and identically distributed (iid) random variables R (r_1, r_2, ...r_n) that represent the daily returns. The excess distribution $F(r)$, which is the probability that R exceeds a fixed threshold v, can be estimated with a GPD fitted by the maximum likelihood method. The tail estimator is as follow:

$$F(\hat{r}) = 1 - \frac{k}{n}\left(1 + \hat{\xi}\frac{r - v}{\beta}\right)^{\frac{-1}{\xi}} \text{ for } r > v, \tag{10}$$

where β is the scale parameter and ξ is the shape parameter, n is the number of observations, and k denotes the number of observations beyond the threshold v.

The determination of threshold value v is crucial. There is a trade-off between variance and bias. According to the Picklands-Balkema-De Haan theorem, a high v value is desired. However a high v also leads to a large variance in the estimators because of few exceedance data. In contrast, a small v might include observations not belonging to the tails and leads to biased estimators, and the Picklands-Balkema-De Haan theorem will not hold.[4]

[3] See Coles (2001) and Beirlant et al. (2004) for detailed treatments of the extreme value theory

[4] Detailed deviation is presented in Appendix II.

Previous studies, such as Neftci (2000), McNeil and Frey (2000), and Longin and Solnik (2001), suggest different levels of threshold or propose methods to determine the optimal level. In this study, we adopted the mean-excess plot approach by McNeil and Frey (2000) to choose the optimal threshold. Consistent with the above papers, our findings indicate that most of the thresholds in the data are around the extreme 5% of the observations.

The value of ξ represents the shape of distribution. For the variable sequence that has a finite tail, i.e., $\xi<0$, the distribution is of the Weibull type, such as the beta and uniform distributions. For the tail that declines exponentially, i.e., $\xi=0$, the distribution is of the Gumbell family, such as the normal, log normal, and exponential distributions. If $\xi>0$, then the tail declines slowly and the distribution is in the Frèchet family such as the Pareto distribution. We estimate parameters in two stages. At the first stage, the scale and sharp parameters were estimated. The GPD (in the distribution tails) and the empirical distribution (in the distribution center) are used to model the marginal distributions of stock index return and change in currency value. At the second stage, we apply an *inference function for the margins* (IFM) method suggested by Joe and Xu (1996). The purpose is to estimate the joint distribution of the two returns. The detailed derivation of this approach is explained in Appendix III.

3.3. The Copula-EVT Semiparametric Models

This paper compares the effectiveness of the above models in estimating correlation. We apply the models comprising Pearson's product-movement coefficient, Gaussian copula, Gumbel copula, and the Clayton copula. Furthermore, we evaluate the usefulness of combining the EVT model with Gaussian, Gumbel, and Clayton copulas. The latter three models use both the concept of the copula in estimating the correlation between stock index return distribution and currency exchange rate return distribution and semi-parametric approach in estimating marginal distribution of the stock index returns and changes in exchange rate respectively. This semi-parametric approach in modeling return distribution is expected to yield a close fit to the empirical data.

IV. Data

The daily data of stock market indices and American term currency exchange rates from six Asian countries are obtained from the Global Financial Data. The sample period is from the first business day in 2000 to the last business day in 2007.[5] Table 1 demonstrates a statistical summary of the change in exchange rate, stock index return, and portfolio return. The negative skewness in the stock index returns for all six countries suggests that the distributions have long tails to the left. The fourth moments of the stock returns and the change in exchange rate indicate volatility clustering in distributions. The results of the Jarque-Bera test suggest obvious departure from normality in stock index return, exchange return, and portfolio return in all six countries. Thus, it is critical to examine the effectiveness of various econometrical models in managing portfolio risk.

[INSERT Table 1 ABOUT HERE]

[5] Since the six countries have their own business days, the number of observations varied in each country. Indonesia had 1957 observations, Korea had 1968 observations, Malaysia had 1967 observations, Singapore had 2004 observations, Taiwan had 1997 observations, and Thailand had 1961 observations.

The other characteristics found in Table 1 support the usage of copulas and EVT. First, because most of these countries impose a certain level of control on currency, the volatility in foreign exchange rate is low. Second, though the U.S. dollar portfolio returns in most countries are positive, the portfolios invested in Taiwan generates loss in both stock index and exchange rate. Third, the portfolios invested in Korea, Malaysia, Singapore, and Thailand benefit from both stock capital gain and positive returns in foreign exchange rate.

Table 1 also presents the one-day VaR in the U.S. dollar portfolio by using the unconditional averages and volatilities. The investor can use the numbers to generate what the maximum loss value would be in its U.S. dollar portfolio at a 99% confidence level. Among the countries, the portfolios invested in Korea have the highest VaR but in Malaysia tend to be of the lowest VaR.

V. Empirical Results

5.1. Shapes of the return distribution tails

Table 2 reports the EVT estimations of the return distribution tails. The log-likelihood values (llv) are also presented to demonstrate the fitness of the model. The shape of distribution of stock return in a country is associated with the price limit policy in some of these countries. The market efficiency is weakened if stock price is constrained to move within a certain percentage in comparison with the previous closing price. The significant negative sharp parameters (ξ) of stock returns in Taiwan and Korea suggest the upper tails are finite. This truncated distribution tail phenomenon is associated with the daily price limit policy.[6] Though

[6] To prevent unreasonably large price movements on the market, several emerging markets impose limitations on daily stock price movement, meaning that the change in an individual stock in each trading day cannot exceed a certain fixed percentage point in comparison with the previous day's closing price. e.g. in Taiwan, the change in a

both countries show truncation in the stock return distributions during bullish periods, Korea does not show significant truncation on the left side of the stock return distribution. For a country of a larger price limit (30%) such as Thailand, stock return distribution has significant fat tails on both sides. The shapes of the upper tail of the stock index return distributions in Indonesia, Malaysia, and Singapore are not significant, but the left-side distributions of the stock returns in these three countries are statistically significant fat-tailed. This finding is associated with the fact that market clustering becomes more significant during bearish periods.

[INSERT Table 2 ABOUT HERE]

The distributions of currency exchange returns also vary across countries. The truncation in the upper and lower tails of Korean Won value implies government intervention when its exchange rate is volatile. Taiwan and Thailand have significant fat tails on both upper and lower sides of the change in foreign exchange rate distribution, while Indonesia, Malaysia, and Singapore only have significant fat tails on right side but not on left side.

