

# Effects of aperture averaging and beam width on a partially coherent Gaussian beam over free-space optical links with turbulence and pointing errors

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**Joint effects of aperture averaging and beam width on the performance of free-space optical communication links, under the impairments of atmospheric loss, turbulence, and pointing errors (PEs) are investigated from an information theory perspective. The propagation of a spatially partially coherent Gaussian-beam wave through a random turbulent medium is characterized, taking into account the diverging and focusing properties of the optical beam, as well as the scintillation and beam wander effects. Results show that a noticeable improvement in the average channel capacity can be achieved with an enlarged receiver aperture in the moderate-to-strong turbulence regime, even without knowledge of the channel state information. In particular, it is observed that the optimum beam width can be reduced to improve the channel capacity, albeit the presence of scintillation and PEs, given that either one or both of these adverse effects are least dominant. We show that under strong turbulence conditions, the beam width increases linearly with the Rytov variance for a relatively smaller PE loss, but changes exponentially with steeper increments for higher PE losses. Our findings conclude that the optimal beam width is dependent on the combined effects of turbulence and PEs, and this parameter should be adjusted according to the varying atmospheric channel conditions. Therefore, we demonstrate that the maximum channel capacity is best achieved through the introduction of a larger receiver aperture and a beam width optimization technique.**

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## 1. INTRODUCTION

Free-space optical (FSO) communication is an emerging low-cost, license-free and high-bandwidth access solution, which has attracted significant attention for a variety of applications, such as the last-mile connectivity, optical-fiber backup and enterprise connectivity [1], [2]. While posing numerous technical advantages over its radio-frequency counterpart [3]–[5], the FSO technology is hampered by several channel impairments, which include vulnerability to visibility-limiting weather conditions (e.g., fog, haze), atmospheric turbulence, and pointing errors (PEs) [6]–[8].

The propagation properties and performance of spatially partially coherent Gaussian beams through random turbulent media have been

substantially explored thus far [9]–[11]. These studies demonstrated the benefits of partially coherent beam (PCB) in mitigating the turbulence-induced scintillations and PEs. In principle, a PCB can be generated by passing an initially spatially coherent information-bearing laser source through a phase diffuser, which partially destroys its spatial coherence. This in turn increases the receiver beam size while still retaining its beamlike (highly directional) properties. Nevertheless, this approach inadvertently results in the reduction of the average received power, thus the need for careful investigation and optimization of PCB parameters. As such, the inevitable trade-off between the reduction in scintillation and PEs, and the reduction in mean received irradiance can be achieved appropriately.

The adverse effects of atmospheric loss, turbulence and PEs on the performance and design of FSO communication links have been extensively investigated from an information theory perspective in a large body of literature, in which various channel fading models were considered [2]. In particular, a statistical model was presented in [7] for the received optical intensity fluctuations affected by the combined effects of turbulence and PEs. It was shown that beam width optimization can provide large gains in the outage capacity of the FSO channel. In [12], several optimization models were proposed to limit the PE loss. Similar approach to [7] was also considered in [13]–[15], whereby the authors evaluated the error performance by taking into account the optical fluctuations resulting from turbulence and PEs. However, the link design criteria were not emphasized in these studies, thus lacking a holistic perspective for optimal design of FSO links. In addition, these theoretical investigations are based upon the classical unbounded plane- or spherical-wave approximations, which are the limiting forms of the lowest-order Gaussian-beam wave model [6]. As a result, these simplified models are insufficient to characterize the propagation properties of optical beam waves through the turbulent channels, particularly for the case of PCB, whereby focusing and diverging characteristics are important [9]. Moreover, the conventional Rytov-based scintillation model associated with these approximations does not accurately reflect the irradiance fluctuations, particularly for transmissions over the strong turbulence regime [16]. This is mainly because the influence of beam wander attributed to the presence of large-scale turbulent eddies, which contributes to a widening of the long-term beam profile and PEs, is not considered in the limiting Rytov theory [6], [16]. Furthermore, while numerous reported studies [17]–[22] have considered the case of PCB propagation, the effects of aperture averaging (AA) on a PCB in mitigating the scintillation and PEs were never thoroughly examined [8], [20].

In [8], the effects of PEs on the ergodic and outage capacities of FSO links affected by turbulence were demonstrated, taking into account the Gaussian-beam wave model and wave-optics-based approach. However, the effects of larger apertures and optimization method for the beam parameters were not considered here. In [17], the problem of maximizing the mean received optical intensity and minimizing the scintillation index (SI) was investigated. It was shown that a fully coherent laser beam maximizes the expected intensity, whereas a PCB mitigates the scintillation effect. In [18], an optimization criterion for the initial coherence degree of lasers was proposed, to maximize the mean received irradiance. A performance metric was presented in [19], to study the optimization of the transverse coherence length, which balances the beam spread and scintillation for achieving near-optimal FSO link performance. In [16], Ren *et al.* investigated the impact of beam wander on the FSO channel capacities, and showed that an optimum transmitter beam radius can be selected for best achievable channel capacities. In [20], the problem of coherence length optimization in the weak turbulence regime was examined, in which the conditions for achieving improvement in the outage probability were described. Nevertheless, the effects of PEs and AA were not taken into account in these optimization studies. In [21], Liu *et al.* proposed a theoretical model to investigate the average capacity optimization in the presence of turbulence and PEs, and showed that the transmitter beam divergence angle and beam waist can be tuned to maximize the average capacity. However, the degree of partial (spatial) coherence of the laser source was not taken into account in their beam model. Cang and Liu observed the achievable capacity of a PCB propagating through the non-Kolmogorov turbulence channel in [22], but did not consider the PEs or any optimization method.

Our aim in this paper is to explore the significance and impact of the AA phenomenon and the relevant link design considerations on a PCB for long-distance horizontal-path (terrestrial) FSO transmissions. The proposed study substantiates the development of a beam width optimization method in mitigating the adverse effects of scintillation and PEs. For this purpose, we investigate the performance of partially coherent FSO links subject to atmospheric loss, turbulence and PEs

from the information theory perspective. In conjunction with a spatially partially coherent Gaussian-beam wave, the important link design criteria are considered here, including the transmitter beam width, receiver aperture size, link range, knowledge of channel state information (CSI) at the receiver, and weather conditions. In particular, we show that the FSO channel capacity can be significantly improved due to the AA of enlarged receiver aperture, which effectively mitigates both the PE loss and scintillation-induced intensity fluctuations, even when the CSI is unavailable at the receiver. Furthermore, we demonstrate that the PCB experiences substantial alteration in its beam characteristics when propagating in free-space. Several interesting observations are made here, whereby it is shown that the optimum beam width can be made smaller to maximize the average capacity, subject to the condition that either one or both the effects of turbulence-induced scintillations and PEs are least dominant. In the strong turbulence regime, the beam width must be increased accordingly with respect to higher PE loss, where such incremental trend changes from a linear to an exponential behavior. These notable results demonstrate that the optimum beam width is susceptible to the combined effects of turbulence and PEs, and conclude that beam width optimization is a promising approach for capacity enhancement in long-distance terrestrial FSO communication links.

The remainder of this paper is structured as follows: Section 2 describes the FSO system, Gaussian-beam wave model, and different components of the optical slow-fading channel model. In Section 3, we review the average channel capacity considered in our analysis. Next, the effects of AA on a PCB are examined in Section 4, whereas the importance of beam width optimization is discussed in Section 5. Finally, our concluding remarks are highlighted in Section 6.

## 2. SYSTEM DESCRIPTION

### A. System Model

We consider an intensity modulation with direct detection (IM/DD) single-input single-output (SISO) horizontal-path FSO link (see Fig. 1), with the non-return-to-zero on-off keying (NRZ-OOK) technique. The information-bearing electrical signals modulate the instantaneous intensity of a collimated Gaussian-beam wave operating in the lowest-order transverse electromagnetic (TEM<sub>00</sub>) mode [6], [23], and then pass through a phase diffuser. The latter alters the divergence of the optical beam while retaining its beamlike (i.e., directionality) properties [10]. Next, the PCB propagates through a turbulent channel in the presence of beam extinction and PEs. At the receiving-end, the optical signals are collected by a circular lens of diameter  $D$  [6], [7] and focused onto a photodetector (PD), which converts the received optical intensities into a resulting photocurrent.

The received signal  $y_\kappa$  can be described by the conventional channel model given as [7], [24]:

$$y_\kappa = h\gamma x_\kappa + n_o ; \quad (1)$$

where  $h$  is the channel state,  $\gamma$  is the PD responsivity (in A/W),  $x_\kappa \in \{0, 2P_{\text{FSO}}\}$  is the optical power of the transmitted signal which corresponds to the OOK signal with binary values 0 or 1,  $P_{\text{FSO}}$  is the average transmit optical power, and  $n_o$  is the signal-independent additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma_n^2$ . For a slow-fading channel with OOK signaling, the received electrical signal-to-noise ratio (SNR) is defined as [7]:

$$SNR(h) = \frac{2P_{\text{FSO}}^2 \gamma^2 h^2}{\sigma_n^2} , \quad (2)$$

which is a fluctuating term due to the influence of  $h$ .

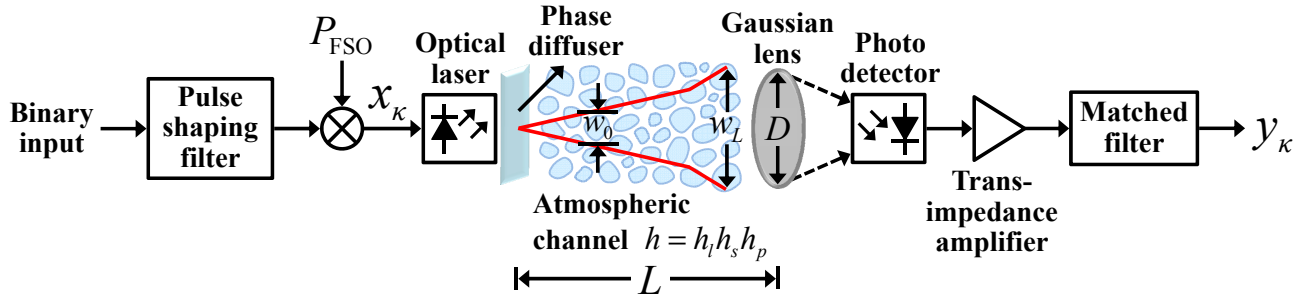


Fig. 1. Block diagram of a SISO horizontal-path FSO communication link.

The channel fading coefficient  $h$  models the atmospheric loss and optical intensity fluctuations caused by turbulence- and misalignment-induced signal fading, which can be described as [7]:

$$h = h_l h_s h_p ; \quad (3)$$

where  $h_l$ ,  $h_s$  and  $h_p$  represent attenuation, scintillation, and PEs, respectively.  $h_l$  is deterministic and does not exhibit randomness in its behavior, thus acting as a fixed scaling factor for a long period of time (on the order of hours) [7]. On the other hand,  $h_s$  and  $h_p$  are time-variant factors, depicting variations in the fading channel on the order of milliseconds; and their stochastic behavior are described by their respective distributions [24]. Since the time scales of these fading processes are much larger than the bit interval, this block fading (or quasi-static fading) channel is commonly known as the slow-fading channel, where  $h$  is assumed constant over a large number of transmitted bits [7], [14].

### B. Beam Model

We adopt the Gaussian Schell beam model as in [6], [10], [11], with a Gaussian amplitude distribution and an effective beam radius (spot size)  $w_0$  (in m) at the exit aperture of the optical transmitter. Following propagation along the atmospheric turbulent channel with a link distance  $L$ , the received beam size is given by:

$$w_L = w_0 \sqrt{\Theta_n^2 + \zeta \Lambda_n^2} , \quad (4)$$

where the normalized components are defined as:

$$\Theta_n = 1 - \frac{L}{F_0} \text{ and } \Lambda_n = \frac{2L}{kw_0^2} ; \quad (5)$$

$F_0$  is the phase front radius of curvature at the transmitter,  $k = 2\pi/\lambda$  is the optical wave number, and  $\lambda$  is the laser wavelength. The global coherence parameter is a measure of the global degree of coherence of light across each transversal plane along the propagation path, and is defined by:

$$\zeta = \zeta_s + \frac{2w_0^2}{\rho_0^2} . \quad (6)$$

The source coherence parameter describes the degree of partial (spatial) coherence of the source laser beam, and is given by:

$$\zeta_s = 1 + \frac{2w_0^2}{l_c^2} , \quad (7)$$

where  $l_c$  is the spatial coherence length. The coherence length of a spherical wave is given by:

$$\rho_0 = [0.55C_n^2 k^2 L]^{-3/5} ; \quad (8)$$

where  $C_n^2$  (in  $m^{-2/3}$ ) is the refractive index structure parameter of the atmosphere, which signifies the strength of atmospheric turbulence. It is associated with the structure function for fluctuations in the index of refraction, and is assumed to be constant for horizontal FSO links [3], [6].

Turbulence-induced scintillations are largely contributed by optical beam fragmentation, which is the decay of the initial optical beam resulting in multiple spatially separated beams. If the PD size is smaller than the characteristic distance between the optical beams, then there is a probability that these separated beams cannot be detected. Therefore, beam wandering significantly contributes to scintillations. It is known that scintillations can be reduced by employing a slow time response PD in combination with a PCB. The SI presents a viable statistical quantitative measure of the level of scintillations. For the case of a point-receiver, the SI is given by [6], [10], [20]:

$$\begin{aligned} \sigma_I^2(0) \cong & 4.42 \sigma_R^2 \Lambda_L^{5/6} \frac{\sigma_{pe}^2}{w_L^2} \\ & + 3.86 \sigma_R^2 \left\{ 0.40 \left[ (1 + 2\Theta_L)^2 + 4\Lambda_L^2 \right]^{5/12} \right. \\ & \left. \times \cos \left[ \frac{5}{6} \tan^{-1} \left( \frac{1 + 2\Theta_L}{2\Lambda_L} \right) - \frac{11}{16} \Lambda_L^{5/6} \right] \right\} ; \quad (9) \end{aligned}$$

where  $\sigma_{pe}^2$  is the jitter-induced PE variance, and  $\sigma_R^2$  is the Rytov variance for a plane wave given by:

$$\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6} . \quad (10)$$

The receiver beam parameters are given as:

$$\Theta_L = 1 + \frac{L}{F_L} \text{ and } \Lambda_L = \frac{2L}{kw_L^2} ; \quad (11)$$

where the phase front radius of curvature for a PCB at the receiver is defined by [9], [10]:

$$F_L = \frac{L(\Theta_n^2 + \zeta \Lambda_n^2)}{\phi \Lambda_n - \zeta \Lambda_n^2 - \Theta_n^2} , \text{ with } \phi \equiv \frac{\Theta_n}{\Lambda_n} - \frac{\Lambda_n w_0^2}{\rho_0^2} . \quad (12)$$