5.2. Correlation coefficient estimates

We further evaluate the effectiveness in modeling the Monte Carlo VaR by using the following seven methods to estimate correlations: Pearson correlation, Gaussian copula, Gaussian copula-EVT, Gumbel copula, Gumbel copula-EVT, Clayton copula, and Clayton copula-EVT. The findings in the previous section indicate the importance of selecting

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stock price cannot exceed 7%, and the ceiling in Korea is 8%. With this imposed narrow price range on its stock market, the country's stock return distribution is easily truncated with no distribution tail.

dependence estimation models in risk management. Specifically, the distributions of the change in foreign currency exchange and stock index return in many countries are skew, leptokurtic, or truncated. Considering the joint effects caused by the above departures, the correlation coefficients modeled by normality-assuming methods, such as Pearson product-moment and Gaussian copula, might be questionable.

Table 3 shows the results of various copulas. The log-likelihood values (llv) are also presented to demonstrate the fitness of the model. The conditional correlations estimated by Gumbel and Clayton copulas as well as the Gumbel-EVT and Clayton-EVT are standardized by converting δ into λ_u and λ_l, the dependences of upper and lower tails, respectively. The stock returns, on average, are positively but weakly associated with the changes in the foreign exchange rate in these countries. The estimations of Gaussian copula are similar to those of Pearson correlation in certain countries, such as Korea, Malaysia, and Singapore. The difference between Gaussian copula and Pearson correlation are trivial for the returns that do not have significant fat tails or truncated distribution. On the other hand, for countries of major divergence from normality on tails, such as Taiwan and Thailand, the correlations estimated by the Gaussian copula vary from those estimated by the Pearson correlation.

[INSERT Table 3 ABOUT HERE]

The findings also suggest that the dependence between asset return and exchange rate vary in different states of economy. The Gumbel copula and Gumbel copula-EVT are used to estimate the upper tail dependence, i.e., the relationship of *excessive* increase in value between asset and currency. On the other hand, Clayton copula and Clayton copula-EVT are used to model the dependence at lower tail. In general, the correlation coefficients of the Gumbel copulas are higher than those of the Clayton copulas. Among the six countries, the correlation coefficients in Indonesia are the highest. This is associated with the fact that Indonesia imposes minimum currency control and stock price daily movement restriction is 30%. The less restrictive regulations provide greater freedom for both stock and foreign exchange markets.

The conditional correlations measured by these non-Gaussian copulas, in general, are smaller than normality-assuming models. The Gumbel copula and Gumbel copula-EVT (Clayton copula and Clayton copula-EVT) indicate that the connection of appreciation (depreciation) in stock and currency values is less strong than those measured by their normality-assuming counterparts. A lower co-movement between equity and currency values is particularly significant for the downfall of markets in these countries. This finding is related with the intervention of governments when financial crisis happens in this region. The only exceptions are the extreme upper returns in Taiwan and Thailand, in which there are fat-tails and truncation in return distributions. This suggests that the international capital flow may play an important role during a period of bullish market in the two countries. On the other hand, the governments tend to implement stabilizing policies during bearish markets.

VI. Backtesting Analysis

To examine effectiveness of the above models, we perform backtesting analysis of the Monte Carlo VaRs by using the correlations estimated from the copulas with and without incorporating the EVT. Suppose the daily VaR, $V\hat{a}R_{\alpha}^{h}$, at time *t-1* can be estimated by a Monte Carlo simulation series, s_{t-1}, s_{t-2},… s_{t-h}. At time *t*, a violation is said to occur if the real historical portfolio return $r_{p,t}$ is greater than $V\hat{a}R_{\alpha}^{h}$. According to the study by McNeil and Frey (2000), the backtesting process can be defined as follows:

$$L_t = l_{r_t > V\hat{a}R_{\alpha}^{h}} \sim Be(1-q)\,. \tag{11}$$

We define the indicator variance as $L_t = l_{r_t > V\hat{a}R_{\alpha}^{h}}$. Thus, the process L_t for $t \in \Re$, is a binomial distribution with a violation probability α. For the backtesting of each country, we use a time rolling window of 250 days to estimate each Monte Carlo VaR. Specifically, based on the correlations estimated by the seven methods, we use the Monte Carlo method to generate seven simulation series which have an identical length like the actual data for each country respectively. Then we take the simulation data s_1 to s_{250} to compute the portfolio VaR with a 1% of α for the next period, $V\hat{a}R_{251}$. A violation is defined as the realized loss L_{251} is greater than that of $V\hat{a}R_{251}$. We repeat the process to compute the VaR for the next day and further compare it with the actual loss on the same day in the portfolio[7], i.e. using simulation data s_2 to s_{251} to compute for the $V\hat{a}R_{252}$ and comparing it with the L_{252} .

Figure1 presents comparison between Monte Carlo simulation VaRs and actual portfolio loss in each country. The results of each country are presented in four graphs that include the actual values vs. Person product moment correlation Monte Carlo simulated VaRs, the actual values vs. the Gaussian family simulated Monte Carlo VaRs, the actual values vs. the Gumbel family simulated Monte Carlo VaRs, and the actual values vs. the Clayton family simulated Monte Carlo VaRs. In general, the copulas with the EVT have greater tendency to generate more

[7] In general there are 250 business days in a year. As explained in footnote 4, the length of the data in each country is around 1950 days; therefore, each model generates around 1700 VaRs.

accurate Monte Carlo VaR estimations than the corresponding ones that do not take into account the EVT.

[INSERT Figure 1 ABOUT HERE]

Table 4 reports the results of the backtesting. Following a binomial distribution, the theoretical expectation is determined by the number of observations and the expected probability of occurrence of violation. Due to difference in return distributions in each country, the backtesting results vary slightly among the seven models. The copulas that include the EVT show better performance than the corresponding methods without incorporating the EVT. The normality-based approaches provide less accurate estimation in the countries with fat tails in return distributions than the copula-based approaches do. For instance, the Pearson correlation coefficient performs not worse than the conditional correlations in Singapore and Malaysia, in which stock and currency returns do not have significant fat tails. The estimates provided by the non-normality copulas with the EVT, particularly the Clayton copula, perform better than the other approaches in Korea, Malaysia, Singapore, Taiwan, and Thailand. These models seem to fit the data more closely than Gaussian-based methods, even the data that has fat, truncated, or thin distribution tails. The superiority of the Clayton copula that suggests estimating the downside distribution is a key element in risk modeling.

[INSERT Table 4 ABOUT HERE]

VII. Conclusion

Rarely has the importance of selecting models of dependence among assets received more attention than during the recent financial crisis. The answer to the puzzle is critical to both financial academics and Wall Street. This paper responds to this crucial issue by empirically exploring the effectiveness of various copula-extreme value based semi-parametric approaches in managing portfolio risks in six markets in Asia. We incorporate extreme value theory (EVT) to model the tails of return distributions and use various copula functions, such as Gaussian, Gumbel, and Clayton, to build the joint distribution of returns. In addition, to evaluate the effectiveness of these models, we perform backtesting analysis by conducting Monte Carlo VaR simulation using the outcomes of different copulas.