The AA effect is taken into account in the aperture-averaged SI:

$$\sigma_I^2(D) = A_G \sigma_I^2(0) , \quad (13)$$

where the AA factor  $A_G$  is given by [10]:

$$\begin{aligned} A_G = \frac{\sigma_I^2(D)}{\sigma_I^2(0)} = & \frac{16}{\pi} \int_0^1 \delta \delta \exp \left[ \frac{-D^2 \delta^2}{\rho_0^2} \left( 2 + \frac{\rho_0^2}{w_0^2 \Lambda_n^2} - \frac{\rho_0^2 \phi^2}{w_L^2} \right) \right] \\ & \times \left[ \cos^{-1}(\delta) - \delta \sqrt{1 - \delta^2} \right] . \quad (14) \end{aligned}$$

## C. Channel Models

### 1. Atmospheric Loss

The atmospheric loss due to beam extinction arising from scattering and absorption can be modeled by the Beers-Lambert law [6]:

$$h_l = e^{-\sigma L}, \quad (15)$$

where  $\sigma$  denotes a wavelength- and weather-dependent attenuation coefficient. For clear and foggy weather conditions, the attenuation coefficient can be determined from the visibility data through the Kim's model [25]:

$$\sigma = \frac{3.91}{V} \left( \frac{\lambda}{550} \right)^{-q}; \quad (16)$$

where  $V$  is the visibility (in km), and  $q$  is a parameter related to the particle size distribution and  $V$ .

### 2. Atmospheric Turbulence-Induced Fading

The log-normal distribution is a widely adopted model for the probability density function (PDF) of the randomly fading irradiance signal under weak-to-moderate turbulence conditions due to its simplicity. However, it underestimates the peak irradiance of the PDF and the behavior in the distribution tails, particularly with increasing turbulence strength [3], [6]. By this model, the PDF of the irradiance intensity in the turbulent medium is given by:

$$f_{h_s}(h_s) = \frac{1}{h_s \sigma_I(D) \sqrt{2\pi}} \exp \left\{ -\frac{\left[ \ln(h_s) + \frac{1}{2} \sigma_I^2(D) \right]^2}{2\sigma_I^2(D)} \right\}. \quad (17)$$

The gamma-gamma distribution is a more recent fading model to address the large- and small-scale scintillations in the moderate-to-strong turbulence regime. In this case, the PDF of  $h_s$  is given by [3], [6]:

$$f_{h_s}(h_s) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} h_s^{(\alpha+\beta)/2-1} K_{\alpha-\beta} \left( 2\sqrt{\alpha\beta h_s} \right); \quad (18)$$

where  $\Gamma(\cdot)$  denotes the gamma function, and  $K_{\alpha-\beta}$  is the modified Bessel function of the second kind of order  $(\alpha-\beta)$ .  $\alpha$  and  $\beta$  are the effective number of large- and small-scale eddies, which relate to the scattering effects of the turbulent environment. In the case of Gaussian-beam wave model with AA at the receiver,  $\alpha$  and  $\beta$  can be calculated according to the expressions provided in [6].

### 3. Pointing Errors

We consider the statistical misalignment-induced fading model proposed in [7], which provides a tractable PDF for describing the stochastic behavior of PEs. This model assumes a circular detection aperture of diameter  $D$ , and a Gaussian spatial intensity profile of beam waist radius  $w_L$  on the receiver plane. In addition, both the elevation and horizontal displacements (sways) are considered as independent and identically Gaussian distributed with variance  $\sigma_s^2$ . Correspondingly, the PDF of  $h_p$  is given by:

$$f_{h_p}(h_p) = \frac{\xi^2}{A_0^{\xi^2}} h_p^{\xi^2-1}, \quad 0 \leq h_p \leq A_0; \quad (19)$$

where  $A_0$  is the fraction of the collected power:

$$A_0 = [\text{erf}(v)]^2, \quad \text{with } v = \frac{\sqrt{\pi}D}{2\sqrt{2}w_L}; \quad (20)$$

and  $\xi$  is the ratio between the equivalent beam radius at the receiver and the PE displacement (jitter) standard deviation at the receiver:

$$\xi = \frac{w_{z_{eq}}}{2\sigma_{pe}}, \quad \text{with } w_{z_{eq}} = w_L \sqrt{\frac{\sqrt{\pi} \text{erf}(v)}{2v \exp(-v^2)}}. \quad (21)$$

### 4. Combined Channel Fading Model

Taking into account  $h_l$  as a scaling factor and the conditional distribution in [7, Eq. (13)], the distributions of  $h = h_l h_s h_p$  for both weak and strong turbulence regimes are given by Eq. (22) and Eq. (23), respectively, as below. For the weak atmospheric turbulence condition, we have:

$$f_h(h) = \frac{\xi^2}{(A_0 h_l)^{\xi^2}} h^{\xi^2-1} \times \int_{h/A_0 h_l}^{\infty} \frac{1}{h_s^{\xi^2+1} \sigma_I(D) \sqrt{2\pi}} \exp \left\{ -\frac{\left[ \ln(h_s) + \frac{1}{2} \sigma_I^2(D) \right]^2}{2\sigma_I^2(D)} \right\} dh_s. \quad (22)$$

In the strong turbulence regime:

$$f_h(h) = \frac{2\xi^2 (\alpha\beta)^{(\alpha+\beta)/2}}{(A_0 h_l)^{\xi^2}} \Gamma(\alpha) \Gamma(\beta) h^{\xi^2-1} \times \int_{h/A_0 h_l}^{\infty} h_s^{(\alpha+\beta)/2-1-\xi^2} K_{\alpha-\beta} \left( 2\sqrt{\alpha\beta h_s} \right) dh_s. \quad (23)$$

## 3. AVERAGE CHANNEL CAPACITY

The average channel capacity represents the practically achievable information carrying rate (in bits per channel use (bpcu)) through the time-varying fading channel with an arbitrarily small probability of detection error. It can be approached by using channel codes of large block lengths, to capture the effects of the channel variations through the codewords [8]. In principle, the average capacity  $\langle C \rangle$  for a binary-input continuous-output channel is defined as the maximum mutual information between the input to the channel  $x$  and output from the channel  $y$ , where the maximum is taken over all input distributions [26]. For a known channel at the receiver, we have [8], [27]:

$$\langle C \rangle = \sum_{x=0}^1 P_X(x) \int_{-\infty}^{\infty} \int_0^{\infty} f(y|x, h) f_h(h) \times \log_2 \left[ \frac{f(y|x, h)}{\sum_{m=0,1} f(y|x=m, h) P_X(m)} \right] dh dy; \quad (24)$$

where  $P_X(x)$  is the probability of the bit being one ( $x=1$ ) or zero ( $x=0$ ), with  $P_X(x=0) = P_X(x=1) = 0.5$ ; and  $f(y|x, h)$  is given by:

$$f(y|x, h) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left[-\frac{y^2}{2\sigma_n^2}\right], & x = 0 \\ \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left[-\frac{(y - 2P_{\text{FSO}}\mathcal{H})^2}{2\sigma_n^2}\right], & x = 1. \end{cases} \quad (25)$$

In the case of an unknown channel at the receiver [8], [27]:

$$\langle C \rangle = \sum_{x=0}^1 P_X(x) \int_{-\infty}^{\infty} f(y|x) \times \log_2 \left[ \frac{f(y|x)}{\sum_{m=0,1} f(y|x=m, h) P_X(m)} \right] dy ; \quad (26)$$

where  $f(y|x)$  can be expressed as:

$$f(y|x) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left[-\frac{y^2}{2\sigma_n^2}\right], & x = 0 \\ \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left[-\frac{(y - 2P_{\text{FSO}}\mathcal{H})^2}{2\sigma_n^2}\right] f_h(h) dh, & x = 1. \end{cases} \quad (27)$$

#### 4. THE APERTURE AVERAGING EFFECT

We investigate the effects of AA in horizontal (terrestrial) FSO links, to examine the distinctive advantages of using enlarged receiver apertures in mitigating the turbulence-induced scintillation and PEs. The relevant link design considerations include the weather conditions (i.e., visibility  $V$ , turbulence strength  $C_n^2$ , and PE-induced jitter  $\sigma_{pe}^2$ ), aperture size, link distance, and availability of CSI at the receiver. In principle, AA reduces the scintillation level through the shifting of the relative frequency content of the irradiance power spectrum towards lower frequencies, thereby averaging out the fastest fluctuations [28], [29]. Such phenomenon also potentially mitigates the PE loss, as can be observed from the reduction in the normalized jitter  $2\sigma_{pe}/D$  [7].

Fig. 2(a) presents the average channel capacity  $\langle C \rangle$  as a function of the average electrical SNR  $\langle \text{SNR} \rangle$  for  $D = \{40, 80, 200, 400\}$  mm, assuming the CSI is known to the receiver. We consider a light fog condition here, with  $V = 0.642$  km and  $C_n^2 = 5.0 \times 10^{-15} \text{ m}^{-2/3}$  at  $L = 1.0$  km, which corresponds to a Rytov variance of  $\sigma_R^2 = 0.1$ . The relevant system parameters and weather-dependent propagation parameters are summarized in Table 1. These are the default parameters that we consider hereafter, unless otherwise specified. Under the visibility-impaired weak turbulence condition, we observe a rapid increase in  $\langle C \rangle$  with respect to  $\langle \text{SNR} \rangle$ , in which the maximum  $\langle C \rangle \sim 1.0$  bpcu is approached for  $\langle \text{SNR} \rangle > 14$  dB. A near-identical behavior is also noted for all the considered  $D$  values, albeit observing an increase in the fraction of collected power  $A_0$  and a reduction in the aperture-averaged SI  $\sigma_I^2(D)$  (see figure legend). We conclude that in this case,  $\langle C \rangle$  is almost independent of  $D$ , since the effects of scintillation and PEs are less dominant, as compared to the beam extinction due to scattering and absorption of atmospheric particles.

**Table 1. Parameters of the FSO system**

System Parameters				
Parameter	Symbol	Value		
Laser wavelength	$\lambda$	1550 nm		
Photodetector responsivity	$\gamma$	0.5 A/W		
Noise variance	$\sigma_n^2$	$10^{14} \text{ A}^2$		
Spatial coherence length	$l_c$	1.38 mm		
Nominal beam width	$w_0$	5 cm		
Link distance	$L$	{1.0, 4.5, 7.5} km		
Receiver diameter	$D$	{40, 80, 200, 400} mm		
PE-induced jitter standard deviation	$\sigma_{pe}$	30 cm		
Weather-Dependent Propagation Parameters				
Condition	$V$ (km)	$C_n^2$ ( $\text{m}^{-2/3}$ )	$L$ (km)	$\sigma_R^2$
Light fog	0.642	$5.00 \times 10^{-15}$	1.0	0.10
Clear air	10.0	$5.00 \times 10^{-14}$	1.0	1.00
		$3.12 \times 10^{-16}$	7.5	0.25
		$1.12 \times 10^{-16}$	7.5	9.00
		$1.87 \times 10^{-14}$	7.5	15.00

The corresponding results for the clear weather scenarios are shown in Fig. 2(b), with  $V = 10$  km,  $C_n^2 = 5.0 \times 10^{-14} \text{ m}^{-2/3}$  at  $L = 1.0$  km ( $\sigma_R^2 = 1.0$ ), and  $C_n^2 = 1.87 \times 10^{-14} \text{ m}^{-2/3}$  at  $L = 7.5$  km ( $\sigma_R^2 = 15.0$ ). In comparison to the light fog condition in Fig. 2(a), it is evident that  $\langle C \rangle$  is more susceptible to the adverse effects of turbulence, which impose higher scintillation levels and PE losses, as indicated by larger  $\sigma_I^2(D)$  and  $\xi$  values, respectively. At  $\sigma_R^2 = 1.0$ ,  $\langle C \rangle$  increases at a relatively slower rate for  $D = 40$  mm, as compared to the other cases, and approaches its maximum for  $\langle \text{SNR} \rangle > 18$  dB. We notice that maximum  $\langle C \rangle$  enhancement can be achieved with larger  $D = \{200, 400\}$  mm, where near-optimal  $\langle C \rangle > 0.99$  bpcu is obtained for  $\langle \text{SNR} \rangle \geq 14$  dB. On the other hand, we observe a larger penalty upon  $\langle C \rangle$  at  $\sigma_R^2 = 15.0$ , which is particularly pronounced for higher  $\langle \text{SNR} \rangle$  values. It is shown that there is a significant variation in  $\langle C \rangle$  for all the considered  $D$  values, with a maximum deviation of 0.31 bpcu at  $\langle \text{SNR} \rangle = 14$  dB. It is also observed that  $\langle C \rangle$  increases at a much slower rate for  $D = \{40, 80\}$  mm, thereby requiring  $\langle \text{SNR} \rangle$  much higher than 20 dB to achieve the maximum capacity. However, an improvement in  $\langle C \rangle$  of more than 25% can be made possible for  $\langle \text{SNR} \rangle > 10$  dB with  $D = 400$  mm.

Fig. 3 depicts  $\langle C \rangle$  against  $\langle \text{SNR} \rangle$  for the weak and moderate-to-strong turbulence regimes, with  $C_n^2 = 3.12 \times 10^{-16} \text{ m}^{-2/3}$  ( $\sigma_R^2 = 0.25$ ) and  $C_n^2 = 1.12 \times 10^{-14} \text{ m}^{-2/3}$  ( $\sigma_R^2 = 9.00$ ), respectively, at  $L = 7.5$  km and  $V = 10$  km. The effect of PEs, as signified by the normalized jitter  $2\sigma_{pe}/D$ , is examined for a range of  $D$  and the two cases of known and unknown CSI at the receiver. For the weak turbulence case (Fig. 3(a)), we observe a faster increase in  $\langle C \rangle$  with respect to  $\langle \text{SNR} \rangle$  for all the considered  $D$  values, where its maximum is approached for  $\langle \text{SNR} \rangle > 16$  dB. In particular, there is a very small variation in  $\langle C \rangle$  between the known and unknown CSI cases. Such variation is much more significant for the moderate-to-strong turbulence scenario, as shown in Fig. 3(b). This is most prevalent under the influence of PEs with  $2\sigma_{pe}/D = 15.0$ , where we notice a  $\langle C \rangle$  variation of  $\sim 0.15$  bpcu

at  $\langle SNR \rangle = 14$  dB. Therefore, greater penalty is imposed upon  $\langle C \rangle$  under the combined effects of turbulence and PEs, which is worsened when the CSI is unknown to the receiver. Nevertheless, the use of an enlarged receiver aperture significantly enhances the capacity, and potentially minimizes the variation in  $\langle C \rangle$  irrespective of the CSI availability. For instance, a noticeable improvement in  $\langle C \rangle$  of  $\sim 0.46$  bpcu can be achieved at  $\langle SNR \rangle = 14$  dB for  $D = 400$  mm, even without knowledge of CSI at the receiver.