Our empirical findings suggest that the sample countries have positive but weak correlations between stock index returns and change in currency value. The difference in foreign exchange rate policies and government intervention affect the distribution and tail dependence in each country. This is particularly significant for the countries (e.g., Indonesia and Thailand) with constrained price movement limits during a period of high volatility. The backtesting analysis of the Monte Carlo simulation of the portfolio VaR suggests that the copulas with the EVT perform better estimations on portfolio VaRs than the pure copulas for the return distributions with fat tails and truncated tails. Among all models we presented in this paper, Clayton copula-EVT approach has the highest significant level regardless of the shapes of return distributions. Our results suggest that asymmetric conditional correlation incorporating the extreme value theory could be useful in modeling portfolio risk.

This paper adds to the current literature by investigating the value of conditional dependence model in managing market risk in developing countries. We estimate portfolio VaRs by applying various copulas incorporating extreme value theory. These functions take into account the shape, tail dependence, and truncation of distributions and more adeptly model the return co-movements among assets. The backtesting results support the benefit of tail-dependent models, such as Gumbel and Clayton copulas, to craft portfolio VaRs simulation. Our analysis also provides insight in determining the usage of portfolio risk management models in different scenarios. Future research of copulas and the EVT can extend to portfolio management and contagion of asset returns and interest rates.

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Appendix I

According to Embrechts, Frey, and McNeil (2005), the coefficient of upper tail dependence (λ_u) of 2 series X and Y can be defined as:

$$\lambda_u(x,y) = \lim_{q-1^-} p\left[Y > \overleftarrow{F_{U_y}}(q) \middle| X > \overleftarrow{F_{U_x}}(q)\right]. \tag{A-I-1}$$

The upper tail dependence presents the probability that Y exceeds its q-th quantile given that X exceeds its q-th quantile, considering the limit as q goes to its infinity. If the limit $\lambda_u \in [0,1]$ exists, then X and Y are said to show upper tail dependence. In the same manner, the coefficient of lower tail dependence (λ_l) of X and Y is described as:

$$\lambda_l(x,y) = \lim_{q-0^+} p\left[Y \le \overleftarrow{F_{U_y}}(q) \middle| X \le \overleftarrow{F_{U_x}}(q)\right]. \tag{A-I-2}$$

Since both F_{U_x} and F_{U_y} are continuous density functions, the upper tail dependence can be presented as:

$$\lambda_u(x,y) = \lim_{q-1^-} \frac{p\left[Y > \overleftarrow{F_{U_y}}(q) \middle| X > \overleftarrow{F_{U_x}}(q)\right]}{p\left[X > \overleftarrow{F_{U_x}}(q)\right]}. \tag{A-I-3}$$

For lower tail dependence, it can be described as:

$$\lambda_l(x,y) = \lim_{q-0^+} \frac{p\left[Y \le \overleftarrow{F_{U_y}}(q) \middle| X \le \overleftarrow{F_{U_x}}(q)\right]}{p\left[X \le \overleftarrow{F_{U_x}}(q)\right]}. \tag{A-I-4}$$

Appendix II

Consider *R*, a sequence of n independent and identically distributed (*iid*) random variables $r_1, r_2, ...r_n$, representing the profits and losses of daily returns. The excess distribution $F_v(r)$, which R exceeds a fixed threshold v, has the following cumulative distribution function:

$$F_v(r) = p(R > r + v | R > v) = \frac{1 - F(r+v)}{1 - F(v)}, \text{ for } r > 0. \qquad \text{(A-II-1)}$$

According to Picklands-Balkema-De Haan theorem, an appropriate distribution to approximate $F_v(r)$ is the Generalized Pareto Distribution (GPD). The Picklands-Balkema-De Haan theorem shows that:

$$\lim_{v \to r_0} \sup_{0 \le r \le r_0 - v} \left| F_v(r) - G_{\xi\beta(v)}(r) \right| = 0, \qquad \text{(A-II-2)}$$

where r_0 is defined as the endpoint of R and $G_{\xi\beta(v)}(r)$ is the function of the GPD. In other words, when *v* is very close to the endpoint, the excess distribution is the approximation of the GPD.

$$\hat{F}_v(r) \approx \hat{G}_{\xi\beta(v)}(r)$$

The GPD has the following analytical form:

$$\hat{G}_{\xi\beta(v)}(r) = \left(1 + \frac{\xi(r-v)}{\beta}\right)^{\frac{-1}{\xi}},$$

(A-II-3)

Where β is the scale parameter and ξ is the shape parameter, such as that if $\xi<0$, the distribution of the variable sequence has a finite tail; if $\xi=0$, then the tail declines exponentially (thin-tailed; and if ξ>0, the tail declines slowly (fat-tailed). The approximation distribution $F_v(r)$ for $r>v$ can be defined as the follows:

$$F_v(r) = \left(1 - (1 - F(v)\right)G_{\xi\beta(v)}(r)\,. \qquad \text{(A-II-4)}$$

The CDF function of $F(v)$ can be defined as:

$$\hat{F}(v) = \frac{(n-k)}{n}$$

Where n denotes the number of total observations and k is the number of exceedance over the threshold v. If n is large enough, then the estimation of $F_v(r)$ is defined as:

$$\hat{F}_v(r) = 1 - \frac{k}{n}\left(1 + \frac{\xi(r-v)}{\beta}\right)^{\frac{-1}{\xi}}. \qquad \text{(A-II-5)}$$

Appendix III

A bivariate joint density function can be represented as:

$$f(x,y) = c(F_X(x_i;\theta_x), F_Y(y_i;\theta_y);\Theta) f_X(x_i;\theta_x), f_Y(y_i;\theta_y)\,. \qquad \text{(A-III-1)}$$

Here θ_x are the parameters for the marginal distribution F_X and θ_y are the parameters for the marginal distribution F_Y. Θ are parameters for the copula density c. Therefore, the exact log-likeligood function of the above joint density function can be presented as

$$l(\eta) = \sum_{i=1}^{n} \ln c(F_X(x_i;\theta_x), F_Y(y_i;\theta_y);\Theta) + \sum_{i=1}^{n}\left[\ln f_X(x_i;\theta_x) + \ln f_Y(y_i;\theta_y)\right]. \qquad \text{(A-III-2)}$$

Then by maximization, we can obtain the exact maximum likelihood estimator as

$$\hat{\eta}_{MLE} = \max_{\eta} l(\eta)$$

According to Joe and Xu (1996), the parameters can be estimated by an *inference for the margins* or IFM method. This method includes two steps:

First, the parameters of the univariate marginal distributions were estimated as:

$$\hat{\theta}_x = ArgMax_{\theta_x} \sum_{i=1}^{n} \ln f_X(x_i;\theta_x)\,, \qquad \text{(A-III-3)}$$

And

$$\hat{\theta}_y = ArgMax_{\theta_y} \sum_{i=1}^{n} \ln f_Y(y_i;\theta_y)\,. \qquad \text{(A-III-4)}$$

Then at the second step, given $\hat{\theta}_x$ and $\hat{\theta}_y$, the dependence parameters Θ were estimated as:

$$\hat{\Theta} = ArgMax_{\Theta} \sum_{i=1}^{n}\left[\ln c(f_X(x_i;\hat{\theta}_x) + \ln f_Y(y_i;\hat{\theta}_y))\right]. \qquad \text{(A-III-5)}$$

Table 1. Statistical Summary

The table presents the time of liberalization of stock market, one day 1% value-at-risk (VaR) of US$ portfolio, mean, standard deviation, skewness, kurtosis of the return. The results of Jarque-Bera test are demonstrated.

Country/Stock Market Index			Liberalization	VaR (1%, 1day)	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
Indonesia	Jakarta SE Composite	$r_{e,t}$			-0.0001416	0.0078	1.0391	24.791	39068.85**
		$r_{i,t}$	Sept. 89		0.0000070	0.0138	-0.7058	7.615	1899.39**
		$r_{p,t}$		5.21%	0.0000056	0.0177	-0.6056	9.035	3089.41**
Korea	Korea Stock Exchange Index	$r_{e,t}$			0.0000923	0.0046	-0.4036	5.475	555.59**
		$r_{i,t}$	Jan. 92		0.0000030	0.0182	-0.5519	6.966	1389.63**
		$r_{p,t}$		5.62%	0.0000039	0.0198	-0.5209	6.295	979.04**
Malaysia	Kuala Lumpur Stock Exchange Composite	$r_{e,t}$			0.0000007	0.0014	2.0930	19.792	23116.07**
		$r_{i,t}$	Dec. 88		0.0000028	0.0092	-0.5709	8.961	3019.51**
		$r_{p,t}$		2.95%	0.0000035	0.0096	-53.0901	8.409	2490.30**
Singapore	Singapore Strait Times Stock Index	$r_{e,t}$			0.0000007	0.0027	0.0555	5.927	716.60**
		$r_{i,t}$	Open		0.0000014	0.0116	-0.5157	7.668	1908.35**
		$r_{p,t}$		3.41%	0.0000021	0.0121	-0.4713	6.784	1269.55**
Taiwan	Taiwan Stock Exchange Capitalization Weighted Index	$r_{e,t}$			-0.0000002	0.0034	0.4196	95.080	705855.05**
		$r_{i,t}$	Jan. 91		-0.0000002	0.0155	-0.1011	4.945	318.37**
		$r_{p,t}$		4.60%	-0.0000004	0.0164	-0.1276	4.733	255.51**
Thailand	Thailand General Index	$r_{e,t}$			0.0000011	0.0060	0.8475	137.702	1481785.00**
		$r_{i,t}$	Sept. 87		0.0000028	0.0145	-0.7406	13.531	9340.70**
		$r_{p,t}$		4.20%	0.0000039	0.0161	-0.4350	10.674	4864.47**

** 1% significance level.

Table 2. Shapes of Distribution Tails

The estimation results of return distribution tails are presented. The parameters are estimated based on the Generalized Pareto Distribution, and the log likelihood values (llv) are presented. The t statistics are given in brackets with asterisks demonstrating the level of statistical significance.

Country		Indonesia	Korea	Malaysia	Singapore	Taiwan	Thailand
Stock Index Return							
Upper tail	ξ	0.0131 (0.1169)	-0.2299 (1.9414)*	0.0468 (0.4955)	-0.0027 (0.0228)	-0.2788 (2.6621)***	0.2087 (1.8101)*
	β	0.0074 (6.3511)***	0.0147 (5.8178)***	0.0058 (7.8426)***	0.0075 (6.1583)***	0.0135 (7.1298)***	0.0063 (6.7876)***
	llv	308.1	210.4	573.8	315.4	379.7	431.9
Lower Tail	ξ	0.2505 (1.8354)*	0.2359 (1.6891)*	0.3383 (2.2113)**	0.1482 (1.3776)	-0.1439 (-1.3596)	0.2787 (2.6082)***
	β	0.0085 (5.9346)***	0.0104 (5.7843)***	0.0059 (5.6042)***	0.0125 (6.7104)***	0.0125 (6.7104)***	0.0085 (5.8577)***
	llv	338.0	303.1	398.3	366.0	313.5	292.7
Foreign Exchange Return							
Upper tail	ξ	0.2897 (2.5364)**	-0.1761 (2.3007)**	0.2465 ((1.6092)	0.2259 (1.9080)*	0.4383 (3.8363)***	0.5542 (3.6700)***
	β	0.0050 (7.1975)***	0.0034 (8.9451)***	0.0014 (5.5971)***	0.0012 (6.8782)***	0.0018 (7.1390)***	0.0027 (5.7995)***
	llv	480.8	696.5	598.7	525.0	602.9	435.3
Lower Tail	ξ	0.0136 (0.1107)	-0.2156 (-1.7427)*	0.0149 (0.1101)	0.1086 (1.0154)	0.6394 (3.7947)***	0.4149 (3.1819)***
	β	0.0076 (6.2212)***	0.0047 (5.8190)***	0.0020 (5.7423)***	0.0015 (6.8173)***	0.0016 (5.3800)***	0.0033 (6.3579)***
	llv	360.0	306.1	432.7	509.1	463.3	460.7

*0.90 statistical significance
**0.95 statistical significance
***0.99 statistical significance

Table 3. Estimation of Correlation Coefficients

The estimation results of correlation coefficient are presented. The parameters are estimated based on an inference function for the margins (IFM) method, and the log likelihood values (llv) are presented. The t statistics are given in brackets. We only present t statistics for the correlation coefficients in Gumbel and Clayton copulas, but not for the parameters, because the correlation coefficients are converted from the parameters as explained in section 4.23 and 4.24, so t- statistics are identical for both the correlation s and the parameters. Based on the t-statistics values, all correlation coefficients have reached the 99% statistical significance.