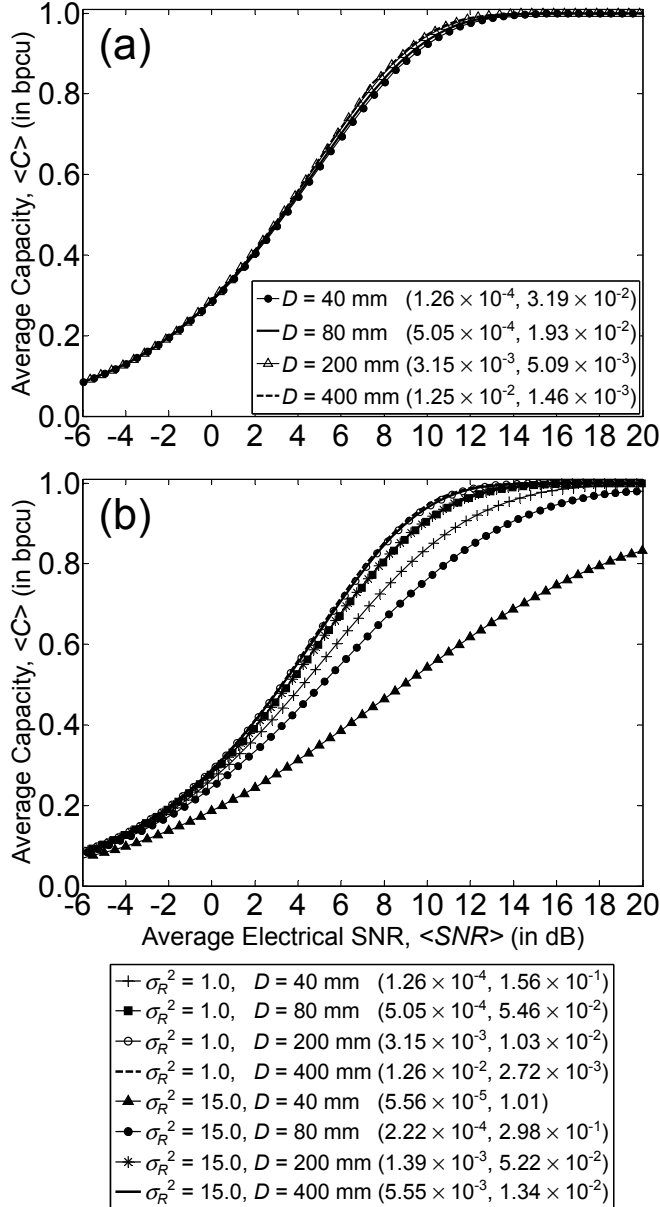


Fig. 2. Average channel capacity in terms of the average electrical SNR for  $D = \{40, 80, 200, 400\}$  mm, under (a) light fog ( $V = 0.642$  km,  $\sigma_R^2 = 0.1$  at  $L = 1.0$  km); and (b) clear weather ( $V = 10.0$  km,  $\sigma_R^2 = 1.0$  at  $L = 1.0$  km,  $\sigma_R^2 = 15.0$  at  $L = 7.5$  km) conditions. The CSI is assumed known to the receiver. The numerical values presented in the figure legend refer to  $(A_0, \sigma_T^2(D))$ .

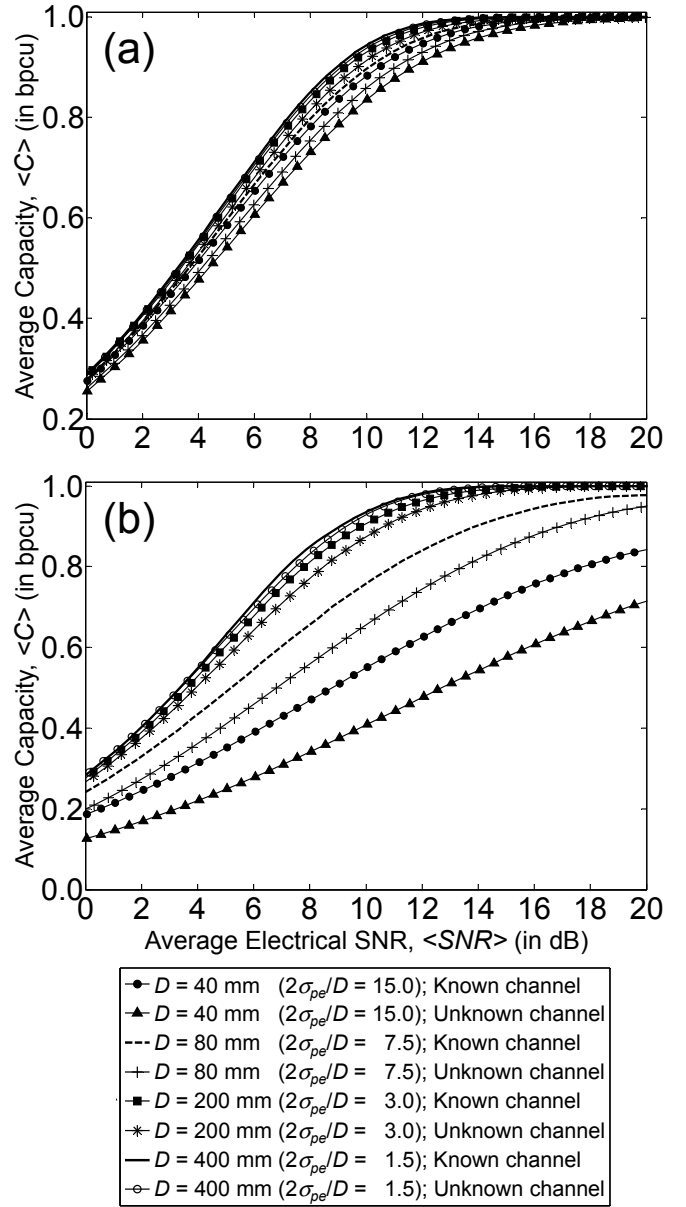


Fig. 3. Average channel capacity as a function of the average electrical SNR for the (a) weak ( $\sigma_R^2 = 0.25$ ) and (b) moderate-to-strong ( $\sigma_R^2 = 9.00$ ) turbulence regimes, where  $V = 10$  km and  $L = 7.5$  km.

## 5. BEAM WIDTH OPTIMIZATION

In Fig. 4, we examine the effects of the transmitter beam width  $w_0$  on the average channel capacity  $\langle C \rangle$ , in the presence of turbulence and PEs, given that the CSI is unknown to the receiver. The weak ( $\sigma_R^2 = 0.25$ ) and moderate-to-strong ( $\sigma_R^2 = 9.00$ ) turbulence regimes are considered here, with  $L = 7.5$  km,  $V = 10$  km and  $\langle SNR \rangle = 14$  dB (corresponding to a mean bit error rate of  $\approx 10^{-6}$ ). It is evident that the FSO channel capacity is susceptible to the adverse effects of scintillation and PEs, particularly for the case of moderate-to-strong turbulence (Fig. 4(b)). For instance, the FSO link suffers a significant  $\langle C \rangle$  reduction of 0.55 bpcu, under the severe PE loss of  $2\sigma_{pe}/D = 19.5$  for  $w_0 = 4$  cm, in contrast to a negligibly small variation of less than 0.05 bpcu for the weak turbulence scenario (Fig. 4(a)). Another interesting observation is the dependence of  $\langle C \rangle$  on  $w_0$ , whereby

changing the beam size affects the achievable capacity under different atmospheric conditions. This reveals the importance of finding an optimal  $w_0$  to maximize  $\langle C \rangle$ ; and hence,  $w_0$  is an important parameter to consider in the FSO link design.