Country		Indonesia	Korea	Malaysia	Singapore	Taiwan	Thailand
Pearson	ρ	0.2973 (13.77)	0.2361 (10.77)	0.1802 (8.12)	0.0852 (3.83)	0.1493 (6.75)	0.0636 (2.95)
Gaussian	δ	0.2857 (14.23)	0.2392 (11.51)	0.1659 (7.62)	0.0925 (4.17)	0.2334 (11.26)	0.1629 (7.46)
	llv	82.29	57.24	27.08	8.50	55.26	26.02
Gaussian-EVT	δ	0.2850 (14.27)	0.2738 (11.49)	0.1429 (6.93)	0.0913 (4.14)	0.2289 (11.13)	0.6000 (7.37)
	llv	82.89	57.22	22.91	8.37	54.27	25.45
Gumbel	δ	1.1873	1.1537	1.0814	1.0504	1.1430	1.1030
	λ_u	0.2072 (60.33)	0.1764 (62.30)	0.1017 (65.90)	0.0655 (70.68)	0.1661 (62.01)	0.1254 (64.61)
	llv	66.48	47.98	16.51	7.10	38.74	24.32
Gumbel-EVT	δ	1.1852	1.1524	1.0781	1.0497	1.1403	1.1016
	λ_u	0.2053 (60.55)	0.1752 (62.47)	0.0980 (67.33)	0.0646 (71.22)	0.1635 (62.26)	0.1239 (64.89)
	llv	65.50	47.49	15.98	7.03	37.78	24.02
Clayton	δ	0.4098	0.3090	0.1818	0.1083	0.2685	0.1758
	λ_l	0.1842 (11.94)	0.1061 (9.51)	0.0221 (6.23)	0.0017 (4.01)	0.0757 (8.47)	0.0194 (5.79)
	llv	97.96	59.33	24.13	9.63	45.56	20.58
Clayton-EVT	δ	0.4030	0.3028	0.1274	0.1060	0.2589	0.1708
	λ_l	0.1791 (11.87)	0.1013 (9.40)	0.0043 (5.13)	0.0014 (3.96)	0.0688 (8.28)	0.0173 (5.69)
	llv	97.90	58.5	15.83	9.46	43.77	19.36

Table 4. Backtesting Results

The number of violations obtained using our approaches have been compared with the theoretically number of VaR violations, which can be calculated by the number of observations multiplying by the expected probability of occurrence of violation, and the P-values for a binomial test are presented in the table.

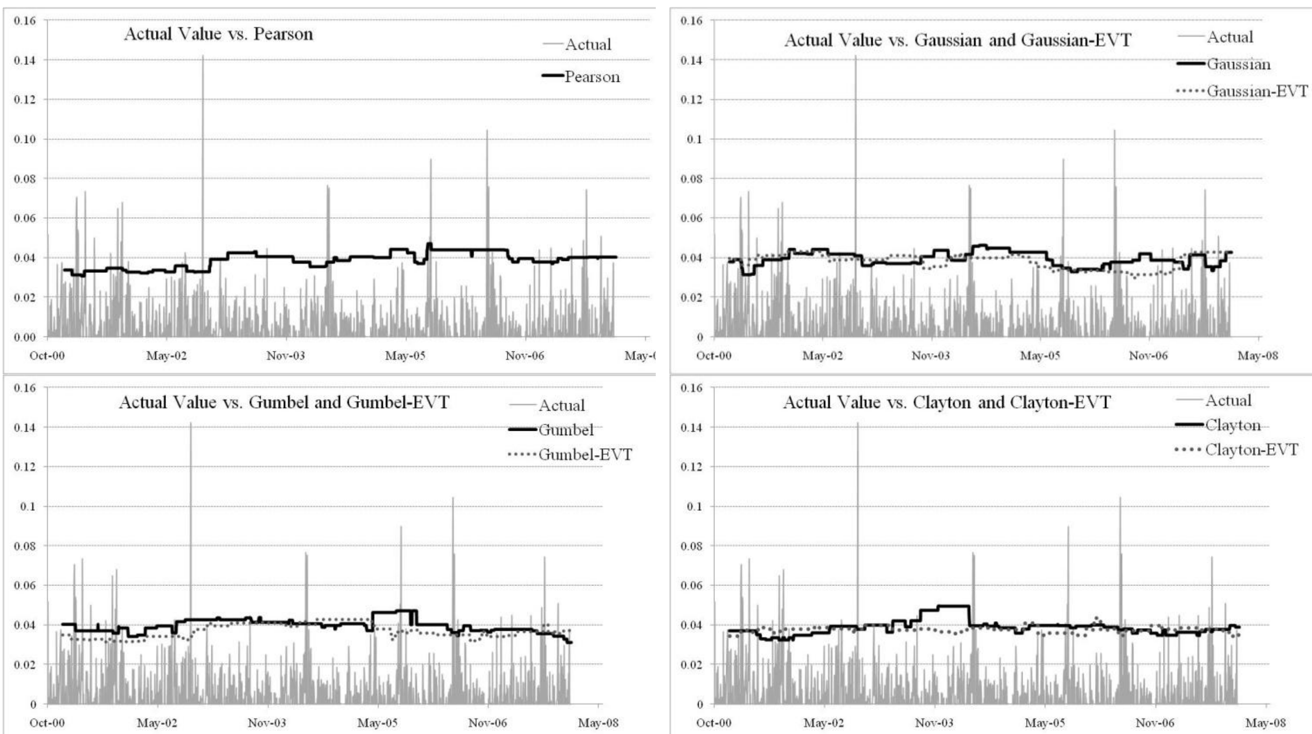
Country	Indonesia	Korea	Malaysia	Singapore	Taiwan	Thailand
Pearson	0.0011***	0.0764*	0.0437**	0.0000***	0.0542*	0.2874
Gaussian Copula	0.0005***	0.7508	0.0238**	0.0954	0.1487	0.9767
Gaussian Capula-EVT	0.0001***	0.7508	0.0123**	0.0565	0.3352	0.4116
Gumbel Copula	0.0005***	0.5755	0.0759	0.0170**	0.1487	0.2874
Gumbel Copula-EVT	0.0000***	0.0124**	0.0123**	0.0008***	0.0161**	0.1212
Clayron Copula	0.0002***	0.0764*	0.1254	0.0003***	0.0000***	0.0730*
Clayton Copula-EVT	0.0000***	0.0124**	0.0028***	0.0000***	0.0001***	0.0057***

*0.90 statistical significance
**0.95 statistical significance
***0.99 statistical significance