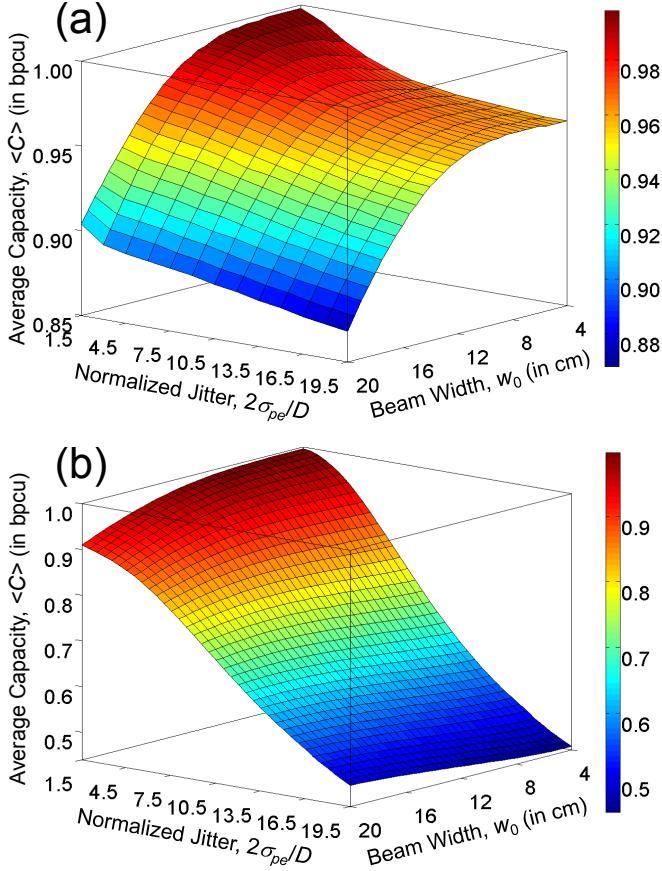


Fig. 4. Dependence of the average channel capacity on the transmitter beam width and PE loss, for  $V = 10$  km,  $L = 7.5$  km,  $\langle SNR \rangle = 14$  dB, and an unknown CSI at the receiver. The (a) weak ( $\sigma_R^2 = 0.25$ ) and (b) moderate-to-strong ( $\sigma_R^2 = 9.00$ ) turbulence cases are considered.

To extend our study on the impact of  $w_0$  on  $\langle C \rangle$ , we present Fig. 5 to investigate the characteristics of  $\langle C \rangle$  in terms of  $\sigma_R^2$ , for a range of  $w_0$  at  $L = \{4.5, 7.5\}$  km, without knowledge of the CSI at the receiver. The clear weather condition is considered here, with  $V = 10$  km,  $2\sigma_{pe}/D = 12.0$  and  $\langle SNR \rangle = 14$  dB. At  $L = 4.5$  km (Fig. 5(a)), near-optimal  $\langle C \rangle > 0.85$  bpcu is approached in the weak turbulence regime ( $\sigma_R^2 \leq 1.0$ ), with optical beams of smaller size ( $w_0 = \{3, 8\}$  cm); whereas performance degradation is observed with increasing  $w_0$ . For instance,  $\langle C \rangle$  is subjected to a noticeable reduction of 0.48 bpcu, when  $w_0$  is intentionally increased from 3 cm to 24 cm at  $\sigma_R^2 = 1.0$ ; and exhibits a rather persistent behavior at  $\langle C \rangle \approx 0.5$  bpcu for  $\sigma_R^2$  ranging from 0.1 to 10.0 and increases thereafter, thus giving the worst performance among all the considered  $w_0$  values.

For the case of  $L = 7.5$  km (Fig. 5(b)), it is observed that smaller  $w_0 = \{5, 10\}$  cm are preferred at  $\sigma_R^2 \leq 1.0$  with  $\langle C \rangle > 0.83$  bpcu, but they result in the lowest  $\langle C \rangle$  in the strong turbulence case of  $\sigma_R^2 \geq 10.0$ . In

particular, we notice that the optimal  $\langle C \rangle$  can be achieved with  $w_0 = 20$  cm in the intermediate turbulence regime of  $\sigma_R^2 = 2.0$  to 7.0. These observations can be explained by the fact that the PCB suffers from a substantial alteration in its beam profile/characteristics when propagating in free-space, which is mainly caused by the combined effects of scintillation and beam wander, thus resulting in the variation of  $\langle C \rangle$  for different  $\sigma_R^2$ . Our results in Fig. 5 also demonstrate that the link distance is another important link design criterion, since a larger  $L$  corresponds to a stronger turbulence. As a result, the optimal  $w_0$  will be larger for the case of  $L = 7.5$  km (Fig. 5(b)), as compared to  $L = 4.5$  km (Fig. 5(a)), under a significant PE loss of  $2\sigma_{pe}/D = 12.0$ , as seen from the observations for  $\sigma_R^2 > 10.0$ .

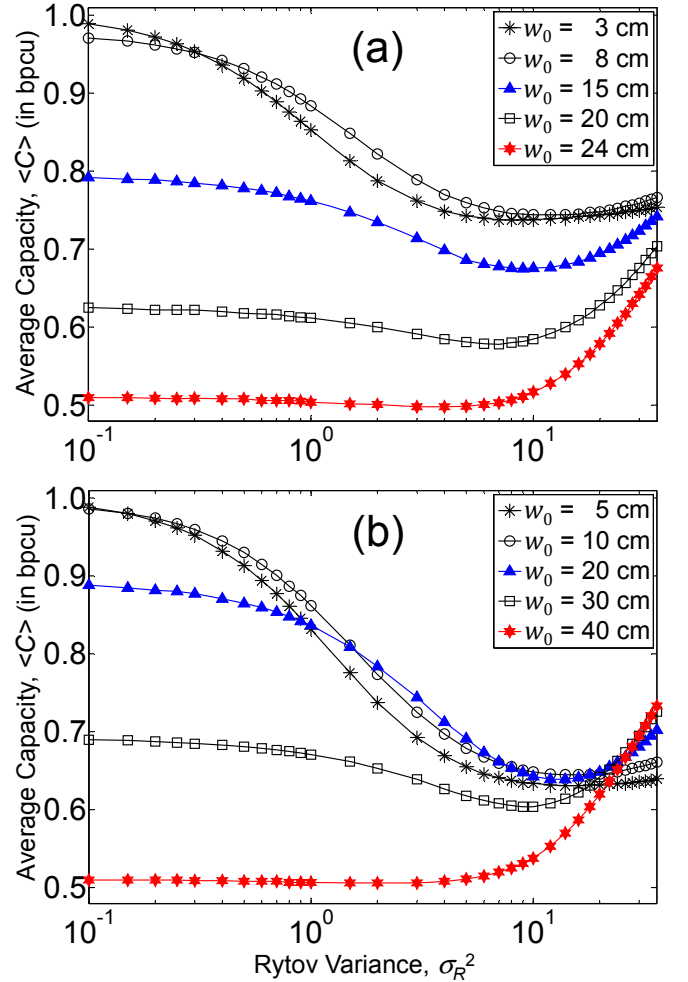


Fig. 5. Average channel capacity versus the Rytov variance for a range of beam width settings, without knowledge of CSI at the receiver. The clear weather scenario is considered at (a)  $L = 4.5$  km and (b)  $L = 7.5$  km, where  $V = 10$  km,  $2\sigma_{pe}/D = 12.0$ , and  $\langle SNR \rangle = 14$  dB.