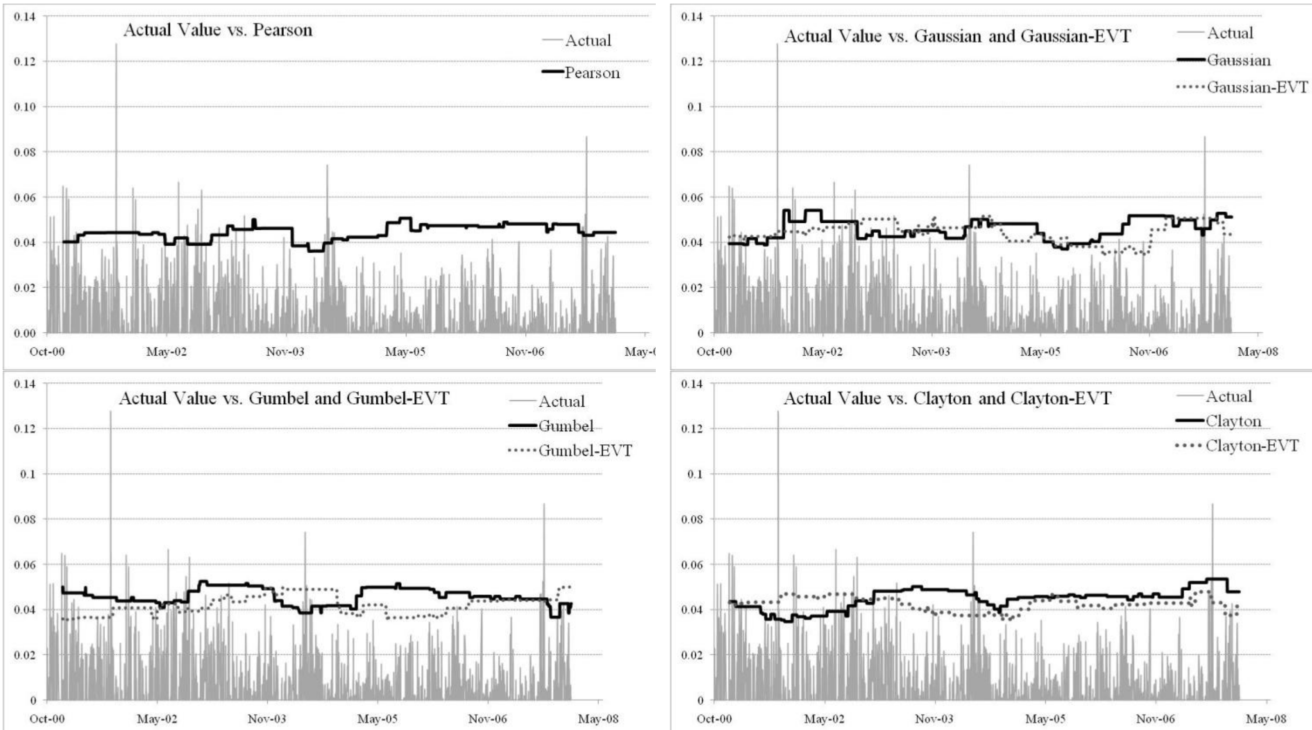
Figure 1: Actual Portfolio Loss and Different Monte Carlo *VaRs*

The graphs present the actual portfolio loss and Monte Carlo VaRs using various copulas in different countries. The copula models used for backtesting simulations are Pearson correlation, Gaussian copula, Gaussian copula-EVT, Gumbel copula, Gumbel copula-EVT, Clayton copula, and Clayton copula-EVT. We use a window of 250 days to generate around 1700 VaRs with a 1% of violation probability (α) from each dependence estimation approach for each country.

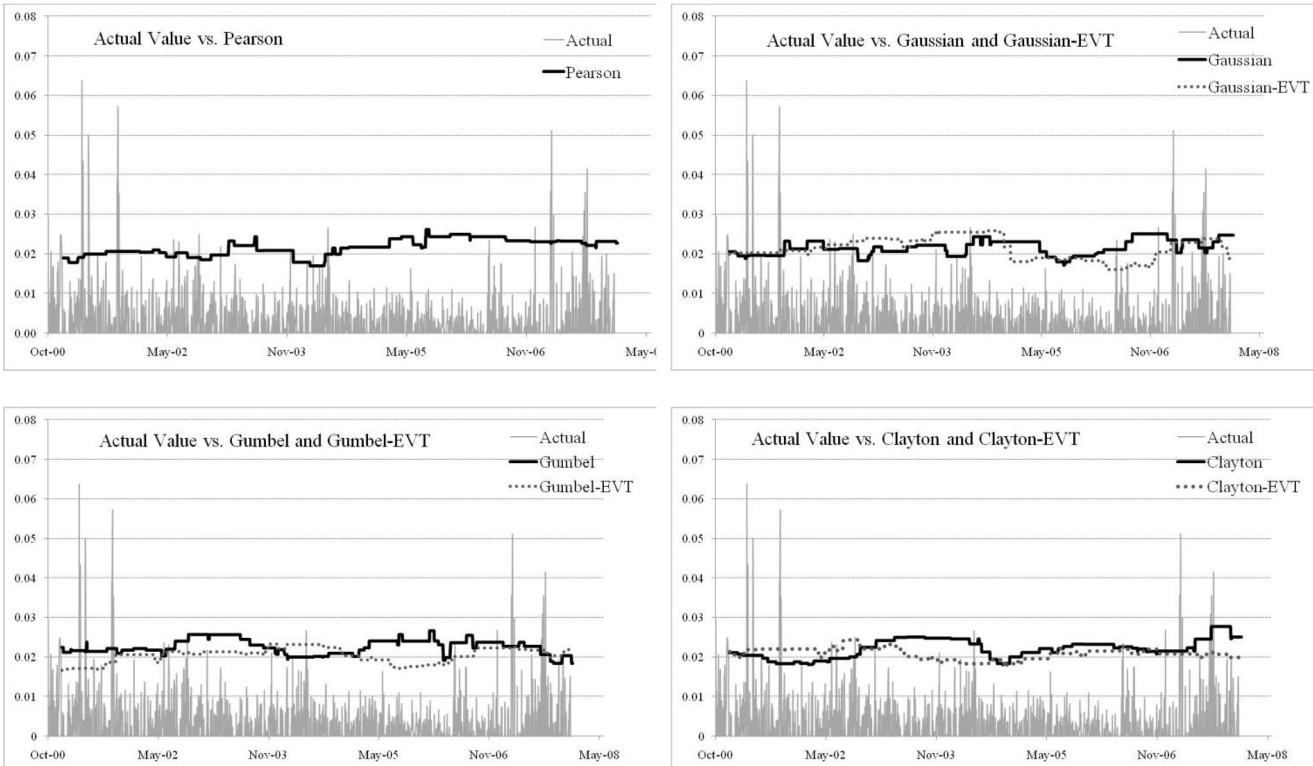
A. Indonesia



B. Korea

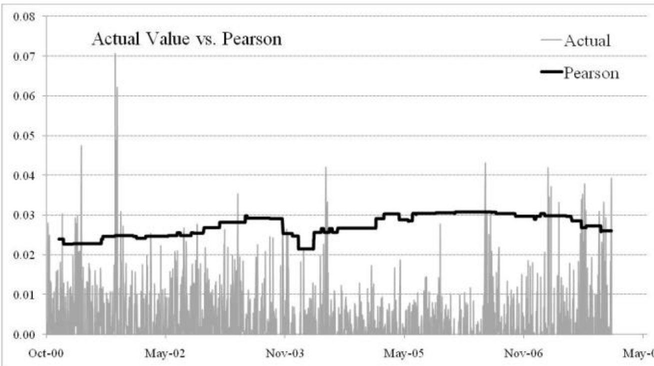

Actual Value vs. Pearson
Actual
Pearson
Actual Value vs. Gaussian and Gaussian-EVT
Actual
Gaussian
Gaussian-EVT
Actual Value vs. Gumbel and Gumbel-EVT
Actual
Gumbel
Gumbel-EVT
Actual Value vs. Clayton and Clayton-EVT
Actual
Clayton
Clayton-EVT
Oct-00
May-02
Nov-03
May-05
Nov-06
May-08

C. Malaysia



Actual Value vs. Pearson
Actual
Pearson
Actual Value vs. Gaussian and Gaussian-EVT
Actual
Gaussian
Gaussian-EVT
Actual Value vs. Gumbel and Gumbel-EVT
Actual
Gumbel
Gumbel-EVT
Actual Value vs. Clayton and Clayton-EVT
Actual
Clayton
Clayton-EVT
Oct-00
May-02
Nov-03
May-05
Nov-06
May-08

D. Singapore


Actual Value vs. Pearson
Actual
Pearson
0.08
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0.01
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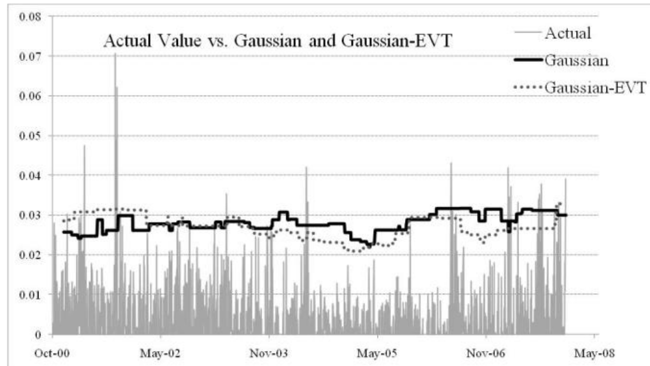
Oct-00
May-02
Nov-03
May-05
Nov-06


Actual Value vs. Gaussian and Gaussian-EVT
Actual
Gaussian
Gaussian-EVT
0.08
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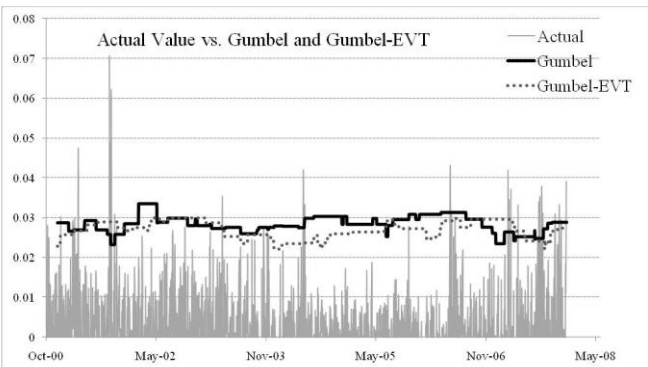
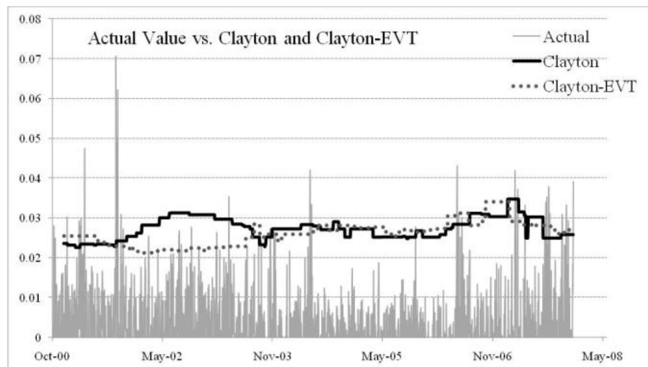
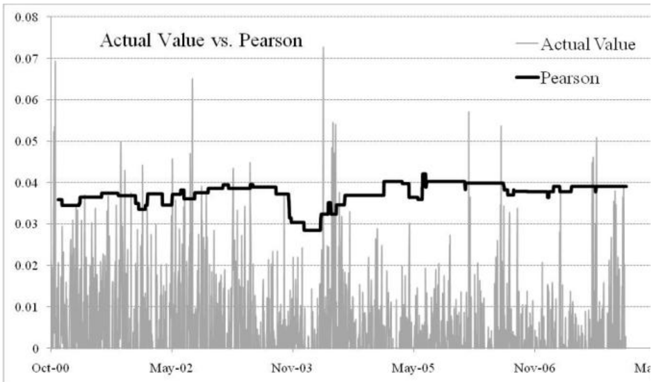
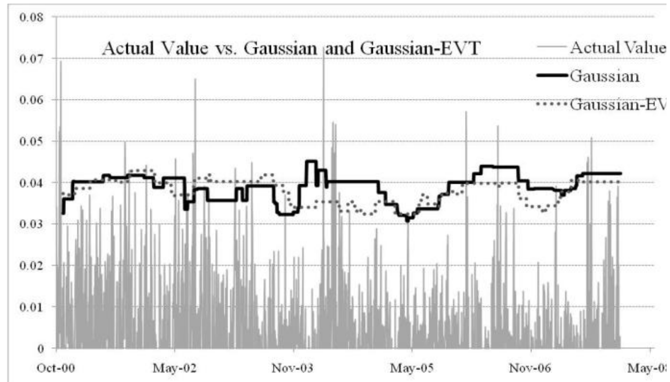
Oct-00
May-02
Nov-03
May-05
Nov-06
May-08