From the results presented thus far, we have demonstrated that beam width optimization is an interesting feature, to achieve capacity enhancement in relatively long-distance terrestrial FSO links. This motivates our proposed study to find the optimum beam width  $w_0^{\text{opt}}$  and examine its characteristics, under the combined influences of different time-variant atmospheric channel conditions that are likely to occur in practice. The  $w_0^{\text{opt}}$  values are determined through an exhaustive search over their discrete sets, and the results are presented in Fig. 6. Fig. 6(a) shows the variation of  $w_0^{\text{opt}}$  versus  $\sigma_R^2$ ,

for  $2\sigma_{pe}/D = \{6.0, 9.0, 12.0, 15.0\}$  at  $L = 7.5$  km, with  $V = 10$  km and  $\langle SNR \rangle = 14$  dB. The corresponding optimized  $\langle C \rangle$  is depicted in Fig. 6(b). In general, the  $w_0^{\text{opt}}$  values are relatively smaller when the effects of turbulence and/or PEs are least significant. It is observed that  $w_0^{\text{opt}}$  is typically less than 15 cm for  $\sigma_R^2 \leq 1.5$  and/or  $2\sigma_{pe}/D \leq 6.0$ , which corresponds to an optimal  $\langle C \rangle > 0.88$  bpcu.

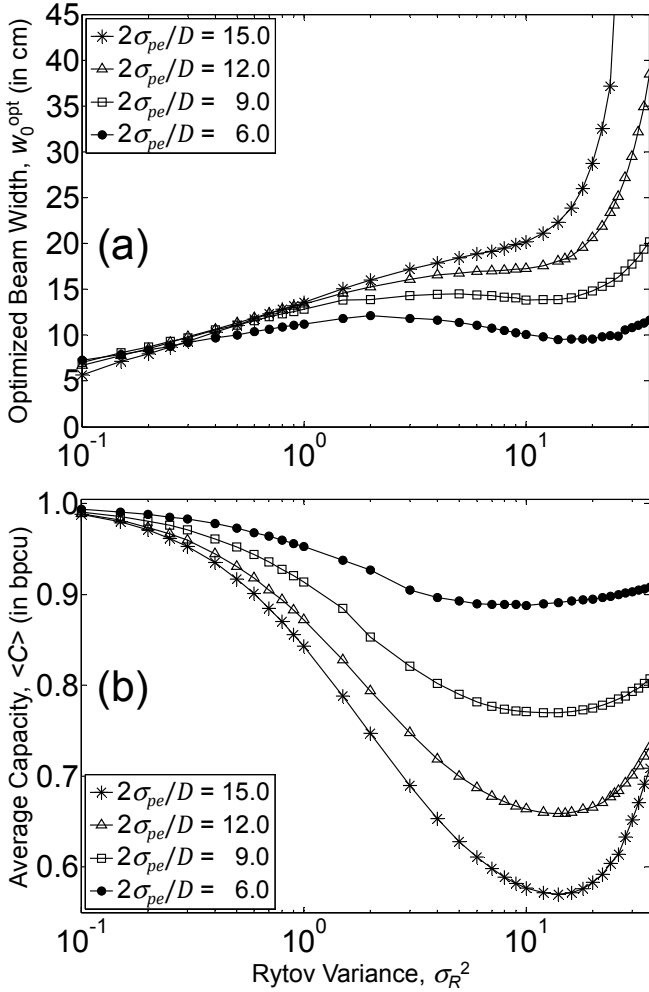


Fig. 6. The optimal (a) beam width and (b) average channel capacity with respect to the Rytov variance under clear weather condition, for  $2\sigma_{pe}/D = \{6.0, 9.0, 12.0, 15.0\}$  at  $\langle SNR \rangle = 14$  dB; where  $V = 10$  km,  $L = 7.5$  km, and the CSI is unknown to the receiver.

In the weak-to-moderate turbulence regime with  $\sigma_R^2 \leq 2.0$ ,  $w_0^{\text{opt}}$  exhibits a somewhat linear incremental trend for all cases of  $2\sigma_{pe}/D$ , and demonstrates vastly contrasting characteristics thereafter. In particular, an appealing observation is made here, showing a reduction in  $w_0^{\text{opt}}$  from 7.25 cm to 5.65 cm, as  $2\sigma_{pe}/D$  varies from 6.0 to 15.0 at  $\sigma_R^2 = 0.1$ . This in turn reveals the fact that the transmitter beam width can be made smaller even with increased PE loss. In the intermediate turbulence regime with  $\sigma_R^2$  ranging from 2.0 to 10.0,  $w_0^{\text{opt}}$  depicts small steps of decrease in its value for  $2\sigma_{pe}/D = \{6.0, 9.0\}$ , but shows a linear increase for the remaining cases, where the PE loss is more pronounced. This reduction in  $w_0^{\text{opt}}$  can be explained by the fact that the combined effects of scintillation and PEs may cause an

increase in the effective beam radius, which reduces the sensitivity to optical intensity fluctuations; and hence, result in a smaller  $w_0^{\text{opt}}$  requirement to maximize  $\langle C \rangle$ . It should be noted that such phenomenon is subjected to the condition that either one or both of these adversities are least dominant (i.e.,  $\sigma_R^2 \leq 1.5$  and/or  $2\sigma_{pe}/D \leq 6.0$ ). On the other hand,  $w_0^{\text{opt}}$  must be increased accordingly for the strong turbulence case with  $\sigma_R^2 > 10.0$ , where we notice a linear increase in  $w_0^{\text{opt}}$  for smaller  $2\sigma_{pe}/D = \{6.0, 9.0\}$ . A steeper incremental trend is observed for the other cases of higher PE loss, as can be seen from the exponential behavior for larger  $2\sigma_{pe}/D = \{12.0, 15.0\}$  in Fig. 6(a). As evident from Fig. 6(b), the FSO channel capacity is highly susceptible to the adverse effects of turbulence, and suffers from significant performance degradation for higher PE loss albeit performing beam width optimization. A notable reduction in  $\langle C \rangle$  is observed for all the considered  $2\sigma_{pe}/D$  values, with a maximum variation of 0.11, 0.22, 0.33 and 0.42 bpcu, respectively. Furthermore, we notice a slight improvement in the optimized  $\langle C \rangle$  for  $\sigma_R^2 > 10.0$ , since increasing the beam width and transmit optical power (to give  $\langle SNR \rangle = 14$  dB) alters the PCB into a relatively coherent laser beam, which is more resilient to strong turbulence.

In a practical implementation of the beam width optimization, the atmospheric channel changes with time and must be continuously monitored, in order to predict the random characteristics of the channel effects resulting from the variation of weather conditions or propagation distances. These channel predictions encompassing the visibility, turbulence strength, and PE-induced jitter are determined from a series of measurements and computations, and used as indicators to adaptively control an electronically variable optical diffuser for adjusting the beam width with good accuracy [18, 30]. As a result, an efficient optimization algorithm and robust multivariate training sequences must be designed and constructed to obtain the optimum beam width, in order to maximize the FSO channel capacities under varying channel conditions, at the expense of incurring excess delay and processing overhead on the system.

## 6. CONCLUSIONS

Jointly considering a spatially partially coherent Gaussian-beam and important link design criteria, we have investigated the performance of FSO communication links from the information theory perspective, impaired by atmospheric loss, turbulence and PEs. The average channel capacity has been evaluated, taking into account the receiver aperture dimension and its resulting AA phenomenon, transmitter beam width, link range, knowledge of CSI at the receiver, and weather conditions. Using the lowest-order Gaussian-beam wave model, we have characterized the propagation properties of the optical signal through random turbulent medium, taking into account the diverging and focusing of the PCB, and the scintillation and beam wander effects. Correspondingly, this study provides a holistic perspective for the optimal design of horizontal FSO links employing PCB.

It has been demonstrated that a larger penalty will be imposed upon the FSO channel capacity under the combined effects of turbulence and PEs, particularly when the CSI is unknown to the receiver. With the introduction of an enlarged receiver aperture, a notable average capacity improvement of up to 0.46 bpcu can be achieved for a mean SNR of 14 dB in the moderate-to-strong turbulence regime, even without knowledge of the channel state conditions. Furthermore, we have shown that the PCB properties are substantially altered when propagating in free-space, and revealed the importance of finding an optimal beam width to maximize the capacity. From this study, several appealing observations were made, leading to the conclusion that the beam width can be reduced to improve the FSO channel capacity, in



the presence of weak scintillation and small PE loss, and valid for the condition that either one or both of these adversities are least dominant. In the strong turbulence regime, the beam width increases linearly with respect to the Rytov variance for smaller PE loss, but demonstrates a steeper increment to give an exponential behavior with increased PE loss. In conclusion, we showed that beam width optimization is a feasible method for capacity enhancement in long-distance terrestrial FSO links. This is mainly due to the fact that the beam parameter is subject to the combined effects of turbulence and PEs; and its optimal value must be adjusted according to the varying channel conditions, in order to maximize the FSO channel capacity.

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## References

- Z. Ghassemlooy, W. Popoola, and S. Rajbhandari, *Optical Wireless Communications: System and Channel Modelling with MATLAB* (Taylor & Francis Group, 2012).
- M. A. Khalighi and M. Uysal, "Survey on free space optical communication: a communication theory perspective," *IEEE Commun. Surveys Tuts.* **16**(4), 2231-2258 (2014).
- A. K. Majumder, "Free-space laser communication performance in the atmospheric channel," in *Free-Space Laser Communications: Principles and Advances*, A. K. Majumder and J. C. Ricklin, eds. (Springer-Verlag, 2008).
- I. E. Lee, Z. Ghassemlooy, W. P. Ng, and M. Uysal, "Performance analysis of free space optical links over turbulence and misalignment induced fading channels," in *Proc. CSNDSP*, 1-6 (2012).
- S. Arnon, "Optimization of urban optical wireless communication systems," *IEEE Trans. Wireless Commun.* **2**(4), 626-629 (2003).
- L. C. Andrews and R. L. Phillips, *Laser Beam Propagation through Random Media*, 2nd ed. (SPIE Optical Engineering Press, 2005).
- A. A. Farid and S. Hranilovic, "Outage capacity optimization for free-space optical links with pointing errors," *J. Lightwave Technol.* **25**(7), 1702-1710 (2007).
- D. K. Borah and D. G. Voelz, "Pointing error effects on free-space optical communication links in the presence of atmospheric turbulence," *J. Lightw. Technol.* **27**(18), 3965-3973 (2009).
- J. C. Ricklin and F. M. Davidson, "Atmospheric turbulence effects on a partially coherent Gaussian beam: implications for free-space laser communication," *J. Opt. Soc. Am. A* **19**(9), 1794-1802 (2002).
- J. C. Ricklin and F. M. Davidson, "Atmospheric optical communication with a Gaussian Schell beam," *J. Opt. Soc. Am. A* **20**(5), 856-866 (2003).
- O. Korotkova, L. C. Andrews, and R. L. Phillips, "Model for a partially coherent Gaussian beam in atmospheric turbulence with application in Lasercom," *Opt. Eng.* **43**(2), 330-341 (2004).
- H. G. Sandalidis, "Optimization models for misalignment fading mitigation in optical wireless links," *IEEE Commun. Lett.* **12**(5), 395-397 (2008).
- H. G. Sandalidis, T. A. Tsiftsis, G. K. Karagiannidis, and M. Uysal, "BER performance of FSO links over strong atmospheric turbulence channels with pointing errors," *IEEE Commun. Lett.* **12**(1), 44-46 (2008).
- H. G. Sandalidis, "Coded free-space optical links over strong turbulence and misalignment fading channels," *IEEE Trans. Commun.* **59**(3), 669-674 (2011).
- H. G. Sandalidis, T.A. Tsiftsis, and G.K. Karagiannidis, "Optical wireless communications with heterodyne detection over turbulence channels with pointing errors," *J. Lightw. Technol.* **27**(20), 4440-4445 (2009).
- Y. Ren, A. Dang, B. Luo, and H. Guo, "Capacities for long-distance free-space optical links under beam wander effects," *IEEE Photon. Technol. Lett.* **22**(14), 1069-1071 (2010).
- T. J. Schulz, "Optimal beams for propagation through random media," *Opt. Lett.* **30**(10), 1093-1095 (2005).
- C. Chen, H. Yang, X. Feng, and H. Wang, "Optimization criterion for initial coherence degree of lasers in free-space optical links through atmospheric turbulence," *Opt. Lett.* **34**(4), 419-421 (2009).
- D. G. Voelz and X. Xiao, "Metric for optimizing spatially partially coherent beams for propagation through turbulence," *Opt. Eng.* **48**(3), 036001-036001 (2009).
- D. K. Borah and D.G. Voelz, "Spatially partially coherent beam parameter optimization for free space optical communications," *Opt. Express* **18**(20), 20746-20758 (2010).
- C. Liu, Y. Yao, Y. X. Sun, J. J. Xiao, and X. H. Zhao, "Average capacity optimization in free-space optical communication system over atmospheric turbulence channels with pointing errors," *Opt. Lett.* **35**(19), 3171-3173 (2010).
- J. Cang and X. Liu, "Average capacity of free-space optical systems for a partially coherent beam propagating through non-Kolmogorov turbulence," *Opt. Lett.* **36**(17), 3335-3337 (2011).
- B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics*, 2nd ed. (John Wiley & Sons, 2007).
- X. M. Zhu and J. M. Kahn, "Free-space optical communication through atmospheric turbulence channels," *IEEE Trans. Commun.* **50**(8), 1293-1300 (2002).
- I. I. Kim, B. McArthur, and E. Korevaar, "Comparison of laser beam propagation at 785 nm and 1550 nm in fog and haze for optical wireless communications," in *Proc. SPIE* **4214**, 26-37 (2001).
- T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. (Wiley-Interscience, 2006).
- E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: information-theoretic and communications aspects," *IEEE Trans. Inf. Theory* **44**(6), 2619-2692 (1998).
- M. A. Khalighi, N. Schwartz, N. Aitamer, and S. Bourennane, "Fading reduction by aperture averaging and spatial diversity in optical wireless systems," *IEEE/OSA J. Opt. Commun. Netw.* **1**(6), 580-593 (2009).
- F. S. Vetelino, C. Young, L. Andrews, and J. Rekolons, "Aperture averaging effects on the probability density of irradiance fluctuations in moderate-to-strong turbulence," *Appl. Opt.* **46**(11), 2099-2108 (2007).
- F. Wang, X. Liu, and Y. Cai, "Propagation of partially coherent beam in turbulent atmosphere: a review (invited review)," *Progress In Electromagnetics Research*, **150**(1), 123-143 (2015).