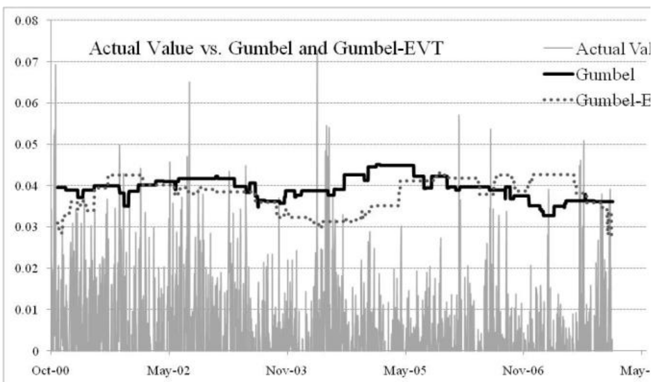

Actual Value vs. Gumbel and Gumbel-EVT
Actual
Gumbel
Gumbel-EVT
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Oct-00
May-02
Nov-03
May-05
Nov-06
May-08

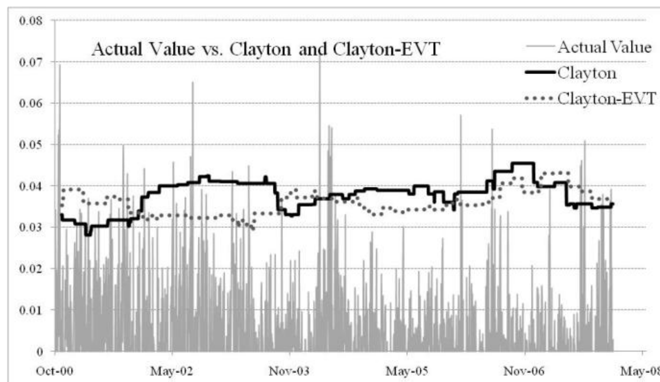

Actual Value vs. Clayton and Clayton-EVT
Actual
Clayton
Clayton-EVT
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Oct-00
May-02
Nov-03
May-05
Nov-06
May-08

E. Taiwan

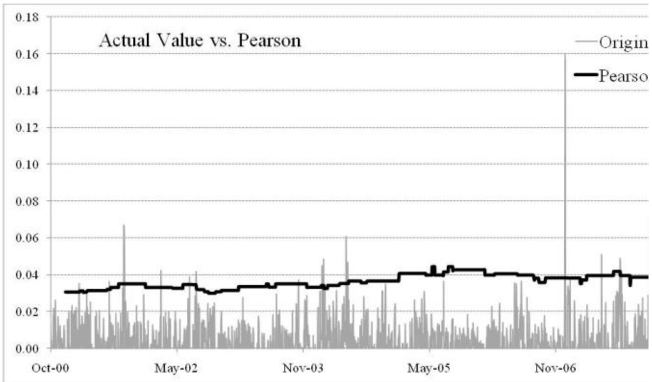

Actual Value vs. Pearson
Actual Value
Pearson
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Oct-00
May-02
Nov-03
May-05
Nov-06

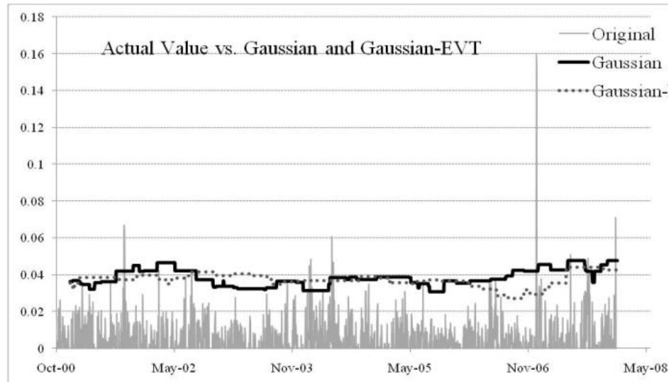

Actual Value vs. Gaussian and Gaussian-EVT
Actual Value
Gaussian
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Oct-00
May-02
Nov-03
May-05
Nov-06

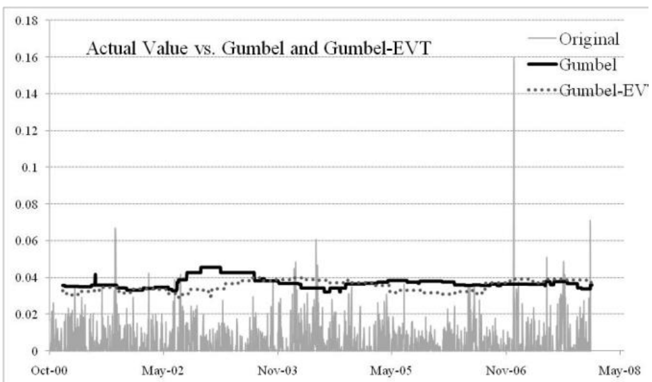

Actual Value vs. Gumbel and Gumbel-EVT
Gumbel
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Oct-00
May-02
Nov-03
May-05
Nov-06

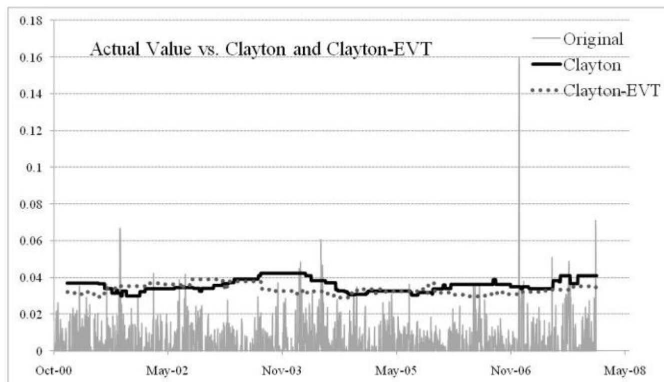

Actual Value vs. Clayton and Clayton-EVT
Actual Value
Clayton
Clayton-EVT
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Oct-00
May-02
Nov-03
May-05
Nov-06
May-08

F. Thailand


Actual Value vs. Pearson
Origin
Pearso
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Oct-00
May-02
Nov-03
May-05
Nov-06


Actual Value vs. Gaussian and Gaussian-EVT
Original
Gaussian
Gaussian-
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Oct-00
May-02
Nov-03
May-05
Nov-06
May-08


Actual Value vs. Gumbel and Gumbel-EVT
Original
Gumbel
Gumbel-EV
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Oct-00
May-02
Nov-03
May-05
Nov-06
May-08


Actual Value vs. Clayton and Clayton-EVT
Original
Clayton
Clayton-EVT
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Oct-00
May-02
Nov-03
May-05
Nov-06
May-08